3. Optimization problem based on an undefined Petri net

Outline

Statement of the problem

- Max/min objective/multiobjective function { Quality measurement of a solution
- Definition of the structure of the feasible solutions.
- Constraints → configuration of the solution space.
- Undefined Petri net:

  1. Disjunctive constraint: only one simple alternative PN must be verified.

  2. It is a complement to the objective function.

3. Optimization problem based on an undefined Petri net

Classical solution

Decomposition of the problem in *n* different cases based on a simple alternative Petri net.

The proposed methodology allows to transform a compound PN into a set of simple alternative PN

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3. Optimization problem based on an undefined Petri net

Classical solution

Decomposition of the problem in *n* different cases based on a simple alternative Petri net

The disjunctive constraint is avoided.

It requires the solution of *n* optimization problems.

It requires a further stage of comparing the results.

If *n* is large…

…selection.

3. Optimization problem based on an undefined Petri net

Proposed methodology

Alternatives aggregation Petri nets

Transformation of an undefined PN into an AAPN.

Efficient exploration of a single solution space.

Solution of the problem by means of classical methodologies (state explosion).

Heuristics / metaheuristics

3. Optimization problem based on an undefined Petri net

Alternatives aggregation Petri nets

Construction of an AAPN

Decomposition of a set of alternative Petri nets in:

- Subnets
- Link transitions

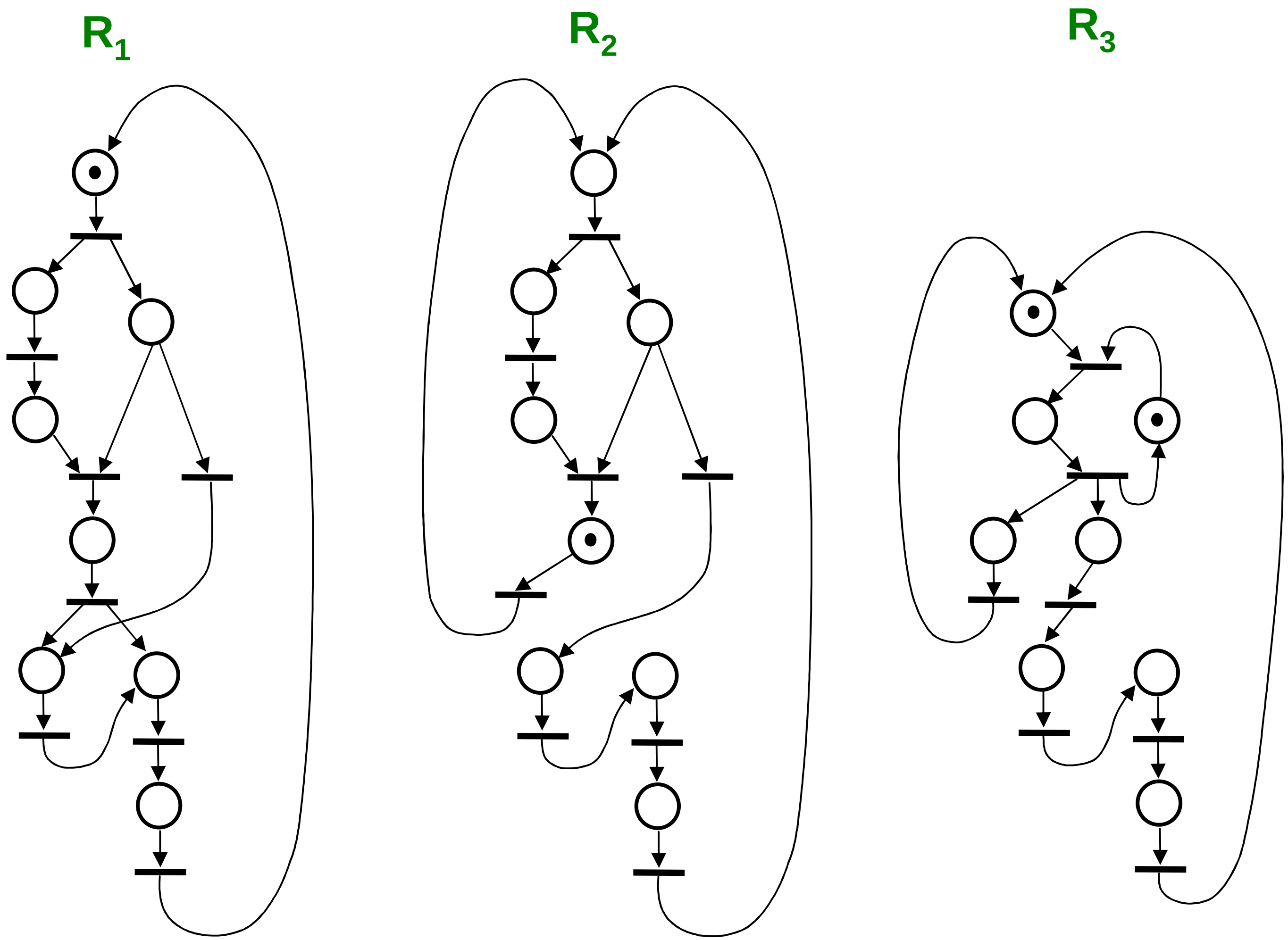
Aggregation and link of the subnets.

Application of the reduction rules.

Application of the simplification rules.

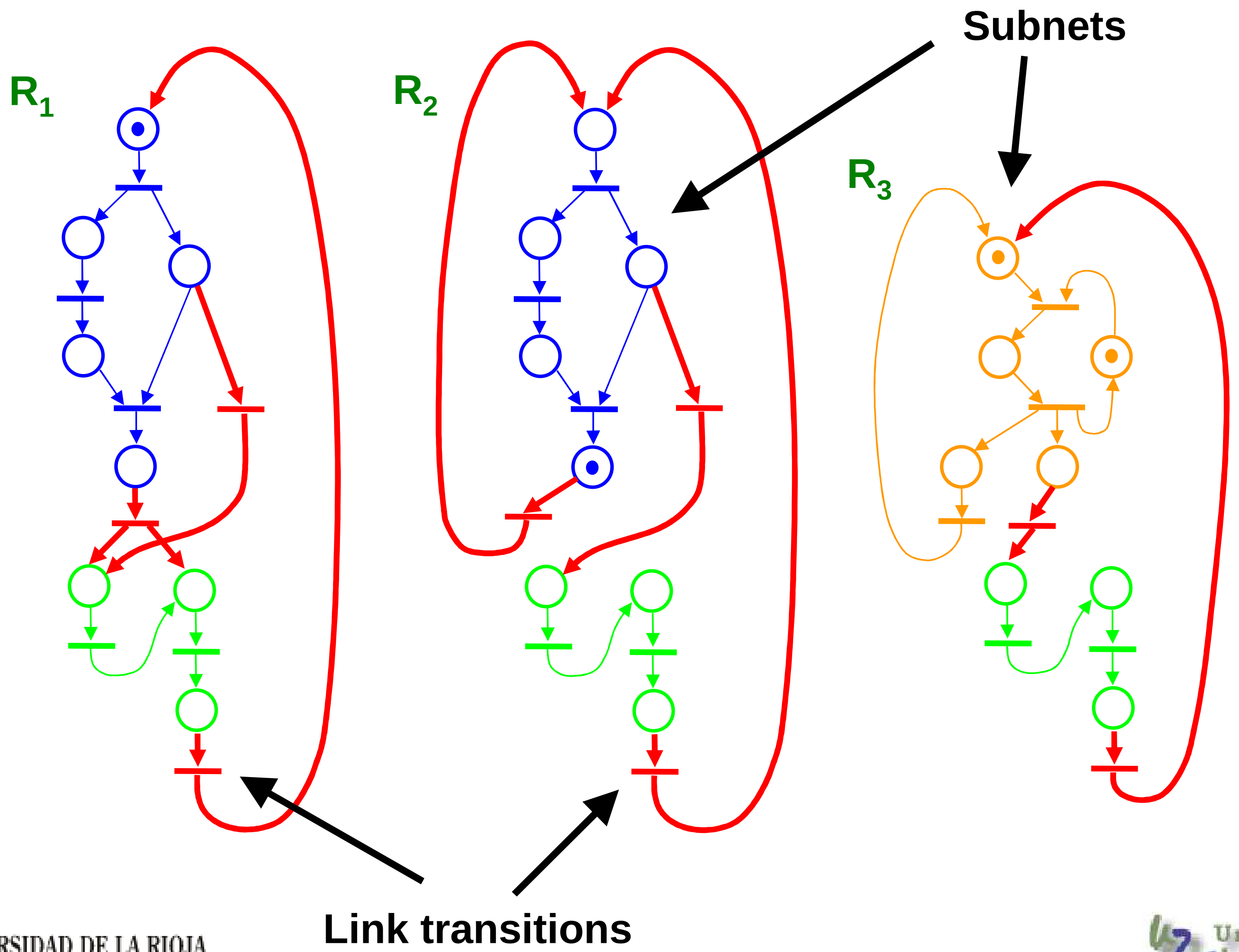
3. Optimization problem based on an undefined Petri net

Example 4 | **Alternatives aggregation Petri nets**



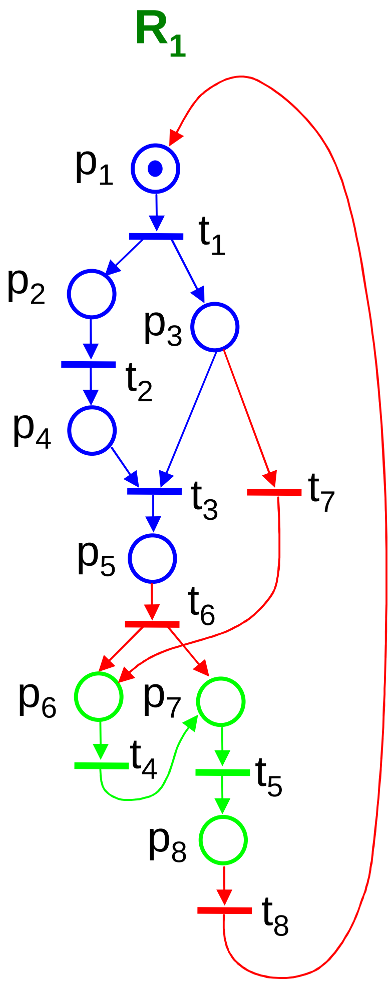
3. Optimization problem based on an undefined Petri net

Example 4 — 1. Decomposition into subnets



3. Optimization problem based on an undefined Petri net

Example 4 — 1. Decomposition into subnets



Incidence matrix

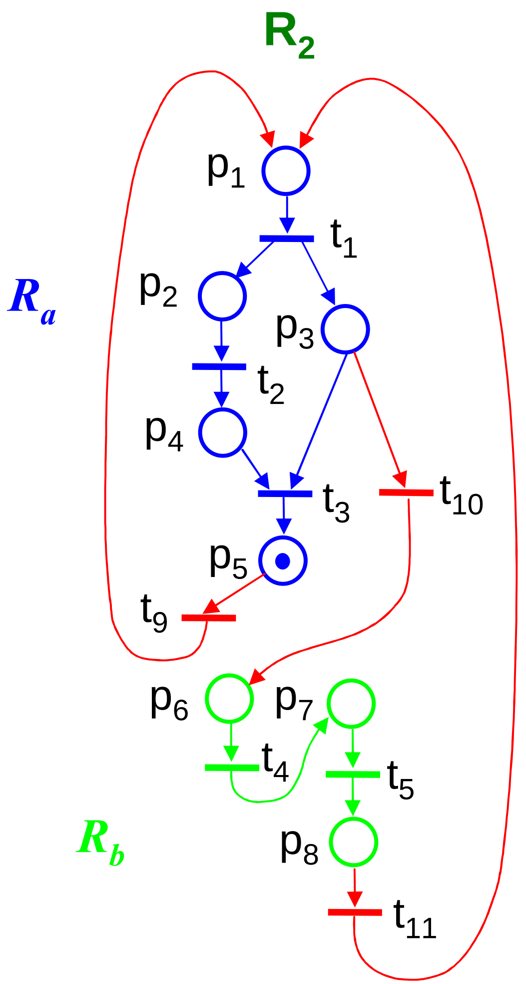
Q_a

$$W_1 = \begin{array}{c} \\ \left(\begin{array}{cccccccc} t1 & t2 & t3 & t4 & t5 & t6 & t7 & t8 \\ \hline -1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & -1 \end{array}\right) \end{array} \begin{array}{l} \\ p1 \\ p2 \\ p3 \\ p4 \\ p5 \\ p6 \\ p7 \\ p8 \end{array}$$

Q_b

3. Optimization problem based on an undefined Petri net

Example 4 — 1. Decomposition into subnets



Incidence matrix

Q_a

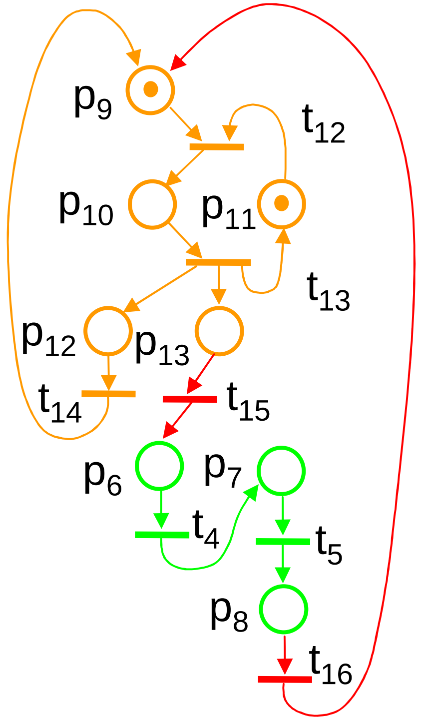
$$W_2 = \begin{array}{c} \\ \left(\begin{array}{cccccccc} -1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & -1 \end{array}\right) \end{array} \begin{array}{c} p1 \\ p2 \\ p3 \\ p4 \\ p5 \\ p6 \\ p7 \\ p8 \end{array}$$

