Fast localization of avalanche victims using Sum of Gaussians

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***Abstract*— The probability of finding alive a person buried by a snow avalanche decreases dramatically with time. The best chance for the victims is to carry an avalanche beacon or ARVA (from the french: *Appareil de Recherche de Victimes d'Avalanche*), that transmits a magnetic field that can be detected by the rescuer's ARVA. However, the signals received are difficult to interpret and require people with good training on the actual searching techniques. In this paper we propose to address the search for victims as a SLAM problem: tracking the rescuer location while building a map of the locations of the victims under the snow, using 3D measurements of the magnetic field produced by their ARVAs. Given the high non-linearity of the problem, we propose a technique based on sum of Gaussians (SOGs) and Extended Kalman Filter. We present preliminary simulation results showing that this technique is faster and more accurate than the classical ones.**

I. Introduction

In this work we are interested in rescuing people buried in the snow by an avalanche. Time is essential in this application: during the first 15 minutes, a victim is found alive with a 93% of probability whereas this probability falls to 25% after 45 minutes [1]. Therefore, a fast, accurate and automatic technique would be desirable in these circumstances.

Nowadays, the most useful device to localize the victims is called ARVA (*Appareil de recherche de Victimes d'Avalanche*). An ARVA is a transceiver that generates an oscillating magnetic field of $457kHz$ whose characteristics are defined on the standard *ETS 300718*, to ensure compatibility between different brands and models. This field is generated by a solenoid antenna and has symmetry of revolution with respect to its axis. To save batteries and make the detection easier, the magnetic field is transmitted in pulses of a tenth of a second every second. People in the mountain must always wear the ARVA in transmission mode just in case. If an avalanche occurs and buries completely one or more victims, the rescue tasks developed by the rest of the group or other neighboring groups become essential. Rescuers must switch their ARVAs to reception mode and search the avalanche area trying to detect the magnetic field of the victim's ARVA and finding its location. To be successful, rescuers must be people trained on the basic search techniques, briefly explained in the next section. Even for trained rescuers, there are some challenging situations such as deep burials or multiple close victims that may cause confusion and delay the rescue.

Our objective is to develop new localization techniques to automate and speed up the search. Our intuition is that finding the victims can be seen as a SLAM problem [2],[3], where the rescuer takes the role of the robot and the victims are the features to be mapped. The observations are measurements of the magnetic field vector generated by the victim's ARVAs taken with a 3D antenna carried by the rescuer. As a proof of concept, in this paper we address a simpler case: assuming that the trajectory of the rescuer is perfectly known (thus, we only address here the mapping problem, not the full SLAM problem).

Even in this case, the main difficulty arising is the high non-linearity of the equations describing the magnetic field. As a result, the probability distribution function of the estimated location of a victim after a few measurements is far from being a Gaussian. Linear estimation techniques as the EKF successfully used in feature-based SLAM, become useless here. In this paper we propose to approximate the victim's location pdf using a Sum of Gaussians [4]. Every time a new measurement is obtained, each Gaussian is updated with an EKF and the coefficients of the mixture are modified. A similar technique has been previously used in underwater SLAM to model complex surfaces from the sea floor [5].

The rest of the paper is arranged as follows. In section II we summarize the basic steps of an ARVA search. In section III a mathematical description of the antenna and the magnetic field it produces is presented. Section IV gives a detailed definition of the problem from the point of view of estimation theory. In section V we propose an estimation method based on mixture of Gaussians and Kalman Filter. Finally, section VI provides a comparison between our method and the best classical technique, using simulated data.

II. Basic ARVA searching techniques

It is important to notice that the magnetic field is a three-dimensional vectorial quantity and thus in each point of the space it has a certain intensity and direction. The main difference between the ARVAs consists on *how much* of this field they can measure. Attending to this criteria three types of ARVA can be found:

- ARVAs with one reception antenna: They are the oldest ones and usually are analog. The same antenna used to transmit the signal is used to measure the magnatic field when the ARVA is in reception mode. As a consequence, they only can measure the projection of the magnetic field on the axis of the solenoid. This type of ARVAs are the most difficult to use and the more time consuming during the search of victims.

