Thermal modeling, analysis and control using an electrical analogy

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Outline

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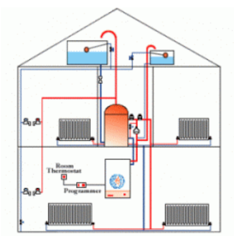
2. Modeling
3. Identification
4. Analysis
5. Control
6. Conclusions

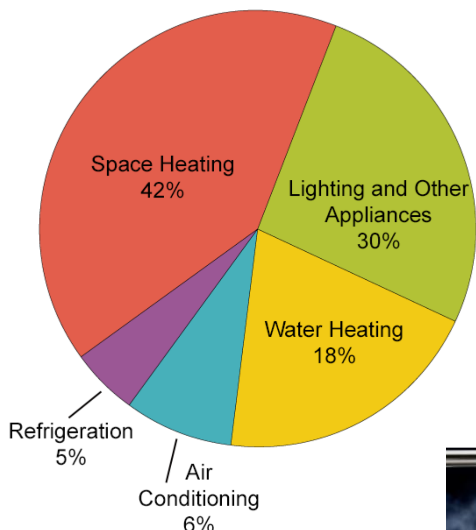
1. Introduction

Reducing world energy consumption has become a major issue of human beings.

A large proportion of this energy is used in thermal processes.



How Energy Is Used in Homes (2009)*



Space Heating	42 %
Refrigeration	5 %
Air conditioning	6 %
Water heating	18 %
TOTAL	**71 %**

* 2009 is the most recent year for which data are available.

Source: U.S. Energy Information Administration, *Residential Energy Consumption Survey (RECS) 2009*.

1. Introduction

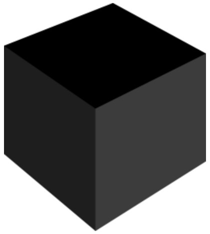
Idea → Use simple dynamic models to analyze the energy behavior of thermal systems.

Goal → Develop a general method to obtain models of thermal systems with energy information.

General conditions: limited knowledge of the system, temperature measurements available.

Modeling options:

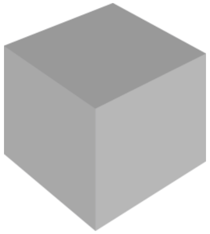
Black-box



Problem:

Not able to model energy variables (not measured).

Grey-box



Heat transfer phenomena equations.

Experimental data to identify the model.

White-box

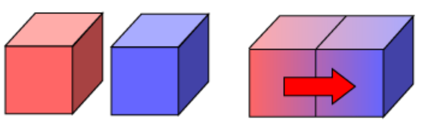
Problem:

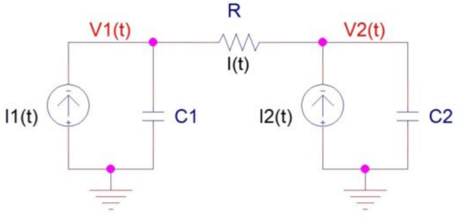
Complete knowledge of the system may be difficult to obtain.

2. Modeling

Our modeling method is based on an electrical analogy:

Heat transfer	**Electricity**
$$m c_P \frac{dT(t)}{dt} = \dot{Q}(t)$$ *Heat equation (homogeneous temperature)*	$$C \frac{dV(t)}{dt} = I(t)$$ *Capacitor load*
$$\dot{Q}(t) = \frac{kA}{l}\left(T_1(t) - T_2(t)\right)$$ *Fourier's law*	$$I(t) = \frac{1}{R}\left(V_1(t) - V_2(t)\right)$$ *Ohm's law*





Advantages:

- Easy analysis (Kirchhoff laws)
- Reduced parameterization (C and R instead of m, c_p, k, l, A…)

Not suitable if:

- Mass movement is not negligible
- Radiation is the dominant phenomenon (nonlinear)

