Active Sensing for Undelayed Range-Only SLAM

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***Abstract*— This paper describes a method to consider active sensing strategies in order to map the position of a set of nodes with a mobile robot using radio signal measurements. The method employs Gaussian Mixtures Models (GMM) for undelayed initialization of the position of the wireless nodes within a SLAM filter. An upper bound of the entropy of the GMM is used as a measurement of information gain, and allows to prioritize control actions of the robot.**

I. Introduction

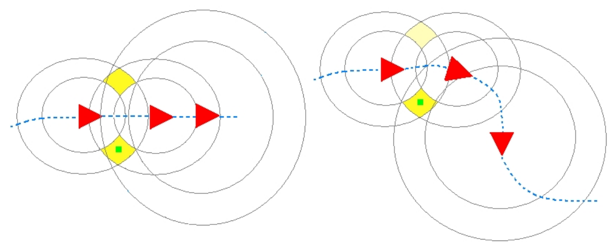
Range-only mapping in wireless sensor networks is an active research area that poses a number of challenges, from range computation to map building. One of the key issues is the lack of bearing information in the measurements. The receiver must receive information in different positions (trilateration) to properly localize the emitter.

Some approaches use delayed initialization through trilateration or employ other means to determine the bearing, like partial directivity [1] or time of arrival [2] information. [3, 4, 5] employ delayed initialization through trilateration. The problems related with multiple hypotheses in the early steps of the estimation in range-only localization approaches have been recently addressed in [6] and [7]. In the latter, the authors make use of Gaussian Mixtures Models (GMM) to represent the non-Gaussian prior distribution of the node position allowing un-delayed initialization for wireless network mapping using range-only measurements.

Most of the previous approaches do not take into account the possibility of controlling the robot, and the robot is just commanded a predefined path. However, active sensing strategies may lead to more efficient exploration and mapping using radio-signals. The robot can adapt its trajectory, avoiding for instance non-observable motions or following those paths which are most informative, in the sense of reducing the uncertainty on the nodes' positions. Fig. 1 illustrates the benefit of considering active sensing strategies.

Active sensing requires a measurement on the information gain obtained when executing a certain task or action. For Bayesian approaches, one possibility is to use the (expected) variation on the entropy of the beliefs on the nodes' positions as a measure of information gain, as for instance in [8]. In [9], the same ideas about active sensing are applied to a set-theoretic framework used to represent and handle uncertainty.

The main contribution of the paper is an extension of the approach presented in [7] for node mapping based on



(a) (b)

Fig. 1: Range-only mapping. The robot (red triangle) receives range data from the beacon (green square) at three different positions. Yellow areas denote possible localizations of the beacon (as more intense is the yellow color, more likely this localization is). (a) Result of the node localization using a straight robot trajectory (there are two possible solutions for the localization). (b) Results of the node localization if the robot trajectory is adapted from active sensing considerations (the localization converges to single correct solution)

a weighted GMM, with an active sensing strategy in order to gather as much information as possible for localizing the nodes. Entropy variation is considered in this paper to measure the information gained by a given robot motion considering all node position hypotheses. As the computation of the entropy of GMM has not a closed form, entropy bounds are used in this approach. The paper will show how these bounds provide an effective way for active sensing.

II. Range-only mapping and SLAM using Gaussian Mixtures

The objective can be summarized as estimating the position of wireless sensor nodes based on the received signal strength on a node attached to the robot. This process is depicted in Fig. 1.

The state vector of the filter will be composed by the estimated 2D position and orientation of the robot, and the estimated 2D position of all the nodes:

$$\mathbf{x} = [x_r, y_r, \theta_r, \mathbf{b}_1^t, \mathbf{b}_2^t, \ldots, \mathbf{b}_n^t]^t \quad (1)$$

where (x_r, y_r, θ_r) represent the Euclidean position and orientation of the robot and $\mathbf{b}_i$ represents the position of the i-th beacon considered in the filter. The beacon position $\mathbf{b}_i$

will be expressed in polar coordinates (ρ_i, θ_i) with respect to the position from which the robot received the very first range information (x_i, y_i). However, non prior information about the angle of arrival of the beacon information is assumed, so the value of θ_i is unknown. We propose quantizing the space of possible values of θ_i into k possible hypotheses. Thus, each beacon will be expressed as follows:

$$\mathbf{b}_i = [x_i, y_i, \rho_i, \theta_{i0}, \theta_{i2}, \ldots, \theta_{i(k-1)}]^t \tag{2}$$

