

Accurate Performance Estimation for Stochastic Marked Graphs by Bottleneck Regrowing

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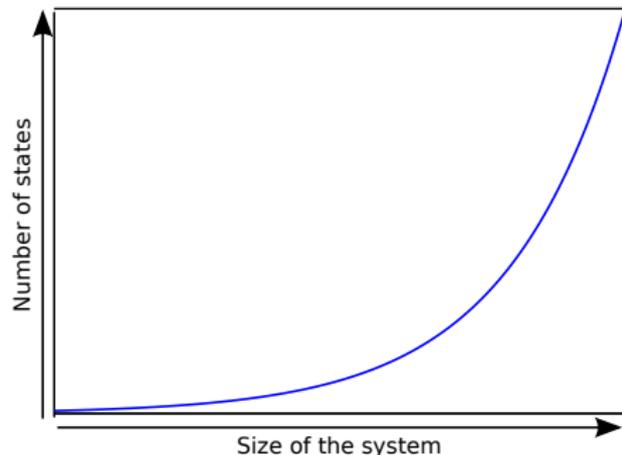
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Bertinoro, Italy

- 1 Motivation
- 2 Some basic concepts
 - Stochastic Marked Graph
 - Critical Cycle
 - Tight Marking
- 3 Graph Regrowing Strategy
- 4 Experiments and Results

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Motivation (I): the need of requirement verification

- New system: problem of **verification of requirements**
- **Performance of an industrial system** → real need
- Many systems modelled as **Discrete Event Systems** (DES)
- **Increasing size** → exact performance computation unfeasible
 - **State explosion problem**



Motivation (II): performance evaluation approaches

- **Exact analytical measures**
 - Need exhaustive state space exploration
- **Performance bounds**: overcoming state explosion problem
 - Reduced running time, BUT **how good** (i.e., accurate) is the bound?

Motivation (II): performance evaluation approaches

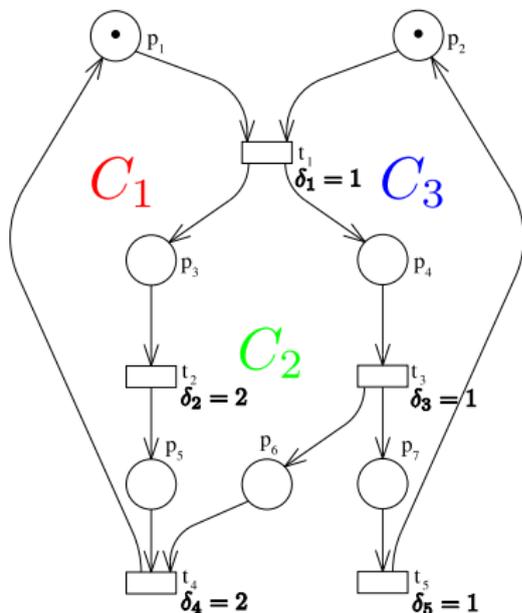
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Our approach

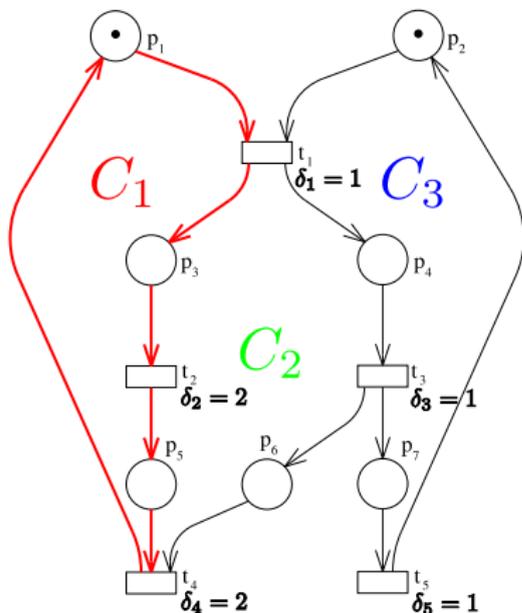
- **Iterative algorithm**
- **Sharper** (i.e., closer) **bounds**
 - 1 **Initial bottleneck cycle** (most restrictive)
 - 2 **Add set of places likely to constraint**
- **Outputs**:
 - **Improved performance bound**
 - **New bottleneck**

Motivation (III): a small example



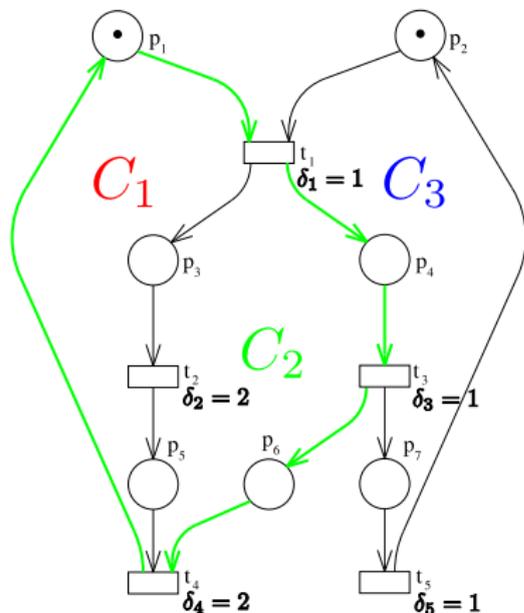
- $r_{C_1} = \frac{1}{5}$, $r_{C_2} = \frac{1}{4}$ and $r_{C_3} = \frac{1}{3}$
- Bottleneck cycle \rightarrow minimum ratio
- Throughput bound: $\frac{1}{5} = 0.2$
- Lowest ratio token/delay $\rightarrow \{p_1, p_4, p_6\}$
- New thr bound: 0.1875 (6.25% lower)
- Seek next constraint cycle non trivial
- *Tight* marking and slack

