A min-max problem for the computation of the cycle time lower bound in interval-based Time Petri Nets *

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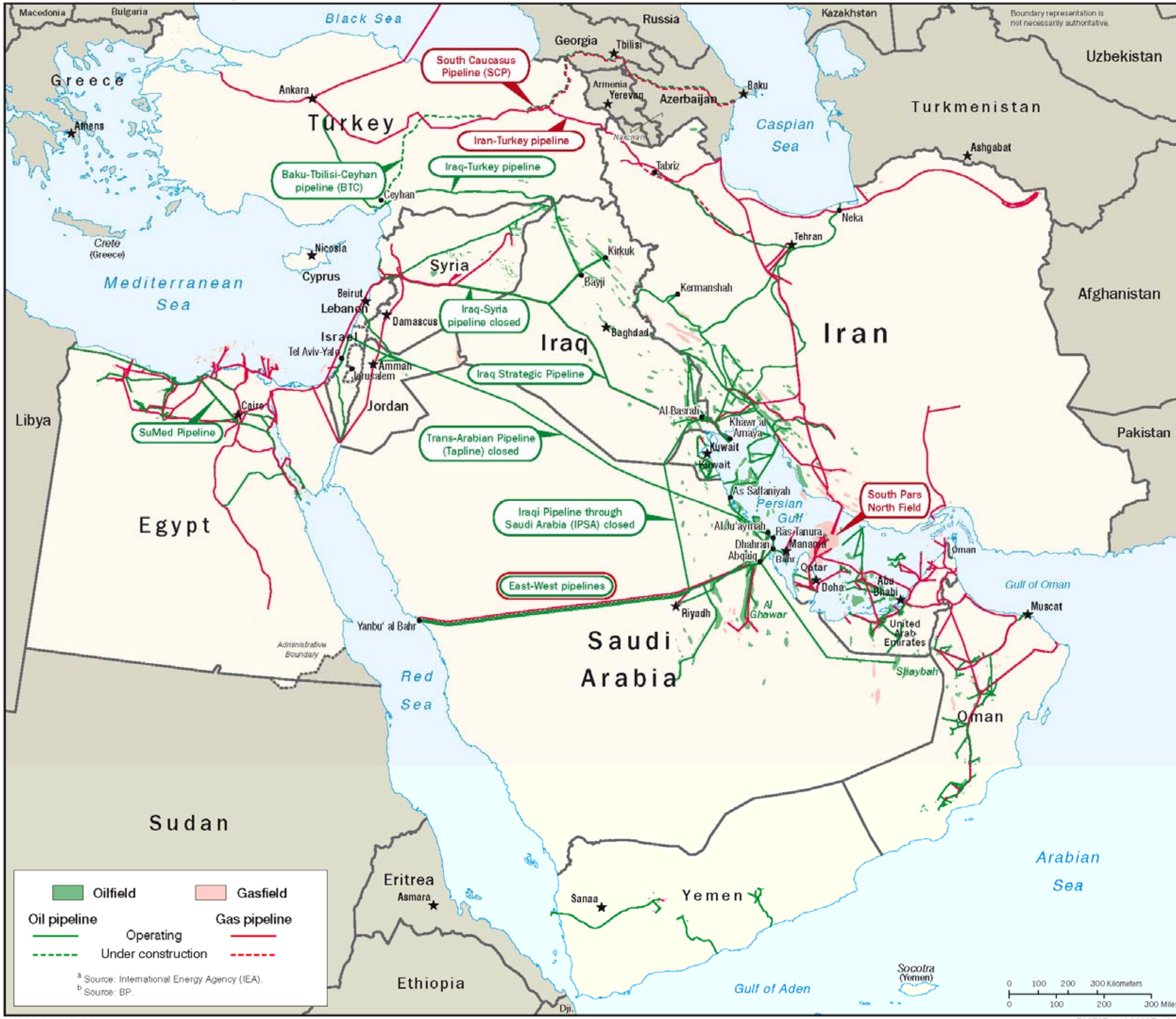
Selected Oil and Gas Pipeline Infrastructure in the Middle East



