Inverse Depth Monocular SLAM

J. Civera, A.J. Davison and J.M.Montiel

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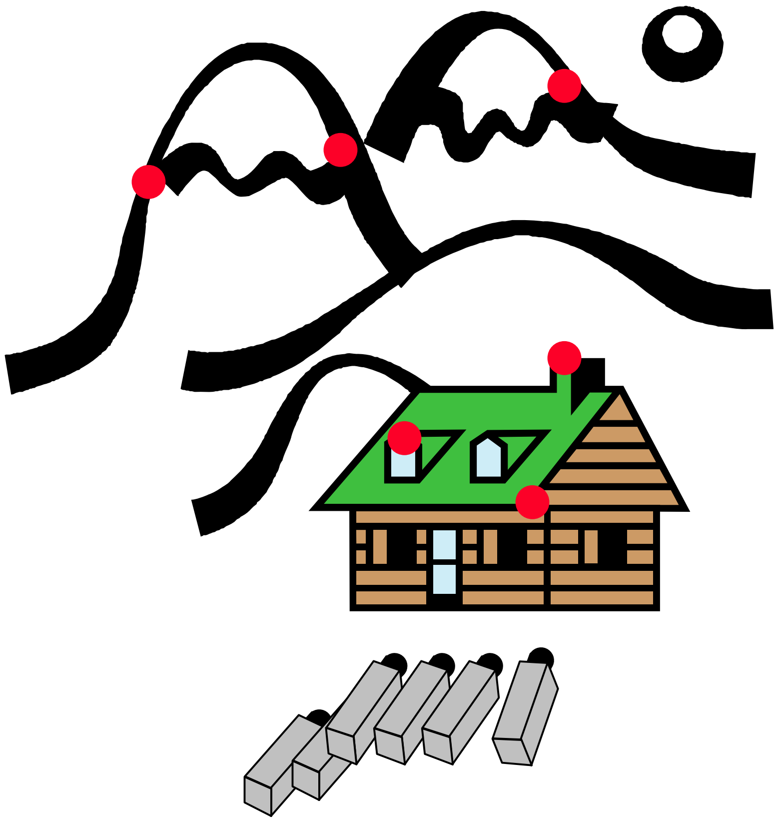
Problem Statement



- Sequential simultaneous sensor location and map building at frame rate, 30Hz.
- Camera moves freely in 3D, 6dof camera motion
- Outdoors real scenes contains close and distant, even at infinity, features
- Main contribution codifying scene points with its inverse depth:
  1. Deals with low parallax cases
  2. Deals with both distant and close points
  3. Map features are initialised just from one image

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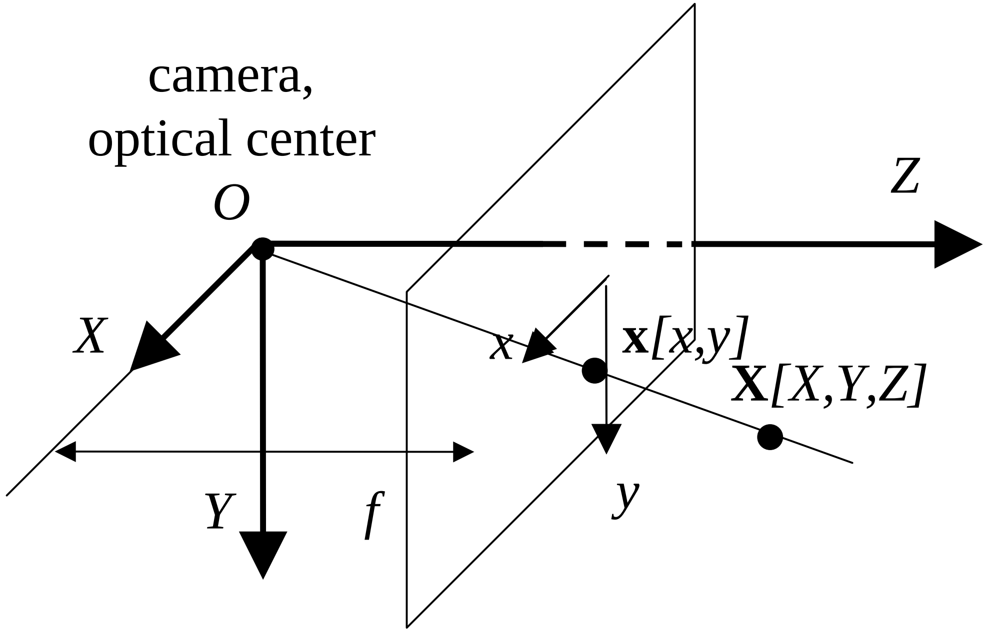
Camera Geometry: Pure Bearing-only Sensor



- Camera detects rays
- A ray is defined by the optical center O and the observed point
- The images is used as the method to determined the detected ray
- Depth is not detected

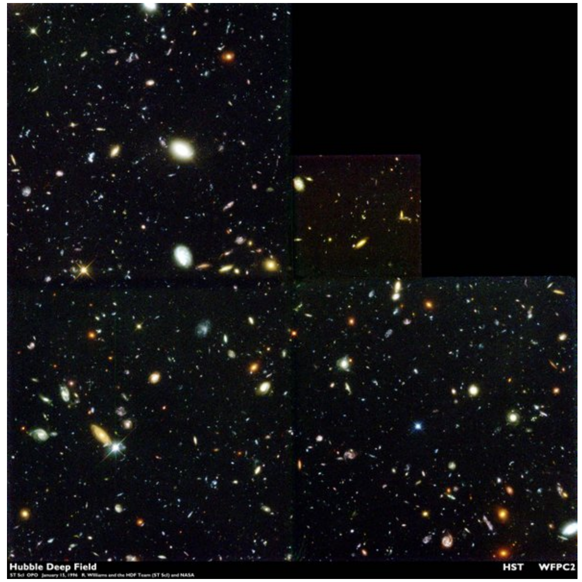


image of gallaxies

at 10^9 years light far from

the Earth

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Points at Infinity

- **projective cameras *do* observe points at infinity**
- **parallel lines meet at infinity, a projective camera does observe this intersection point as vanishing point**
- **we intend to code and exploit this points at infinity in the monocular SLAM problem**

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Parallax

- **No parallax geometries**
  - **Camera rotation**
  - **Camera observing a scene plane**
- **Low parallax cases**
  - **Distant features compared with camera translation**
  - **Initial feature observation**

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State of the Art

- **SLAM, initially proposed by Smith and Cheesman, 1986, widespread usage in robotics for multisensor fusion [Casetellanos 1999], [Ferder 1999] [Thrun 2004]**
  - **Sequential approach**
  - **Ability to close loops, identifying features previously observed as reobserved. Complexity is linked to the scene not to the number of observations processed.**
- **SLAM used for computer vision, [Castellanos 2000], [Davison 1998] combined with odometry**
- **Monocular SLAM vision [Davison 2003]**
  - **Camera "following the laws of mechanics" motion model**
  - **Vision as the only sensor, no odometry.**
  - **Synergic usage of vision geometry and vision photometric map**
  - **Low parallax points avoided:**
    - » **Points represented as XYZ, only works with points close to the camera**
    - » **Delayed initialization**
- **Monocular SLAM Fast SLAM, inverse depth delayed initialization [Eade 2006]**

SFM,computer vision methods

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State of the Art

SLAM methods

- **Photogrammetric bundle adjustment, 60's**
  - **Normally only close points**
- **Computer vision geometry Hartley & Zisserman [Hartley 2003]**
  - **Robust statistics**
  - **Matching between several shots enforcing a coherence with a projective camera model**
  - **Applied to individual shots, to sequences with varying camera parameters**
  - **Applied for robot navigation [Nister 2003, Mouragnon 2006]**
  - **Not sequential**
  - **Wide-baseline performance**
  - **Routine usage of points at infinity**
- **Model selection problem [Torr ICCV98]**
  - **No parallax, homography model**
  - **Parallax epipolar geometry model**
  - **Increasing the frame rate, the interframe motion closes to a homography**