Columns: t1, t2, t3, t4, t5, t9, t10, t11

Q_b

3. Optimization problem based on an undefined Petri net

Example 4 — 1. Decomposition into subnets



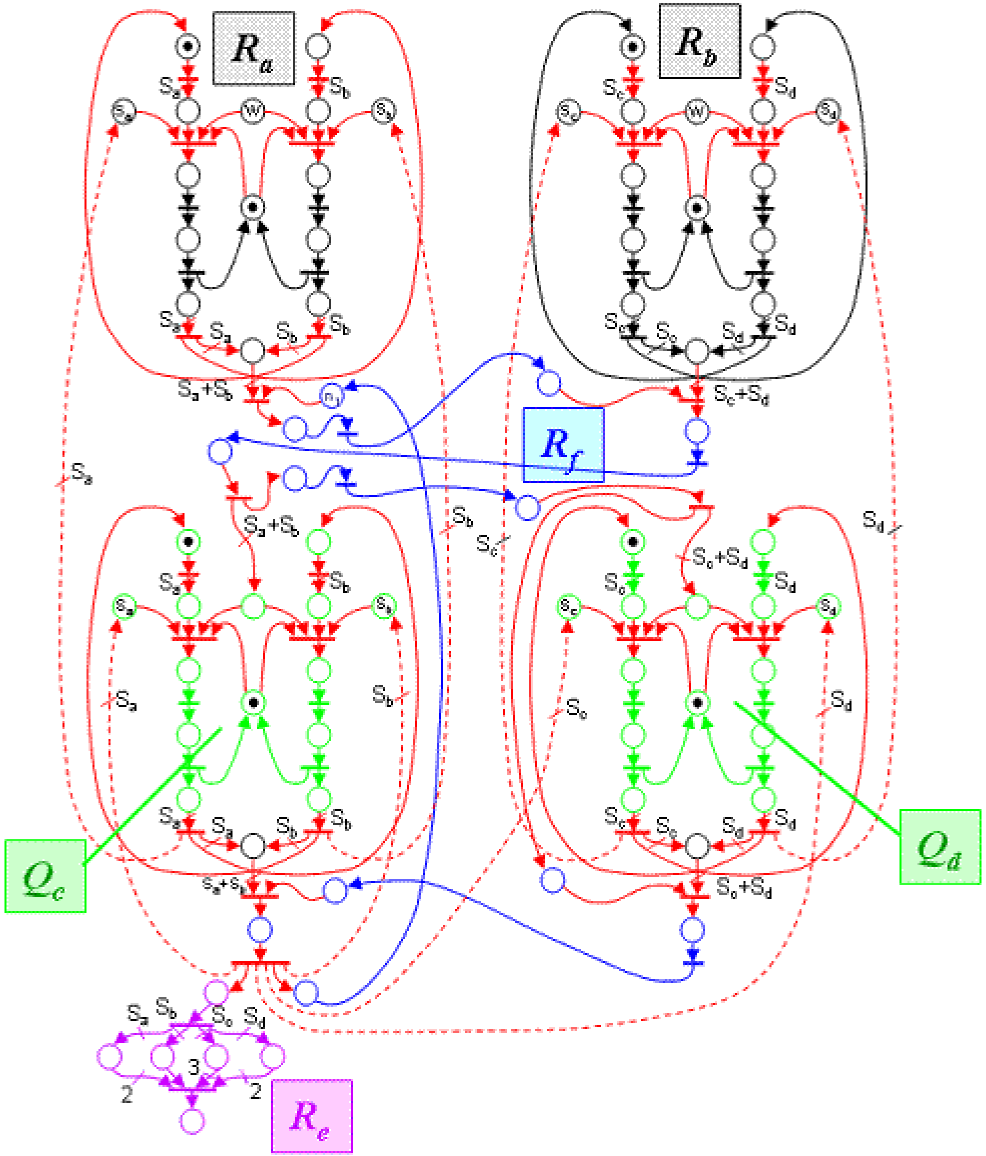
Incidence matrix

$$W_3 = \begin{array}{c} \\ \left(\begin{array}{ccccccc} t4 & t5 & t12 & t13 & t14 & t15 & t16 \\ -1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 & 0 \end{array}\right) \end{array} \begin{array}{c} \\ p1 \\ p2 \\ p3 \\ p4 \\ p5 \\ p6 \\ p7 \\ p8 \end{array}$$

Subnet R_a | Subnet R_1^3 | Link transitions t_{15}, t_{16}

3. Optimization problem based on an undefined Petri net

Example 2 | **1. Decomposition into subnets**



$R^c{}_1$

Manufacturing strategy pull.

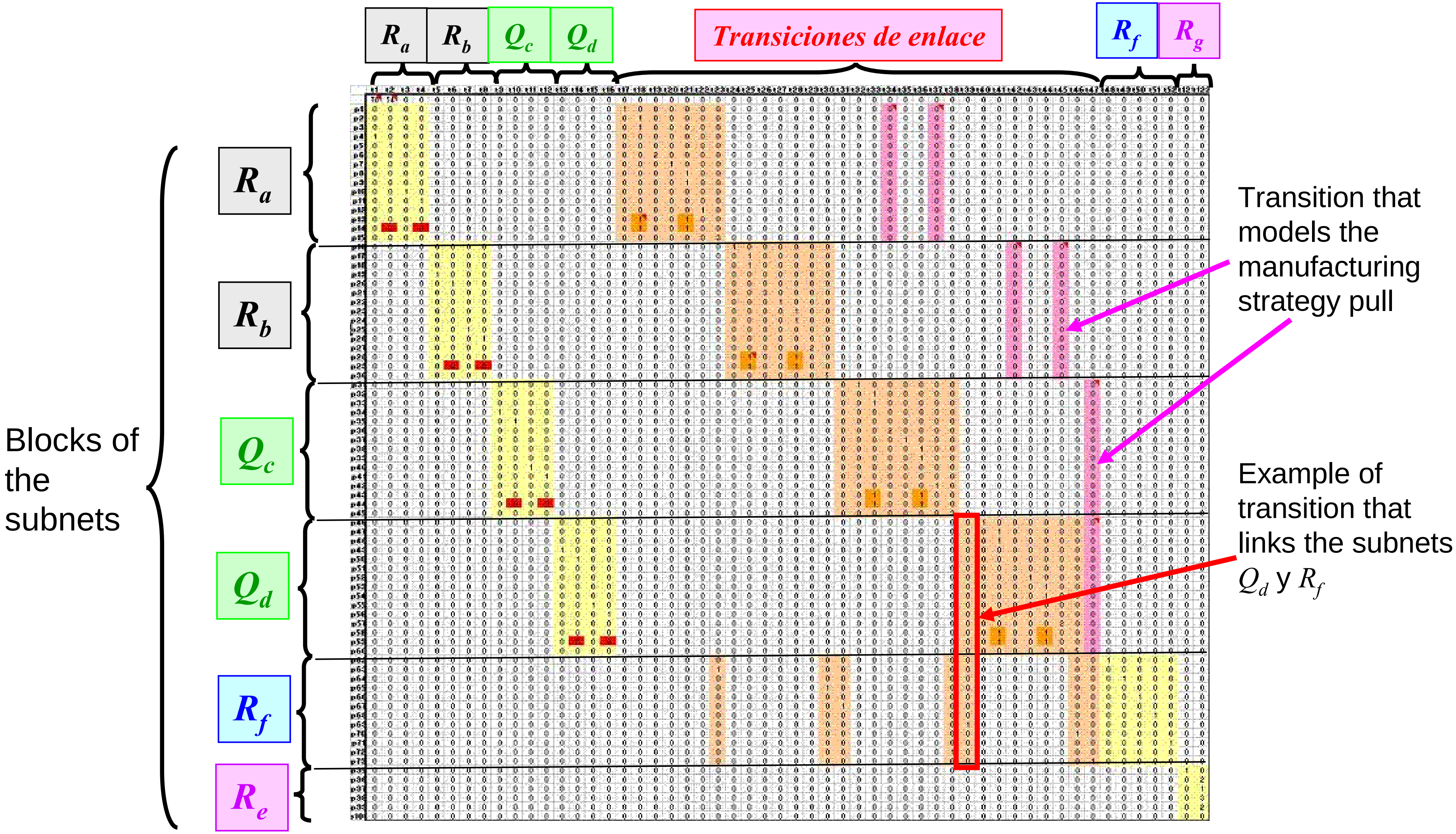
1 route of AGV.

Undefined structural parameters

Lot sizes.

3. Optimization problem based on an undefined Petri net

Example 2 — 1. Decomposition into subnets



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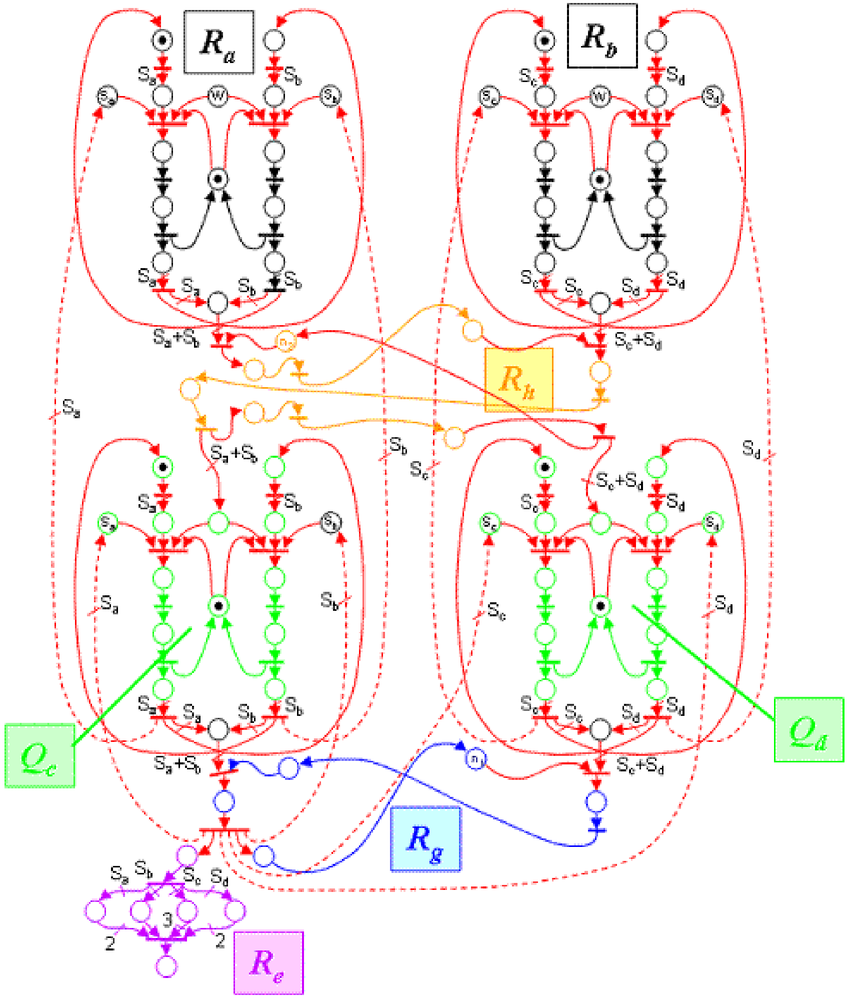
3. Optimization problem based on an undefined Petri net

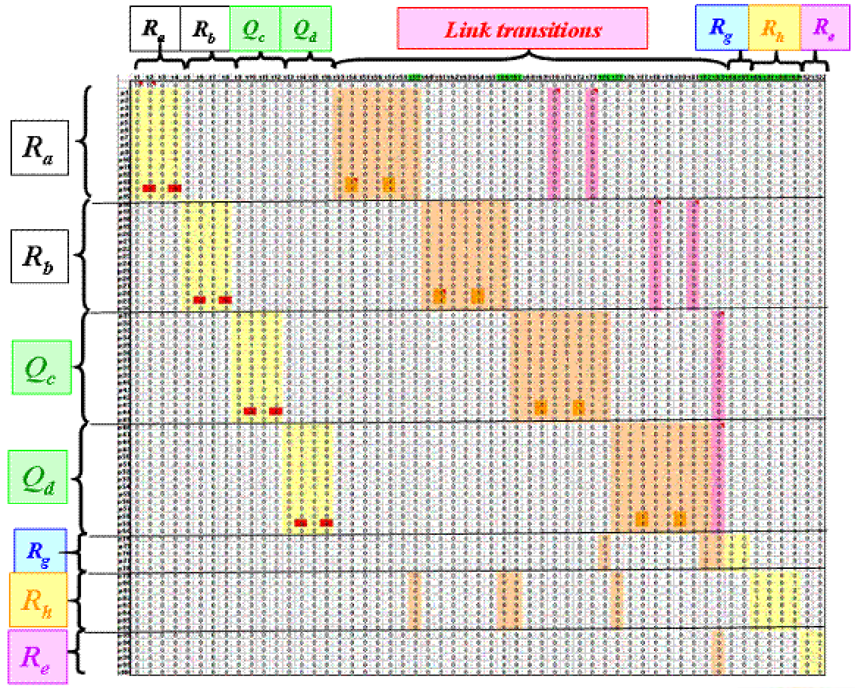
Example 2 | 1. Decomposition into subnets



R^c_2

Manufacturing strategy pull.

2 routes of AGV.



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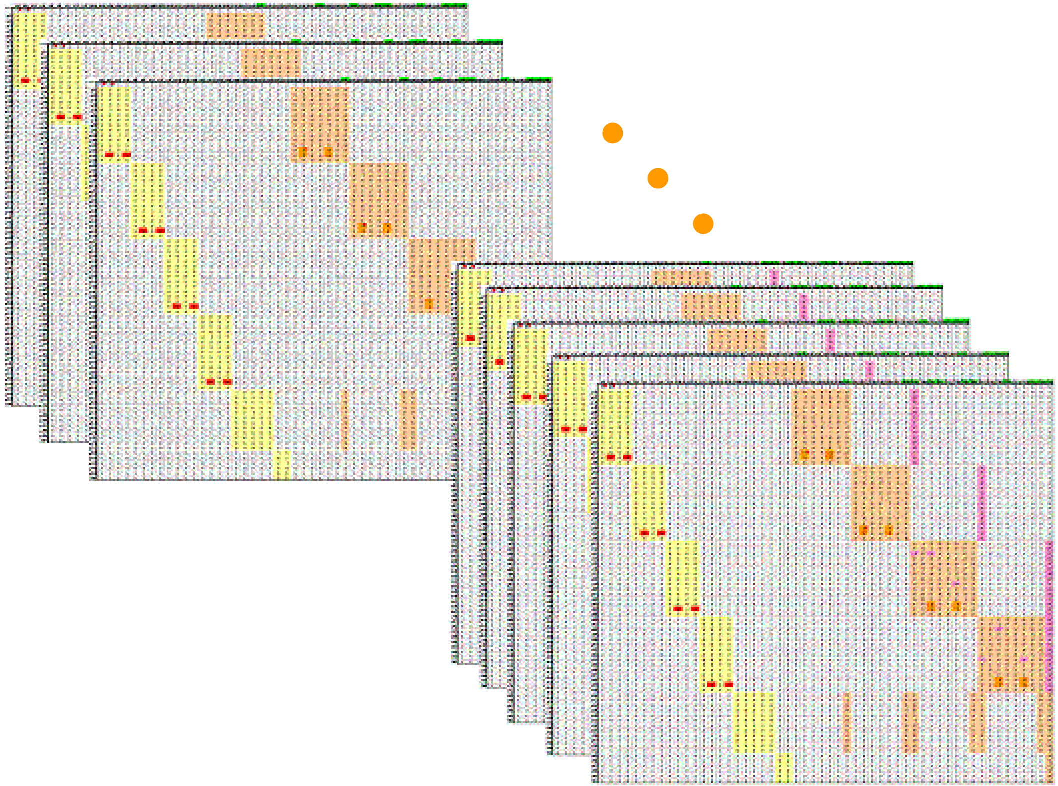
3. Optimization problem based on an undefined Petri net

Example 2	**1. Decomposition into subnets**



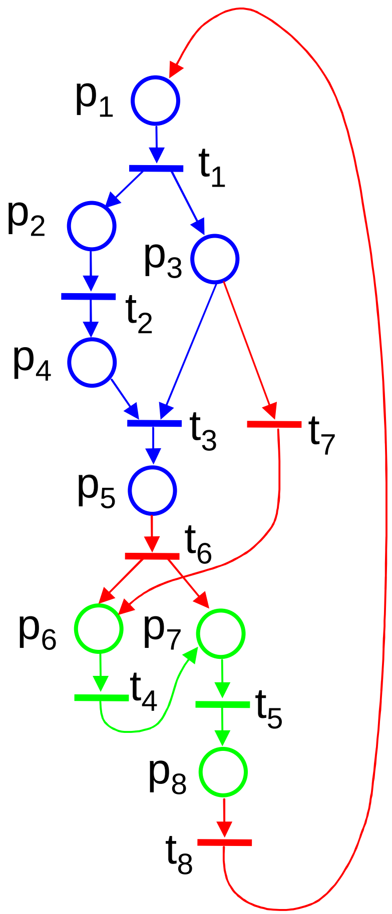
Incidence matrices

24 compound alternative PN.

Size of each matrix ≈ 78 x 54.