- ARVAs with two perpendicular reception antennas: All of them are based on digital technology (microprocessors or DSPs). When they are hold in horizontal position they can measure the intensity and direction of the horizontal component of the magnetic field.
- ARVAs with three mutually perpendicular reception antennas: They are also based on digital technology. Because of the three perpendicular antennas they are able to measure the complete vector field. Therefore, in this case the relative orientation of the ARVA with respect to the magnetic field is not important.

The search techniques used to localize the victims depend on the type of ARVA available. The easiest and fastest technique is obviously related to the ARVA with three antennas. To compare our results with the best classical technique we focus in the next subsection on the search algorithms based on these ARVAs.

A. Searching

There are three main steps to localize the victim.

- Primary search: The rescuer(s) sweep the avalanche as fast as possible searching for the first signal from a victim's ARVA. The effective range at which the signal can be detected varies from 20m for a single-antenna receptor to 60m or 80m for a triple-antenna receptor. The position where the first signal is received is marked on the snow and the secondary search begins.
- Secondary Search: Approaching the victim. The most used method nowadays is the directional search that consists in following the direction of the magnetic field, because all the flux lines must converge to the victim's antenna. If after following the initial direction along 5m the signal decreases, the rescuer must turn and follow the opposite direction. Using this method the trajectories followed are usually curved as the flux lines are. This steps ends when the intensity of signal received corresponds to a distance of 2 to 4 meters to the victim.
- Tertiary Search: Precise victim location. With the receptor just above the snow surface, perform a cross-search looking for the point of maximum signal intensity. An additional difficulty in case of ARVAs with single or double antenna, is that, depending on the transmitter orientation, they may appear two maxima separated by a distance equal to the burial depth. In this case, the victim is located somewhere between them.

Once the ARVA search is finished, the rescuer must search with a snow probe until contacting the victim's body and, without removing the probe, begin shoveling the snow.

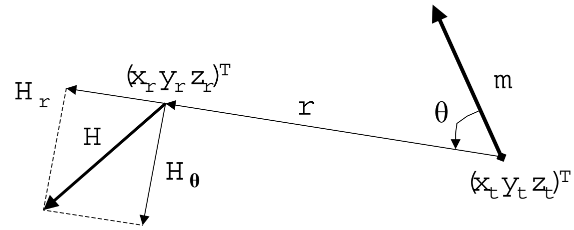

Fig. 1. Plain that contains $\mathbf{m}$, $\mathbf{r}$, $\mathbf{H}$ and its relations

TABLE I
NAMES AND UNITS OF THE MAGNETIC VARIABLES USED

Variable	Name	Units
B	magnetic induction	T (tesla), Wb/m^2
H	magnetic field strength	A/m
m	magnetic moment	Am^2
μ_0	free space permeability	$Wb/(Am)$

III. Fundamentals

Along the text, bold letters are for vectors whereas standard typography is for scalars or modulus of vectors. Table I contains important magnetic variables and their units.

The receiver (rescuer's antenna) and the transmitter (victim's ARVA) are described by

$$\begin{aligned} \mathbf{x}_r &= (x_r, y_r, z_r, \psi_r, \theta_r, \phi_r)^T \\ \mathbf{x}_t &= (x_t, y_t, z_t, m_x, m_y, m_z)^T \end{aligned} \tag{1}$$

The first three components of both vectors represent the cartesian coordinates of the receiver and the transmitter respectively. The last three components of $\mathbf{x}_r$ are the roll, pitch and yaw angles which define the orientation of the receiver antenna. The last three components of $\mathbf{x}_t$ define the magnetic moment $\mathbf{m}$ which encloses the information about the orientation and power of the transmitter solenoid antenna.

The radio vector $\mathbf{r}$ is defined as the vector that goes from the dipole to the point where the magnetic field is calculated (Fig.1).

There are two important characteristics of the magnetic field generated by a magnetic dipole. One is that it has symmetry of revolution with respect to the magnetic moment $\mathbf{m}$ and the other is that $\mathbf{H}$ is contained in the plane defined by $\mathbf{m}$ and $\mathbf{r}$.