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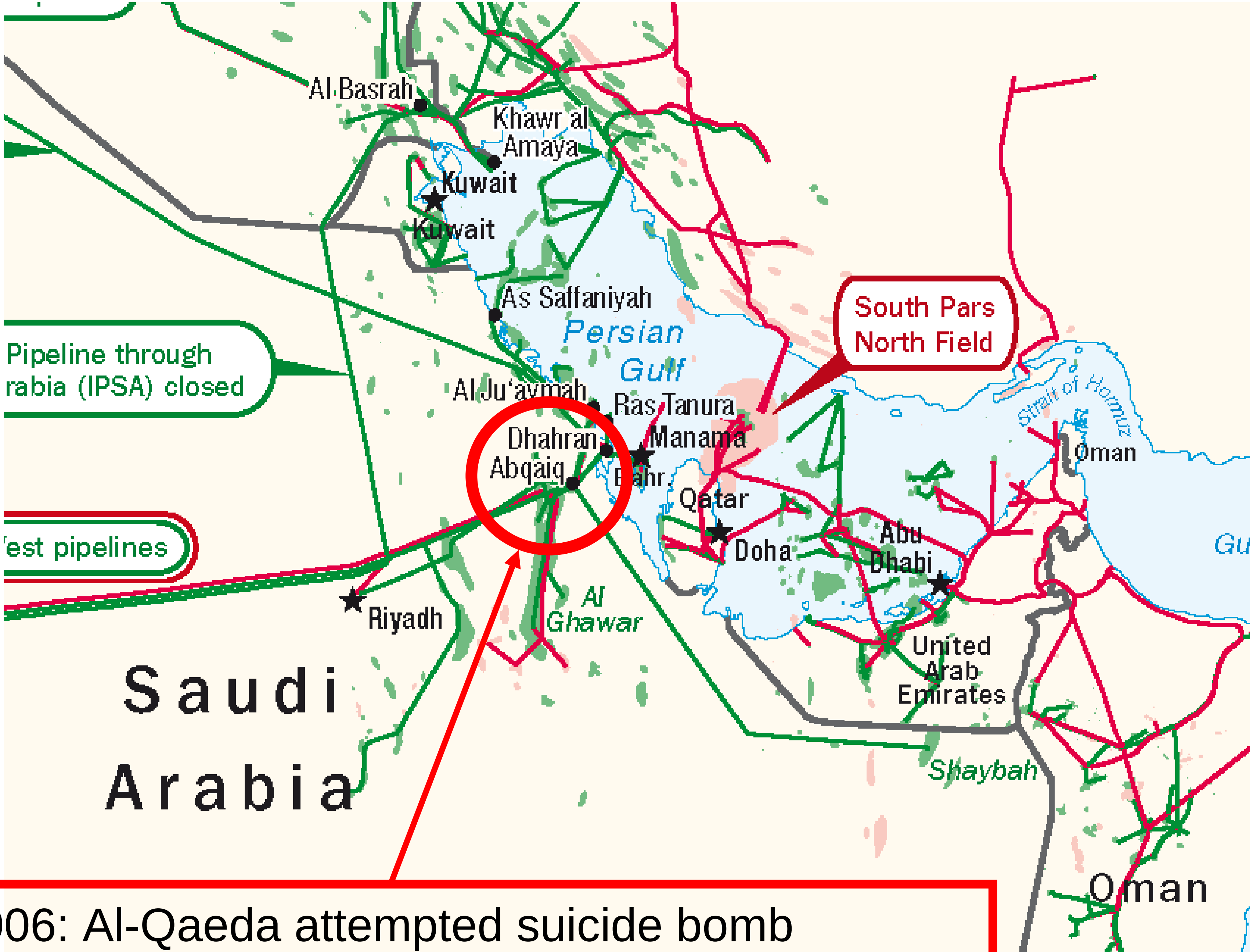


2006: Al-Qaeda attempted suicide bomb attack at the Abqaiq petroleum processing facility