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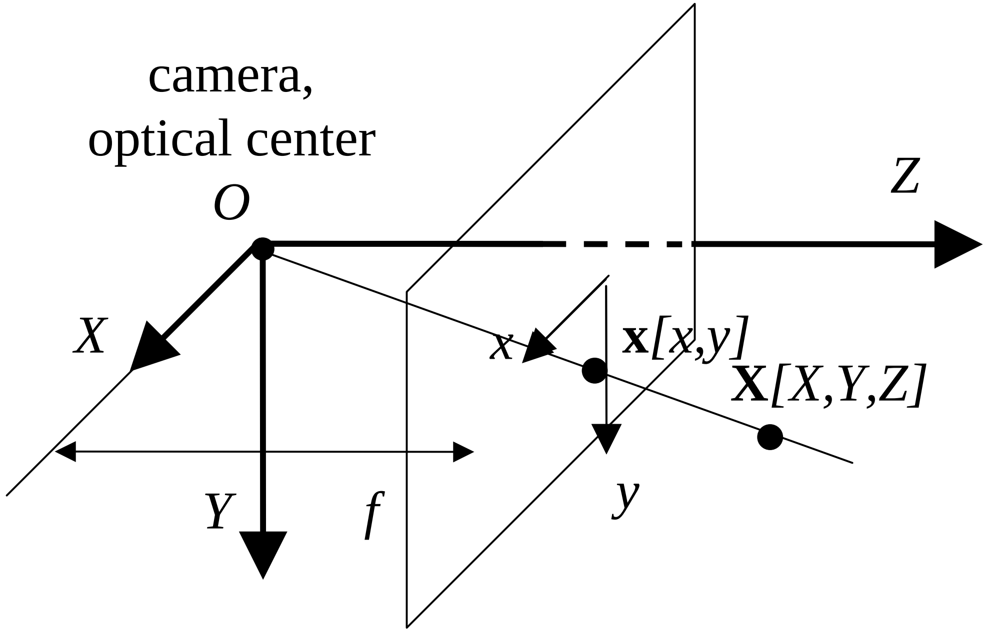
Camera Geometry: Pure Bearing-only Sensor



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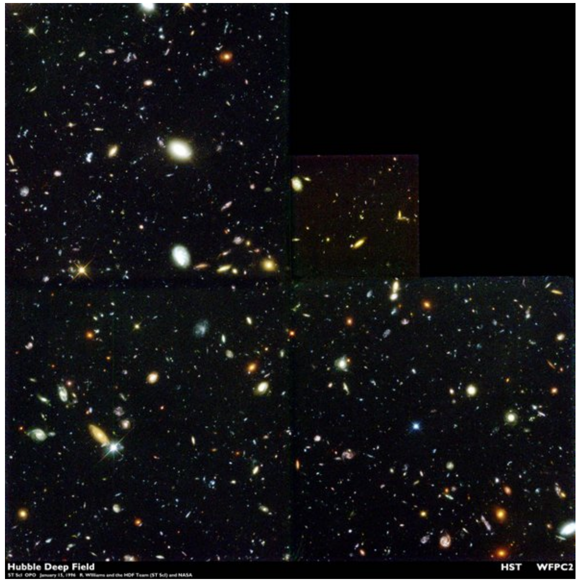

image of gallaxies

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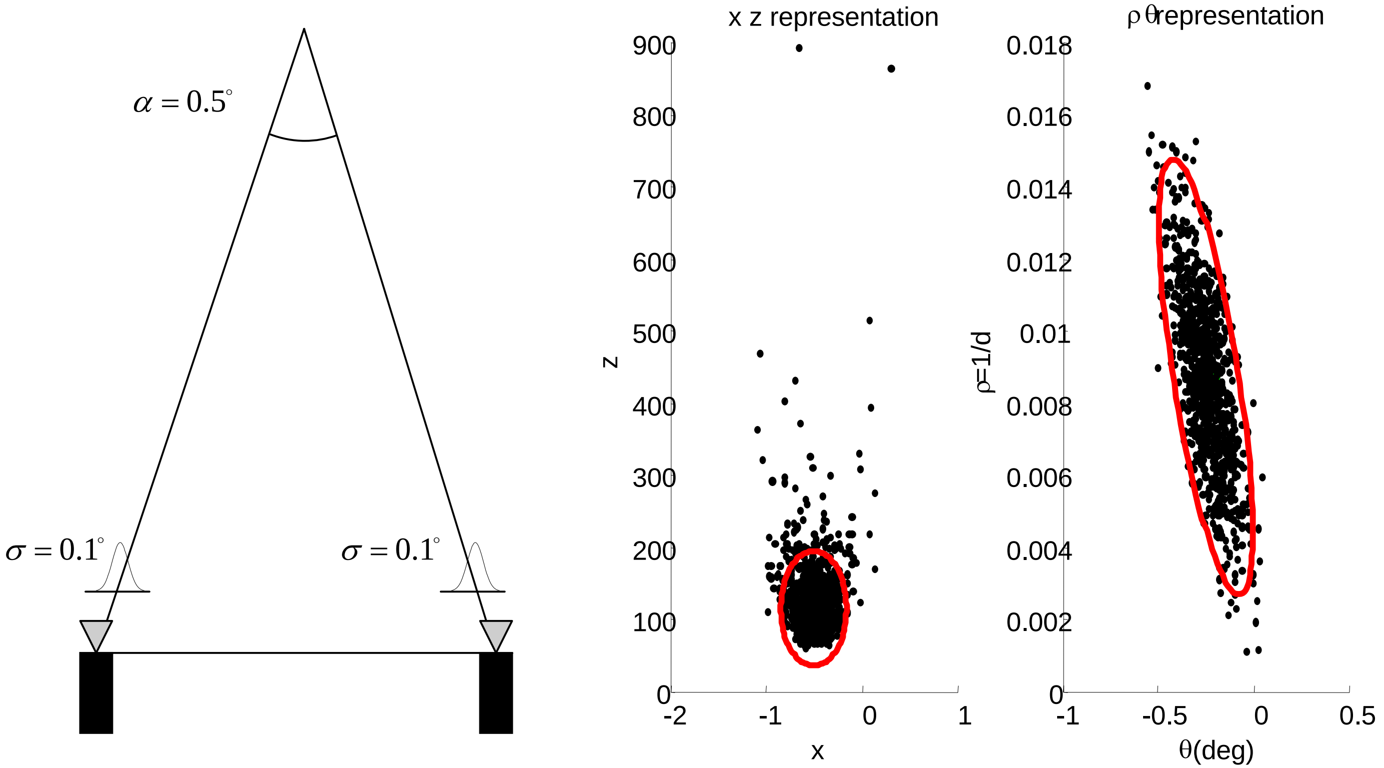
Gaussianity of the Inverse Depth Coding



Simulation: computing depth of a point from 2 views at known camera locations

- Non Gaussian in XZ
- Gaussian in 1/d, theta

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Camera Motion Priors

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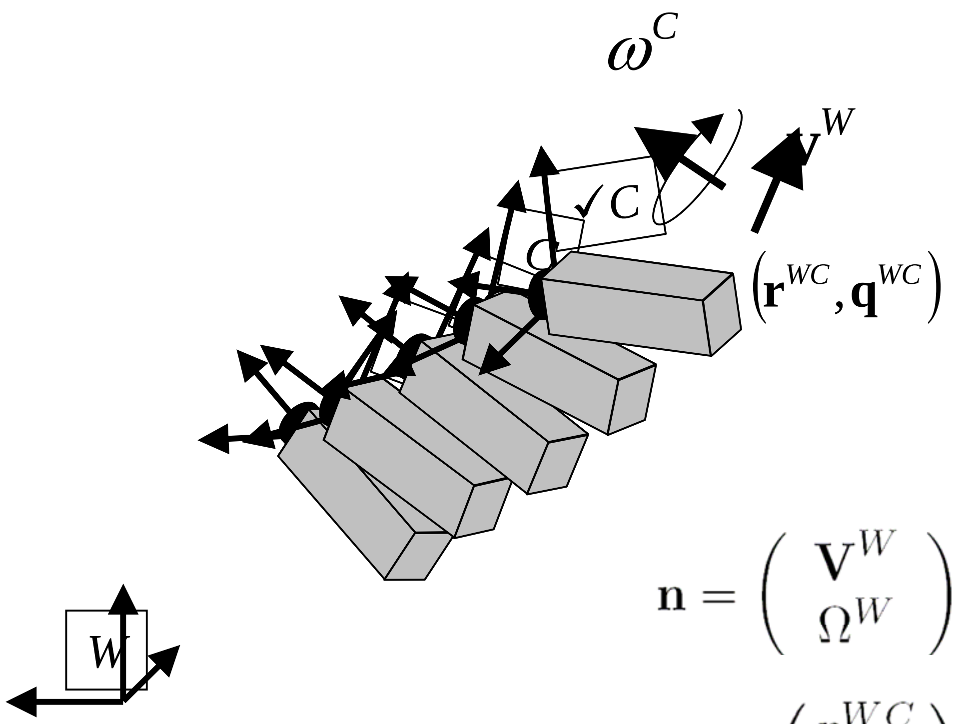