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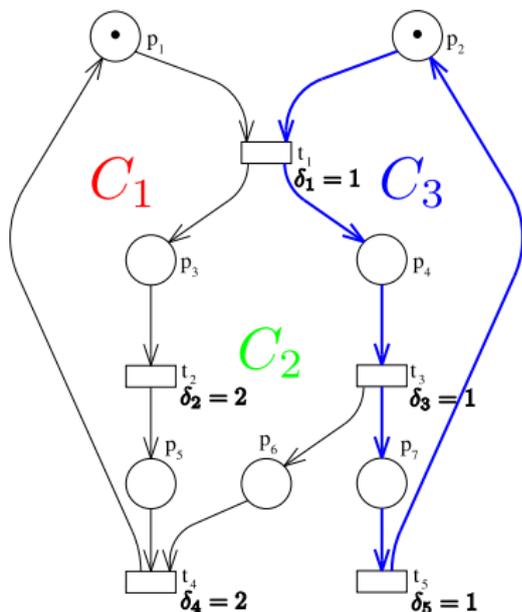
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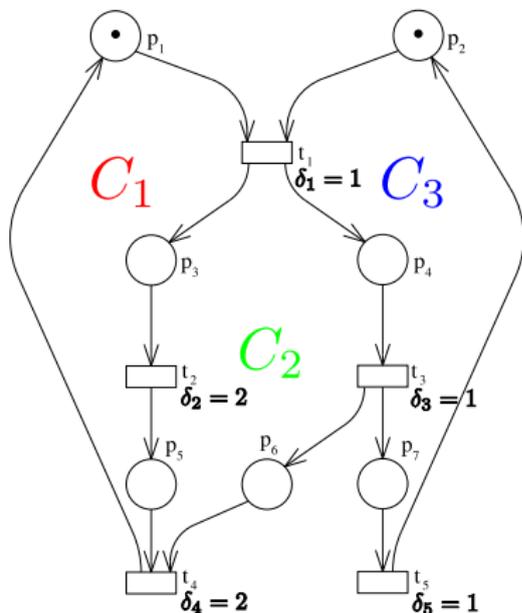
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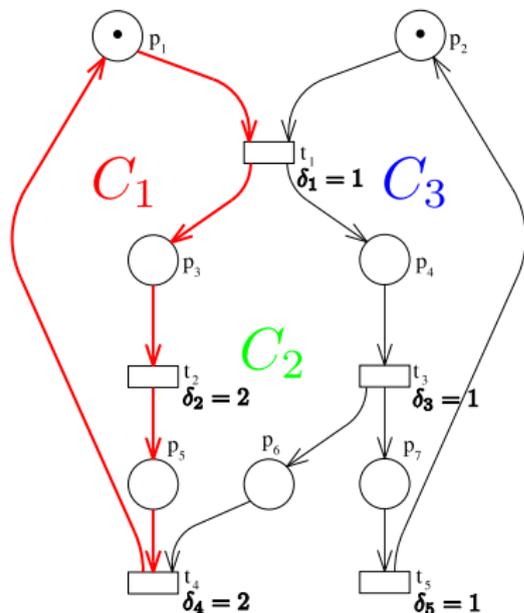
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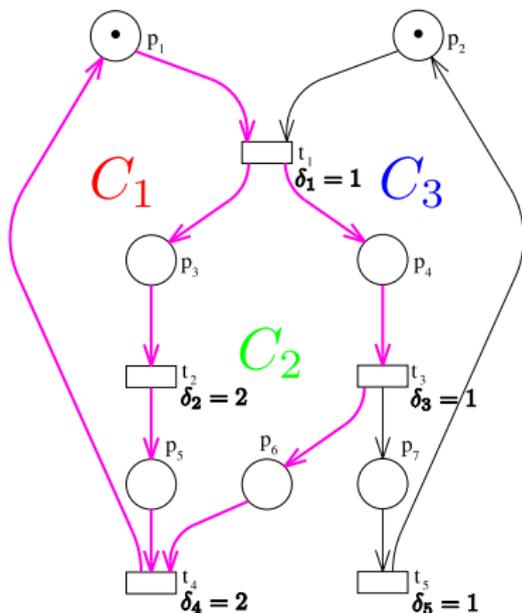
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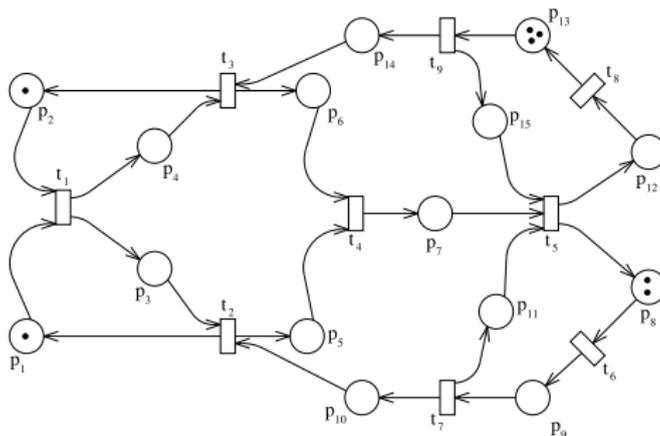
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Motivation (IV): running example

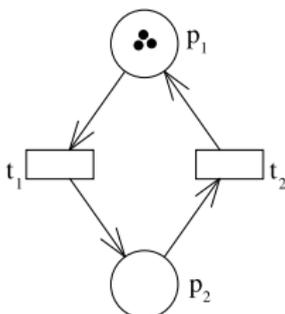


- **Performance bound 12.9% lower** than the initial one
 - More iterations: a bound just 0.3% greater than the real performance
- **Benefits** of the proposed method:
 - **Efficient** (uses linear programming)
 - **Accurate** (converges in few iterations)

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Some basic concepts (I): Stochastic Marked Graph

- **Petri Net system**: $\mathcal{S} = \langle P, T, \mathbf{Pre}, \mathbf{Post}, \mathbf{m}_0 \rangle$
- **Marked graph** (MG): ordinary PN such that each place has exactly one input and exactly one output arc
- **Stochastic Marked Graph** (SMG): MG and a vector δ , where $\delta(t)$ is the mean of the exponential firing time distribution associated to each transition $t \in T$
- SMG's transitions work under **infinite server semantics** (assumed)
- **Steady state throughput** χ : average number of firing counts per u.t.