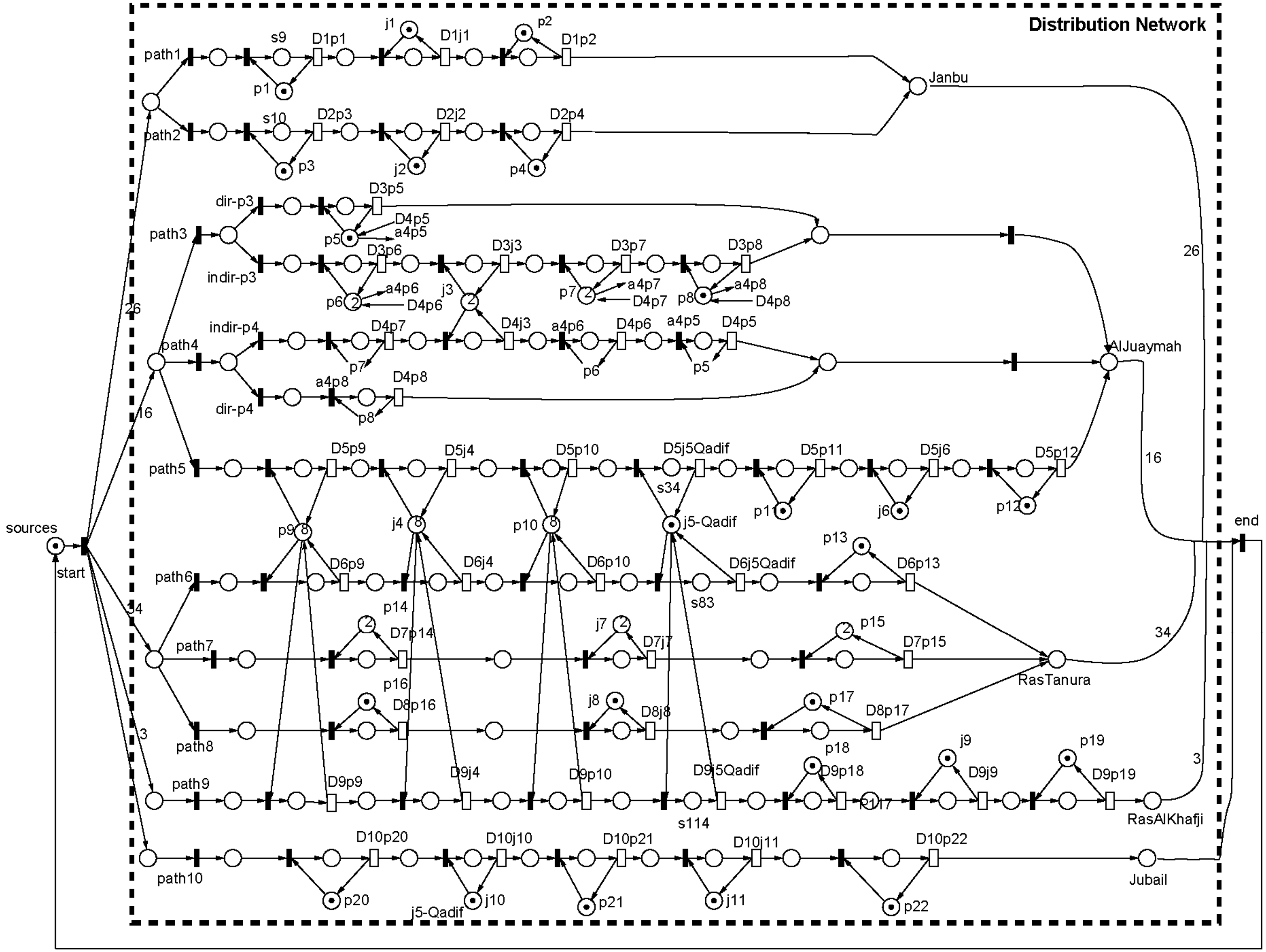
Vulnerability analysis of the network

- Analyze the impact of a coordinated attack on the network throughput

  - Maximize throughput after attack
  - Identify critical paths
  - How to distribute the oil-flow between the alternative paths to maximize throughput

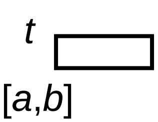
  - Reduce the economic loss due to an attack

Distribution network

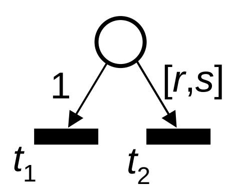


Transition	a[t] (day)
D10p20, D10p21, D10p22	0.005
D10j10, D10j11	0.006
D9j5Qadif, D9j9	0.02
D9p19	0.01
D9p18	0.04
D9p10	0.08
D9j4	0.16
D9p9	0.24
D8p16, D8p17	0.005
D8j8	0.006
D7p14, D7p15	0.005
D7j7	0.006
D6p13	0.01
D6j5Qadif	0.02
D6p10	0.08
D6j4	0.16
D6p9	0.24
D5p12	0.01
D5j6, D5j5Qadif	0.02
D5p11	0.04
D5p10	0.08
D5j4	0.16
D5p9	0.24
D4p5, D4p6, D4p7, D4p8	0.005
D4j3	0.006
D3p5, D3p6, D3p7, D3p8	0.005
D3j3	0.006
D2p4	0.03
D2j2	0.04
D2p3	0.07
D1p2	0.05
D1j1	0.05
D1p1	0.07

Transition	R
path1	[1,1]
path2	[0.75, 1.25]
path3	[1, 1.5]
path4	[1, 1.5]
path5	[1,1]
path6	[1,1]
path7	[0.75, 1.25]
path8	[1, 1.5]
dir-p4	[1,1]
indir-p4	[0.5, 0.75]
dir-p3	[1,1]
indir-p3	[0.5, 0.75]