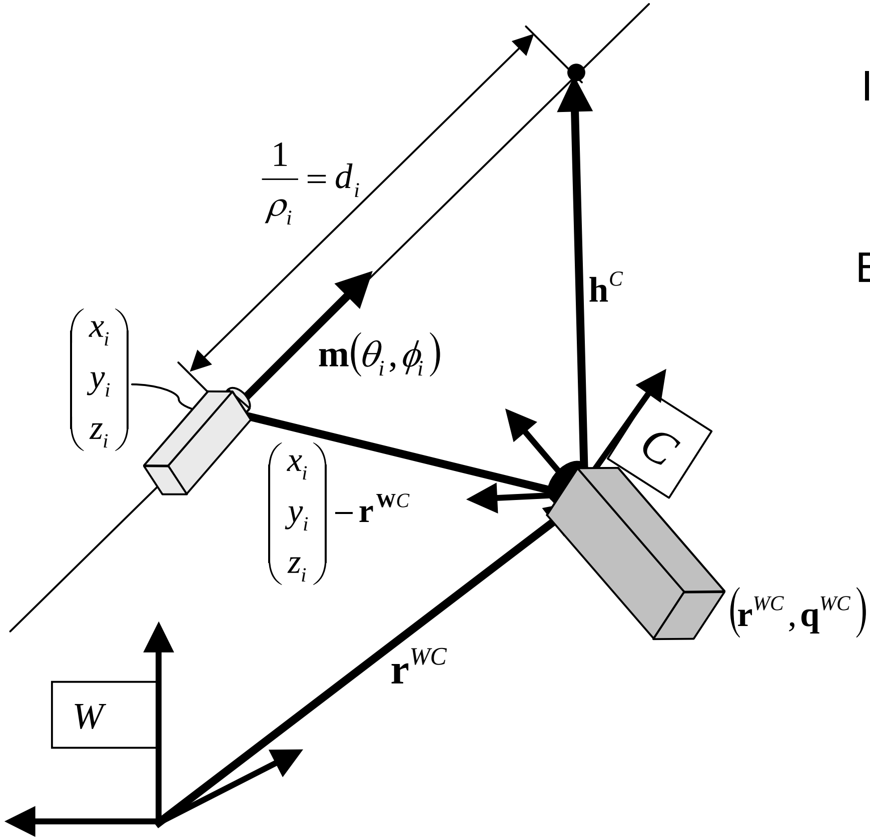
$$\mathbf{x}_v = \begin{pmatrix} \mathbf{r}^{WC} \\ \mathbf{q}^{WC} \\ \mathbf{V}^W \\ \omega^C \end{pmatrix}$$

Constant Velocity Motion Model
smooth camera motion
impulse acceleration noise

$$\mathbf{n} = \begin{pmatrix} \mathbf{V}^W \\ \Omega^W \end{pmatrix} = \begin{pmatrix} \mathbf{a}^W \Delta t \\ \alpha^W \Delta t \end{pmatrix}$$

$$\mathbf{f_v} = \begin{pmatrix} \mathbf{r}^{WC}_{k+1} \\ \mathbf{q}^{WC}_{k+1} \\ \mathbf{v}^W_{k+1} \\ \omega^C_{k+1} \end{pmatrix} = \begin{pmatrix} \mathbf{r}^{WC}_k + v^W_k \Delta t + a^W_k \Delta t^2 \\ \mathbf{q}^{WC}_k \times \mathbf{q}(\omega^C_k \Delta t + \alpha^C_k \Delta t^2) \\ \mathbf{v}^W_k + a^W_k \Delta t \\ \omega^C_k + \alpha^C_k \Delta t \end{pmatrix}$$

Scene Point Coding in Inverse Depth. Measurement Equation



Inverse dept point coding

$$\mathbf{y}_i = \begin{pmatrix} x_i & y_i & z_i & \theta_i & \phi_i & \rho_i \end{pmatrix}^\top$$

Bearing only camera measurement

$$u = u_0 - \frac{f}{d_x}\frac{h_x}{h_z} \qquad v = v_0 - \frac{f}{d_y}\frac{h_y}{h_z}$$

Measurement equation

$$\mathbf{h}^C = \begin{pmatrix} h_x \\ h_y \\ h_z \end{pmatrix} = \mathbf{R}^{CW}\left(\rho_i\left(\begin{pmatrix} x_i \\ y_i \\ z_i \end{pmatrix} - \mathbf{r}^{WC}\right) + \mathbf{m}(\theta_i, \phi_i)\right)$$

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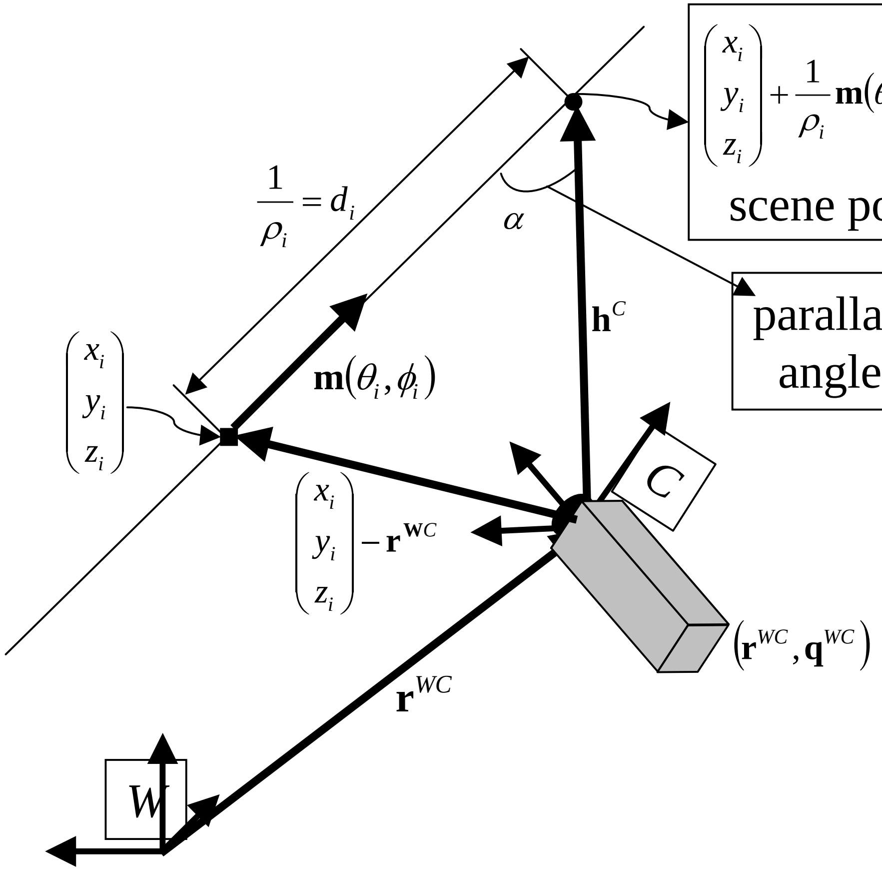
Scene Point Coding in Inverse Depth. Measurement Equation



$$u = u_0 - \frac{f}{d_x}\frac{h_x}{h_z} \qquad v = v_0 - \frac{f}{d_y}\frac{h_y}{h_z}$$

$$\mathbf{h}^C = \begin{pmatrix}h_x\\h_y\\h_z\end{pmatrix} = \mathbf{R}^{CW}\left(\begin{pmatrix}x_i\\y_i\\z_i\end{pmatrix} + \frac{1}{\rho_i}\mathbf{m}(\theta_i,\phi_i) - \mathbf{r}^{WC}\right) =$$

$$\mathbf{R}^{CW}\left(\rho_i\left(\begin{pmatrix}x_i\\y_i\\z_i\end{pmatrix} - \mathbf{r}^{WC}\right) + \mathbf{m}(\theta_i,\phi_i)\right)$$