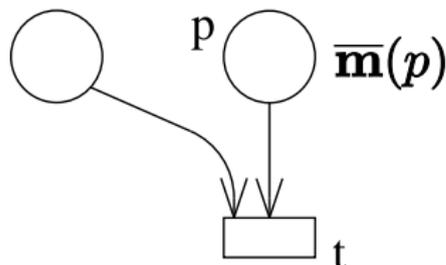
Some basic concepts (II): Critical Cycle (1)

Little's law

- Average number of customers L in a queue: $L = \lambda \cdot W$
- In a SMG: each pair $\{p, t\}$, where $p^\bullet = \{t\}$, can be seen as a simple queueing system

$$\bar{m}(p) = \chi(p^\bullet) \cdot \bar{s}(p) \quad (1)$$

- $\bar{s}(p)$ = average waiting time + average service time ($\delta(p^\bullet)$ in our case)
 $\rightarrow \delta(p^\bullet) \leq \bar{s}(p)$



$$\bar{m}(p) \geq \chi(p^\bullet) \cdot \delta(p^\bullet) \quad (2)$$

Some basic concepts (II): Critical Cycle (2)

- Note that **MGs** have a single minimal t-semiflow equal to **1**
 → same steady state throughput for every transition

Maximize Θ :

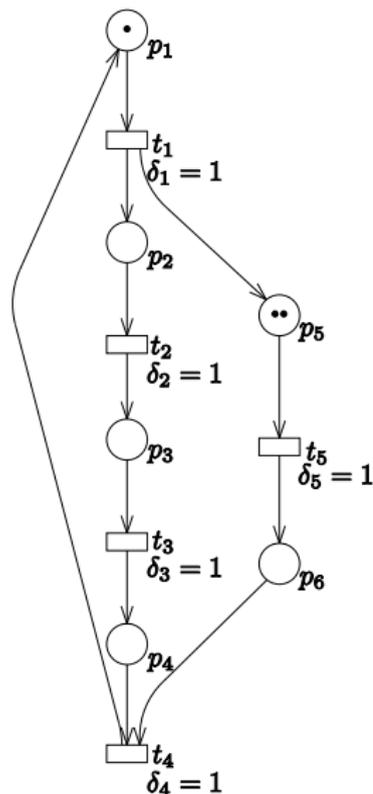
$$\hat{\mathbf{m}}(p) \geq \delta(p^\bullet) \cdot \Theta \quad \forall p \in P \quad (3a)$$

$$\hat{\mathbf{m}} = \mathbf{m}_0 + \mathbf{C} \cdot \sigma \quad (3b)$$

$$\sigma \geq 0 \quad (3c)$$

- Θ is an upper throughput bound

Some basic concepts (II): Critical Cycle (3)

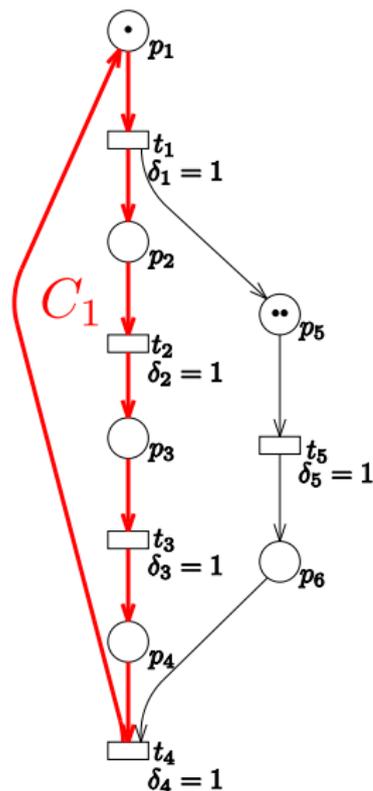


Concept of *slack*: μ

$$\hat{m}(p) \geq \delta(p^\bullet) \cdot \Theta \longrightarrow m(p) = \delta(p^\bullet) \cdot \Theta + \mu(p)$$

- $\mu(p) = 0$ if p belongs to critical cycle
- Value of vector μ will depend on the algorithm used by the LP solver
- The lower the slack, the higher the probability that place will constraint

Some basic concepts (II): Critical Cycle (3)

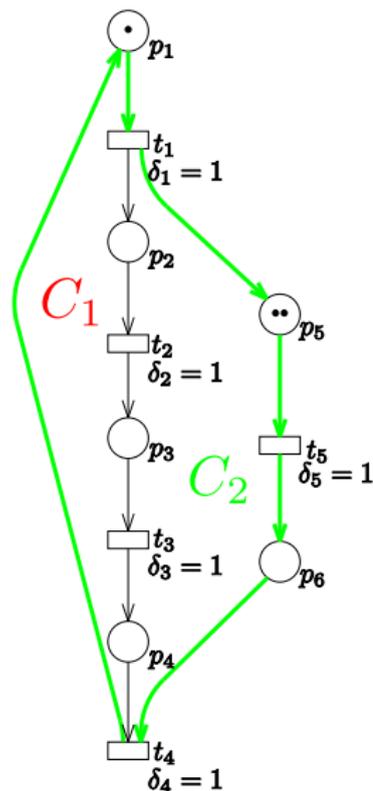


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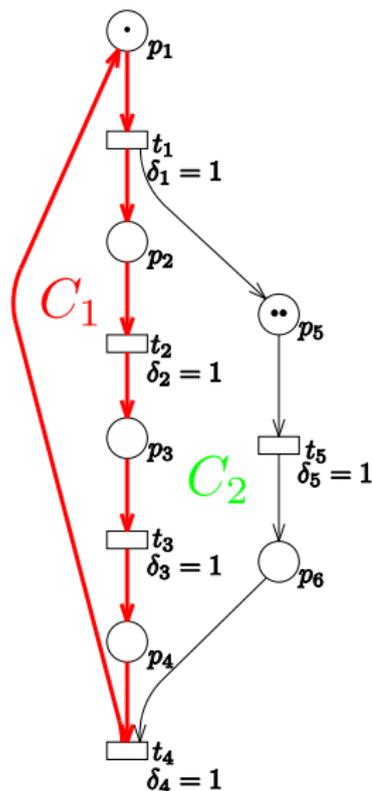