3. Optimization problem based on an undefined Petri net

Example 4 — 2. Aggregation of subnets

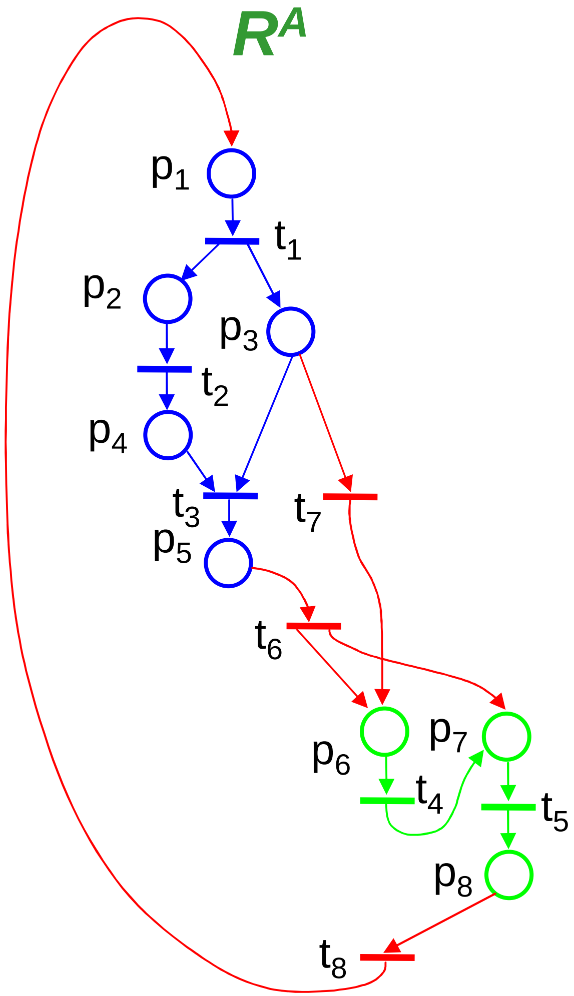


R_1

$$
W_1 = \begin{array}{c} \\ \\ \\ \\ \\ \\ \\ \\ \\ \end{array}
\begin{array}{c}
\begin{array}{cccccccc} t1 & t2 & t3 & t4 & t5 & t6 & t7 & t8 \end{array} \\
\begin{pmatrix}
-1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & -1 & 0 & 0 & 0 & -1 & 0 \\
0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & -1 & 0 & 0 \\
0 & 0 & 0 & -1 & 0 & 1 & 1 & 0 \\
0 & 0 & 0 & 1 & -1 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & -1
\end{pmatrix}
\end{array}
\begin{array}{c} \\ p1 \\ p2 \\ p3 \\ p4 \\ p5 \\ p6 \\ p7 \\ p8 \end{array}
$$

3. Optimization problem based on an undefined Petri net

Example 4 | **2. Aggregation of subnets**

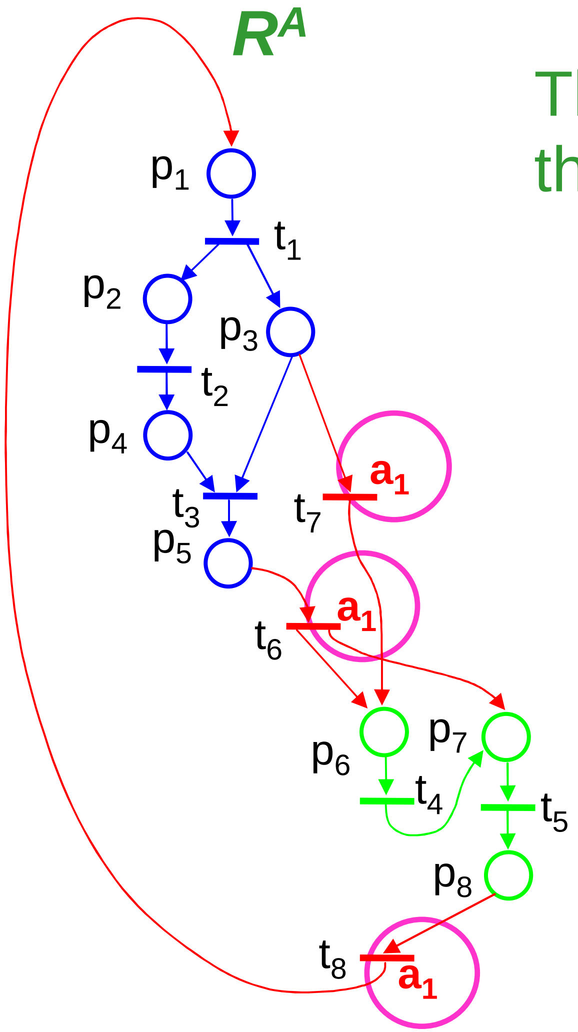


Step 1. Aggregation of R_1

$$
W^A = \begin{array}{c}
\begin{array}{cccccccc} t1 & t2 & t3 & t4 & t5 & t6 & t7 & t8 \end{array} \\
\left(\begin{array}{cccccccc}
-1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & -1 & 0 & 0 & 0 & -1 & 0 \\
0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & -1 & 0 & 0 \\
0 & 0 & 0 & -1 & 0 & 1 & 1 & 0 \\
0 & 0 & 0 & 1 & -1 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & -1
\end{array}\right)
\end{array}
\begin{array}{c} \\ p1 \\ p2 \\ p3 \\ p4 \\ p5 \\ p6 \\ p7 \\ p8 \end{array}
$$

3. Optimization problem based on an undefined Petri net

Example 4 — 2. Aggregation of subnets



The choice variable a_1 is associated to the link transitions of R_1.

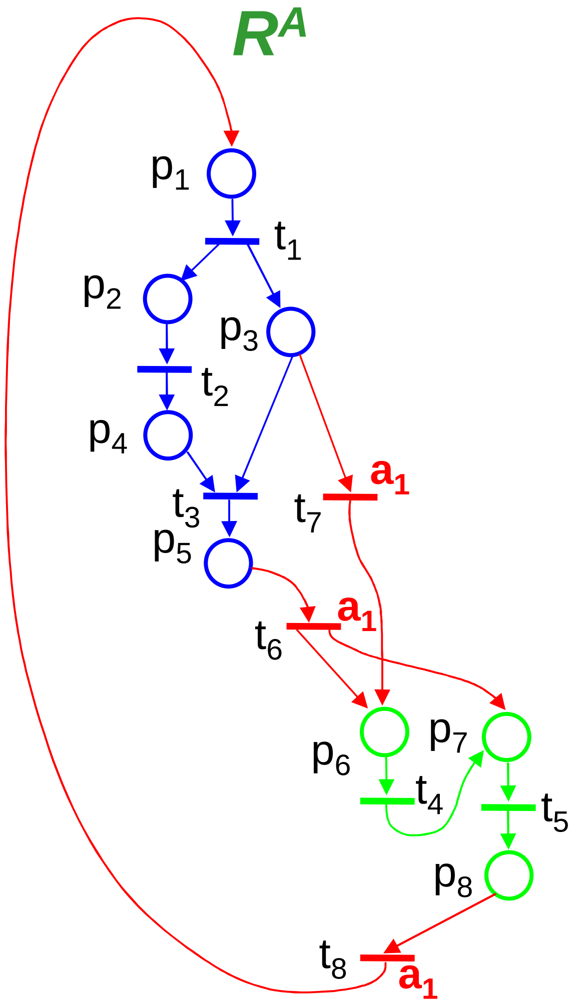
$$W^A = \begin{array}{c} \\ \\ \end{array}\begin{array}{cccccccc} t1 & t2 & t3 & t4 & t5 & t6 & t7 & t8 \end{array}$$

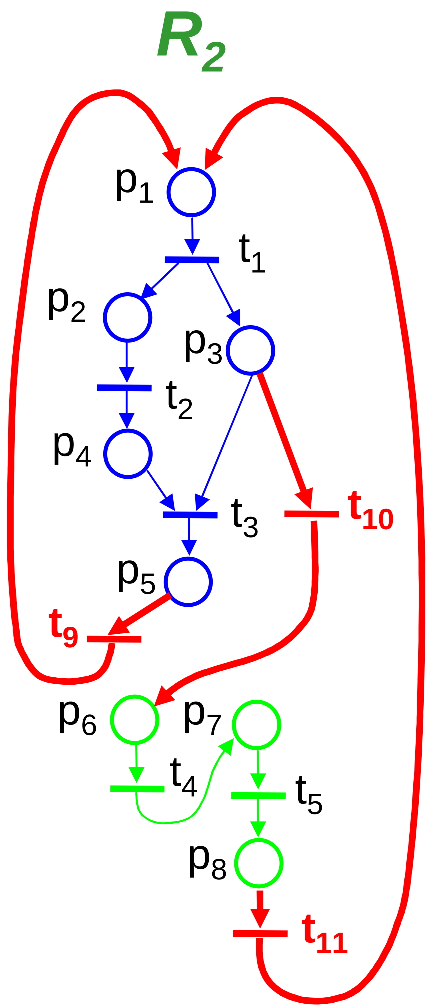
$$W^A = \left(\begin{array}{ccccc|ccc} -1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & -1 \end{array}\right)\begin{array}{c} p1 \\ p2 \\ p3 \\ p4 \\ p5 \\ p6 \\ p7 \\ p8 \end{array}$$

Columns t6, t7, t8: $\underbrace{t6\;t7\;t8}_{a_1}$

3. Optimization problem based on an undefined Petri net

Example 4 — 2. Aggregation of subnets





R_2

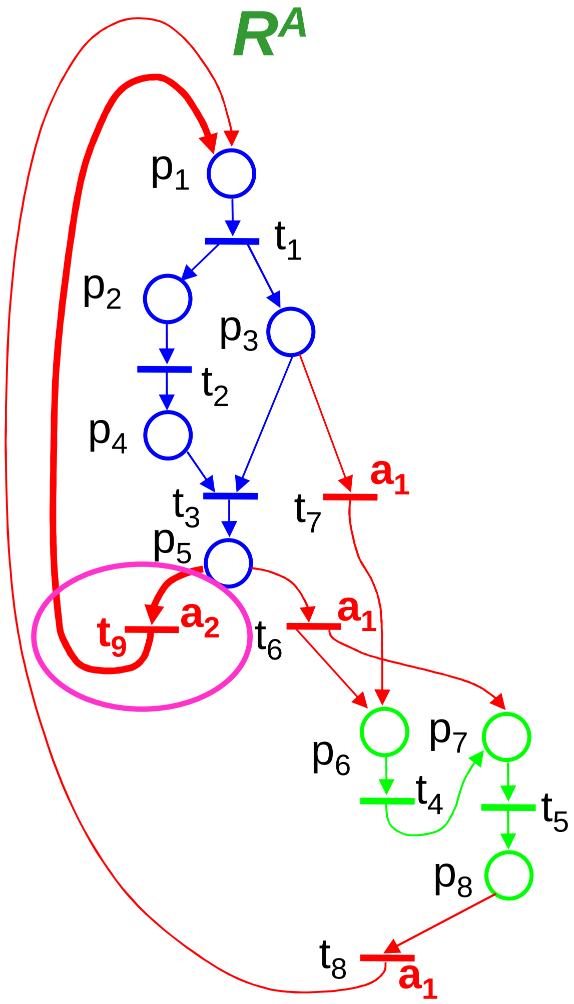
It does not provide with new subnets.

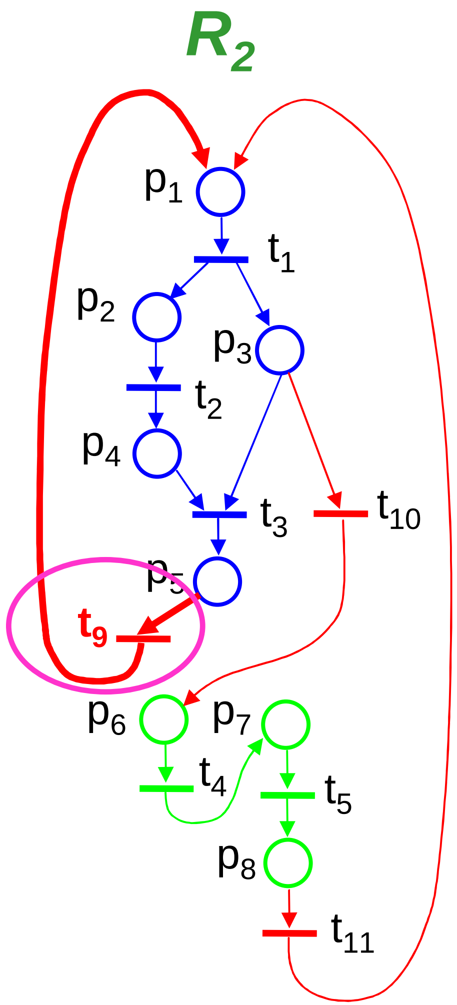
It has three link transitions.

t_9, t_{10}, t_{11}.

3. Optimization problem based on an undefined Petri net

Example 4 | 2. Aggregation of subnets





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3. Optimization problem based on an undefined Petri net

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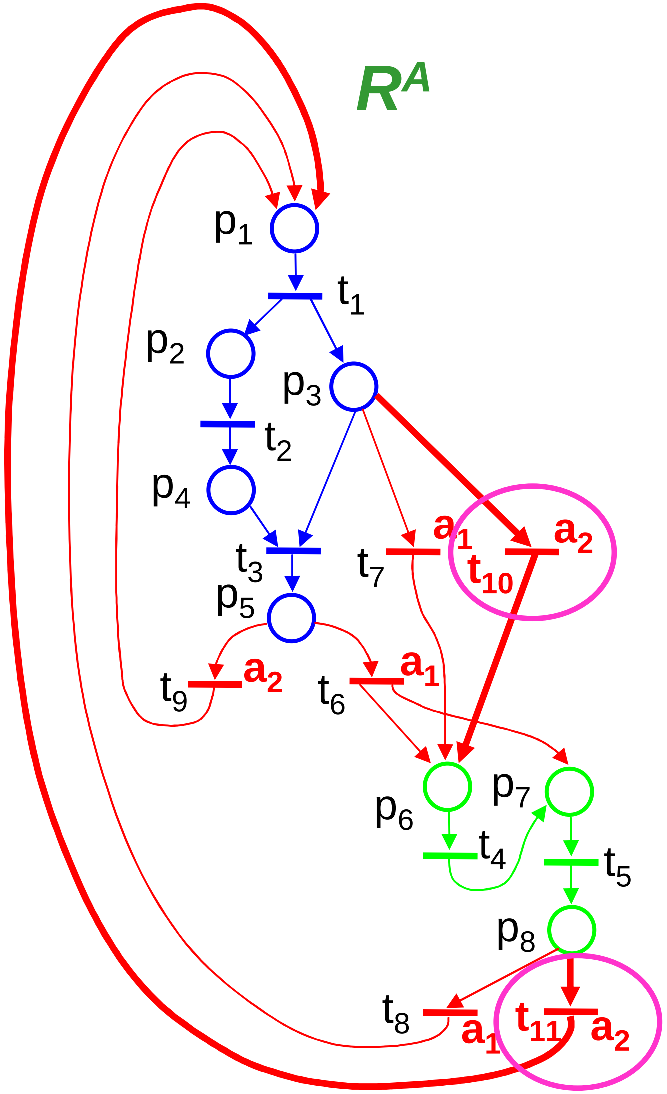