$\begin{pmatrix}x_i\\y_i\\z_i\end{pmatrix}$, $\mathbf{m}(\theta_i,\phi_i)$, those of the first time the feature is observed

$\rho_i\left(\begin{pmatrix}x_i\\y_i\\z_i\end{pmatrix} - \mathbf{r}^{WC}\right)$, parallax

distant point, $\rho_i \to 0, \Rightarrow$ parallax goes to zero

close camera locations, low baseline, $\begin{pmatrix}x_i\\y_i\\z_i\end{pmatrix} - \mathbf{r}^{WC}$, goes to zero

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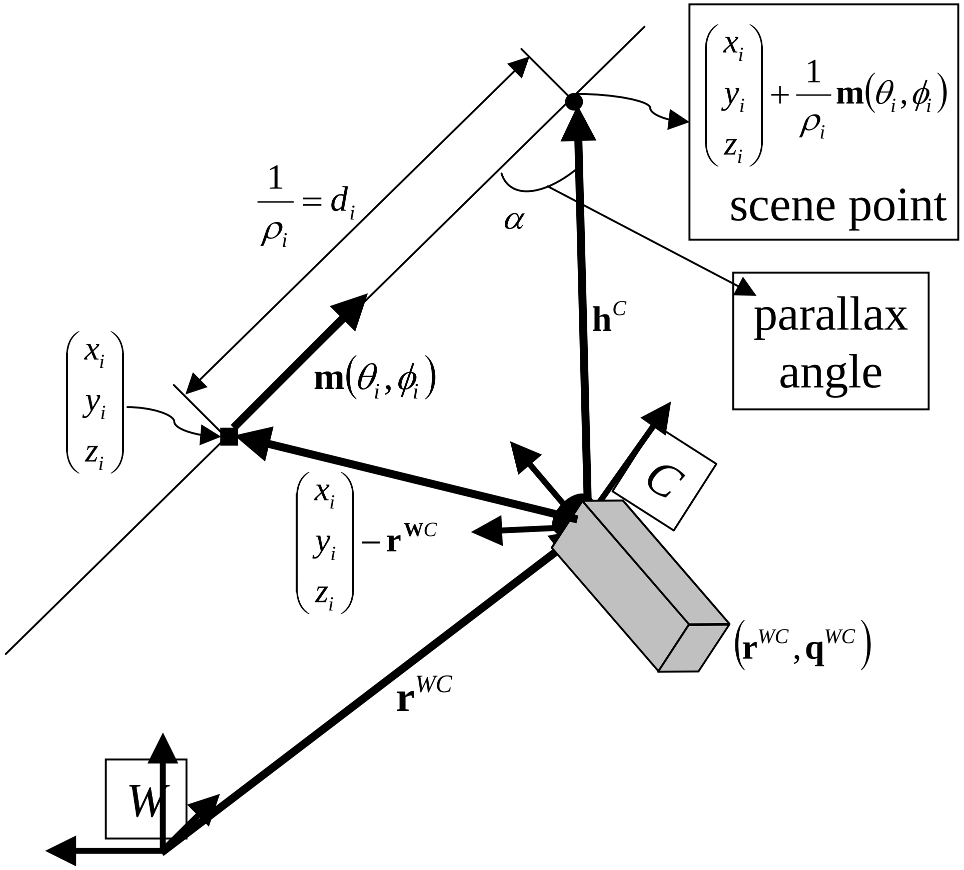
Scene Point Coding in Inverse Depth. Measurement Equation



$$u = u_0 - \frac{f}{d_x}\frac{h_x}{h_z} \qquad v = v_0 - \frac{f}{d_y}\frac{h_y}{h_z}$$

$$\mathbf{h}^C = \mathbf{R}^{CW}\left(\rho_i\left(\begin{pmatrix} x_i \\ y_i \\ z_i \end{pmatrix} - \mathbf{r}^{WC}\right) + \mathbf{m}(\theta_{i,}\phi_i)\right)$$

low parallax, only point at infinity is observed:

$$\mathbf{h}^C \approx \mathbf{R}^{CW}(\mathbf{m}(\theta_{i,}\phi_i))$$

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EKF sequential processing

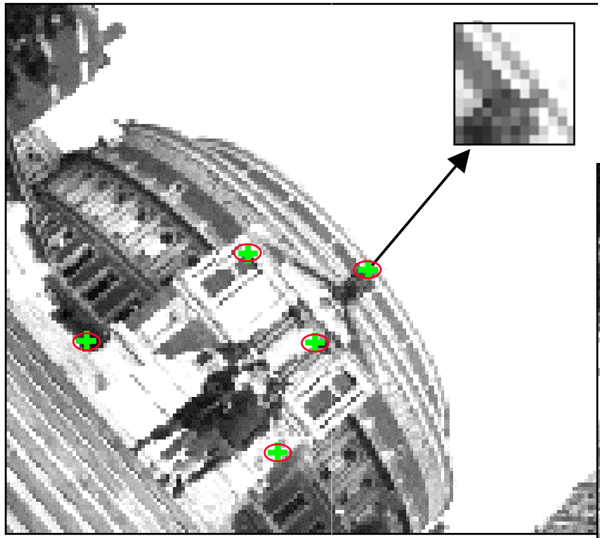
$$\hat{\mathbf{x}}_{k+1|k} = \begin{pmatrix} \mathbf{f}_v\left(\hat{\mathbf{x}}_{v,k|k}, 0\right) \\ \hat{\mathbf{y}}_{1,k|k} \\ \vdots \\ \hat{\mathbf{y}}_{n,k|k} \end{pmatrix}$$

$$\hat{\mathbf{P}}_{k+1|k} = \mathbf{F}\hat{\mathbf{P}}_{k+1|k}\mathbf{F}^\top + \mathbf{G}\begin{pmatrix} (\Delta t)^2\,\mathbf{P}_\alpha & 0 \\ 0 & 0 \end{pmatrix}\mathbf{G}^\top$$

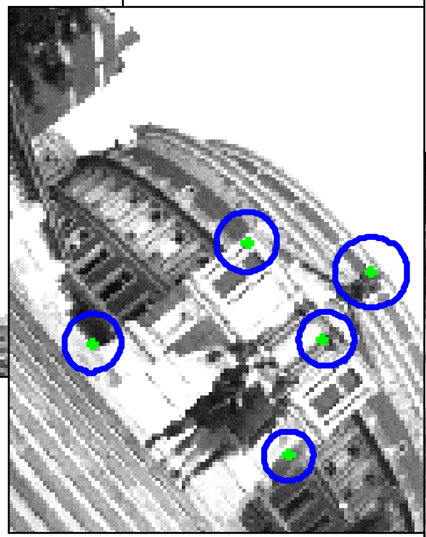
$$\hat{\mathbf{h}}_{i,k+1|k} = \mathbf{h}\left(\hat{\mathbf{y}}_{i,k+1|k}, \hat{\mathbf{x}}_{k+1|k}\right)$$

$$\mathbf{S}_{k+1|k} = \mathbf{H}\mathbf{P}_{k+1|k}\mathbf{H}^\top + \mathrm{diag}\left(\mathbf{R}_1, \ldots, \mathbf{R}_n\right)$$