Because of the low frequency the mathematical equations of the magnetic field follow the field structure of a magnetic dipole under *near field condition* [6]. The sinusoidal component is dropped in the equations for clarity.

$$\mathbf{H} = \frac{m}{4\pi r^3}(2\cos\theta \cdot \mathbf{u}_r + \sin\theta \cdot \mathbf{u}_\theta) = H_r \cdot \mathbf{u}_r + H_\theta \cdot \mathbf{u}_\theta \tag{2}$$

$$|\mathbf{H}| = \frac{m}{4\pi r^3}\sqrt{1 + 3 \cdot \cos^2\theta} \tag{3}$$

where θ is the angle between the dipole moment $\mathbf{m}$ and the radio vector $\mathbf{r}$, H_r and H_θ are the components of $\mathbf{H}$.

A disadvantage of the representation showed in eq.(2) is the difficulty to express H_r and H_θ in an arbitrary absolute reference system. We can achieve a more convenient expression taking into account:

$$\begin{aligned} \mathbf{u}_r &= \frac{\mathbf{r}}{r} \\ \mathbf{u}_\theta &= \frac{(\mathbf{m}\times\mathbf{r})\times\mathbf{r}}{|(\mathbf{m}\times\mathbf{r})\times\mathbf{r}|} \end{aligned} \tag{4}$$

Using these equations in eq.(2) and after some transformations we obtain:

$$\mathbf{H} = \frac{1}{4\pi r^3}\left[\frac{3(\mathbf{m}\cdot\mathbf{r})\cdot\mathbf{r}}{r^2} - \mathbf{m}\right] \tag{5}$$

We can do a last transformation with eq.(5) to express it in matrix form. The final result is:

$$\mathbf{H} = \frac{1}{4\pi r^5} \quad A \quad \mathbf{m} \tag{6}$$

where:

$$A = \begin{pmatrix} 2r_x^2 - r_y^2 - r_z^2 & 3r_xr_y & 3r_xr_z \\ 3r_xr_y & 2r_y^2 - r_x^2 - r_z^2 & 3r_yr_z \\ 3r_xr_z & 3r_yr_z & 2r_z^2 - r_x^2 - r_y^2 \end{pmatrix} \tag{7}$$

IV. Problem definition

We are interested in *estimating* the location of a person buried by an avalanche having as information the magnetic field produced by the victim's ARVA.

From our SLAM perspective the state vector to be estimated is $\mathbf{x} = (\mathbf{x}_r, \mathbf{x}_{t_1}, \ldots, \mathbf{x}_{t_n})^T$, where $\mathbf{x}_r$ represents the rescuer's antenna (robot) and $\mathbf{x}_{t_n}$ characterizes the nth transmitter (feature to be mapped). In this paper however, we will address a simpler case assuming that the trajectory of the rescuer is perfectly known. In this case the unknown state is $\mathbf{x} = \mathbf{x}_t$ which can be partially observed from measurements of the magnetic field. The typical process and observation equations are:

$$\begin{aligned} \mathbf{x}_{k+1} &= \mathbf{f}_k(\mathbf{x}_k) + \mathbf{w}_k \quad \text{(process equation)} \\ \mathbf{z}_k &= \mathbf{h}_k(\mathbf{x}_k) + \mathbf{v}_k \quad \text{(observation equation)} \end{aligned} \tag{8}$$

where k represents time.

Since the system is static and does not depend on k we can just write $\mathbf{x}_{k+1} = \mathbf{x}_k = \mathbf{x}$. The observation equation, $\mathbf{h}_k(\mathbf{x})$ returns a three-dimensional vector representing the components of the magnetic field with respect to a reference system attached to the antennas of the receptor. As the location of the receptor is perfectly known, the measurements can be expressed directly in the absolute reference system by means of $\mathbf{H}$. The vector $\mathbf{v}_k$ represents the noise in the measurement. This noise is supposed to be formed by two components. The first one is a background term which encloses meteorological disturbances of the magnetic field and thus is independent of the receiver characteristics. The second one is chosen proportional to the modulus of the magnetic field $\mathbf{H}$ measured and models the sensor defects. For each component of $\mathbf{H}$ the noises are considered independent. Therefore $\mathbf{v}_k$ is modeled as a white noise with diagonal covariance:

$$\mathbf{v}_k \sim N(0_{3\times1}, R_k) \tag{9}$$

where

$$\begin{aligned} R_k &= diag(\sigma^2, \sigma^2, \sigma^2) \\ \sigma^2 &= \sigma_{bkg}^2 + \sigma_{sen}^2 \end{aligned} \tag{10}$$