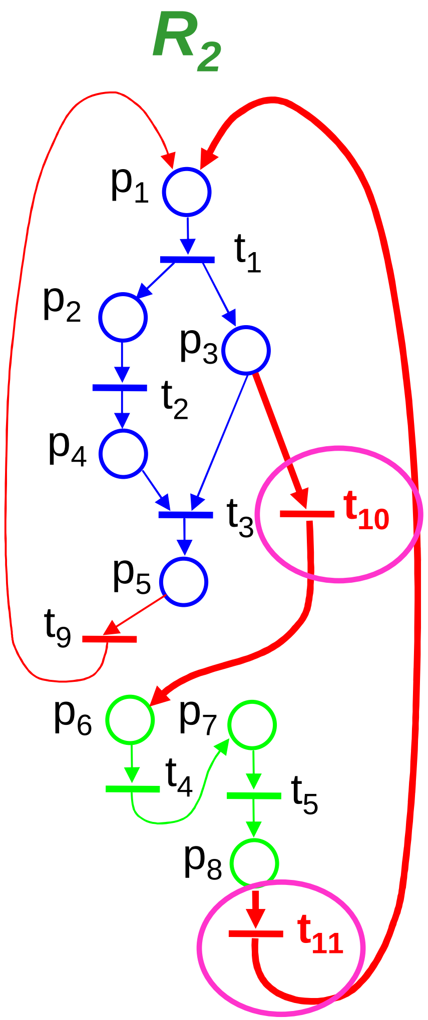

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3. Optimization problem based on an undefined Petri net

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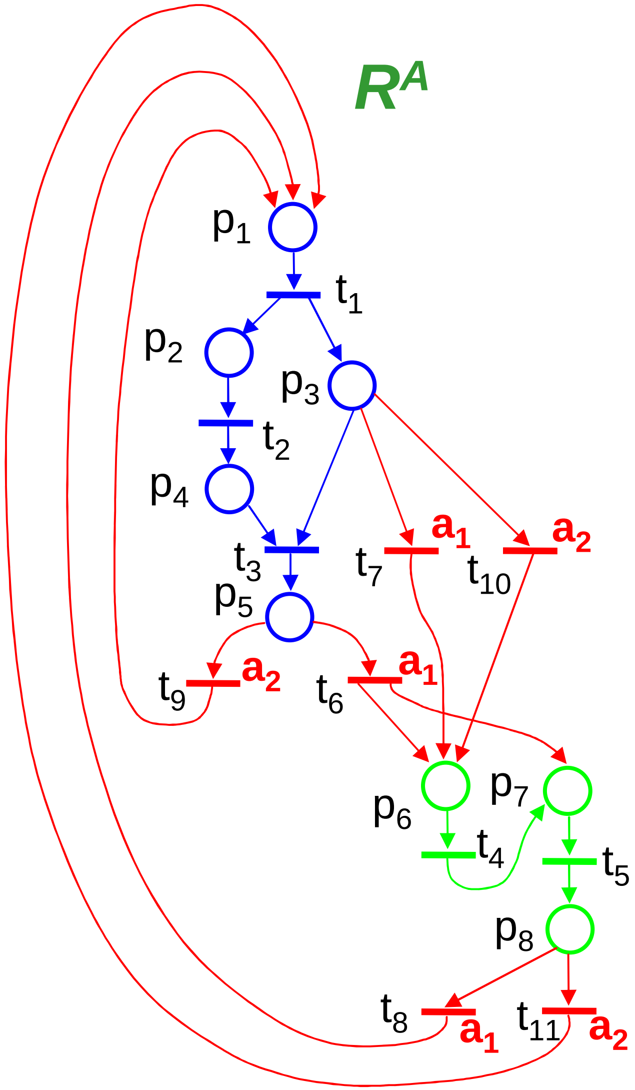
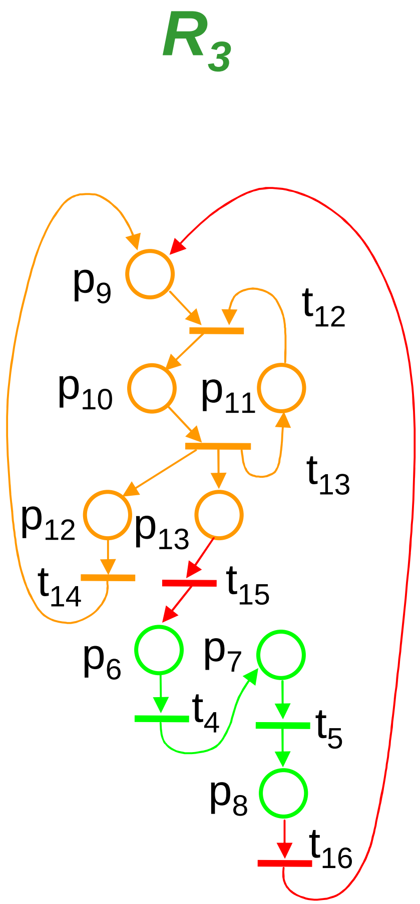




R_3

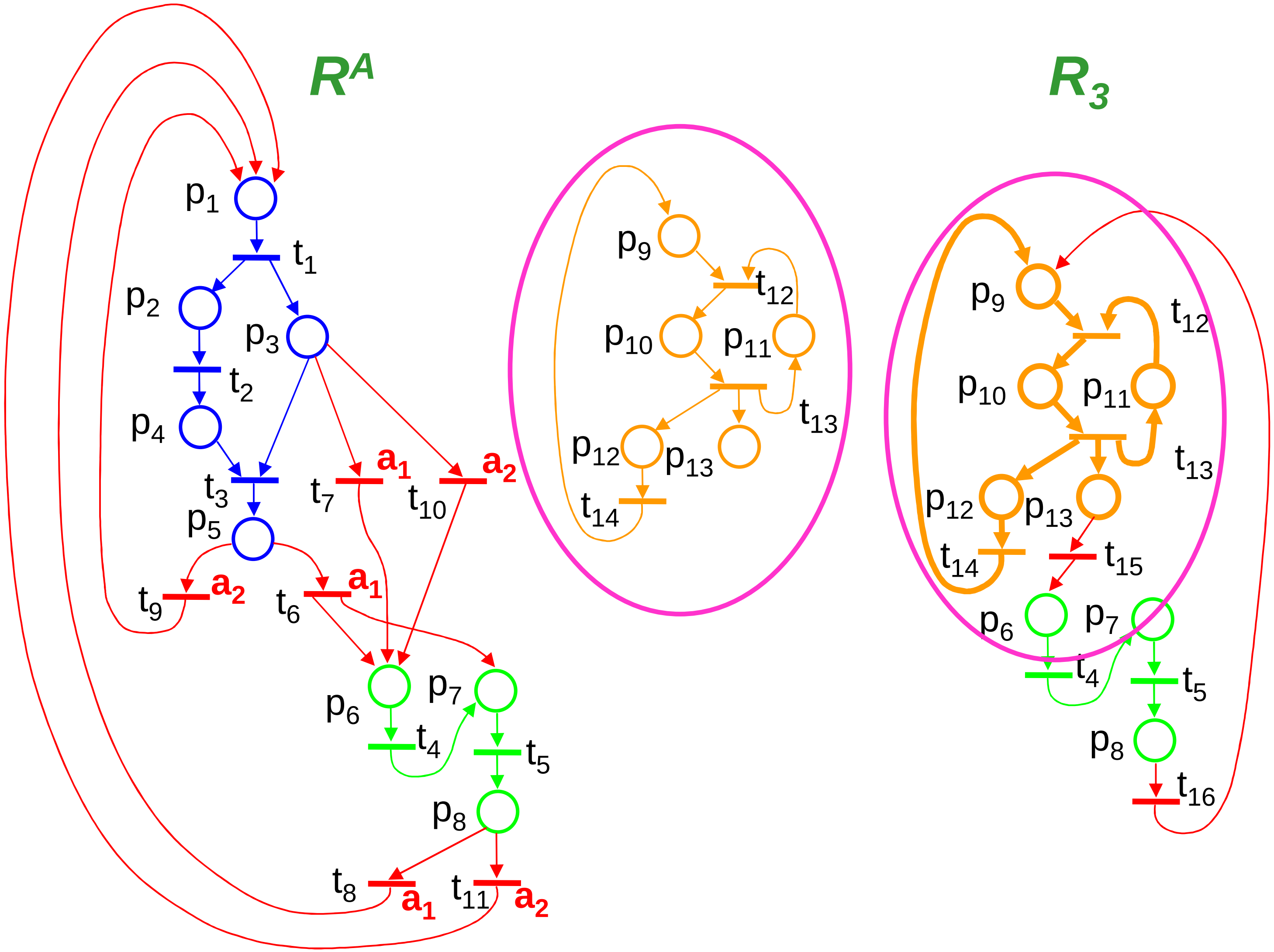
It provides with a new subnet.

It has two link transitions.

t_{15} and t_{16}

3. Optimization problem based on an undefined Petri net

Example 4 — 2. Aggregation of subnets



R_3

It provides with a new subnet.

It has two link transitions.

t_{15} and t_{16}

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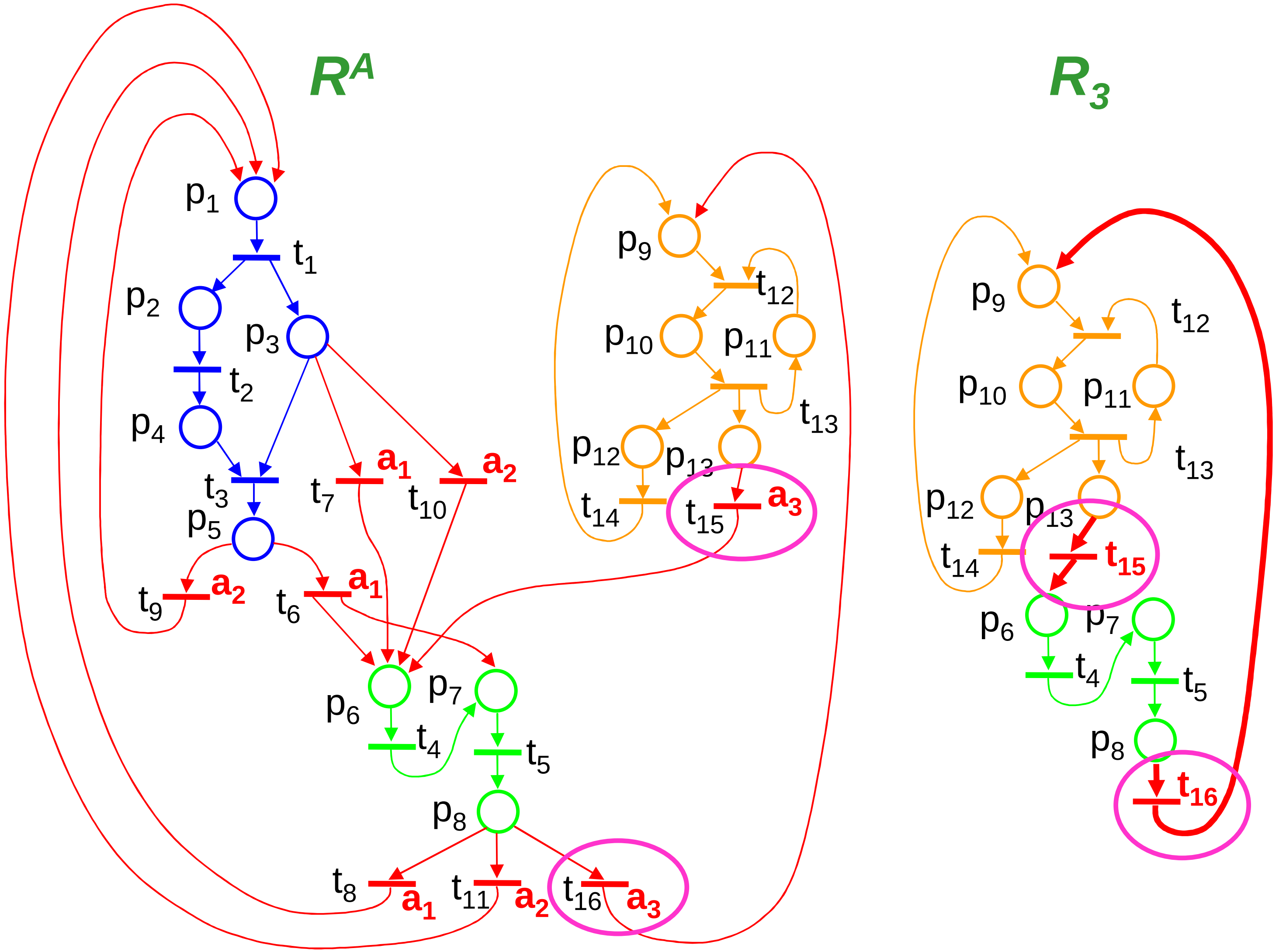
3. Optimization problem based on an undefined Petri net

Example 4	**2. Aggregation of subnets**



R_3

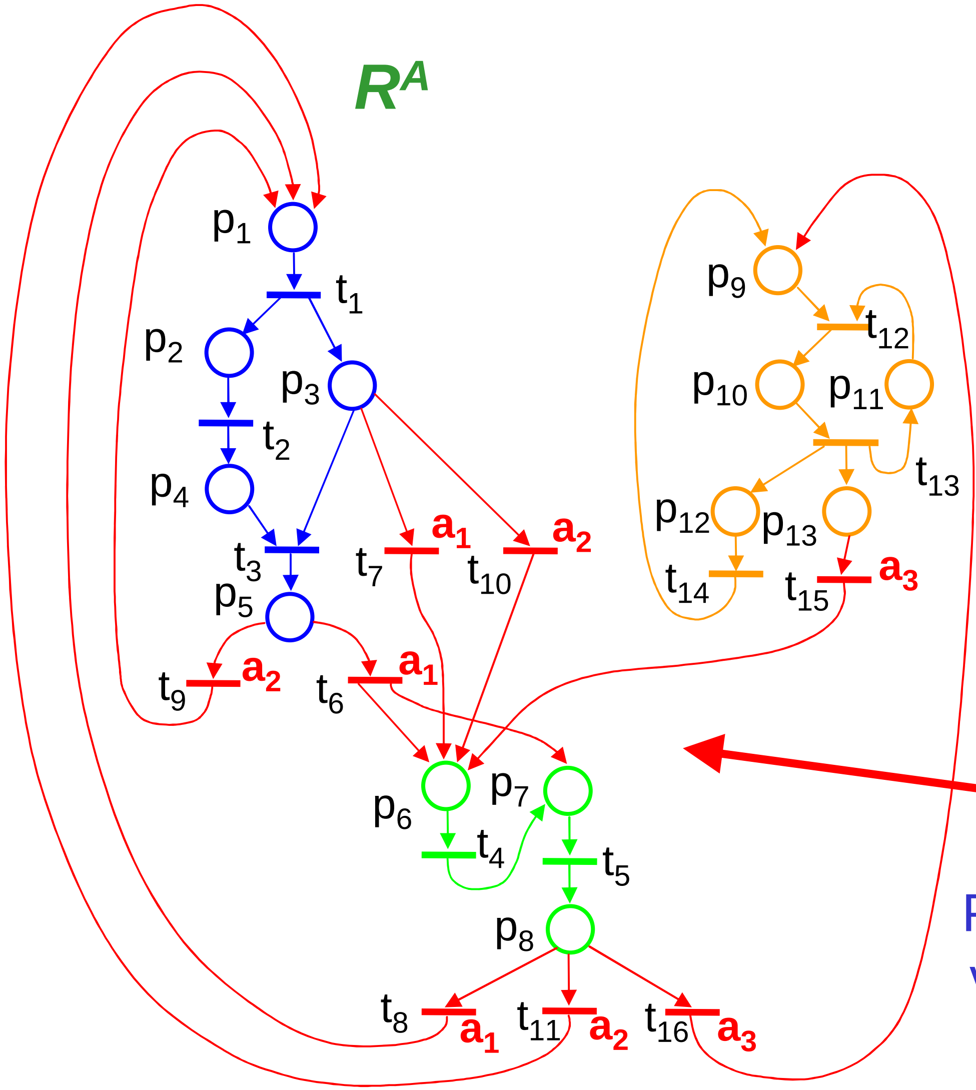
It provides with a new subnet.