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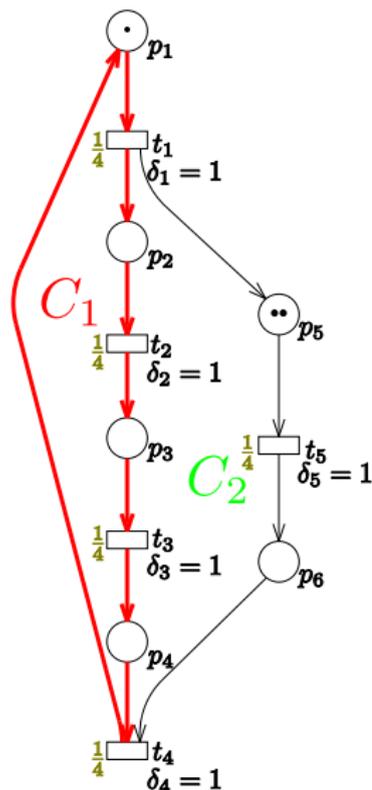
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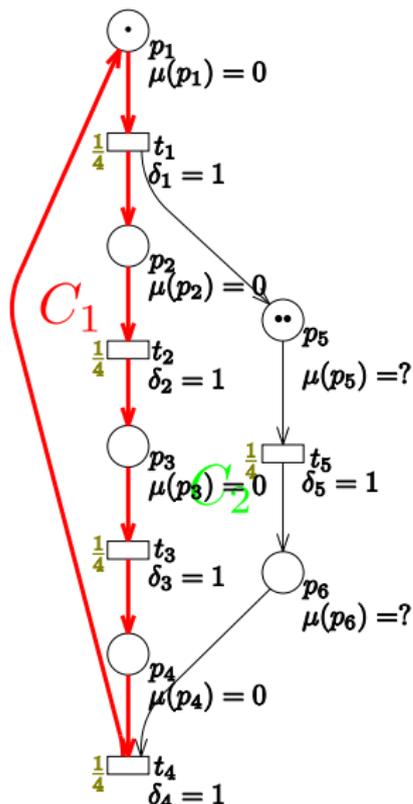
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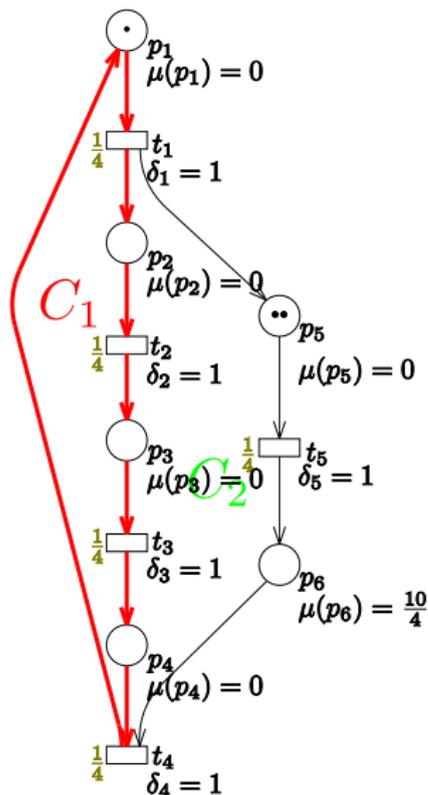
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Some basic concepts (III): Tight Marking (1)

Tight marking vector ($\tilde{\mathbf{m}}$)

$$\tilde{\mathbf{m}} = \mathbf{m}_0 + \mathbf{C} \cdot \sigma \quad (4a)$$

$$\forall p: \tilde{\mathbf{m}}(p) \geq \delta(p^\bullet) \cdot \Theta \quad (4b)$$

$$\forall t \exists p \in \bullet t: \tilde{\mathbf{m}}(p) = \delta(p^\bullet) \cdot \Theta \quad (4c)$$

- Computed **by solving the following LPP:**

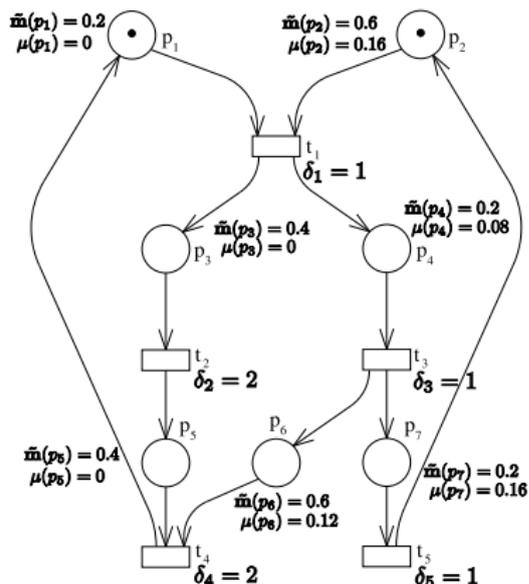
Maximize $\Sigma \sigma$:

$$\delta(p^\bullet) \cdot \Theta \leq \tilde{\mathbf{m}}(p) \quad \text{for every } p \in P \quad (5)$$

$$\tilde{\mathbf{m}} = \mathbf{m}_0 + \mathbf{C} \cdot \sigma$$

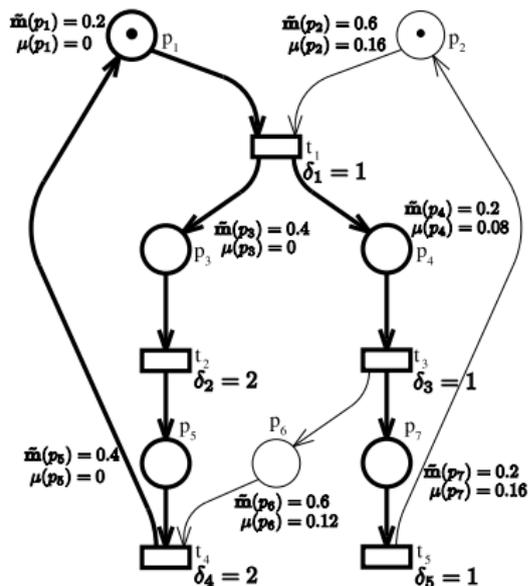
$$\sigma(t_p) = k$$

Some basic concepts (III): Tight Marking (2)