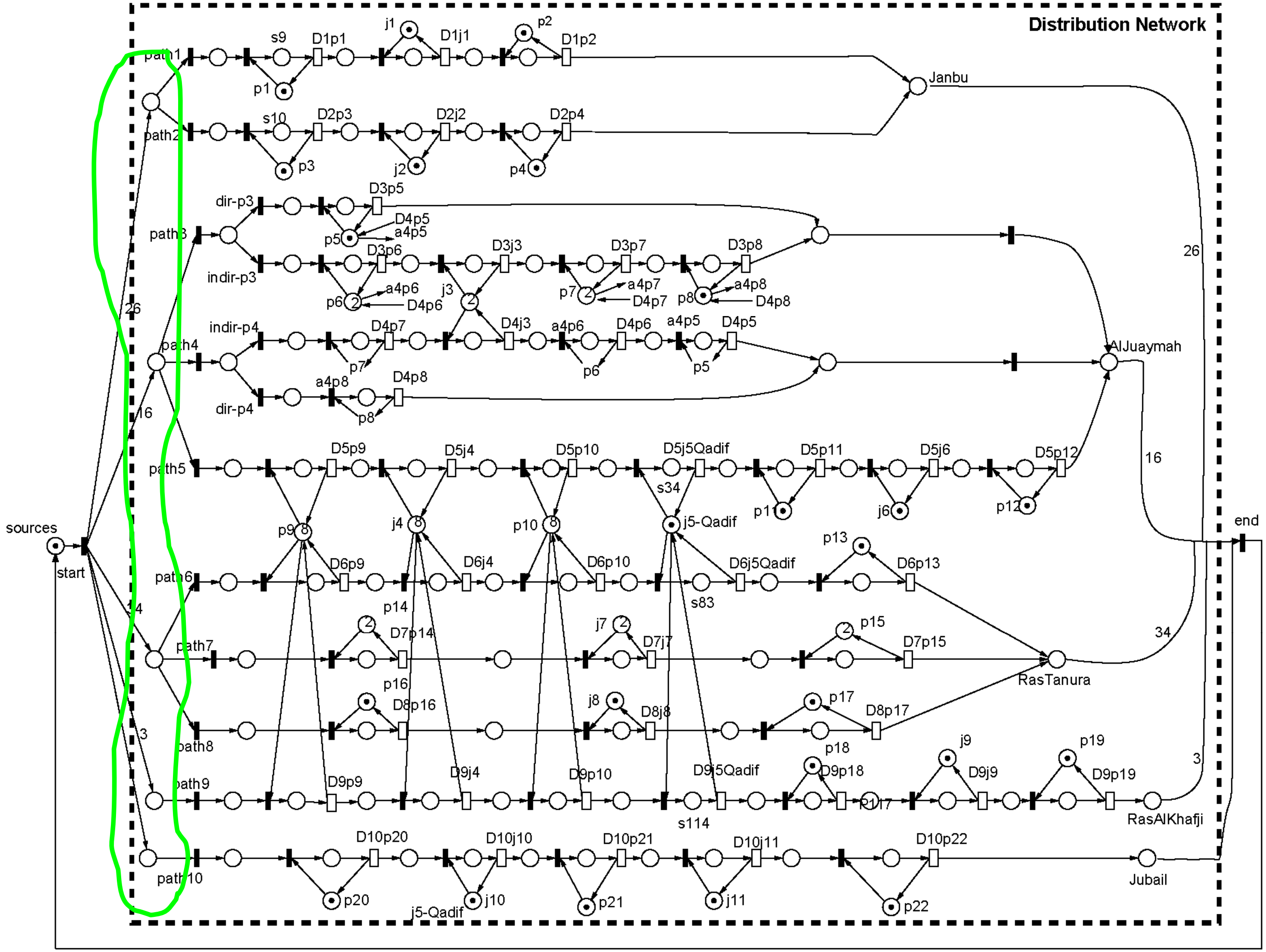


Interval time Petri net, i.e. when enabled, t fires within interval $[a,b]$ of time.



Routing intervals, i.e. per each firing of t_1, t_2 must fire in average a number of times in $[r,s]$

Distribution network



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D5p12	0.01
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D5p9	0.24
D4p5, D4p6, D4p7, D4p8	0.005
D4j3	0.006
D3p5, D3p6, D3p7, D3p8	0.005
D3j3	0.006
D2p4	0.03
D2j2	0.04
D2p3	0.07
D1p2	0.05
D1j1	0.05
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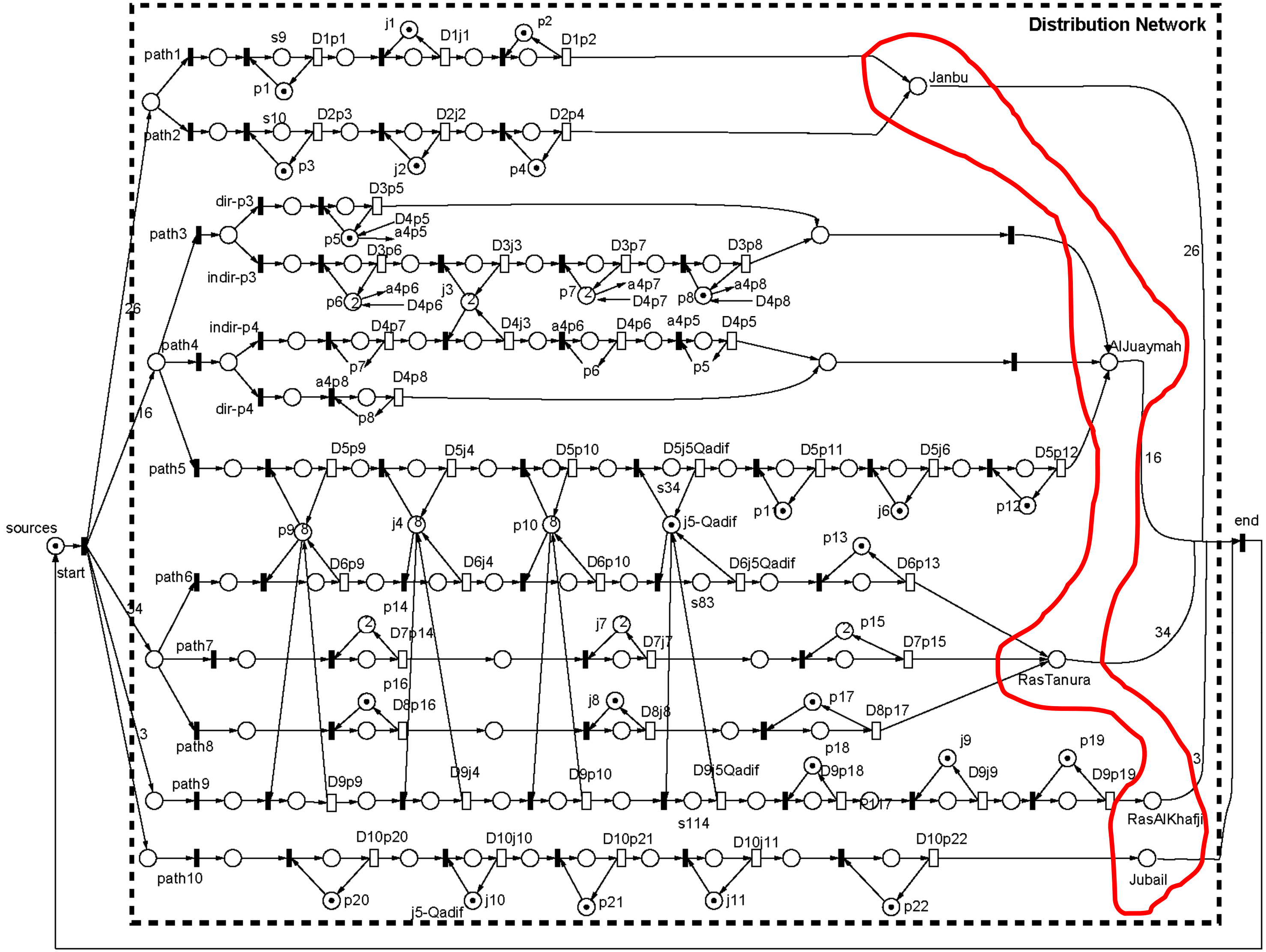
indir-p4	[0.5, 0.75]
dir-p3	[1,1]
indir-p3	[0.5, 0.75]