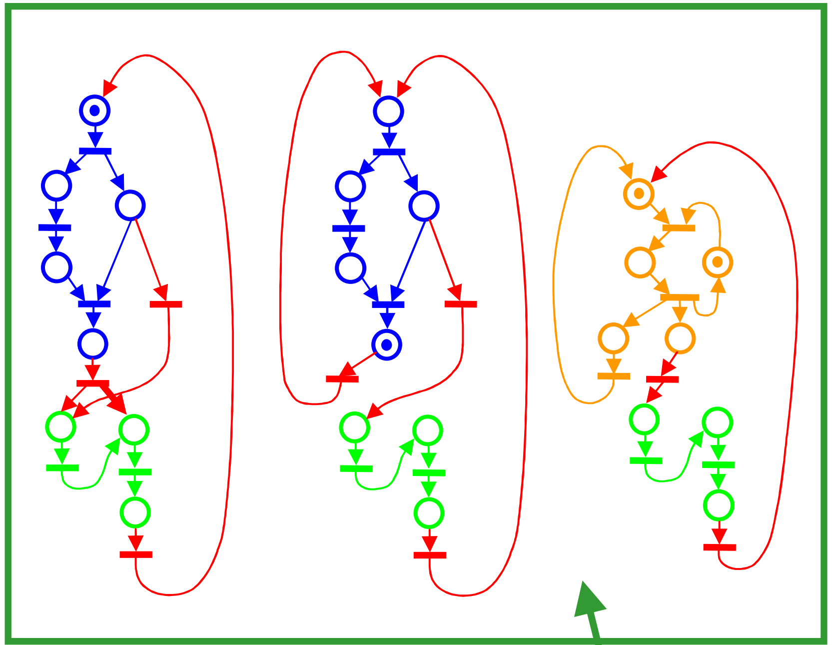
It has two link transitions.

t_{15} and t_{16}

3. Optimization problem based on an undefined Petri net

Example 4 | 2. Aggregation of subnets





Basic AAPN

Presents a choice variable for every link transition.

Equivalent set of simple alternative PN

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3. Optimization problem based on an undefined Petri net

Example 4 — 2. Aggregation of subnets

Incidence matrix of the AAPN

Q_a

Q_b

$$W^A = \begin{array}{c} \\ \left(\begin{array}{cccccccccccccccc} t1 & t2 & t3 & t4 & t5 & t6 & t7 & t8 & t9 & t10 & t11 & t12 & t13 & t14 & t15 & t16 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 & 0 & -1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & -1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 \end{array}\right) \end{array} \begin{array}{c} \\ p1 \\ p2 \\ p3 \\ p4 \\ p5 \\ p6 \\ p7 \\ p8 \\ p9 \\ p10 \\ p11 \\ p12 \\ p13 \end{array}$$

Choice variables associated to the link transitions: a_1 (t6, t7, t8), a_2 (t9, t10, t11), Q^3_1 (t12, t13, t14), a_3 (t15, t16)

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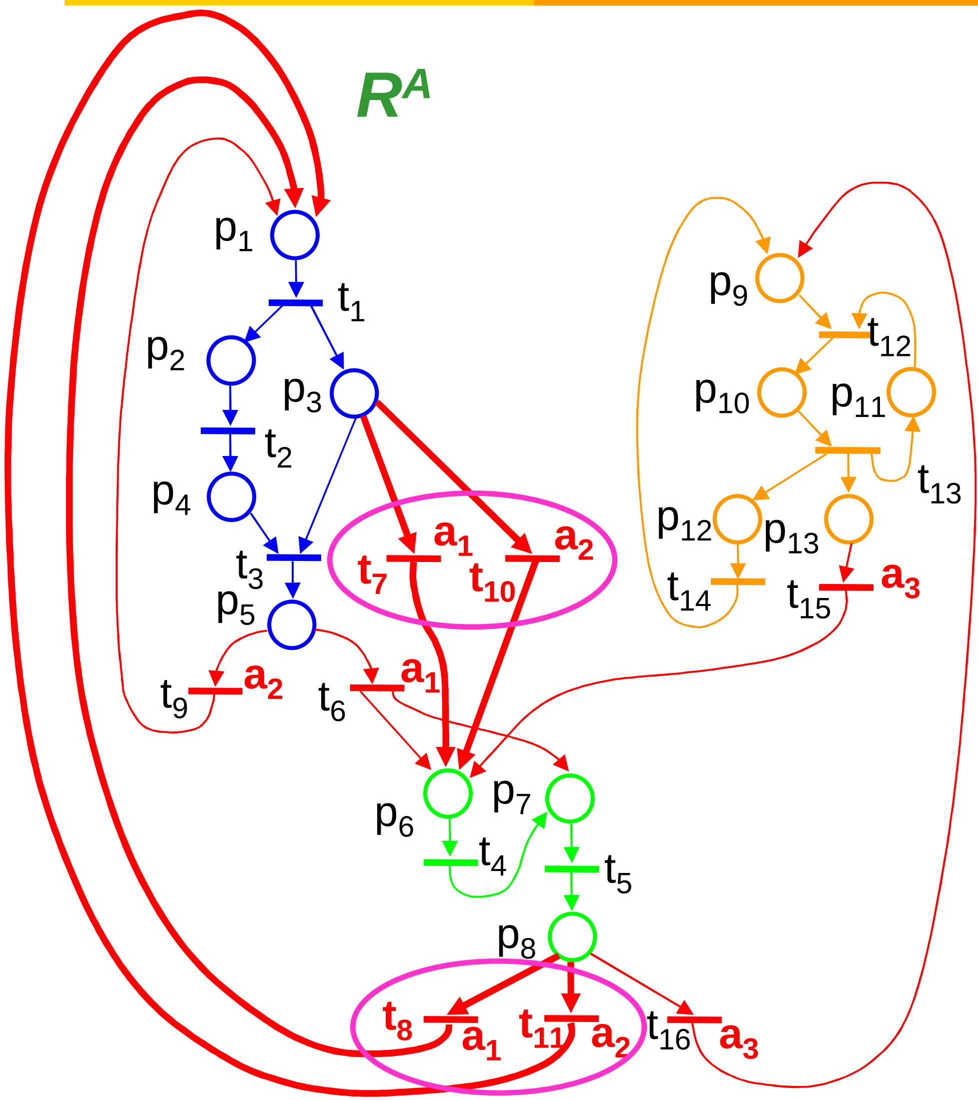
3. Optimization problem based on an undefined Petri net

Example 4 | **3. Reduction of the link transitions**



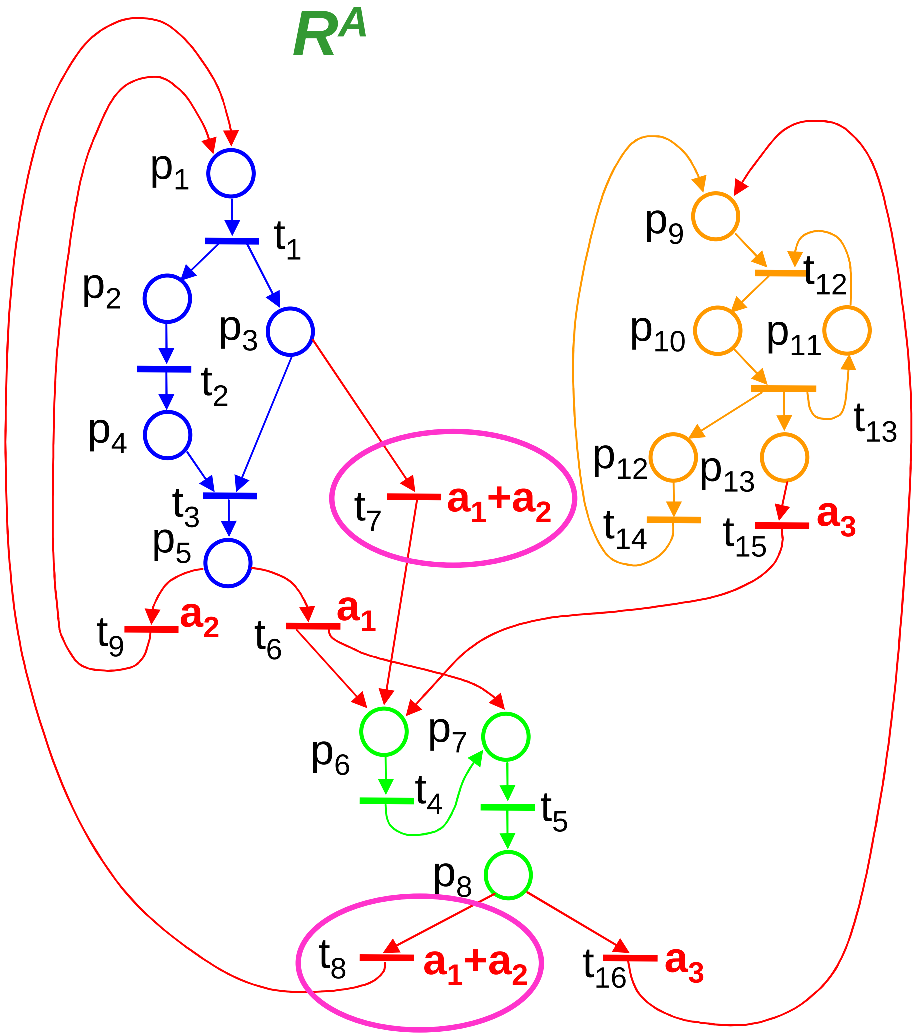
Application of the reduction rule

$\{t_7, t_{10}\}$ reduced to t_7 by associating the next function of choice variables $f_7=a_1+a_2$

$\{t_8, t_{11}\}$ reduced to t_8 by associating the next function of choice variables $f_8=a_1+a_2$

3. Optimization problem based on an undefined Petri net

Example 4 | **3. Reduction of the link transitions**



Application of the reduction rule

$\{t_7, t_{10}\}$ reduced to t_7 by associating the next function of choice variables $f_7=a_1+a_2$

$\{t_8, t_{11}\}$ reduced to t_8 by associating the next function of choice variables $f_8=a_1+a_2$

The incidence matrices are reduced proportionally.

3. Optimization problem based on an undefined Petri net

Example 4 — 3. Reduction of the link transitions

In the incidence matrix of the AAPN there exist several identical columns:

1) Every identical columns can be reduced to a single one.
2) A function of choice variables is associated to the column which is equivalent to the reduced transitions.

Q_a

Q_b

$$W^A = \begin{array}{c|cccccccccccccccc|c}
 & t1 & t2 & t3 & t4 & t5 & t6 & t7 & t8 & t9 & t10 & t11 & t12 & t13 & t14 & t15 & t16 & \\
\hline
 & -1 & 0 & 0 & 0 & 0 & 0 & \boxed{0} & \boxed{1} & 1 & \boxed{0} & \boxed{1} & 0 & 0 & 0 & 0 & 0 & p1 \\
 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & p2 \\
 & 1 & 0 & -1 & 0 & 0 & 0 & -1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & p3 \\
 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & p4 \\
 & 0 & 0 & 1 & 0 & 0 & -1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & p5 \\
 & 0 & 0 & 0 & -1 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & p6 \\
 & 0 & 0 & 0 & 1 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & p7 \\
 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & -1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & -1 & p8 \\
 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 & 1 & p9 \\
 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & p10 \\
 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & p11 \\
 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & p12 \\
 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 & p13
\end{array}$$

Choice variables associated to the link transitions: a_1 a_2 Q^3_1 a_3

3. Problema de 3. Optimization problem based on an undefined Petri net basado en una RdP indefinida

Example 4 — 3. Reduction of the link transitions

In the incidence matrix of the AAPN there exist several identical columns:

1) Every identical columns can be reduced to a single one.

2) A function of choice variables is associated to the column which is equivalent to the reduced transitions.

Q_a

Q_b

$W^A =$

	t1	t2	t3	t4	t5	t6	t7	t8	t9	t10	t11	t12	t13	t14	t15	t16
p1	-1	0	0	0	0	0	0	1	1	0	1	0	0	0	0	0
p2	1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
p3	1	0	-1	0	0	0	-1	0	0	-1	0	0	0	0	0	0
p4	0	1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0
p5	0	0	1	0	0	-1	0	0	-1	0	0	0	0	0	0	0
p6	0	0	0	-1	0	1	1	0	0	1	0	0	0	0	1	0
p7	0	0	0	1	-1	1	0	0	0	0	0	0	0	0	0	0
p8	0	0	0	0	1	0	0	-1	0	0	-1	0	0	0	0	-1
p9	0	0	0	0	0	0	0	0	0	0	0	-1	0	1	0	1
p10	0	0	0	0	0	0	0	0	0	0	0	1	-1	0	0	0
p11	0	0	0	0	0	0	0	0	0	0	0	-1	1	0	0	0
p12	0	0	0	0	0	0	0	0	0	0	0	0	1	-1	0	0
p13	0	0	0	0	0	0	0	0	0	0	0	0	1	0	-1	0

Choice variables associated to the link transitions: a_1 a_2 Q^3_1 a_3

3. Optimization problem based on an undefined Petri net

Example 4 — 3. Reduction of the link transitions

In the incidence matrix of the AAPN there exist several identical columns:

1) Every identical columns can be reduced to a single one.
2) A function of choice variables is associated to the column which is equivalent to the reduced transitions.