All the hypotheses θ_{ij}, together with their weights w_{ij}, will compose a GMM representing the probability mass functions of θ_i, $f_{\theta_i}(x)$:

$$f_{\theta_i}(x) = U(0, 2\pi) \simeq \sum_{j=1}^{k} w_{ij}\mathcal{N}(\theta_{ij}, \sigma_{ij}) \tag{3}$$

After the first range information of a beacon is considered, the probability mass function of its position will be uniformly distributed around the robot location. This uniform probability function will be approximated by a GMM using (3), and each of these Gaussians will be considered as an independent hypothesis into the localization filter. Once the beacon has been initialized into the filter with the first range information, next measurements will be used to update the estimation of each hypothesis and also to refine the weights w_{ij} associated to them.

The measurements provided by the system are the distances of the robot to the set of nodes that are in communication range. The question now is how to deal with the variance associated with the measurement, σ_i^2. A single measurement is available but it has to be applied to all the existing hypotheses for beacon i. The solution to this problem consist of dividing the actual measurement into k new measurements with the same mean and with covariances proportional to the likelihood of each hypothesis such as in [10].

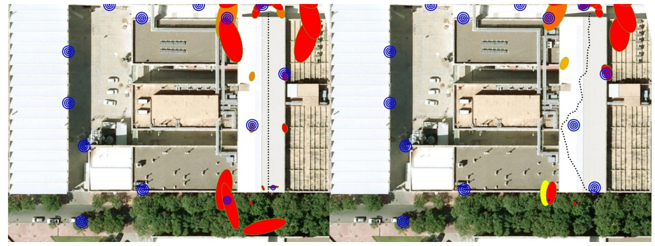
Finally, some heuristic rules are applied to remove those hypotheses with very small weight (see [7] for further details).

III. Active sensing for WSN mapping

The benefit of using a mobile robot to estimate the position of a set of nodes is that its motion can be adapted in order to take the most informative actions. In one hand, from the set of possible motions of the robot, it should take those that allows to estimate the position of the nodes more accurately. On the other hand, the robot should try to avoid motions that decrease the observability of the node position.

Our robot uses a combination of behaviors: basically the robot tries to follow a given path as accurately as possible, but at the same time minimizing a combined cost related to obstacle avoidance, etc. The idea is to include, in the computation of the control commands, a cost related to the gain of information.

A general measure about the information of a probability distribution $p(x)$ is its entropy, $H(p(x))$, defined as the expected value of the information $-\log[p(x)]$. The information gain is defined as the variation in the entropy of the distribution after carrying an information gathering action $\mathbf{u}_t$. When executing this action, a new distribution $p(\mathbf{x}_{t+\Delta t}|\mathbf{u}_t, \mathbf{z}_{t+\Delta t})$ will be obtained from the future measurement $\mathbf{z}_{t+\Delta t}$, by using the filtering algorithm described in Section II. The entropy of this new distribution will be denoted by $H(p(\mathbf{x}_{t+\Delta t}|\mathbf{z}_{t+\Delta t}, \mathbf{u}_t))$.



(a) (b)

Fig. 2: Comparison between the paths followed by the robot. The hypotheses maintained by the filter are represented as ellipses. The colors indicate the weight of the hypothesis (the redder, the lower). (a) non-active control. (b) active sensing control.

However, only the action $\mathbf{u}_t$ can be controlled. Then, we should take the expectation of the entropy for all potential measurements $\mathbf{z}_{t+\Delta t}$ that can be obtained after executing the action. Therefore, the (expected) information gain associated to action $\mathbf{u}_t$ is defined as follows:

$$\Delta(\mathbf{u}_t) = H(p(\mathbf{x}_t)) - E_{\mathbf{z}_{t+\Delta t}}[H(p(\mathbf{x}_{t+\Delta t}|\mathbf{z}_{t+\Delta t}, \mathbf{u}_t))] \tag{4}$$

This metric can be used to establish preferences among actions, favoring those that maximize the value $\Delta(\mathbf{u}_t)$.

However, a closed form for the entropy of a Gaussian Mixture does not exist. One option is to obtain it numerically (for instance by sampling), but this is discarded because of computational requirements. The proposed approach uses upper bounds of the entropy as an approximation to the actual entropy value. Thus, instead of analyzing the expected variation in entropy for a particular action, the expected variation of the entropy bound will be considered.