stored patch



Estimation at step

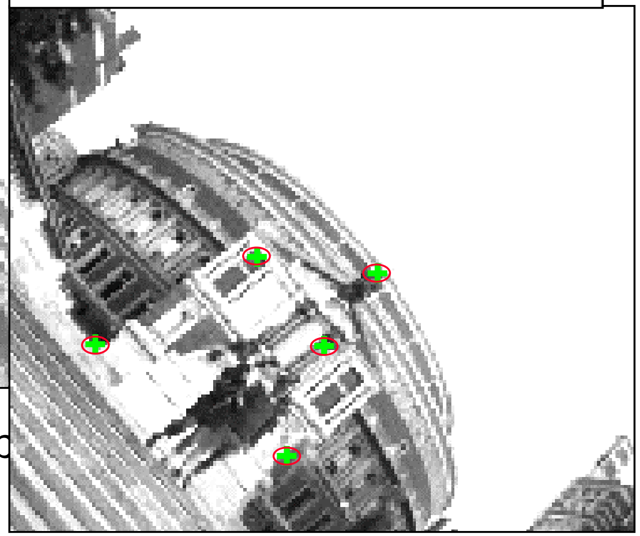


Prediction at step

$$\hat{\mathbf{x}}_{k+1|k+1} = \hat{\mathbf{x}}_{k+1|k} + \mathbf{K}\left(\mathbf{z}_m - \hat{\mathbf{h}}_m\right)$$

$$\mathbf{K} = \hat{\mathbf{P}}_{k+1|k}\mathbf{H}_m^\top\left(\mathbf{H}_m\hat{\mathbf{P}}_{k+1|k}\mathbf{H}_m^\top\right)^{-1}$$

$$\hat{\mathbf{P}}_{k+1|k+1} = \left(I - \mathbf{K}\mathbf{H}_m\right)\hat{\mathbf{P}}_{k+1|k}$$



Update at step k

stored patch is searched in all acceptance region for the feature prediction

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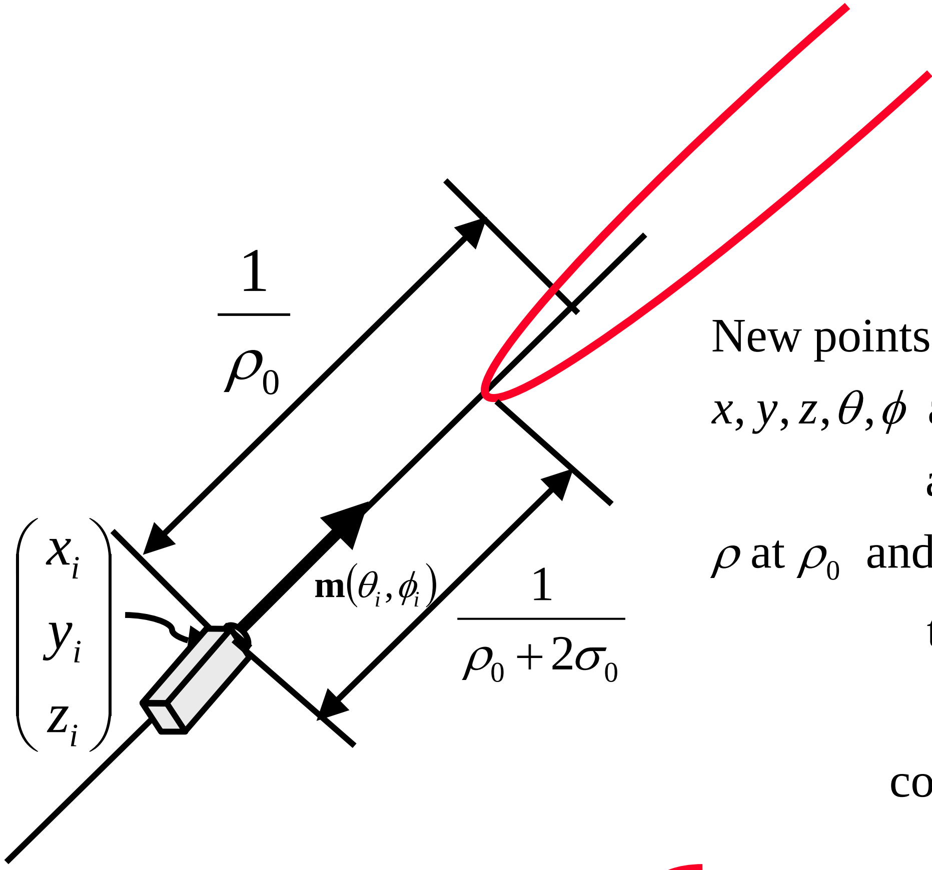
Inverse Depth Feature Initialization Priors



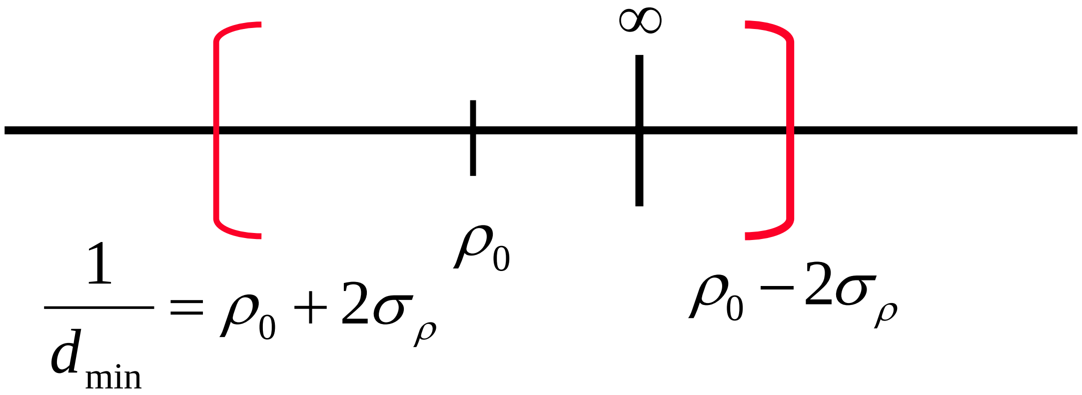
New points are observed from just an observation

x, y, z, θ, ϕ and the corresponding covariance are initialized from the first feature observation

ρ at ρ_0 and its covarinace σ_ρ is initialized so that the interval

$$\left[\rho - 2\sigma_\rho, \rho + 2\sigma_\rho\right]$$

covers a region from $d_{\min}$ and including infinite



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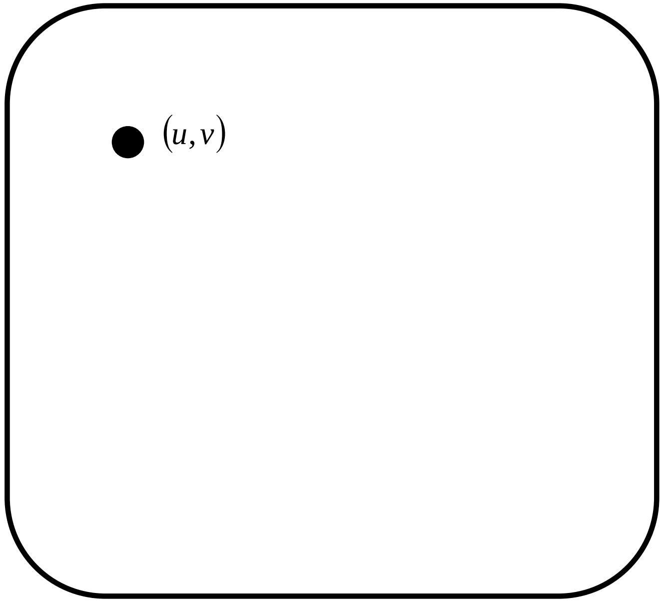
Inverse Depth Feature Initialization



$$\begin{pmatrix} \hat{x}_i & \hat{y}_i & \hat{z}_i & \hat{\theta}_i & \hat{\phi}_i & \hat{\rho}_i \end{pmatrix}^\top$$

$$\begin{pmatrix} \hat{x}_i \\ \hat{y}_i \\ \hat{z}_i \end{pmatrix} = \hat{\mathbf{r}}^{WC}$$

$$\rho_i = \rho_0$$

$$\begin{pmatrix} \theta_i \\ \phi_i \end{pmatrix} = \begin{pmatrix} \arctan\left(-\mathbf{h}_y^W, \sqrt{\mathbf{h}_x^{W\,2} + \mathbf{h}_z^{W\,2}}\right) \\ \arctan\left(\mathbf{h}_x^W, \mathbf{h}_z^W\right) \end{pmatrix}$$

$$\mathbf{h}^W = \mathbf{R}_{WC}\left(\mathbf{q}_{k|k}^{WC}\right)\begin{pmatrix} \upsilon \\ \nu \\ 1 \end{pmatrix}$$

$$\hat{\mathbf{x}}_v = \begin{pmatrix} \hat{\mathbf{r}}_{k|k}^{WC} \\ \hat{\mathbf{q}}_{k|k}^{WC} \\ \hat{\mathbf{V}}_{k|k}^{W} \\ \hat{\omega}_{k|k}^{C} \end{pmatrix} \begin{bmatrix} \mathbf{y}_1 \\ \vdots \\ \mathbf{y}_n \end{bmatrix} \hat{\mathbf{P}}_{k|k}$$

$$\hat{\mathbf{P}}_{k|k}^{\text{new}} = \mathbf{J}\begin{pmatrix} \hat{\mathbf{P}}_{k|k} & 0 & 0 \\ 0 & \mathbf{R}_j & 0 \\ 0 & 0 & \sigma_\rho^2 \end{pmatrix}\mathbf{J}^\top$$