Central to a nonlinear estimation problem is the determination of the probability density function of the state conditioned on the available measurement data $\mathbf{p}(\mathbf{x}|\mathbf{z}_1, \mathbf{z}_2, ..., \mathbf{z}_k)$. If this a posteriori density function were known, an estimate of the state for any performance criteria could be determined. However, only when the process an observation equations are linear and the noise in both is Gaussian a closed form of the posterior can be determined. In this case the posterior is perfectly described by a Gaussian distribution (unimodal function), whose parameters are calculated by the Kalman Filter. To solve nonlinear problems the EKF and smart variations of the KF (UKF [7]) have been applied with positive results. The success of these techniques depends strongly on how well the real posterior can be approximated by a Gaussian and thus by a unimodal pdf. For nonlinear problems with multimodal posteriors other approximations must be investigated.

To know whether the posterior of our problem is represented by a unimodal or multimodal function we have analyzed a simplified situation. Suppose the receiver is 40 meters away from the transmitter and we know that both are on the same plane. We want to estimate the transmitter location taking two measurements (minimum number required to make the system solvable) of the magnetic field generated by the transmitter. The measurements are one meter apart from each other and are affected by noise. Applying a nonlinear optimization technique we solve the problem using as initial seed the true parameters of the transmitter. This procedure is repeated a hundred times for the same problem but for different realizations of the measurement error. The graphical representation of the solutions obtained can be seen on (Fig. 2).

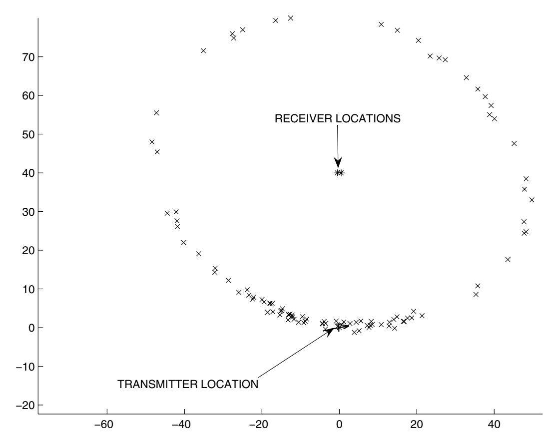

Fig. 2. Solutions obtained using nonlinear optimization

From this figure it can be deduced that the posterior pdf after two measurements $\mathbf{p}(\mathbf{x}|\mathbf{z_1}, \mathbf{z_2})$ follows a ring shape distribution and therefore a multimodal approximation of the pdf is required.

V. The Sum of Gaussian method and its implementation

A. Motivation of the method adopted

The problem presented will generate, in their sequential process, posterior pdfs that are multimodal. This is specially true in the initial steps of the localization since there are several regions of the state space that are compatible with the measurements form a statistical point of view. Several methods can be found in the literature to approximate this multimodal pdfs: grid sampling, particle filters, and Sum of Gaussians (SOGs) [8].

We have implemented grid sampling techniques (Markov Localization) for a simplified $2D$ version of the problem obtaining good results. However, the time requirements of this method has discouraged us from implementing it in $3D$ where the number of variables increases significantly. In addition, the high variability of the signal with the distance requires the implementation of a multi-resolution grid that has to be more and more refined as closer to the transmitter, incrementing the computational cost.

Particle filters provide a faithful approximation to the true shape of the posterior using a large set of sampling points. This filter requires a dynamic system and its process noise to maintain particle diversity after the resampling step. However, our state vector is static and as a consequence there is no process noise. In addition, our sensor is more precise as nearer to the victim because it is more sensitive to changes in the relative location between transmitter and receiver in these cases. However, as it is said in [8], particle filters performance degrades with very precise sensors. These reasons seem to make this filter not well suited to our problem.

Because of the explanations mentioned above, the solution adopted is based on the SOGs. The advantage of the SOGs over the particles is that they allow to cover the state space with less number of Gaussians since we can make use of their covariances. The advantage over the grid sampling methods in this case is its velocity.