- Tight place p : $\tilde{m}(p) = \delta(p^\bullet) \cdot \Theta$
- Considering tight places (and their input and output transitions)
 - kind of tree
 - Critical cycle is the root
 - All transitions are reached

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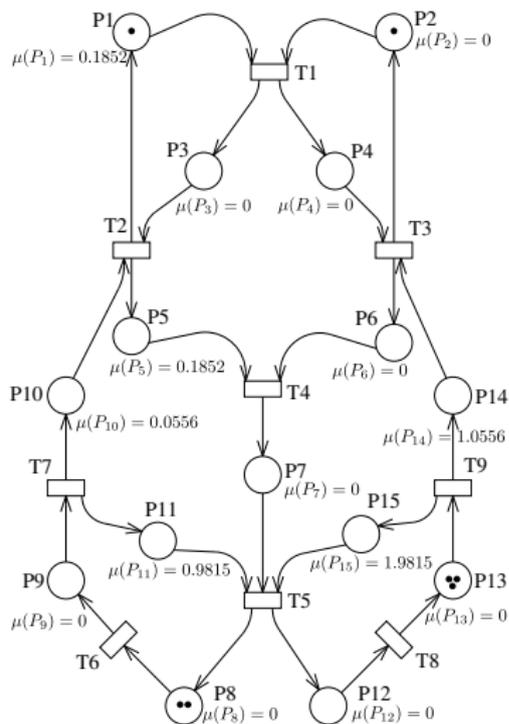
Graph regrowing strategy (I): algorithm

- **Input** data: **SMG**, **accuracy**
- **Output** data: **sharper performance bound**, **bottleneck**

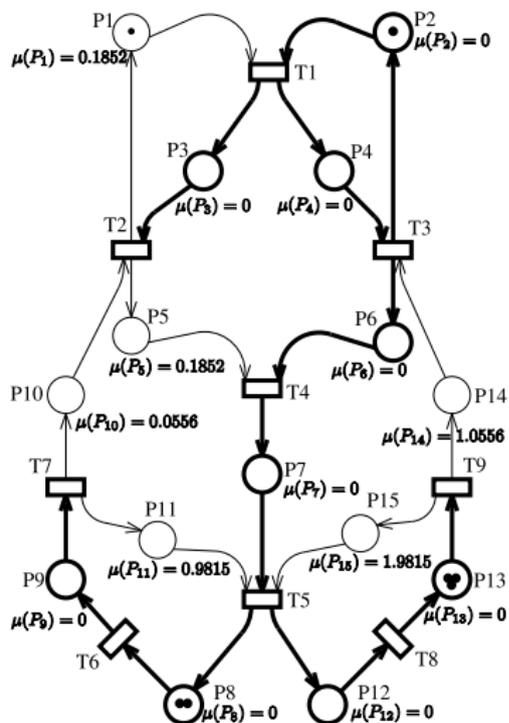
Algorithm steps

- 1 Calculate initial upper throughput bound and initial bottleneck cycle
- 2 Calculate tight marking and slacks
- 3 Iterate until no significant improvement is achieved
 - 1 Look for place with minimum slack and add it
 - 2 Calculate new throughput bound

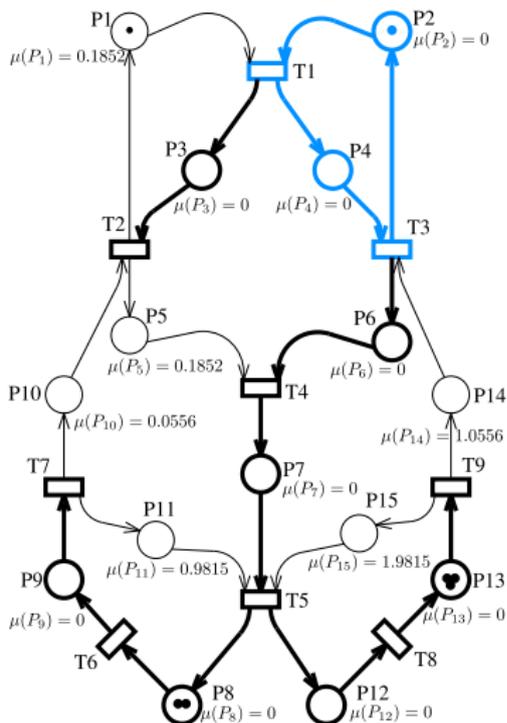
Graph regrowing strategy (II): running example



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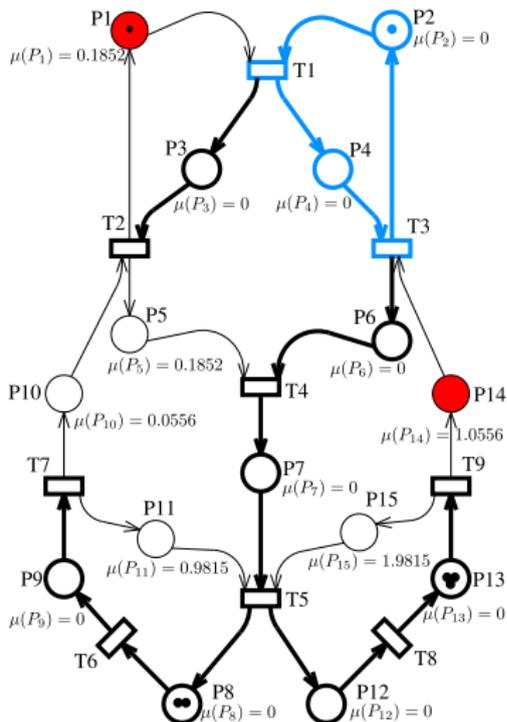