Current camera and map location estimate

$$\mathbf{J} = \left(\begin{array}{c|c} I & 0 \\ \hline \frac{\partial \mathbf{y}}{\partial \mathbf{r}^{WC}}, \frac{\partial \mathbf{y}}{\partial \mathbf{q}^{WC}}, 0, \ldots, 0, & \frac{\partial \mathbf{y}}{\partial \mathbf{h}}, \frac{\partial \mathbf{y}}{\partial \rho} \end{array}\right)$$

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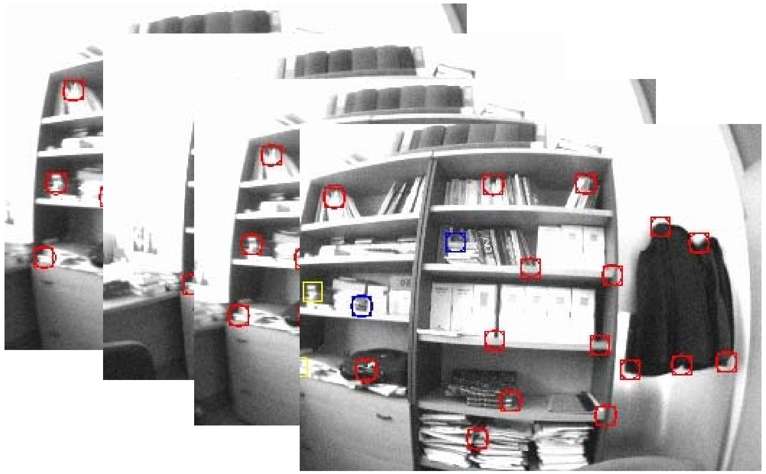
Dimensional Monocular SLAM

Calibrated image measurements $\mathbf{z}_i, \sigma_z$

Frame Rate Δt



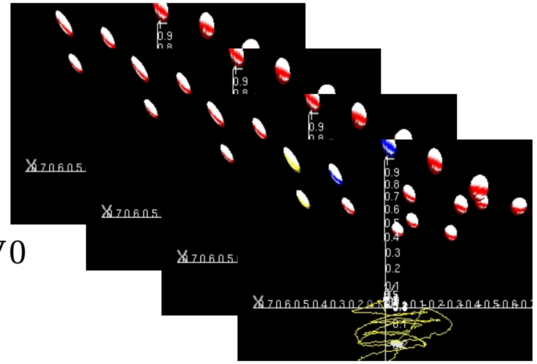
Monocular SLAM Processing

Camera Motion Priors σ_a, σ_α ; $\sigma_{\omega 0}, \sigma_{v0}$

Intial depth Prior ρ_0, σ_ρ

camera location **scene map**

$$\left(\mathbf{r}^{WC}, \mathbf{q}^{WC}, \mathbf{v}^{W}, \omega^{C}, \mathbf{y}_1 \cdots \mathbf{y}_n\right)^T$$



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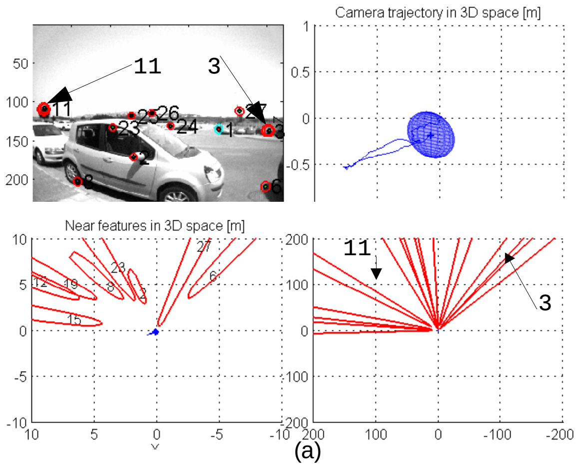
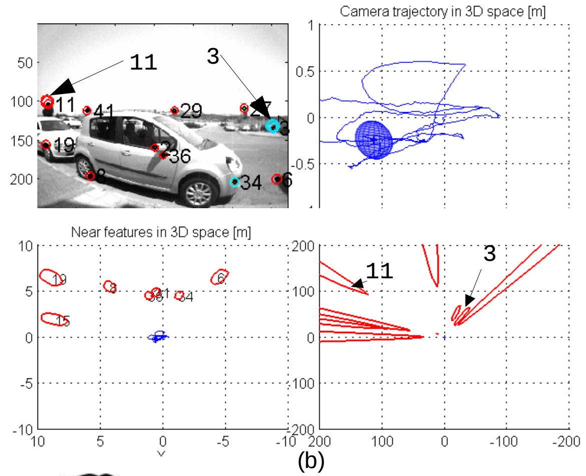
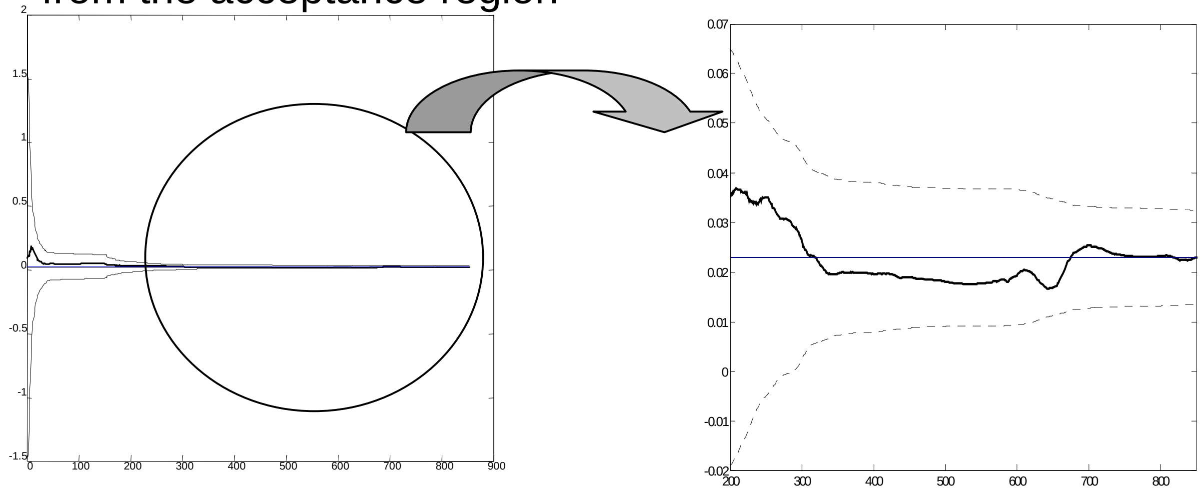
Inverse Depth Estimation History