B. Fundamentals of the SOGs

The Gaussian sum representation $\hat{\mathbf{p}}(\mathbf{x})$ of a density function $\mathbf{p}(\mathbf{x})$ of a vector random variable $\mathbf{x}$ is defined as [4]

$$\hat{\mathbf{p}}(\mathbf{x}) = \sum_{i=1}^{n} w^{(i)} N(\mathbf{x}; \bar{\mathbf{x}}^{(i)}, P^{(i)}) \tag{11}$$

where

$$\sum_{i=1}^{n} w^{(i)} = 1, \qquad w^{(i)} \geq 0 \tag{12}$$

$\hat{\mathbf{p}}(\mathbf{x})$ converge uniformly to $\mathbf{p}(\mathbf{x})$ if the number of terms n increases and the covariance $P^{(i)}$ tends to zero.

When a new measurement is obtained, the state update of each Gaussian is processed using the standard update equations of the EKF (since the sensor model is nonlinear), i.e., we have a bank of EKFs.

The linearized measurement function is

$$J_H = \left[\frac{\partial \mathbf{H}}{\partial x_t}, \frac{\partial \mathbf{H}}{\partial y_t}, \frac{\partial \mathbf{H}}{\partial z_t}, \frac{\partial \mathbf{H}}{\partial m_x}, \frac{\partial \mathbf{H}}{\partial m_y}, \frac{\partial \mathbf{H}}{\partial m_z}\right]_{3\times 6} \tag{13}$$

The innovation covariance $S^{(i)}$ of the Gaussian i at instant k is

$$S_k^{(i)} = J_{H_i} P_k^{(i)} J_{H_i}^T + R^k \tag{14}$$

where $P_k^{(i)}$ is the covariance of the ith Gaussian and R^k is the noise covariance of the measurement k.

The importance weights of the Gaussians are also recalculated using the following formula [4]

$$w_k^{(i)} = \frac{w_{k-1}^{(i)} N(\mathbf{z}^k; \mathbf{h}(\mathbf{x}^{(i)}), S_k^{(i)})}{\sum_j w_{k-1}^{(j)} N(\mathbf{z}^k; \mathbf{h}(\mathbf{x}^{(j)}, S_k^{(j)})} \tag{15}$$

As it is said on [9] the consistency problems of the Kalman Filter are produced by the linealization errors. However, in the SOGs approximation this problem can be partially reduced. Sizes of the Gaussian covariances can be initially chosen in such a way that the local linear approximation of the nonlinear function around the means are better as smaller are the covariances.

The main problem of Gaussian approximations is that its component functions do not form a basis for other density functions [5]. As a consequence there is no unique way to obtain a function description and it is difficult to obtain error bounds of the approximation.

C. Initialization of the Gaussians

Once the first signal is found, the first magnetic field measurement $\mathbf{z}_1 = \mathbf{H}_1$ is used to initialize the sum of Gaussians. From eq. (3), the minimum and maximum range at which the transmitter can be located are:

$$\begin{aligned} r_{min} &= \left(\frac{m_{min}}{4\pi|\mathbf{H}_1|}\right)^{\frac{1}{3}} \\ r_{max} &= \left(\frac{2 \cdot m_{max}}{4\pi|\mathbf{H}_1|}\right)^{\frac{1}{3}} \end{aligned} \tag{16}$$

A fixed number of Gaussians are distributed radially in the x_t, y_t space inside the region that goes from r_{min} to r_{max}. The z_t component is uniformly sampled. Given that the burial depth of avalanche victims is less than $3m$ in 95% of the cases [10], the z_t component is limited to $5m$. The covariances are selected in such a way that any point of the interest region verifies the chi-squared test for at least one of the Gaussians. The value of $\mathbf{m}$ for each Gaussian is initialized using the measured value $\mathbf{H_1}$ and eq.(6). All Gaussians are given the same initial weight $w_i = 1/n$, where n is the total number of Gaussians. To speed up the process, after each EKF run, Gaussians with negligible weights are eliminated.