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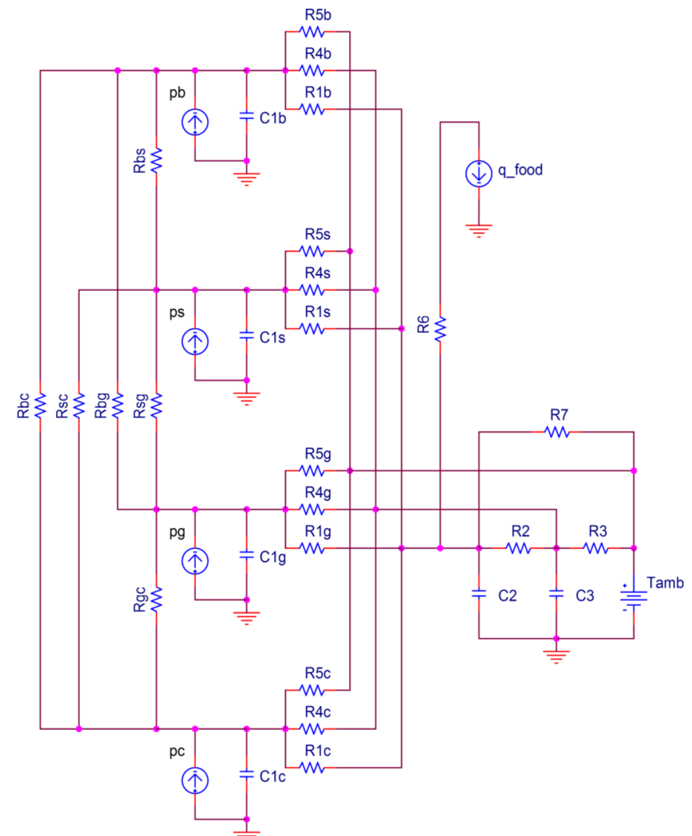
2. Modeling

Diagram building method:

1. Select the system components whose temperatures are to be modeled.
2. Include a capacitor for each temperature.
3. Include a thermal resistance for each feasible thermal connection.
4. Add inputs and disturbances and connect them to the capacitors.

Grey-box model

Electric oven example



2. Modeling

State space representation:

$$\dot{\mathbf{x}}(t) = A\mathbf{x}(t) + B\mathbf{u}(t) + B_P\mathbf{p}(t).$$

State variables: $T_1, T_2, \ldots, T_n$

(Temperatures of the thermal capacitors)

Definitions:

a_{ij}	→	*Element in row i and column j of matrix A*
b_{ij}	→	*Element in row i and column j of matrix B*
b_{Pij}	→	*Element in row i and column j of matrix BP*
$C_1, C_2, \ldots, C_n$	→	*Thermal capacitors of the diagram*
$u_1, u_2, \ldots, u_m$	→	*Inputs*
$p_1, p_2, \ldots, p_q$	→	*Disturbances*
R_{ij}	→	*Thermal resistance between C_i and C_j*
R_{iuj} (R_{ipj})	→	*Thermal resistance between C_i and u_j (p_j)*
α_{iuj} (α_{ipj})	→	*1 if C_i and u_j (p_j) are connected, 0 otherwise*

→ General expressions:

$$a_{ij} = \begin{cases} -\frac{1}{C_i}\left(\sum_{\substack{k=1\\k\neq i}}^{n}\frac{1}{R_{ik}} + \sum_{k=1}^{m}\frac{1}{R_{iuk}} + \sum_{k=1}^{q}\frac{1}{R_{ipk}}\right), & \text{if } i = j \\ \frac{1}{C_i R_{ij}}, & \text{if } i \neq j \end{cases}$$

$$b_{ij} = \begin{cases} \frac{1}{C_i R_{iuj}}, & \text{if } u_j \text{ is a temperature} \\ \frac{\alpha_{iuj}}{C_i}, & \text{if } u_j \text{ is a heat flux} \end{cases}$$

$$b_{Pij} = \begin{cases} \frac{1}{C_i R_{ipj}}, & \text{if } p_j \text{ is a temperature} \\ \frac{\alpha_{ipj}}{C_i}, & \text{if } p_j \text{ is a heat flux.} \end{cases}$$

3. Identification

Continuous-time state-space model with physical basis → Nonlinear identification method.

Method: Interior Point Algorithm.

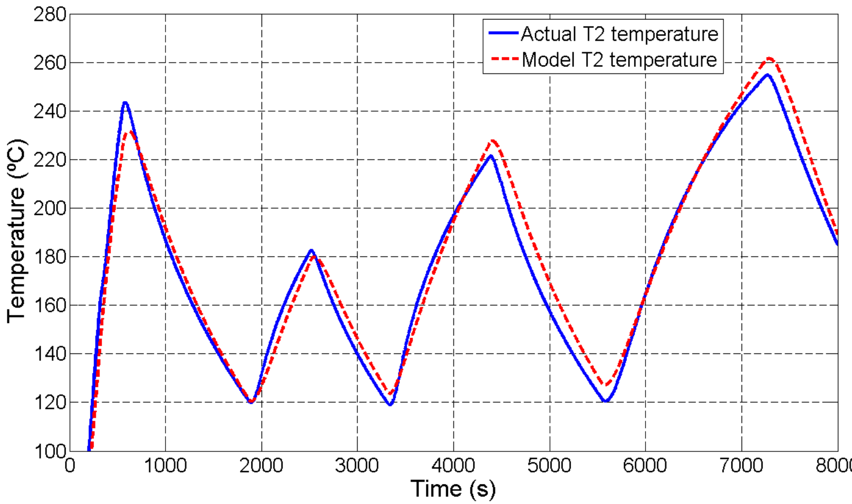
Advantages:

- Implemented in most mathematical softwares
- Permits the use of constraints
  (Cs and Rs must be positive)

Cost function: weighted sum of root mean square errors of state variables.