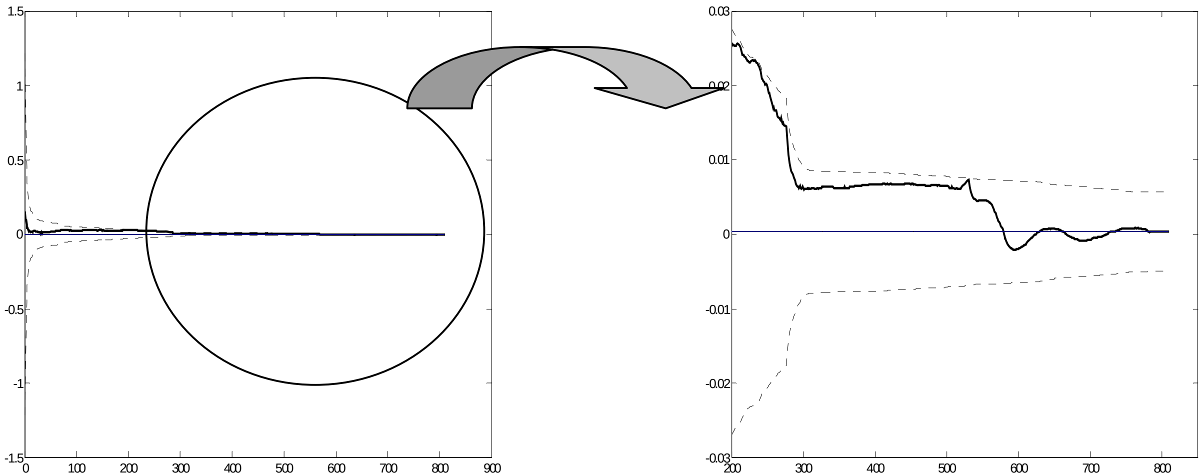




near feature (3), eventually excludes infinite
from the acceptance region



distant feature (11), infinite
always included in the acceptance region



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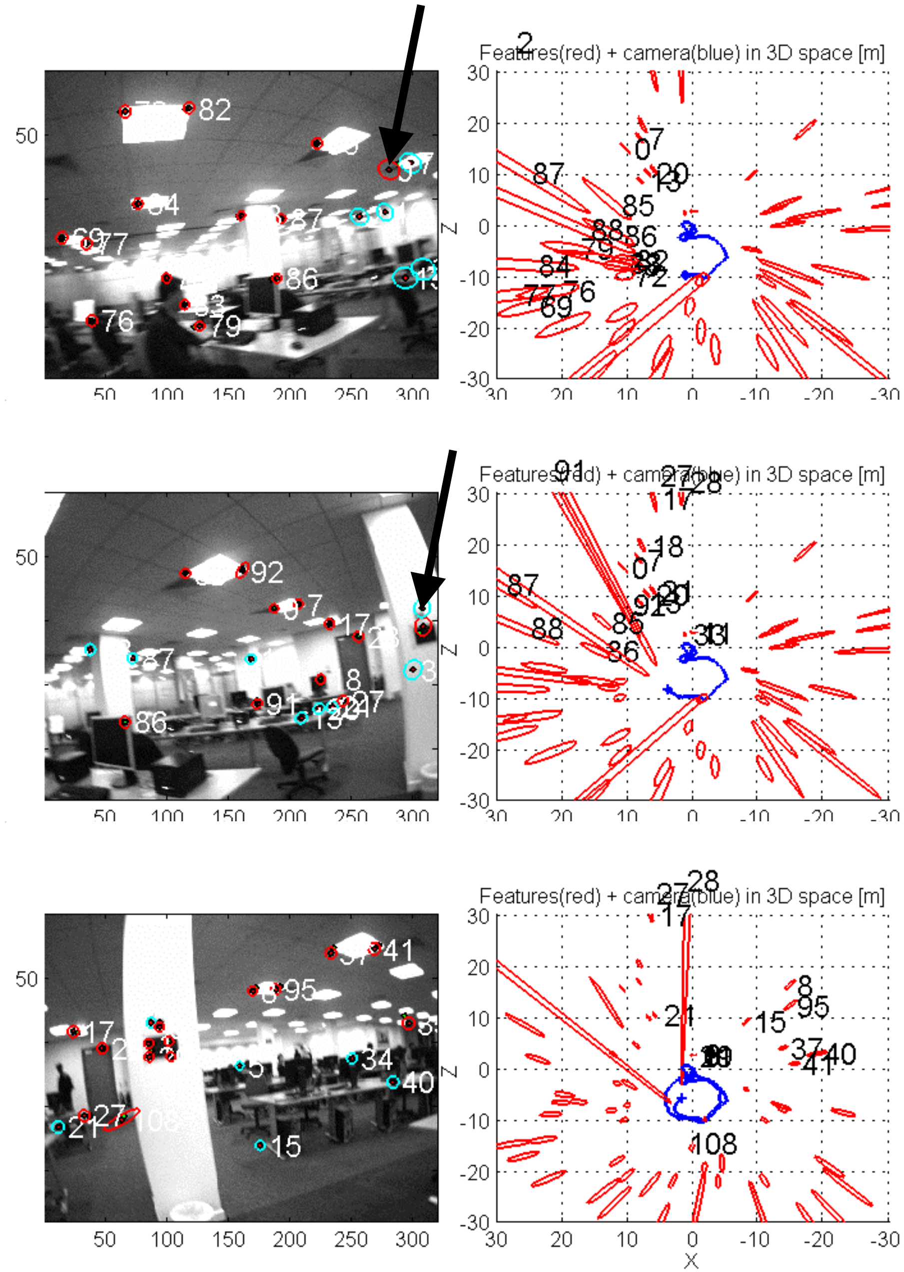
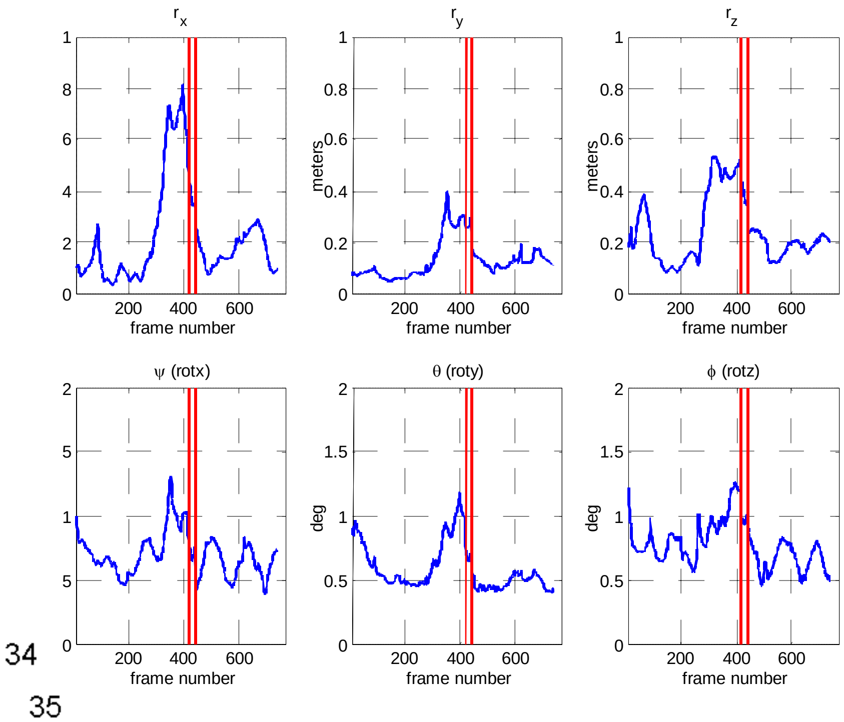
Loop Closing