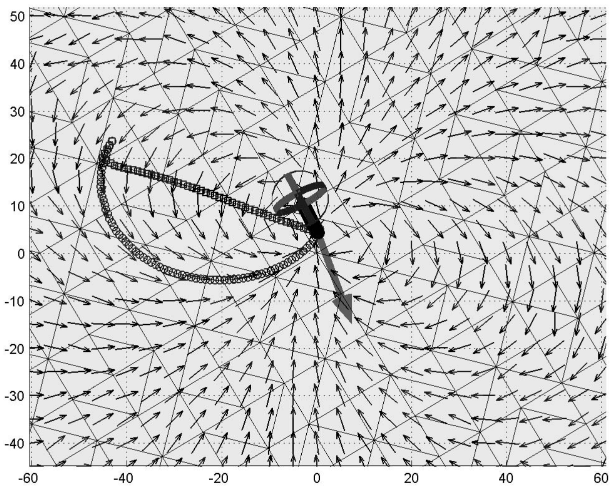

Fig. 4. Path followed using Directional Search (curved trajectory) and the proposed SOGs method (straight trajectory)

D. Search Control

The mean and covariance of the SOGs distribution are easily calculated from the mean and covariance of its Gaussian components [11]:

$$\begin{aligned} \bar{\mathbf{x}} &= \sum_{i=1}^{n} w^{(i)} \bar{\mathbf{x}}^{(i)} \\ P_x &= \sum_{i=1}^{n} w^{(i)} \{ P^{(i)} + (\bar{\mathbf{x}} - \bar{\mathbf{x}}^{(i)})(\bar{\mathbf{x}} - \bar{\mathbf{x}}^{(i)})^T \} \end{aligned} \quad (17)$$

where $\bar{\mathbf{x}}^{(i)}$ and $P^{(i)}$ are the mean and covariance of the ith Gaussian.

The automatization of the searching algorithm is as follows. Initially, the rescuer is directed to follow the direction of the magnetic flux lines, as in the classical directional search. When the receiver position is far enough (in terms of mahalanobis distance) from the mean of the SOGs distribution, the rescuer is directed towards this mean.

VI. Simulations

This section presents simulation results showing the performance of the proposed algorithm. It is assumed that the receiver is moving at $1m/s$ taking a measurement per second. For comparison we have also simulated the classical directional search. Figure 3 (left) shows the initial cylinder-ring distribution of the Gaussians. After some steps (center) the posterior pdf is reduced to a smaller region that contains the real location of the transmitter. At this point the rescuer is going straightforward to the victim's location. When the receiver is still 30 meters apart from the transmitter (right) the uncertainty in the location of the victim has been greatly reduced and the posterior begins to be similar to a single Gaussian.

Figure 4 shows an example of the trajectory followed by both techniques. The initial rescuer position is $(-43, 23.5, 0.6)$ and the transmitter is at $(0.1, 4, -1.35)$, both in meters. The big arrow represents the magnetic moment of the transmitter antenna. The classical technique describes a curved path because the flux lines are also curved as can be seen on the figure (small arrows). However, our method goes almost in a straight line from the beginning. Figure 5 depicts the evolution of the distance between the receiver and the transmitter for both searching techniques, and the evolution of the error in our estimation. This error represents the difference between the mean of the SOGs distribution and the true location of the victim. An important result of this simulation is that when the receiver is 10 meters apart from the transmitter, the estimation error is in the order of a few centimeters.

TABLE II
Comparisons between Directional Search, SOG technique and the ideal case

	MAX	MEAN	MEDIAN
DS error	2.62	0.52	0.42
SOG error	0.07	0.02	0.02
Steps DS	128	71	70
Steps SOG	100	63	62
Steps Ideal	69	54	53

To analyze the performance of the proposed algorithm 100 tries have been performed. On each try the position, orientation and power of the transmitter is randomly chosen. The initial distance from the rescuer to the victim varies from 40 to 70 meters. Table II summarizes the results of the 100 tries. The error rows represent the difference in the horizontal plane (in meters) between the true victim's location and the estimated location obtained with the directional search and the SOGs. Results are clearly favorable to our method.

The same occurs with the number of steps (or number of seconds) needed to find the victim. The ideal procedure represents the case when the victim's location is known from the beginning and the trajectory traversed is then completely straight (minimum number of steps). It can be seen that the time overhead introduced by the SOGs with respect to the ideal case is very similar to the one introduced by the directional search with respect to our method. Nevertheless, the implementation of the directional search represents an optimistic realization of this technique since in our simulations the rescuer follows perfectly the flux lines. More realistic figures for the directional search are reported in [12], where the median time needed by novices using different ARVA brands range from 109 to 214 seconds, including the probing time needed to find the victim after the tertiary search. However, in a few cases, the rescuer needed more than 15 minutes or gave up without finding the victim. The true advantage of the proposed techniques is the high precision on the victim's localization that completely eliminates the tertiary search and reduces the time needed to probe the snow to contact the victim's body to a minimum. Furthermore, the automatization of the search reduces the chance of an human error during the rescue.