Experimental data: temperature measurements (state variables).

Electric oven example



3. Identification

The model fits properly to temperatures, but… Is it able to provide energy information?

We must assure that there only exists an optimum parameter set (the model is identifiable).

"On global identifiability for arbitrary model parametrizations" Ljung and Glad (1994) .

Method based on Ritt's algorithm (Differential algebra).

Recommendations:

1. Use conductances (G) instead of resistances (R) → Linearity in parameters.
2. Use this ranking: $u^{(\mu)} < y^{(\nu)} < G^{(o)} < C^{(\pi)} < x^{(\sigma)}$

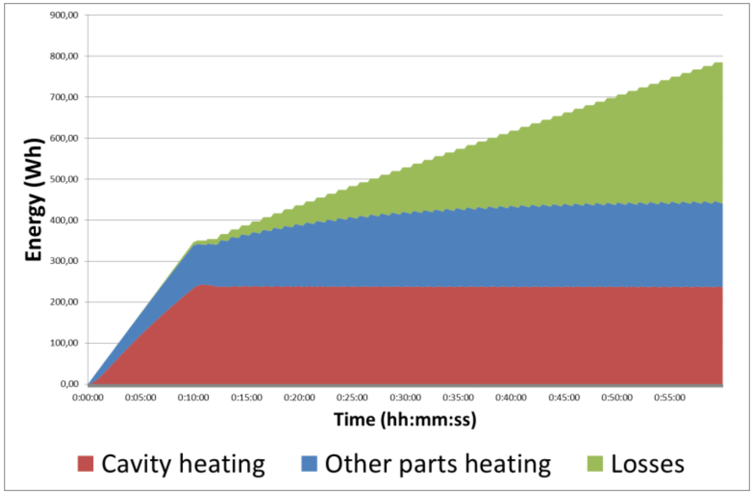
The parameterization in resistances (R) and capacitances (C) is identifiable in most cases.

4. Analysis

Heat fluxes and stored energy

We can use the heat fluxes through thermal resistances or the energy stored in capacitors to analyze the energy behavior of the system

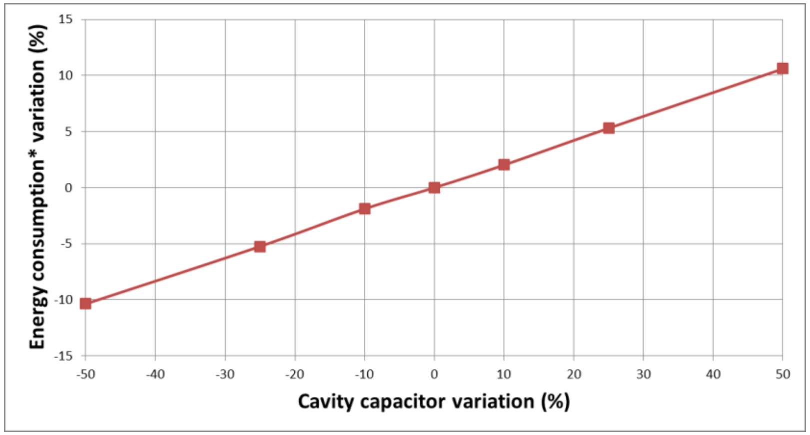
$$\dot{Q}(t) = \frac{1}{R_t}\Delta T(t) \qquad Q(t) = C_t T(t)$$



Sensitivity analysis

We can analyze how a parameter affects to the energy consumption of the system.

Remember that resistances and capacitors are directly related to the real components of the system.