Sources: oil fields (jointly produce 8-9 mmbbl/day of crude oil)

Distribution network



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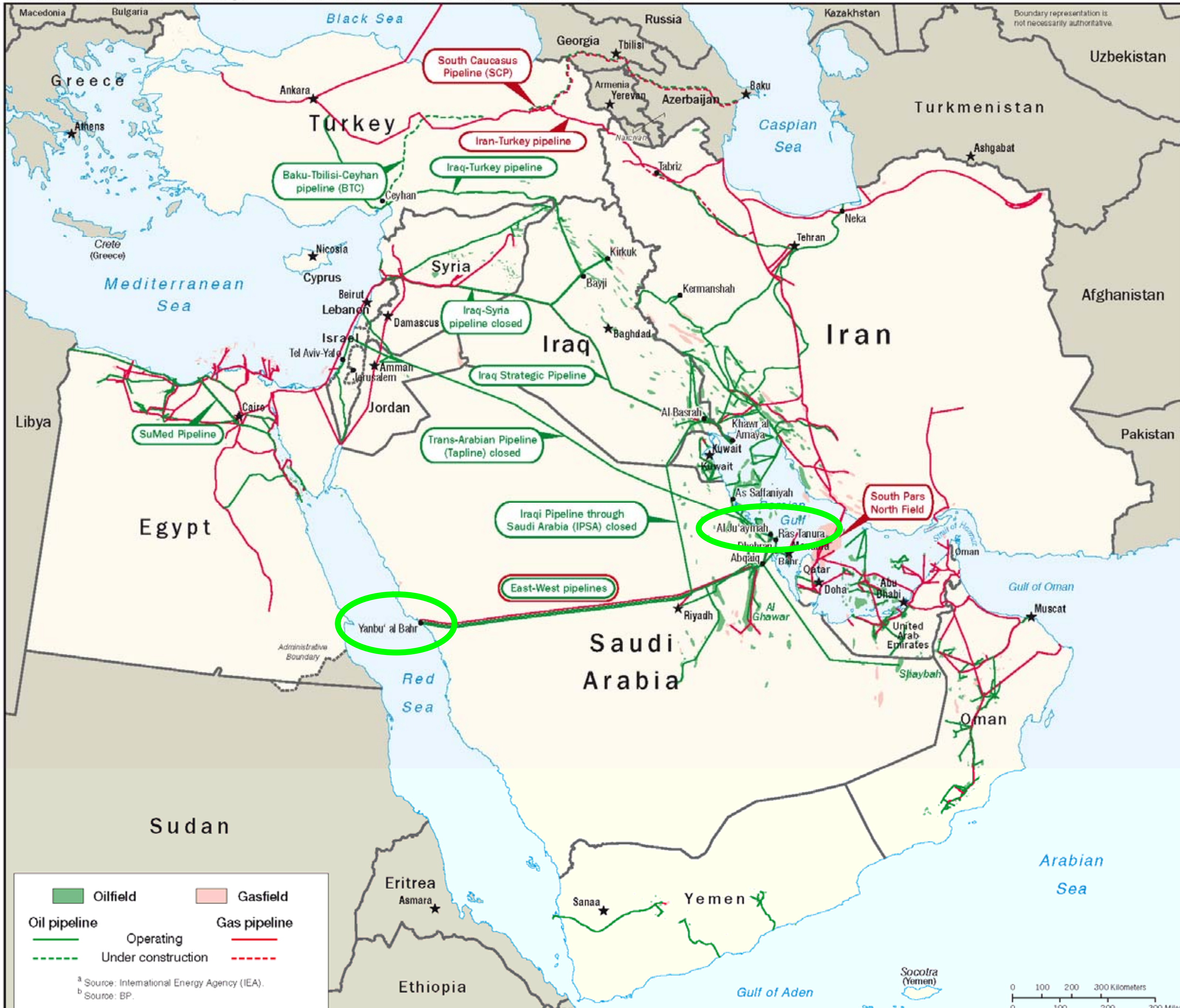
indir-p4	[0.5, 0.75]
dir-p3	[1,1]
indir-p3	[0.5, 0.75]