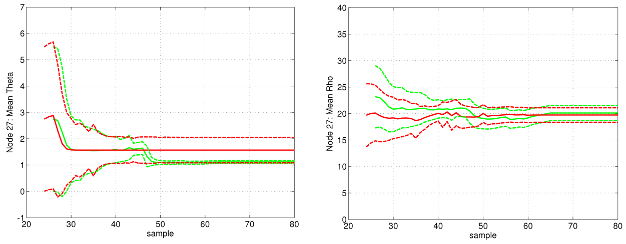
In [11], Huber *et al.* derive analytical approximations to the entropy of a Gaussian mixture; moreover, some analytical upper and lower bounds of the entropy of a Gaussian Mixture are presented as well. Among them, the following expression gives an upper bound of the entropy of a Gaussian Mixture $f(\mathbf{x}) = \sum_{i=1}^{k} w_i \mathcal{N}(\mu_i, \Sigma_i)$, which is very cheap to compute:

$$H(f(\mathbf{x})) \leq \sum_i \omega_i(-\log\omega_i + \frac{1}{2}\log((2\pi e)^N|\Sigma_i|)) \tag{5}$$

with N the dimension of $\mathbf{x}$.

Therefore, a possible strategy is to compare actions taking into account how they affect not the entropy itself, but the upper bound.

Figures 2 and 3 show some results using such an strategy. While in theory a decreasing in the bound could not reflect



(a) (b)

Fig. 3: (a) Estimated value of θ (solid line) and its 3σ confidence interval (dashed) for node 27. (b) Estimated value of ρ (solid line) and its 3σ confidence interval (dashed line) for node 27. The active sensing strategy in green, and the non-active in red.

on a decreasing of the actual entropy, it can be seen that the procedure is effective reducing the actual entropy of the distributions.

References

[1] D. Kurth, G. A. Kantor, and S. Singh, "Experimental results in range-only localization with radio," in *2003 IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS '03)*, vol. 1, October 2003, pp. 974 – 979.

[2] R. Moses, D. Krishnamurthy, and R. Patterson, "A self-localization method for wireless sensor networks," *EURASIP Journal on Applied Signal Processing*, no. 4, pp. 348–358, 2003.

[3] E. Olson, J. Leonard, and S. Teller, "Robust range-only beacon localization," in *Proceedings of Autonomous Underwater Vehicles, 2004*, 2004.

[4] E. Menegatti, A. Zanella, S. Zilli, F. Zorzi, and E. Pagello, "Range-only slam with a mobile robot and a wireless sensor networks," in *Proceedings of the IEEE International Conference on Robotics and Automation, ICRA*, 2009, pp. 8–14.

[5] F. Caballero, L. Merino, I. Maza, and A. Ollero, "A particle filter method for wireless sensor network localization with an aerial robot beacon," in *Proceedings of the International Conference on Robotics and Automation*, Pasadena, CA, USA, May 19-23 2008, pp. 596–601.

[6] J. Djugash and S. Singh, "A robust method of localization and mapping using only range," in *International Symposium on Experimental Robotics*, July 2008.

[7] F. Caballero, L. Merino, and A. Ollero, "A general gaussian-mixture approach for range-only mapping using multiple hypotheses," in *Proceedings of the IEEE International Conference on Robotics and Automation, ICRA*, Anchorage, Alaska (USA), May 2010, pp. 4404–4409.

[8] J. Djugash, S. Singh, and B. P. Grocholsky, "Decentralized mapping of robot-aided sensor networks," in *IEEE International Conference on Robotics and Automation*, May 2008.

[9] E. Stump, V. Kumar, B. Grocholsky, and P. Shiroma, "Control for localization of targets using range-only sensors," *International Journal of Robotics Research*, vol. 28, no. 6, pp. 743–757, 2009.

[10] J. Sola, A. Monin, M. Devy, and T. Lemaire, "Undelayed initialization in bearing only slam," in *Proceedings of the 2005 IEEE/RSJ International Conference on Intelligent Robots and Systems, IROS*, August 2005, pp. 2499–2504.

[11] M. F. Huber, T. Bailey, H. Durrant-Whyte, and U. D. Hanebeck, "On entropy approximation for gaussian mixture random vectors," in *Multisensor Fusion and Integration for Intelligent Systems, MFI. IEEE International Conference on*, 2008, pp. 181–188.