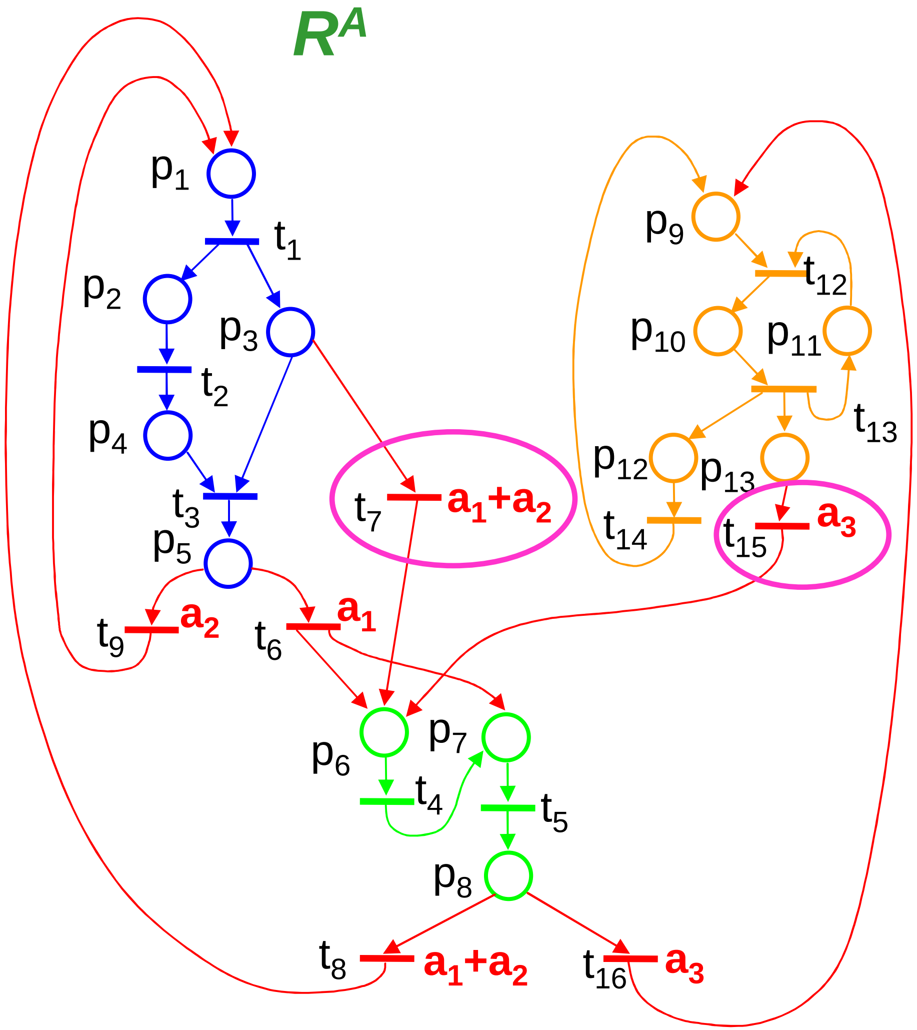
$$
W^A = \begin{array}{c}
\begin{array}{cccccccccccccc} t1 & t2 & t3 & t4 & t5 & t6 & t7 & t8 & t9 & t12 & t13 & t14 & t15 & t16 \end{array} \\
\left(\begin{array}{cccccccccccccc}
-1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\
1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & -1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & -1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & -1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0
\end{array}\right)
\end{array}
\begin{array}{c} \\ p1 \\ p2 \\ p3 \\ p4 \\ p5 \\ p6 \\ p7 \\ p8 \\ p9 \\ p10 \\ p11 \\ p12 \\ p13 \end{array}
$$

Q_a Q_b

Choice variables associated to the link transitions: a_1 (t6), a_1+a_2 (t7, t8), a_2 (t9), Q^3_1 (t12, t13, t14), a_3 (t15, t16)

3. Optimization problem based on an undefined Petri net

Example 4 — 4. Simplification of functions

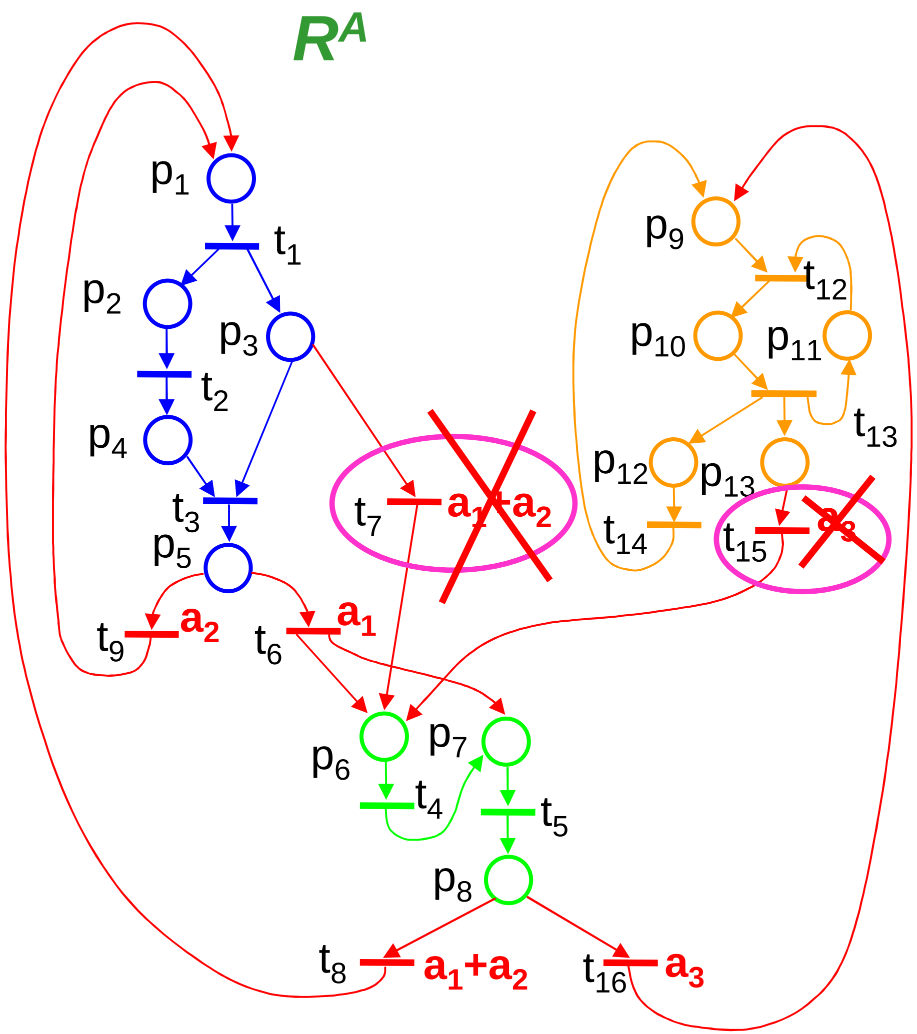


Application of the rule of simplification

t_7 are t_{10} are only enabled when one of the choice variables of the associated function is active.

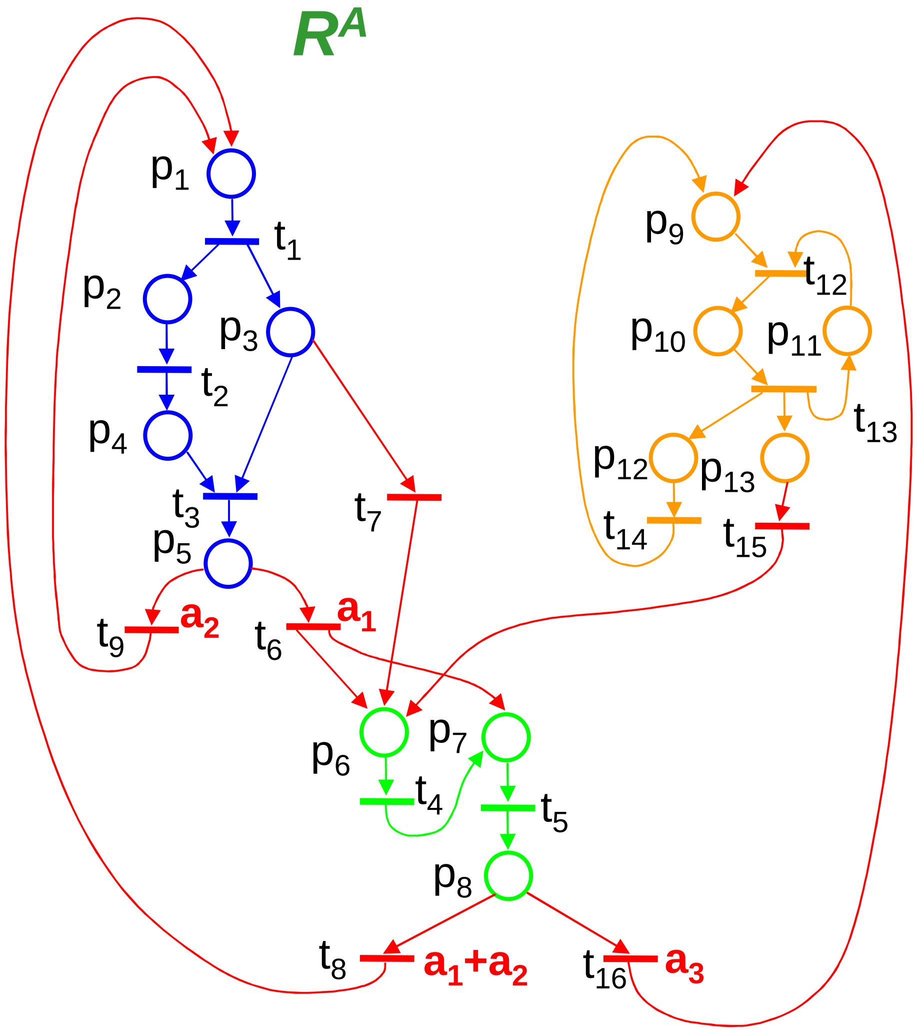
3. Optimization problem based on an undefined Petri net

Example 4 | **4. Simplification of functions**



Application of the rule of simplification

t_7 are t_{10} are only enabled when one of the choice variables of the associated function is active.



Alternatives aggregation PN

PN equivalent to the original set of simple alternative PN

Incidence matrix of the AAPN

$$
W^A = \begin{array}{c}
\begin{array}{cccccccccccccc}
t1 & t2 & t3 & t4 & t5 & t6 & t7 & t8 & t9 & t12 & t13 & t14 & t15 & t16
\end{array} \\
\left(\begin{array}{cccccccccccccc}
-1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\
1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & -1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & -1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & -1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0
\end{array}\right)
\end{array}
\begin{array}{c}
\\ p1 \\ p2 \\ p3 \\ p4 \\ p5 \\ p6 \\ p7 \\ p8 \\ p9 \\ p10 \\ p11 \\ p12 \\ p13
\end{array}
$$

Q_a

Q_b

a_1 $\quad$ $\underbrace{\qquad}_{a_1 + a_2}$ $\quad$ a_2 $\quad$ $Q^3{}_1$ $\quad$ a_3

Choice variables associated to the link transitions:

3. Optimization problem based on an undefined Petri net

Example 2 — 3 y 4. Reduction and simplification

The number of link transitions is reduced from 741 to 92.

Equivalent transition

Reduced transitions

Merged transition	R1	R2	R3	R4	R5	R6	R7	R8	R9	R10	R11	R12	R13	R14	R15	R16	R17	R18	R19	R20	R21	R22	R23	R24
t17	t17	t53	t90	t127	t158	t189	t220	t251	t282	t313	t344	t375	t406	t437	t468	t499	t530	t561	t592	t623	t654	t685	t716	t747
t18	t18	t54	t91	t128	t159	t190	t221	t252	t283	t314	t345	t376	t407	t438	t469	t500	t531	t562	t593	t624	t655	t686	t717	t748
t19	t19	t55	t92	t129	t160	t191	t222	t253	t284	t315	t346	t377	t408	t439	t470				t594	t625	t656	t687	t718	t749
t501																t501	t532	t563						
t20	t20	t56	t93	t130	t161	t192	t223	t254	t285	t316	t347	t378	t409	t440	t471	t502	t533	t564	t595	t626	t657	t688	t719	t750
t21	t21	t57	t94	t131	t162	t193	t224	t255	t286	t317	t348	t379	t410	t441	t472	t503	t534	t565	t596	t627	t658	t689	t720	t751
t22	t22	t58	t95	t132	t163	t194	t225	t256	t287	t318	t349	t380	t411	t442	t473				t597	t628	t659	t690	t721	t752
t504																t504	t535	t566						
t23	t23						t226									t505			t598			t691		
t59		t59						t257									t536			t629			t722	
t96			t96						t288									t567			t660			t753
t133				t133						t319														
t164					t164						t350													
t195						t195						t381												
t412													t412											
t443														t443										
t474															t474									
t24	t24	t60	t97	t134	t165	t196	t227	t258	t289	t320	t351	t382	t413	t444	t475	t506	t537	t568	t599	t630	t661	t692	t723	t754
t25	t25	t61	t98	t135	t166	t197	t228	t259	t290	t321	t352	t383	t414	t445	t476	t507	t538	t569	t600	t631	t662	t693	t724	t755
t26	t26	t62	t99	t136	t167	t198	t229	t260	t291	t322	t353	t384	t415	t446	t477				t601	t632	t663	t694	t725	t756
t508																t508	t539	t570						
t27	t27	t63	t100	t137	t168	t199	t230	t261	t292	t323	t354	t385	t416	t447	t478	t509	t540	t571	t602	t633	t664	t695	t726	t757
t28	t28	t64	t101	t138	t169	t200	t231	t262	t293	t324	t355	t386	t417	t448	t479	t510	t541	t572	t603	t634	t665	t696	t727	t758
t29	t29	t65	t102	t139	t170	t201	t232	t263	t294	t325	t356	t387	t418	t449	t480				t604	t635	t666	t697	t728	t759
t511																t511	t542	t573						
t30	t30						t233									t512			t605			t698		
t66		t66						t264									t543			t636			t729	
t103			t103						t295									t574			t667			t760
t140				t140						t326														
t171					t171						t357													
t202						t202						t388												
t419													t419											
t450														t450										
t481															t481									
t31	t31			t141			t234			t327			t420			t513			t606					
t67		t67			t172			t265			t358			t451			t544			t637				
t104			t104			t203			t296			t389			t482			t575			t668			
t699																						t699		
t730																							t730	
t761																								t761
t32	t32	t68	t105	t142	t173	t204	t235	t266	t297	t328	t359	t390	t421	t452	t483	t514	t545	t576	t607	t638	t669	t700	t731	t762
t33	t33	t69	t106	t143	t174	t205	t236	t267	t298	t329	t360	t391				t515	t546	t577						
t422													t422	t453	t484							t701	t732	t763
t608																			t608	t639	t670			
t34	t34	t70	t107				t237	t268	t299															
t144				t144	t175	t206				t330	t361	t392	t423	t454	t485				t609	t640	t671	t702	t733	t764

Merged transition	R1	R2	R3	R4	R5	R6	R7	R8	R9	R10	R11	R12	R13	R14	R15	R16	R17	R18	R19	R20	R21	R22	R23	R24
t144				t144	t175	t206				t330	t361	t392	t423	t454	t485				t609	t640	t671	t702	t733	t764
t516																t516	t547	t578						
t35	t35	t71	t108	t145	t176	t207	t238	t269	t300	t331	t362	t393	t424	t455	t486	t517	t548	t579	t610	t641	t672	t703	t734	t765
t36	t36	t72	t109	t146	t177	t208	t239	t270	t301	t332	t363	t394				t518	t549	t580						
t425													t425	t456	t487							t704	t735	t766
t611																			t611	t642	t673			
t37	t37	t73	t110				t240	t271	t302															
t147				t147	t178	t209				t333	t364	t395	t426	t457	t488				t612	t643	t674	t705	t736	t767
t519																t519	t550	t581						
t38	t38									t334						t520			t613			t706		
t74		t74	t111								t365	t396					t551	t582		t644	t675		t737	t768
t148				t148			t241																	
t179					t179	t210		t272	t303															
t427													t427											
t458														t458	t489									
t39	t39			t149			t242			t335			t428			t521			t614					
t75		t75			t180			t273			t366			t459			t552			t645				
t112			t112			t211			t304			t397			t490			t583			t676			
t707																						t707		
t738																							t738	
t769																								t769
t40	t40	t76	t113	t150	t181	t212	t243	t274	t305	t336	t367	t398	t429	t460	t491	t522	t553	t584	t615	t646	t677	t708	t739	t770
t41	t41	t77	t114	t151	t182	t213	t244	t275	t306	t337	t368	t399				t523	t554	t585						
t430													t430	t461	t492							t709	t740	t771
t616																			t616	t647	t678			
t42	t42	t78	t115				t245	t276	t307															
t152				t152	t183	t214				t338	t369	t400	t431	t462	t493				t617	t648	t679	t710	t741	t772
t524																t524	t555	t586						
t43	t43	t79	t116	t153	t184	t215	t246	t277	t308	t339	t370	t401	t432	t463	t494	t525	t556	t587	t618	t649	t680	t711	t742	t773
t44	t44	t80	t117	t154	t185	t216	t247	t278	t309	t340	t371	t402				t526	t557	t588						
t433													t433	t464	t495							t712	t743	t774
t619																			t619	t650	t681			
t45	t45	t81	t118				t248	t279	t310															
t155				t155	t186	t217				t341	t372	t403	t434	t465	t496				t620	t651	t682	t713	t744	t775
t527																t527	t558	t589						
t46	t46									t342						t528			t621			t714		
t82		t82	t119								t373	t404					t559	t590		t652	t683		t745	t776
t156				t156			t249																	
t187					t187	t218		t280	t311															
t435													t435											
t466														t466	t497									
t47	t47									t343									t622					
t83		t83	t120								t374	t405								t653	t684			
t157				t157			t250						t436			t529								
t188					t188	t219		t291	t312					t467	t498		t560	t591						
t715																						t715		
t746																							t746	t777