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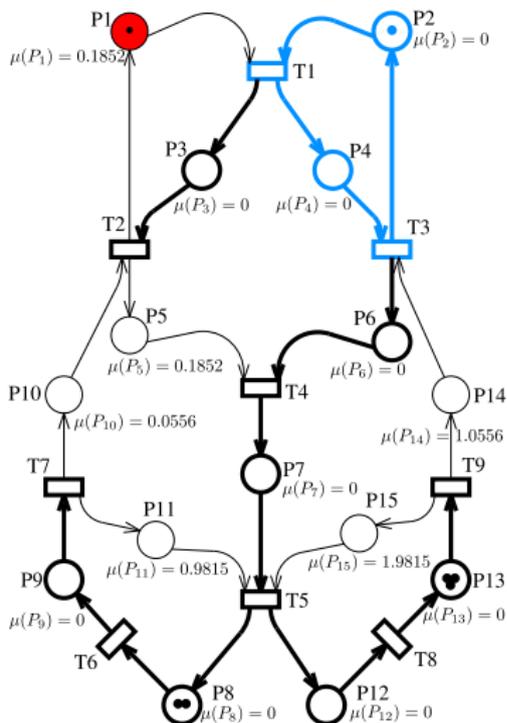
Iteration step	Candidates places	Added	Θ	% _{last}	% _{initial}
0	-	-	0.3704	-	-

Graph regrowing strategy (II): running example



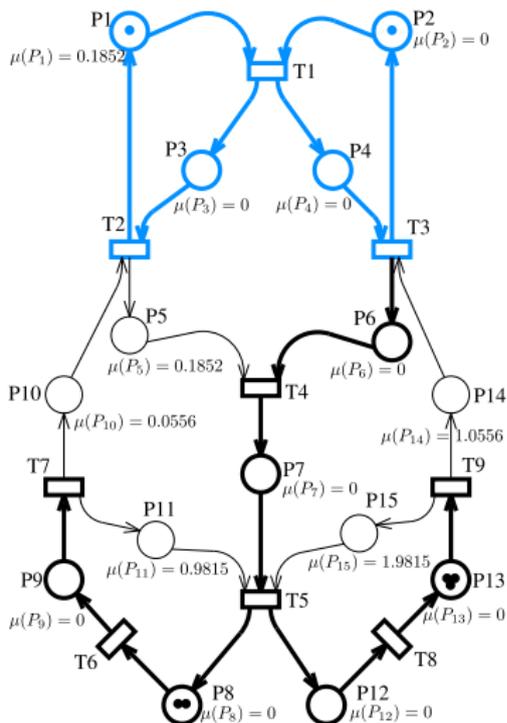
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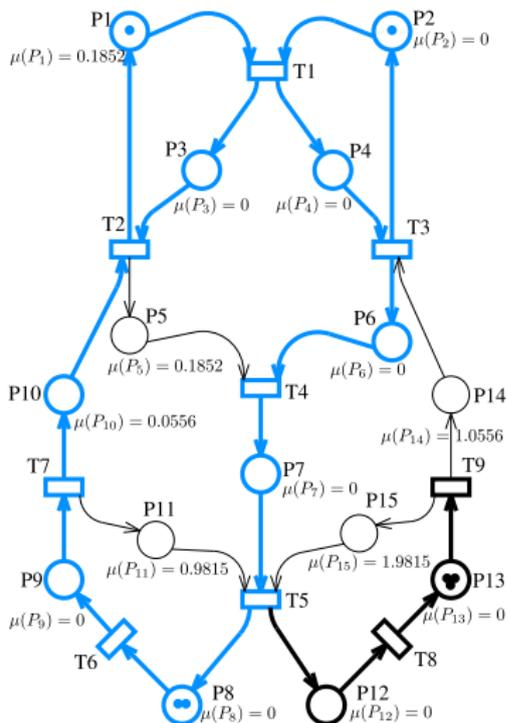
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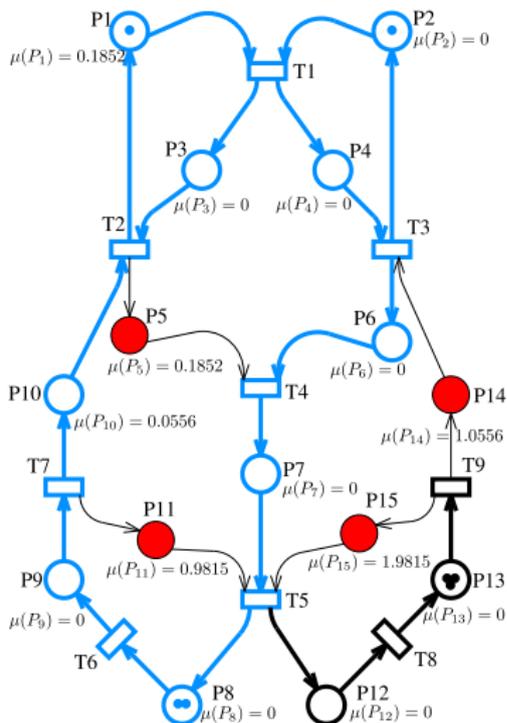
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1	-	-	0.322581	12.9%	12.9%

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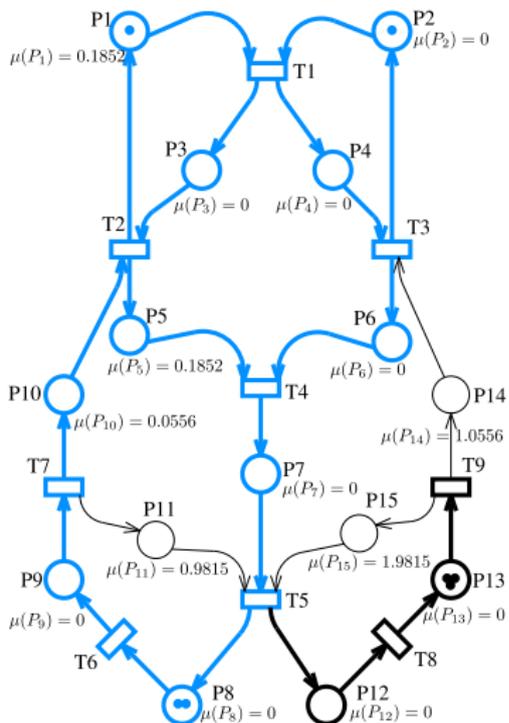
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1	P_{10}, P_{14}	P_{10}	0.322581	12.9%	12.9%
2	-	-	0.297914	7.647%	19.563%