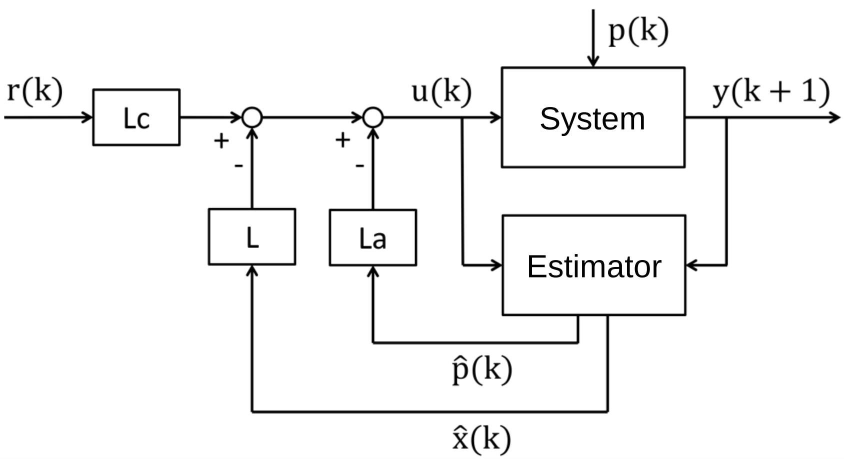
(*) Energy consumption during a simulated 1 hour test

5. Control

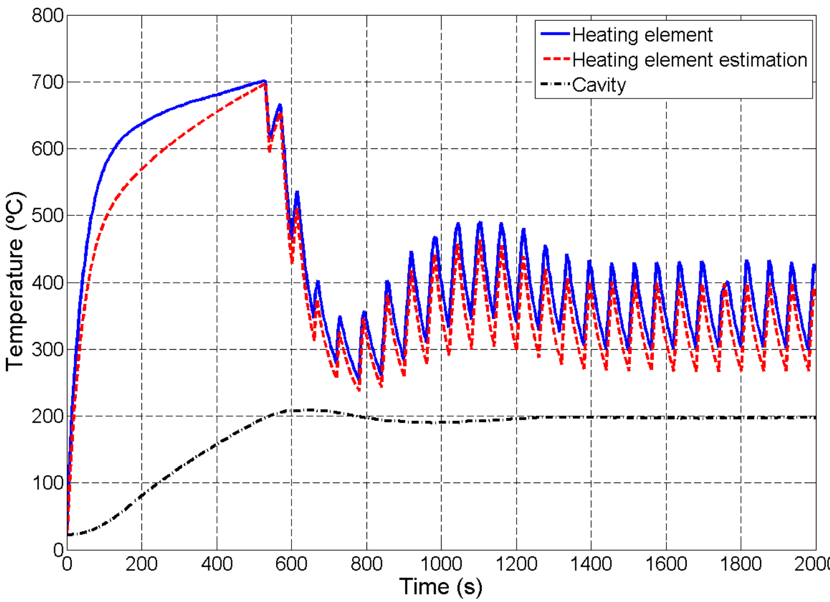
The models are also suitable to design state space controllers and observers.

Electric oven example

We used the cavity temperature to estimate the rest of temperatures and to control the oven.



Control and estimation scheme.



Oven control using one of the heating elements.

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6. Conclusions

- In the paper we have presented a general method to build models of thermal systems. Completely valid for ovens, fridges, buildings, etc.
- We have explained in a didactic way how to build the diagram and we have obtained the general expressions for its state-space description.
- Some notion about identification has been presented: method and identifiability.
- Propose / highlight some analyses using the model.
- The model is also suitable to design controllers for the system.

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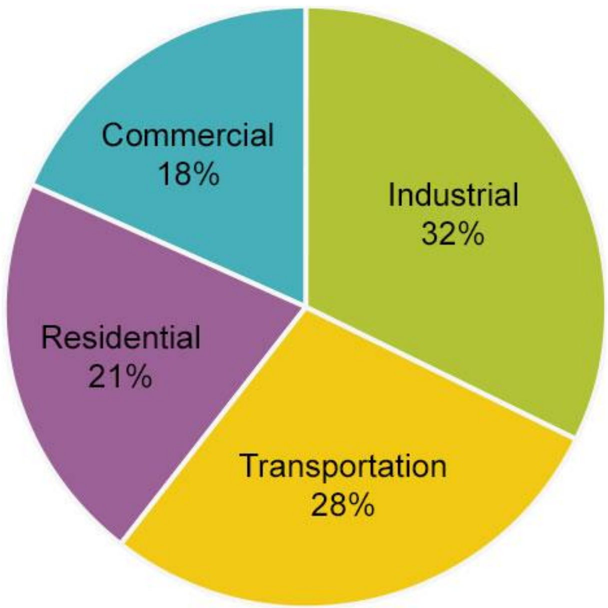
Share of total energy consumed by major sectors of the economy, 2012[1]



[1]Includes electricity consumption.
Source: U.S. Energy Information Administration, *Monthly Energy Review,* Table 2.1 (April 2013),preliminary 2012 data.

Residential sector energy consumption (21% of total)