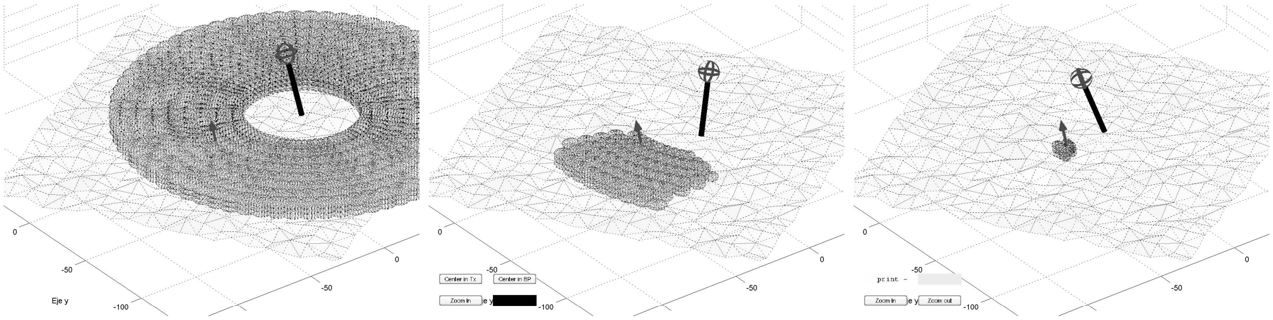

Fig. 3. Initial distribution of the Gaussians in a cylinder-ring (left), After some steps the main hypothesis tend to the real location of the transmitter (middle), Although the receiver is still far from the transmitter its location is almost known (right).

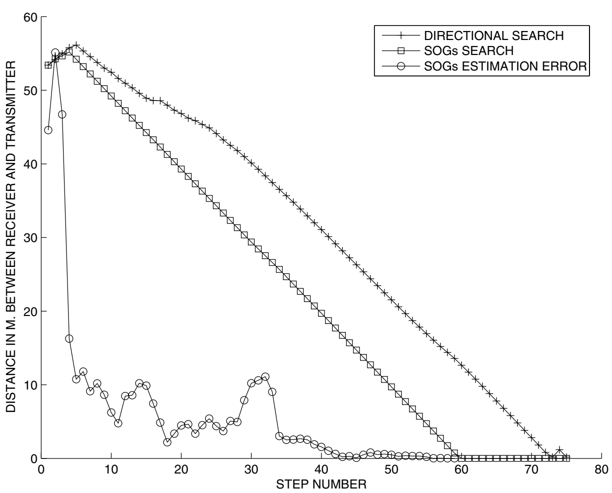

Fig. 5. Faster convergence of our algorithm (almost in a straightforward manner compared to the directional search method. The precision obtained with the SOGs is also presented

VII. Conclusion

Although the problem presented can be interpreted as a SLAM problem, our attention in this paper has been focused to the mapping part. Even in this simplified case, the highly nonlinear measurement equation and the symmetries of the magnetic field makes the posterior pdf be multimodal, specially in the initial steps of the localization. The method presented is based on approximating the multimodal posterior with a Sum of Gaussians. Results obtained in simulations with this method are really promising. Compared with the classical technique, the new algorithm is faster in the secondary search (straight vs curved trajectory) and avoids the significant extra time needed in the tertiary search step. This is of fundamental importance in an application where time reduction is critical.

Future work will include real experiments with a 3D antenna currently under development. To localize the rescuer with high precision (as in the assumptions made in this paper) differential GPS and inertial devices will be used. The method will be generalized to take into account the uncertainty in the rescuer motion. As a consequence, the location of the rescuer will be included in the vector state and therefore it will be also estimated. At this point, the SLAM problem will have to be solved completely.

Finally, the problem of finding several victims will be addressed. Hardware and signal processing techniques will be needed in this case to distinguish between the signals of different ARVAs. It will be also interesting to study the best trajectory of the rescuer to reduce the uncertainty as fast as possible of the location of all the victims.

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