3. Optimization problem based on an undefined Petri net

Example 2 | 3 y 4. Reduction and simplification

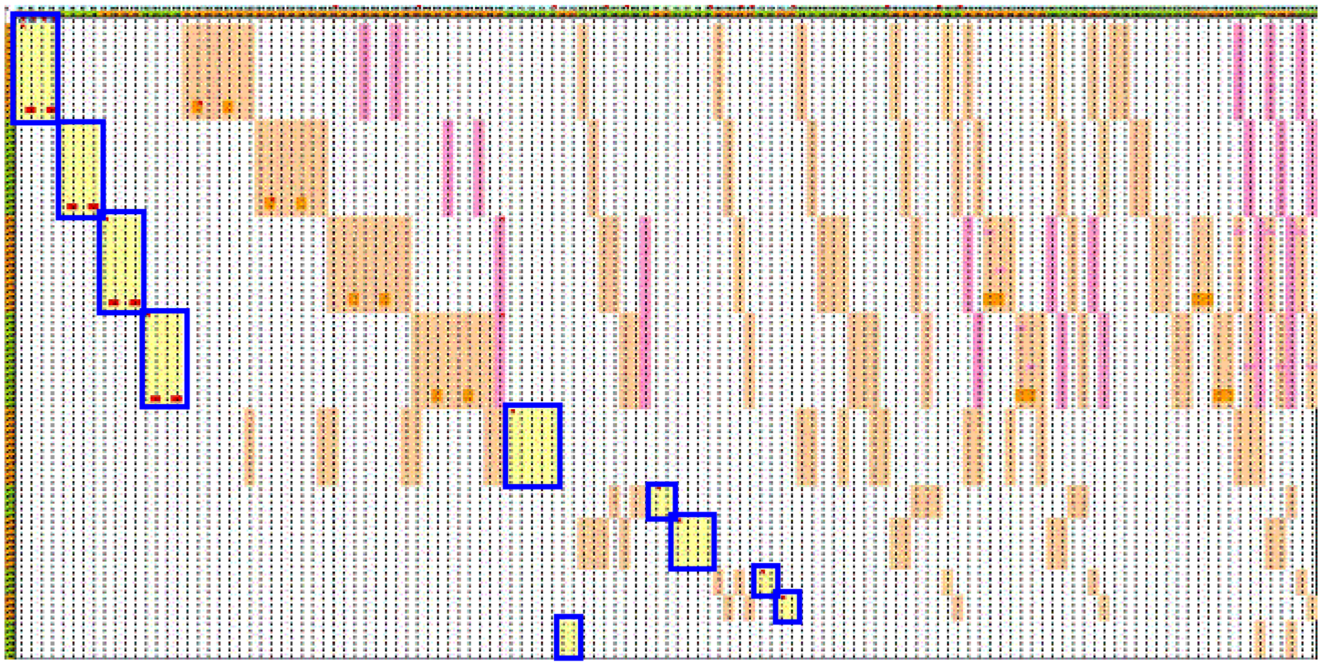
Incidence matrix of the AAPN (Same information than in the original alternative PN).

Size = 99x125 versus 78x54 in the ones from everyone of the 24 alternative PN.

Blocks associated to the subnets.

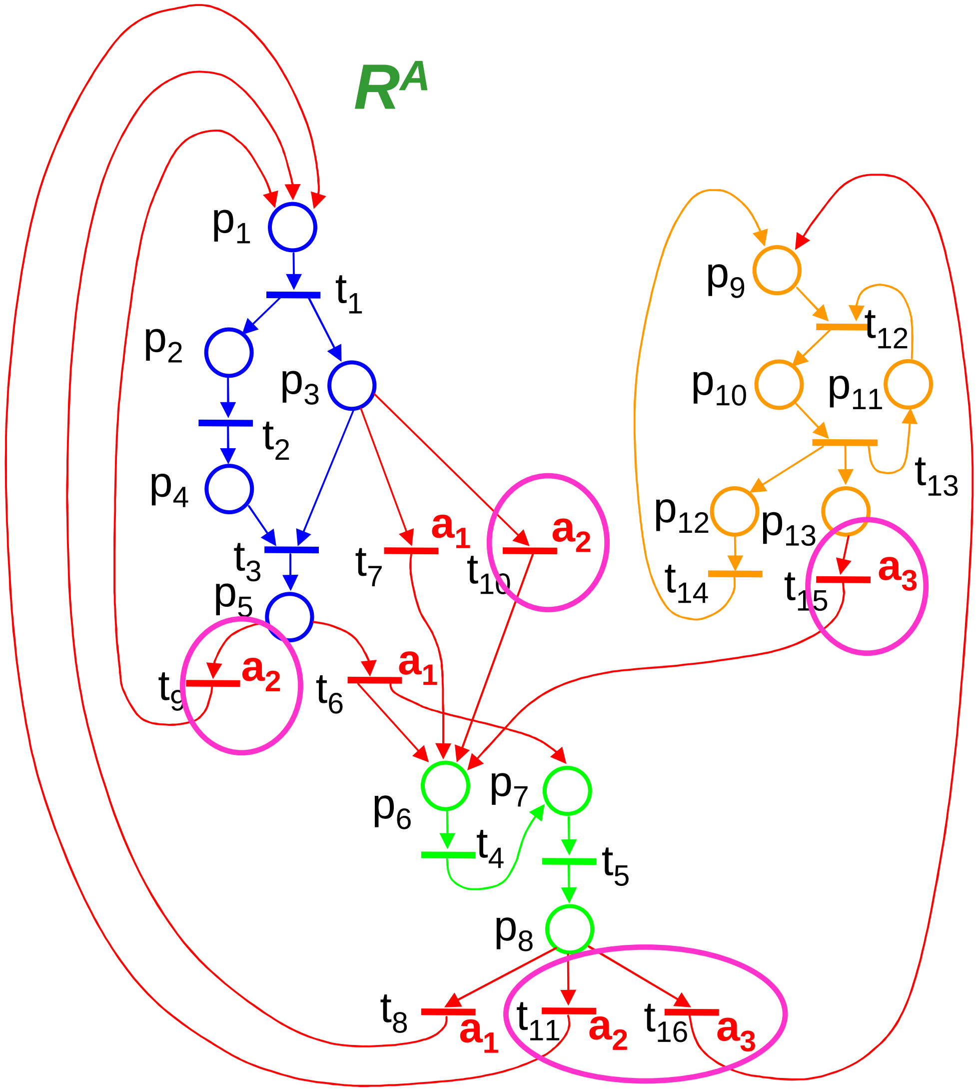
Columns associated to the link transitions.

Columns associated to the link transitions that represent work orders (push/pull).



3. Optimization problem based on an undefined Petri net

Ejemplo 4 — Behaviour of the AAPN



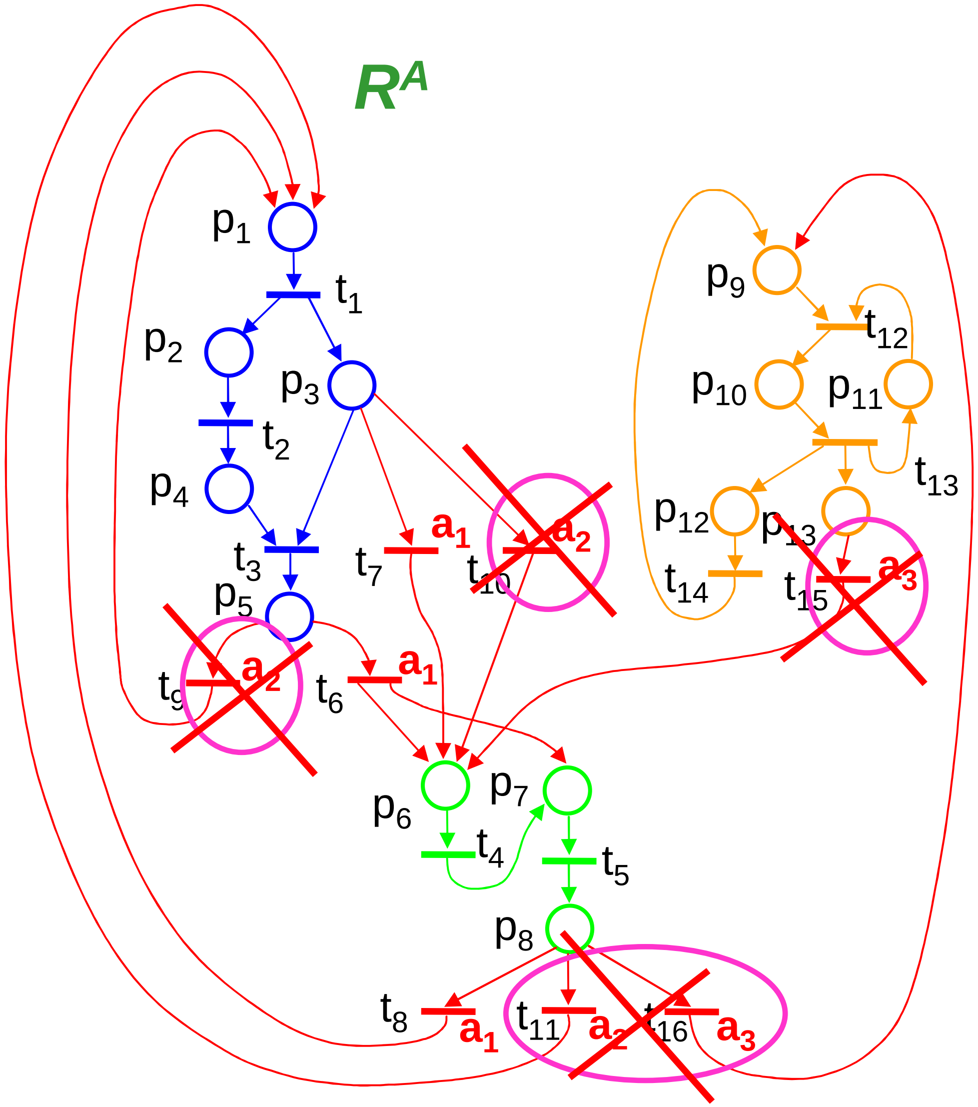
Let us suppose that a_1=1

By definition $\boldsymbol{a_j}$=0, $i \neq j$, $1 \leq j \leq n$, (n = number of alternative PN).

a) Removing the transitions that are not allowed to fire, a PN with the same reachability graph than the original alternative PN $\boldsymbol{R_1}$ is obtained. This behaviour is shared for a basic and a simplified AAPN.

3. Optimization problem based on an undefined Petri net

Ejemplo 4	**Behaviour of the AAPN**



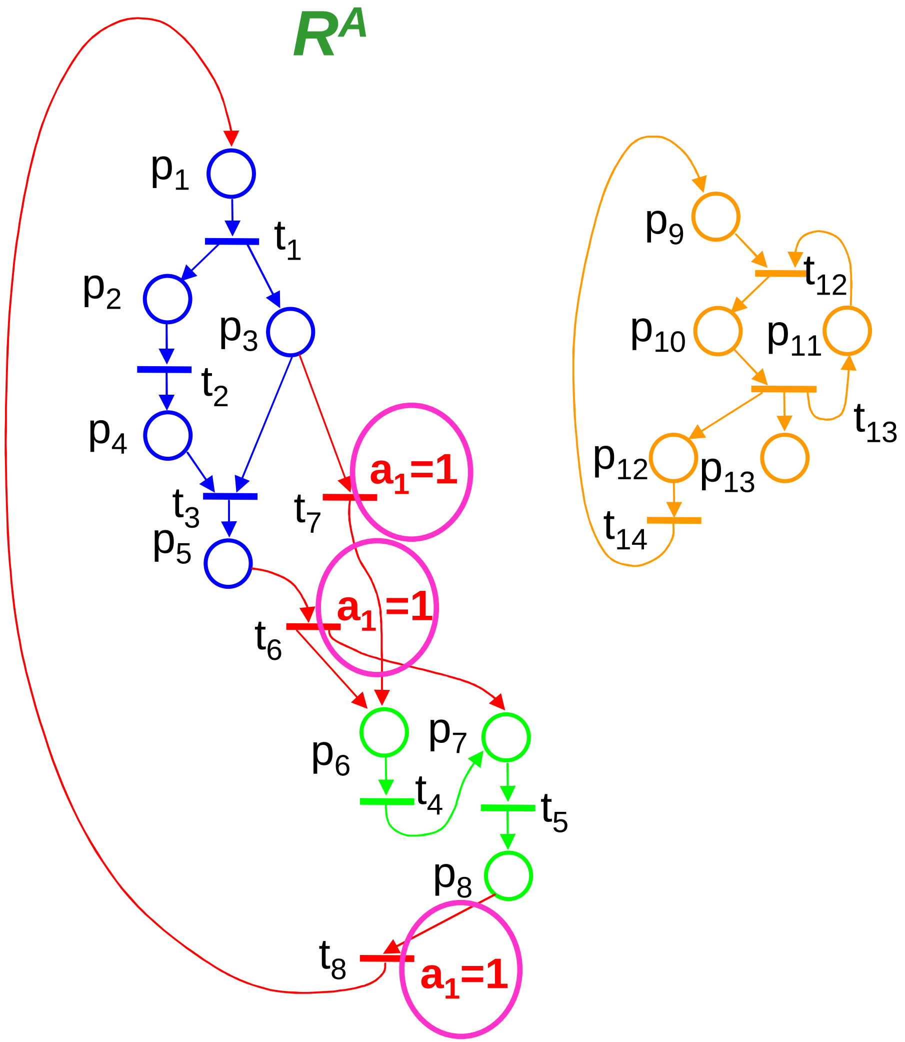
Let us suppose that a_1=1

By definition a_j=0, $i \neq j$, $1 \leq j \leq n$,
(n = number of alternative PN).

a) Removing the transitions that are not allowed to fire, a PN with the same reachability graph than the original alternative PN R_1 is obtained. This behaviour is shared for a basic and a simplified AAPN.