Seaport terminals: Ras Tanura (6 mmbbl), Janbu (4.5 mmbbl), Al Ju’aymah (3 mmbbl)

Selected Oil and Gas Pipeline Infrastructure in the Middle East



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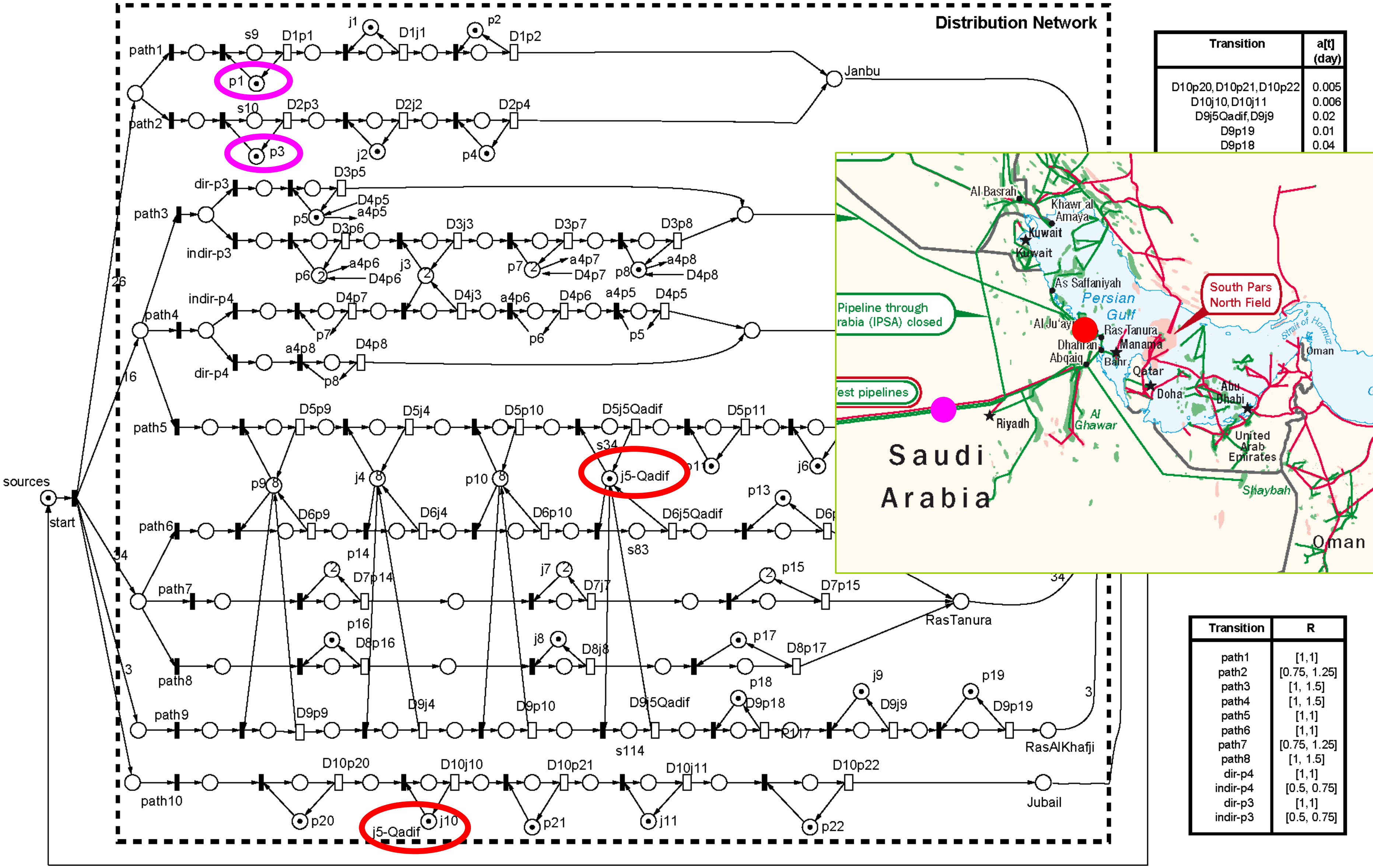