Graph regrowing strategy (II): running example



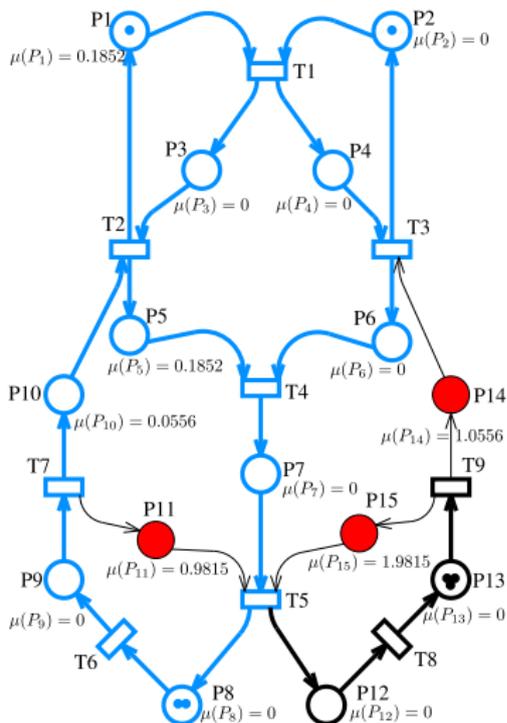
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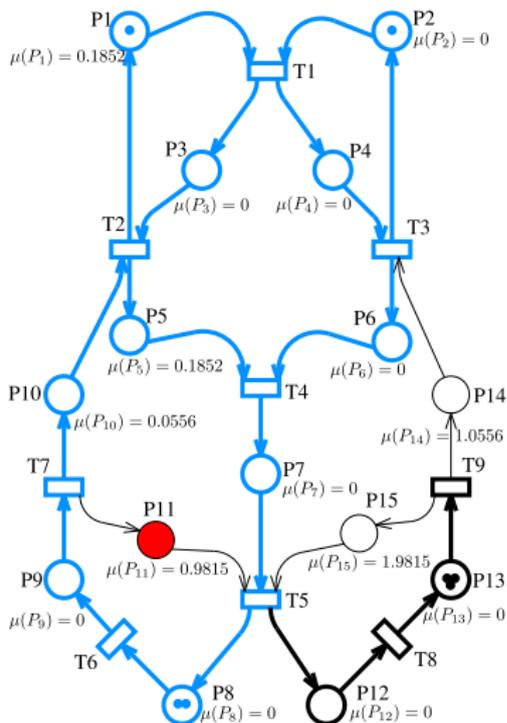
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3	-	-	0.288401	3.193%	22.137%

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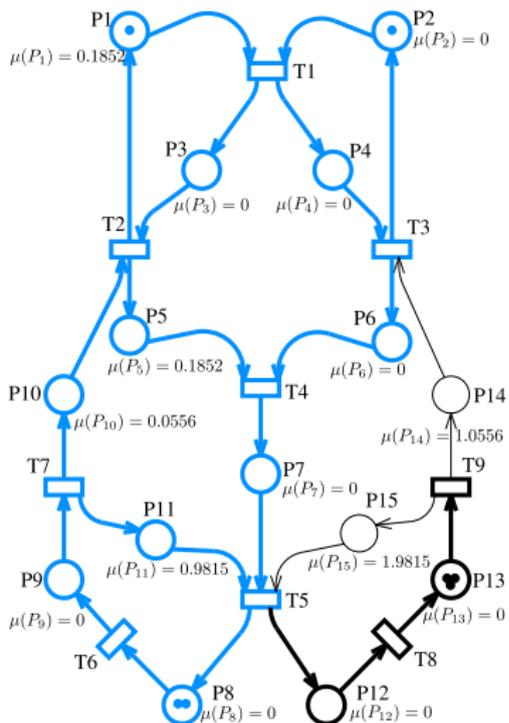
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4	-	-	0.288401	0%	22.137%

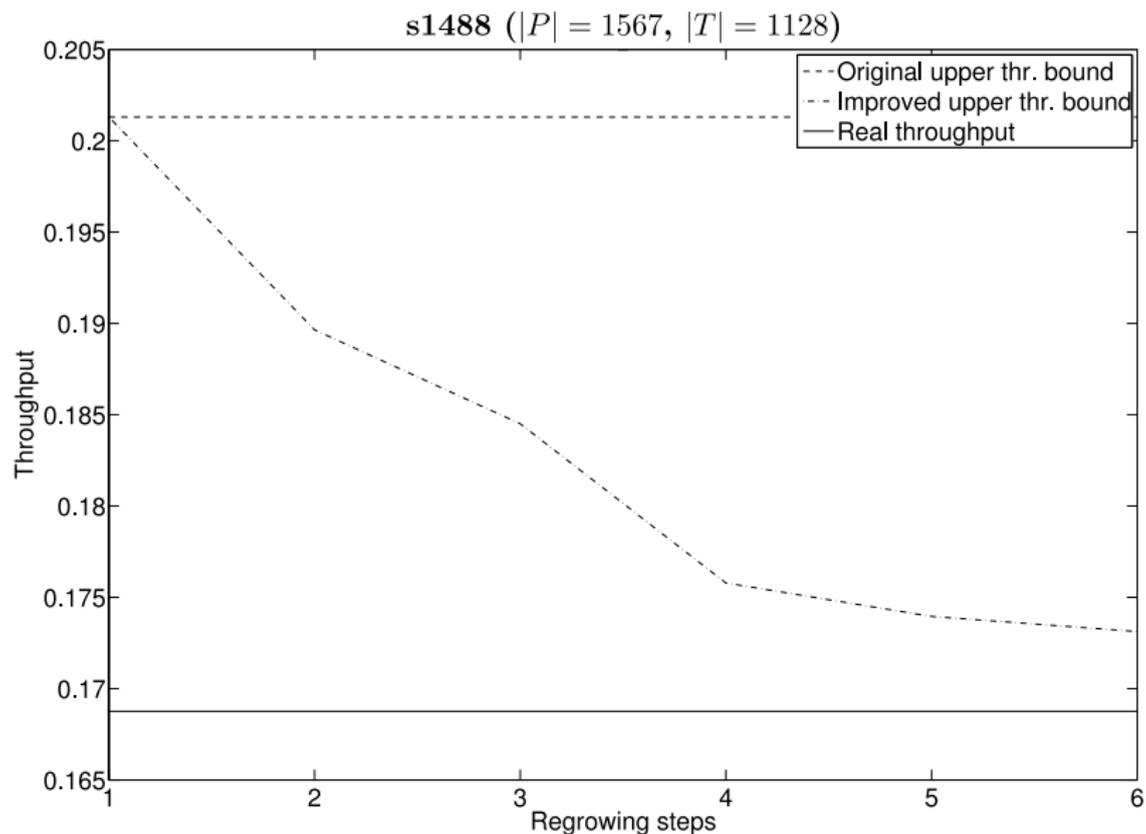
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Experiments (I): description of the experiments