3. Optimization problem based on an undefined Petri net

Ejemplo 4 | Behaviour of the AAPN



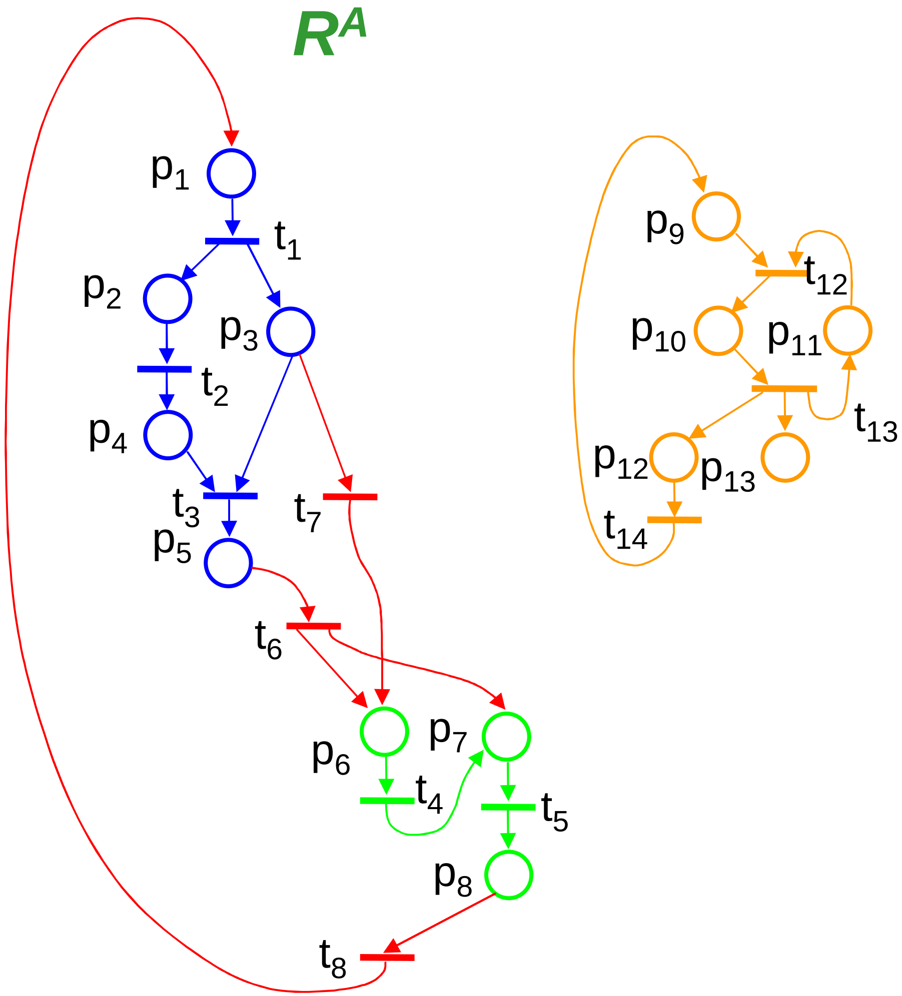
Let us suppose that a_1=1

By definition a_j=0, $i \neq j$, $1 \leq j \leq n$, (n = number of alternative PN).

a) Removing the transitions that are not allowed to fire, a PN with the same reachability graph than the original alternative PN R_1 is obtained. This behaviour is shared for a basic and a simplified AAPN.

3. Optimization problem based on an undefined Petri net

Ejemplo 4 | Behaviour of the AAPN



Let us suppose that a_1=1

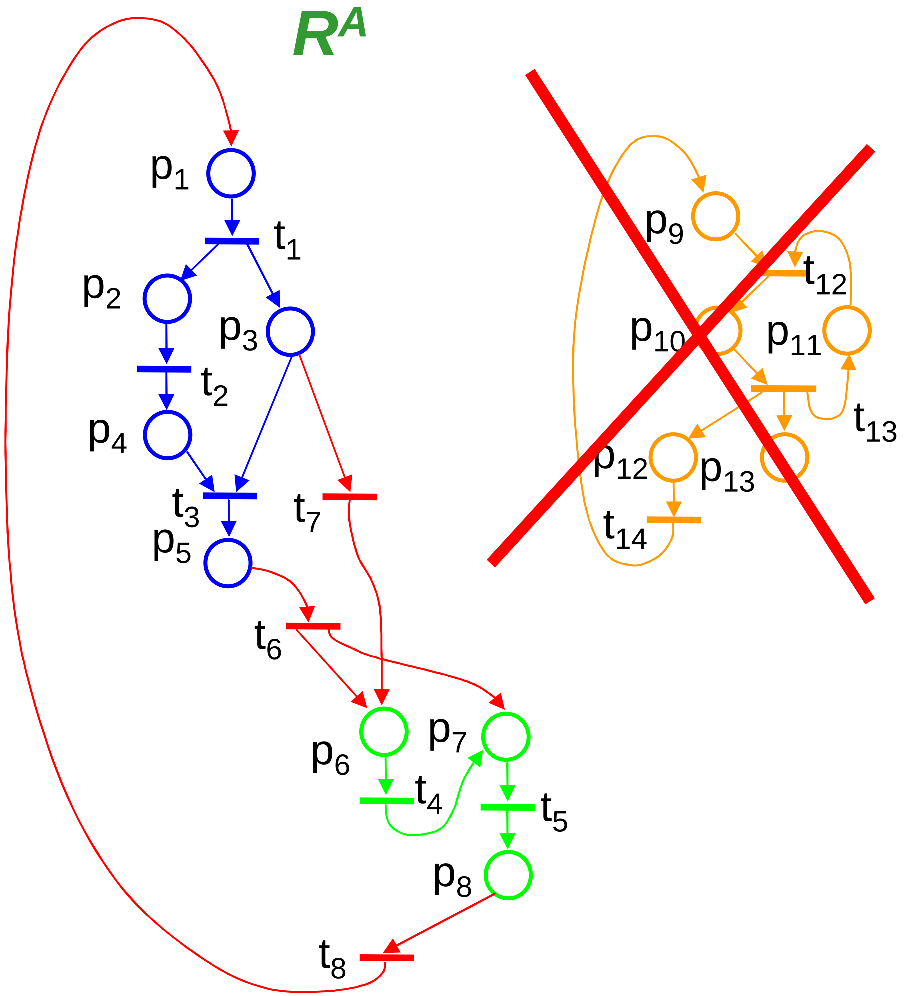
By definition a_j=0, $i \neq j$, $1 \leq j \leq n$, (n = number of alternative PN).

a) Removing the transitions that are not allowed to fire, a PN with the same reachability graph than the original alternative PN R_1 is obtained. This behaviour is shared for a basic and a simplified AAPN.

b) According to the definition of extended marking $m_0^{E1}(p_k) = 0$ with $9 \leq k \leq 13 \Longrightarrow$ the subnet R_1^3 does not participate in the evolution of the PN.

3. Optimization problem based on an undefined Petri net

Ejemplo 4 | Behaviour of the AAPN



Let us suppose that a_1=1

By definition $\boldsymbol{a_j}$=0, $i \neq j$, $1 \leq j \leq n$, (n = number of alternative PN).

a) Removing the transitions that are not allowed to fire, a PN with the same reachability graph than the original alternative PN $\boldsymbol{R_1}$ is obtained. This behaviour is shared for a basic and a simplified AAPN.

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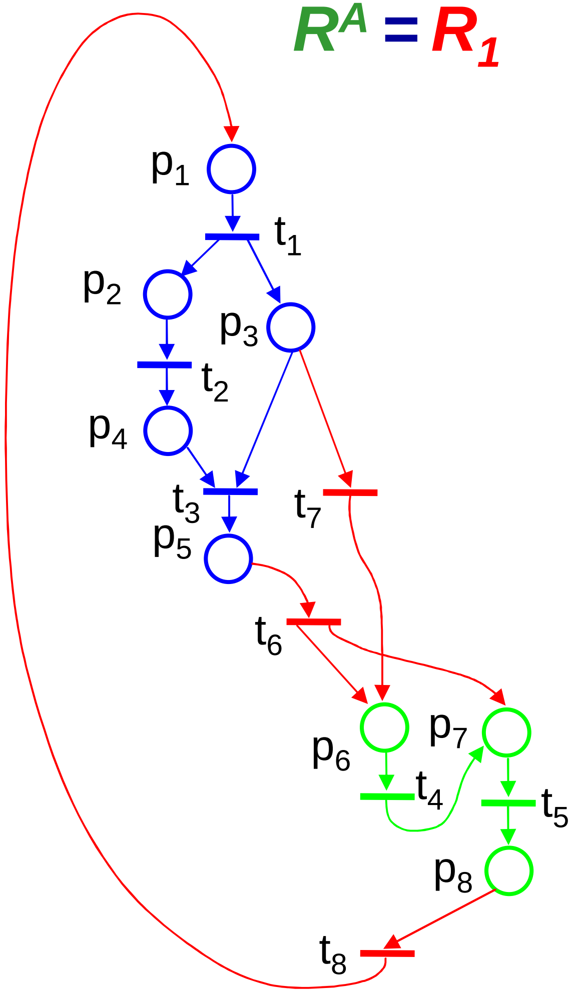
3. Optimization problem based on an undefined Petri net

Ejemplo 4 | Behaviour of the AAPN

R^A = R_1



Let us suppose that a_1=1

By definition $\boldsymbol{a_j}$=0, $i \neq j$, $1 \leq j \leq n$, (n = number of alternative PN).

a) Removing the transitions that are not allowed to fire, a PN with the same reachability graph than the original alternative PN $\boldsymbol{R_1}$ is obtained. This behaviour is shared for a basic and a simplified AAPN.

b) According to the definition of extended marking $m_0^{E1}(p_k) = 0$ with $9 \leq k \leq 13 \Longrightarrow$ the subnet R_1^3 does not participate in the evolution of the PN.

3. Optimization problem based on an undefined Petri net

Outline

New statement of the problem

- Max/min objective/multiobjective function $\left\{\right.$ Quality measurement of a solution
- Definition of the structure of the feasible solutions.
- Constraints → configuration of the solution space.
- Alternatives aggregation Petri net:

  1. Obtained from a disjunctive constraint but it is handled as a non-disjunctive constraint.

  2. It is a complement to the objective function.

3. Optimization problem based on an undefined Petri net

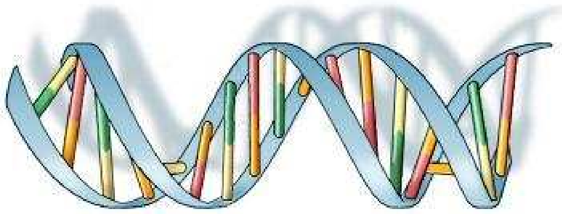
Solution by means of a genetic algorithm

Steps of the classical methodology

1) Random selection of the initial set of feasible solutions.

2) Evaluation of the objective function for every feasible solution

3) Evaluation of the stop criterion.



4) Calculation of the quality of every solution.

5) Removal of the less apt solutions.

6) Obtention of the new generation of solutions from the crossover of the surviving solutions.

7) Return to step 2.

3. Optimization problem based on an undefined Petri net

Comparison

Optimization based on *n* alternative PN.

☹ *n* optimizations based on a single Petri net.

☹ Worst alternative PNs → waste of time.

☹ Additional stage for the comparation of the results.

Optimization based on an AAPN

☺ *1* single optimization process, based on an AAPN.

☺ Computational effort focussed on the most promising regions of the solution space.

☹ Larger size of the incidence matrices.

☺ It profits from the shared subnets.

Example 2	Comparison

General characteristics

15 generations.

Population composed by 50 feasible solutions.

Adjustable parameters (mortality rate, mutation rate, type of crossover, etc).

Option 1: 24 optimizations based on a compound alternative PN.

Option 2: 1 optimization based on a single AAPN.

Example 2	**Comparison**

Solution obtained from the AAPN

Quality of the solution

The value of the objective function is the **98.85%** of the value obtained from the process of 24 optimizations

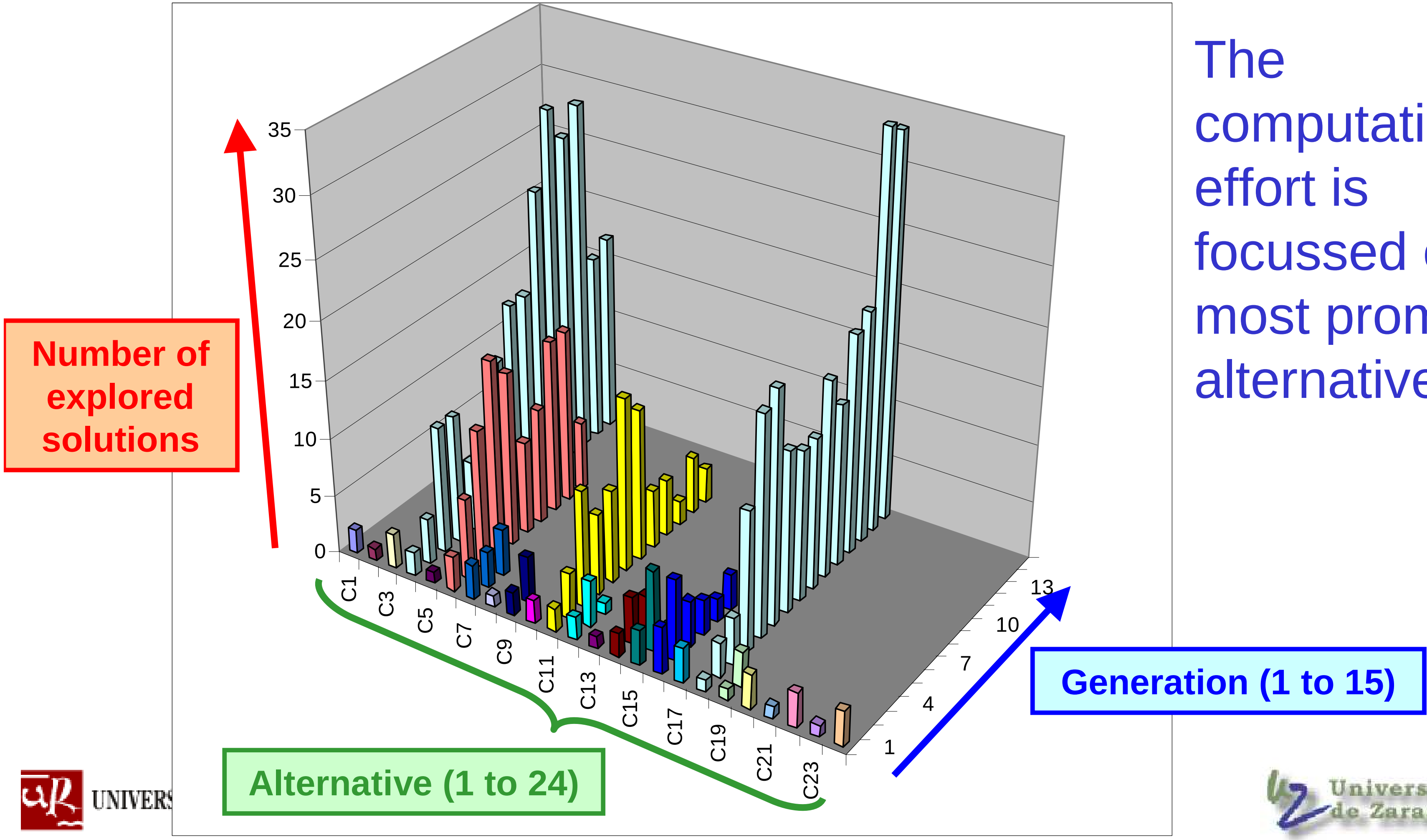
Computational working time

The time needed to develop is the **10.07%** of the time needed to perform the 24 optimization (note: the stage of comparing the results has not been included).

3. Optimization problem based on an undefined Petri net

Example 2	**Comparison**

Explanation of the results



The computational effort is focussed on the most promising alternatives

4. Conclusions

Methodology for decision taking based on alternatives aggregation Petri nets

1. Decision problem based on a DES

2. Decision problem based on an undefined Petri net

Representation by means of
- Simple or compound alternative PN.
- AAPN or CPN.

3. Optimization problem based on an undefined PN.

4. Optimization problem based on an alternatives aggregation Petri net.

Transforms a disjunctive constraint.

The AAPN is not unique. It depends on →
- Set of alternative PN
- Decomposition in subnets and link transitions

Efficient application of a classical methodology

Equivalent to a coloured PN which allows the use of its software

4. Conclusions

Methodology for decision taking based on alternatives aggregation Petri nets

Open research fields

Increase the efficiency in the application of the genetic algorithm:

Data storage / Adjustable characteristics of the algorithm

Extend the analysis of the performance of the optimization based on different representations of an undefined PN (compound PN, etc).

Extend the analysis of the performance of an AAPN obtained from different sets of alternative PN (simple, compound, mixed).

Extend the aplication of this methodology to other metaheuristics.

Analysis the applicability to other problems and solve them:

Problem of the design of a PN (large number of undefined structural parameters)

Problem of the preventive maintenance (timed sequence of decisions that modify the structure of the Petri net)