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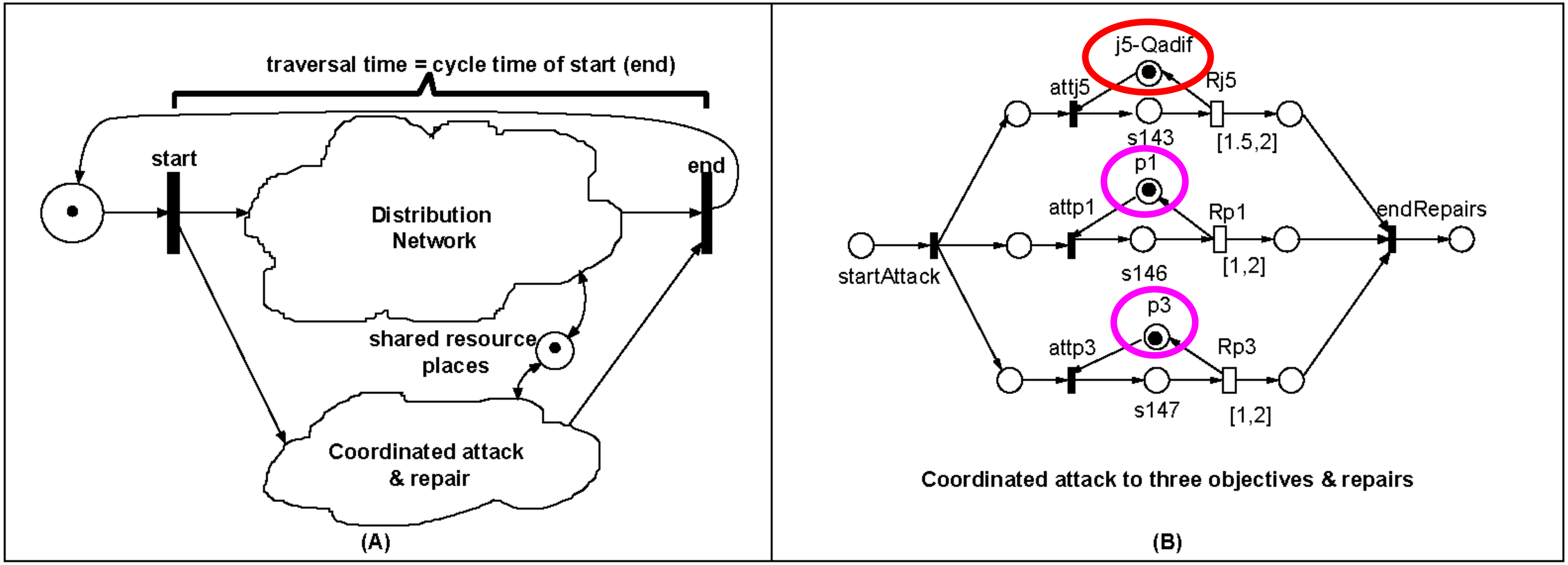
dir-p3	[1,1]
indir-p3	[0.5, 0.75]

Example of two critic pipes (to Janbu) and a critic junction (Qadif)



(A) Petri net model of the attacked network.

traversal time = 1/throughput of transition ‘end’

(B) Subnet of a coordinated attack on three targets (Qadif junction and Janbu pipes) and repairs.

- The Problem in *PN model terms*:

  - Test different coordinated attack scenarios.
  - Identify worst coordinated attack
    = *find attacked network with minimum throughput, and for it*:
    1. Compute the throughput
       = *throughput of transition* "end"
    2. Identify critical points
       = *bottleneck subnets*
    3. Compute optimum oil paths
       = *optimum routing ratios at choice places*

Before this work

- Problem 1. Compute the throughput
  = *throughput of transition* "end", efficiently solved:

Proposition 1(*) *Let $\mathcal{TF} = (\mathcal{T}, \mathcal{R})$ be a live and bounded TPNF system. A throughput upper bound $x[t_1]$ of a transition $t_1 \in T$, can be computed by solving the LPP:*

$$
\begin{aligned}
P_0 = \ & \text{maximize } x[t_1] \qquad (1) \\
s.t.\ & \mathbf{M} = \mathbf{M_0} + \mathbf{C^T}\sigma \\
& \sum_{t \in {}^\bullet p} x[t]F[t,p] = \sum_{t \in p^\bullet} x[t]B[t,p], \forall p \in P \\
& M[p] \geq x[t]a[t]B[t,p], \forall t \in T \text{ and } \forall p \in {}^\bullet t \\
& \underline{r}^j x[t_k] \leq \overline{r}^k x[t_j],\ \underline{r}^k x[t_j] \leq \overline{r}^j x[t_k], \forall t_j, t_k \in ECS \\
& \mathbf{x}, \sigma \geq \mathbf{0}_T,\ \mathbf{M} \geq \mathbf{0}_P
\end{aligned}
$$

(*) Bernardi & Campos: "Computation of performance bounds for real-time systems using Time Petri Nets", *IEEE Transactions on Industrial Informatics,* 5(2):168-180, May 2009.