Benchmarking and used tools

- **ISCAS benchmarking**
 - Strongly connected components of the ISCAS graphs
 - Initial marking randomly selected in $[1 \dots 10]$
 - Delay of transitions randomly selected in $[0.1 \dots 1]$
- **Strategy implemented in MATLAB** (linprog)
- **Simulation tool: GreatSPN**
 - Confidence level 99%; accuracy 1%
- Host: Pentium IV 3.6GHz, 2GB DDR2 533MHz RAM

Experiments (II): Gets close to the real thr. after few steps



Experiments (III): results of improvement

Graph	Size		% Size		Regrowing steps	Initial thr. bound	Θ
	$ P $	$ T $	$ P' $ (%)	$ T' $ (%)			
s1423	1107	792	79 (7.13%)	76 (9.59%)	3	0.236010	0.235213 (0.34%)
s1488	1567	1128	91 (5.8%)	86 (7.62%)	6	0.201300	0.173127 (13.99%)
s208	27	24	27 (100%)	24 (100%)	3	0.409390	0.377683 (7.75%)
s27	54	44	19 (35.18%)	18 (40.9%)	1	0.305960	0.304987 (0.31%)
s349	187	146	26 (13.9%)	24 (16.44%)	2	0.340320	0.327867 (3.66%)
s444	92	68	14 (15.21%)	12 (17.64%)	2	0.181670	0.181260 (0.22%)
s510	1038	734	45 (4.33%)	40 (5.45%)	5	0.133030	0.117819 (11.43%)
s526	113	92	18 (15.93%)	16 (17.39%)	2	0.313490	0.305860 (2.43%)
s713	271	208	11 (4.06%)	10 (4.8%)	1	0.428720	0.427840 (0.2%)
s820	1162	848	40 (3.44%)	38 (4.48%)	2	0.161060	0.147483 (8.43%)
s832	1293	948	84 (6.5%)	78 (12.04%)	5	0.239429	0.208798 (12.79%)
s953	415	312	88 (11.36%)	82 (26.28%)	6	0.369214	0.337811 (8.50%)

- Sharper upper bound in few regrowing steps
 - Improvement varies from 0.2% to 14%
- Uses a very low percentage of the size of the original graph
 - Lower than 10% (in most of cases)

Experiments (III): results of improvement

Graph	Size		% Size		Regrowing steps	Initial thr. bound	Θ
	$ P $	$ T $	$ P' $ (%)	$ T' $ (%)			
s1423	1107	792	79 (7.13%)	76 (9.59%)	3	0.236010	0.235213 (0.34%)
s1488	1567	1128	91 (5.8%)	86 (7.62%)	6	0.201300	0.173127 (13.99%)
s208	27	24	27 (100%)	24 (100%)	3	0.409390	0.377683 (7.75%)
s27	54	44	19 (35.18%)	18 (40.9%)	1	0.305960	0.304987 (0.31%)
s349	187	146	26 (13.9%)	24 (16.44%)	2	0.340320	0.327867 (3.66%)
s444	92	68	14 (15.21%)	12 (17.64%)	2	0.181670	0.181260 (0.22%)
s510	1038	734	45 (4.33%)	40 (5.45%)	5	0.133030	0.117819 (11.43%)
s526	113	92	18 (15.93%)	16 (17.39%)	2	0.313490	0.305860 (2.43%)
s713	271	208	11 (4.06%)	10 (4.8%)	1	0.428720	0.427840 (0.2%)
s820	1162	848	40 (3.44%)	38 (4.48%)	2	0.161060	0.147483 (8.43%)
s832	1293	948	84 (6.5%)	78 (12.04%)	5	0.239429	0.208798 (12.79%)
s953	415	312	88 (11.36%)	82 (26.28%)	6	0.369214	0.337811 (8.50%)

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Experiments (IV): graph throughput and time comparative

Graph	Original graph thr. CPU time (s)	Θ CPU time (s)	Original graph thr.	Θ	% thr.
s1423	59948.980	8.283	0.222720	0.235270	5.63%
s1488	36717.156	7.165	0.168760	0.172154	2.01%
s208	0.492	0.492	0.376892	0.376892	0%
s27	2166.002	0.954	0.305082	0.306166	0.35%
s349	141.210	0.441	0.328340	0.327398	-0.28%
s444	2278.231	0.205	0.181069	0.181260	0.11%
s510	13669.814	1.358	0.117500	0.118040	0.46%
s526	129.181	0.344	0.270010	0.305860	13.27%
s713	628.503	0.405	0.411630	0.427840	3.94%
s820	20775.811	0.788	0.144770	0.147699	2.02%
s832	16165.863	1.914	0.196920	0.208873	6.07%
s953	453.850	19.155	0.327910	0.338644	3.27%

- Θ CPU time insignificant respect to original thr CPU time
- Improvement varies from very close value to 13% over the real thr
 - Slow cycles far away from critical cycle?
- Negative relative error caused by simulation parameters

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Summary

- Proposed approach based on an **iterative algorithm**
 - Takes **initial thr bound** and **refines it** in each iteration
- **Accurate upper bound** in few iterations
- **Efficient** and **good *accuracy-computational complexity load* trade-off**
- **Outputs:**
 - **Accurate estimate** for the steady state thr
 - **Subnet representing bottleneck** of the system

Accurate Performance Estimation for Stochastic Marked Graphs by Bottleneck Regrowing

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