Before this work

- Problem 2. Identify critical points
  = *bottleneck subnets*

Not (efficiently) solved

- Problem 3. Compute optimum oil paths
  = *optimum routing ratios at choice places*

Not (efficiently) solved

After this work

- Problem 1. Compute the throughput = *throughput of transition* "end", efficiently solved:

Proposition 2 *Let $\mathcal{TF} = (\mathcal{T}, \mathcal{R})$ be a live and bounded TPNF system where all the timed transitions are persistent (that is, once enabled they eventually fire). A cycle time lower bound of a transition t_1 (i.e., the inverse of its throughput upper bound) can be computed by solving the min-max problem:*

$$P_1 = \min_{\mathbf{v} \in D_v} \max_{\mathbf{y} \in D_y} \mathbf{y}^{\mathbf{T}} (\mathbf{B}^{\mathbf{T}} \odot \mathbf{a})\ \mathbf{v} \quad (3)$$

$$s.t.\ D_y : \{\mathbf{C}\mathbf{y} = \mathbf{0}_T, \mathbf{M}_{\mathbf{0}}^{\mathbf{T}}\mathbf{y} = 1, \mathbf{y} \geq \mathbf{0}_P\}$$

$$D_v : \{\mathbf{R}\mathbf{v} \leq \mathbf{0}_K, \mathbf{C}^{\mathbf{T}}\mathbf{v} = \mathbf{0}_P, v[t_1] = 1, \mathbf{v} \geq \mathbf{0}_T\}$$

After this work

- Problem 2. Identify critical points
  = *bottleneck subnets*, efficiently solved:

  Support of vector **y***
  of the optimal solution of the previous problem

- Problem 3. Compute optimum oil paths
  = *optimum routing ratios at choice places*,
  efficiently solved:

  Vector **v***
  of the optimal solution of the previous problem

Results (in this case study)

Worst coordinated attack with minimum risk for terrorists

Facilities	Throughput	$\|\|\mathbf{y}^*\|\|$	Path ratios: To Janbu $\frac{v^*_{path_i}}{v^*_{path_1}}, i = 1, 2$	Path ratios: To Al-Ju'aymah $\frac{v^*_{path_i}}{v^*_{path_5}}, i = 3, 4, 5$	Path ratios: To Ras Tanura $\frac{v^*_{path_i}}{v^*_{path_6}}, i = 6, 7, 8$
j_5	0.549048	$\{j_5, s34, s83, s114, s145\}$	$\{1; 0.75\}$	$\{1.5; 1.5; 1\}$	$\{1; 1.25; 1.5\}$
p_1	0.552826	$\{p_1, s9, s146\}$	$\{1; 1.25\}$	$\{1.5; 1.5; 1\}$	$\{1; 1.25; 1.5\}$
p_3	0.561798	$\{p_3, s10, s147\}$	$\{1; 0.75\}$	$\{1.5; 1.5; 1\}$	$\{1; 1.25; 1.5\}$
p_1, p_3	0.523560	$\{p_1, s9, s146\}$	$\{1; 1\}$	$\{1.5; 1.5; 1\}$	$\{1; 1.25; 1.5\}$
j_5, p_1	0.549048	$\{j_5, s34, s83, s114, s145\}$	$\{1; 1.25\}$	$\{1.5; 1.5; 1\}$	$\{1; 1.25; 1.5\}$
j_5, p_3	0.549048	$\{j_5, s34, s83, s114, s145\}$	$\{1; 0.75\}$	$\{1.5; 1.5; 1\}$	$\{1; 1.25; 1.5\}$
j_5, p_1, p_3	0.523560	$\{p_1, s9, s146\}$	$\{1; 1\}$	$\{1.5; 1.5; 1\}$	$\{1; 1.25; 1.5\}$

targets attacked

inverse of traversal time

bottleneck subnets

optimum routing ratios at choice places

Technically speaking

The main result of the paper is in the 5-pages Appendix:

$$P_1 = 1 / P_0$$

where

$$\begin{aligned}
P_0 \;=\; & \text{maximize } x[t_1] \\
& s.t.\ \mathbf{M} = \mathbf{M_0} + \mathbf{C^T}\sigma \\
& \sum_{t \in {}^{\bullet}p} x[t]F[t,p] = \sum_{t \in p^{\bullet}} x[t]B[t,p], \forall p \in P \\
& M[p] \geq x[t]a[t]B[t,p], \forall t \in T \text{ and } \forall p \in {}^{\bullet}t \\
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& \mathbf{x}, \sigma \geq \mathbf{0}_T,\ \mathbf{M} \geq \mathbf{0}_P
\end{aligned}$$

$$\begin{aligned}
P_1 \;=\; & \min_{\mathbf{v} \in D_v} \max_{\mathbf{y} \in D_y} \mathbf{y^T}(\mathbf{B^T} \odot \mathbf{a})\ \mathbf{v} \\
& s.t.\ D_y : \{\mathbf{Cy} = \mathbf{0}_T, \mathbf{M_0^T}\mathbf{y} = 1, \mathbf{y} \geq \mathbf{0}_P\} \\
& D_v : \{\mathbf{Rv} \leq \mathbf{0}_K, \mathbf{C^T}\mathbf{v} = \mathbf{0}_P, v[t_1] = 1, \mathbf{v} \geq \mathbf{0}_T\}
\end{aligned}$$

$\mathbf{y}^*$

$\mathbf{v}^*$

Additional results

- Proposal of two algorithms to solve the new programming problem (P_1):
  - An approximate sub-gradient method
  - Another exact method that previously requires the solution of P_0
- A benchmark of PN models to compare both solution algorithms
  - 40 Time PN models, several of them being case studies taken from the literature

Questions?