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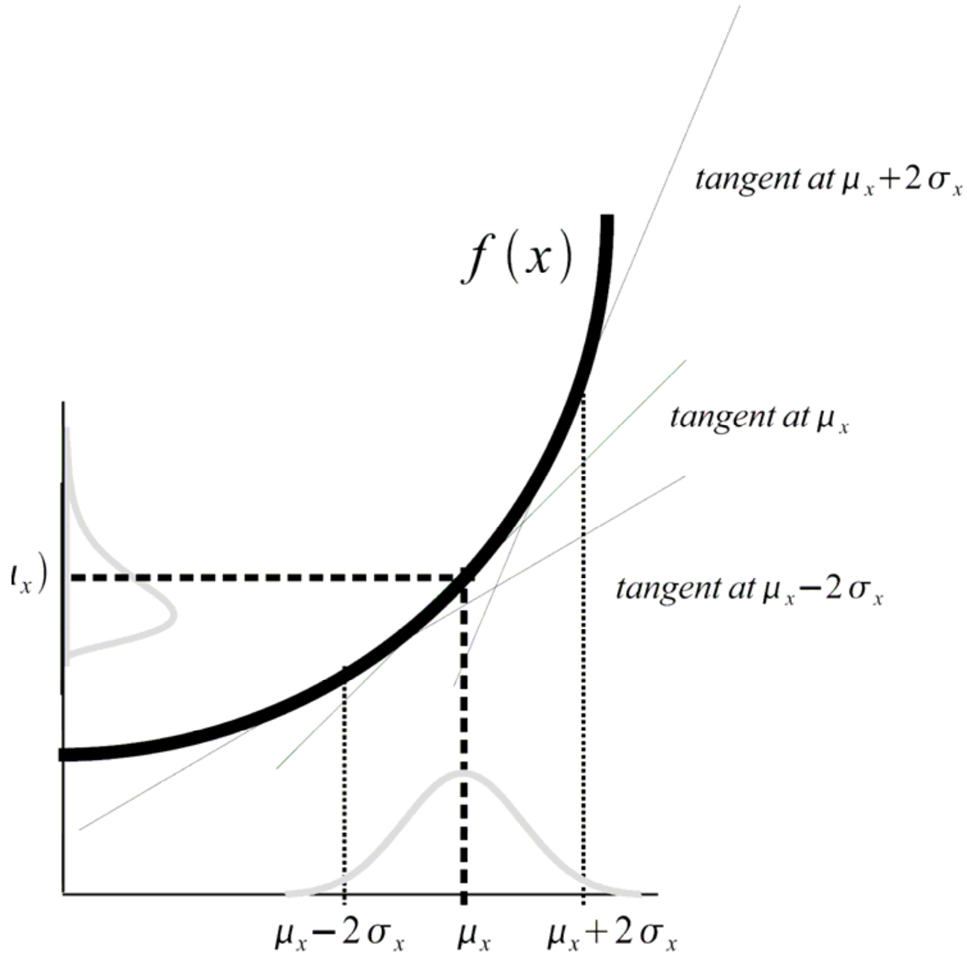
Linearity Index



$$L = \left| \frac{\left.\frac{\partial^2 f}{\partial x^2}\right|_{\mu_x} 2\sigma_x}{\left.\frac{\partial f}{\partial x}\right|_{\mu_x}} \right|$$

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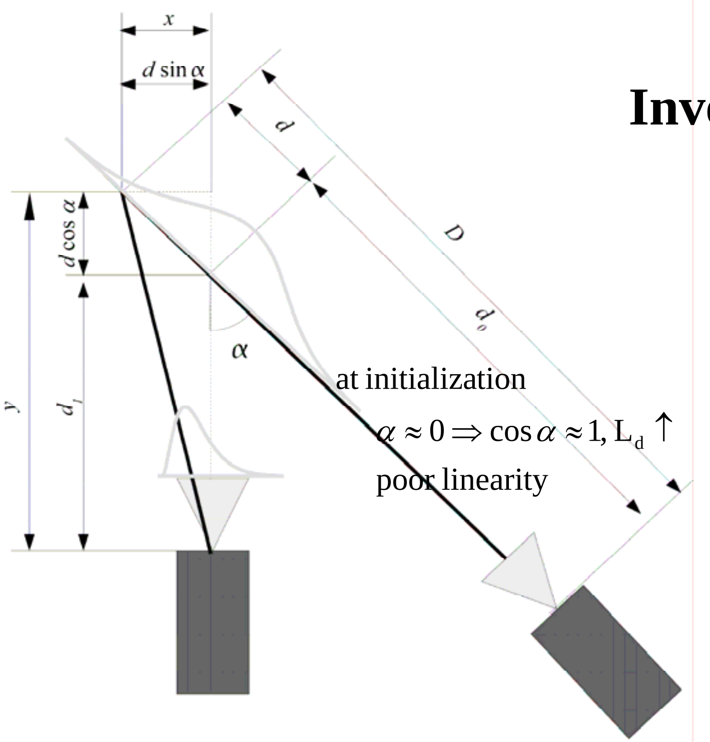
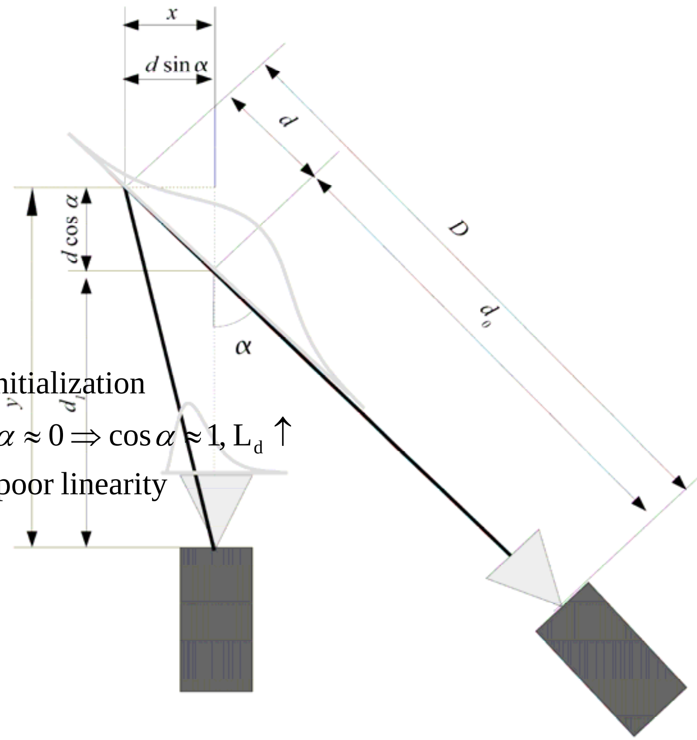
Inverse depth linearity analysis





$$L_d = \frac{4\sigma_d}{d_1} |cos\alpha|$$

at initialization

$\alpha \approx 0 \Rightarrow \cos\alpha \approx 1, \mathrm{L_d} \uparrow$

poor linearity

$$L_\rho = \frac{4\sigma_\rho}{\rho_0} \left|1 - \frac{d_0}{d_1} cos\alpha\right|$$

at initialization

$\alpha \approx 0 \Rightarrow 1 - \cos\alpha \approx 0, \mathrm{L}_\rho \downarrow$

linearity

after parallax gathering,

$1 - \cos\alpha$, but σ_ρ

linearity

good performance along the whole estimation

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Inverse depth to XYZ conversion

- Inverse depth good performance along the whole estimation process
- Inverse depth coding needs 6 parameters
- XYZ coding good performance for reduced depth uncertainty
- So, it is not mandatory to switch from inverse depth to XYZ, but computing overhead can be reduce
- Switching criteria based on the linearity index

$$L_d = \frac{4\sigma_d}{d_1} \left|\cos\alpha\right| < 10\%$$

$$\begin{aligned} d_i &= \left\| \mathbf{h}^C \right\|, \quad \mathbf{h}^C = \mathbf{x}_i - \mathbf{r}^{WC} \\ \sigma_d &= \frac{\sigma_\rho}{\rho_i^2}, \quad \sigma_\rho = \sqrt{\mathbf{P}_{\mathbf{y}_i \mathbf{y}_i}(6,6)} \\ \cos\alpha &= \mathbf{m}^\top \mathbf{h}^C \left\| \mathbf{h}^C \right\|^{-1}. \end{aligned}$$

$$\mathbf{x}_i = \begin{pmatrix} X_i \\ Y_i \\ Z_i \end{pmatrix} = \begin{pmatrix} x_i \\ y_i \\ z_i \end{pmatrix} + \frac{1}{\rho_i}\mathbf{m}(\theta_i, \phi_i) \qquad \mathbf{P}_{\text{new}} = \mathbf{J}\mathbf{P}\mathbf{J}^\top, \quad \mathbf{J} = \text{diag}\left(\mathbf{I}, \frac{\partial \mathbf{x}_i}{\partial \mathbf{y}_i}, \mathbf{I}\right)$$

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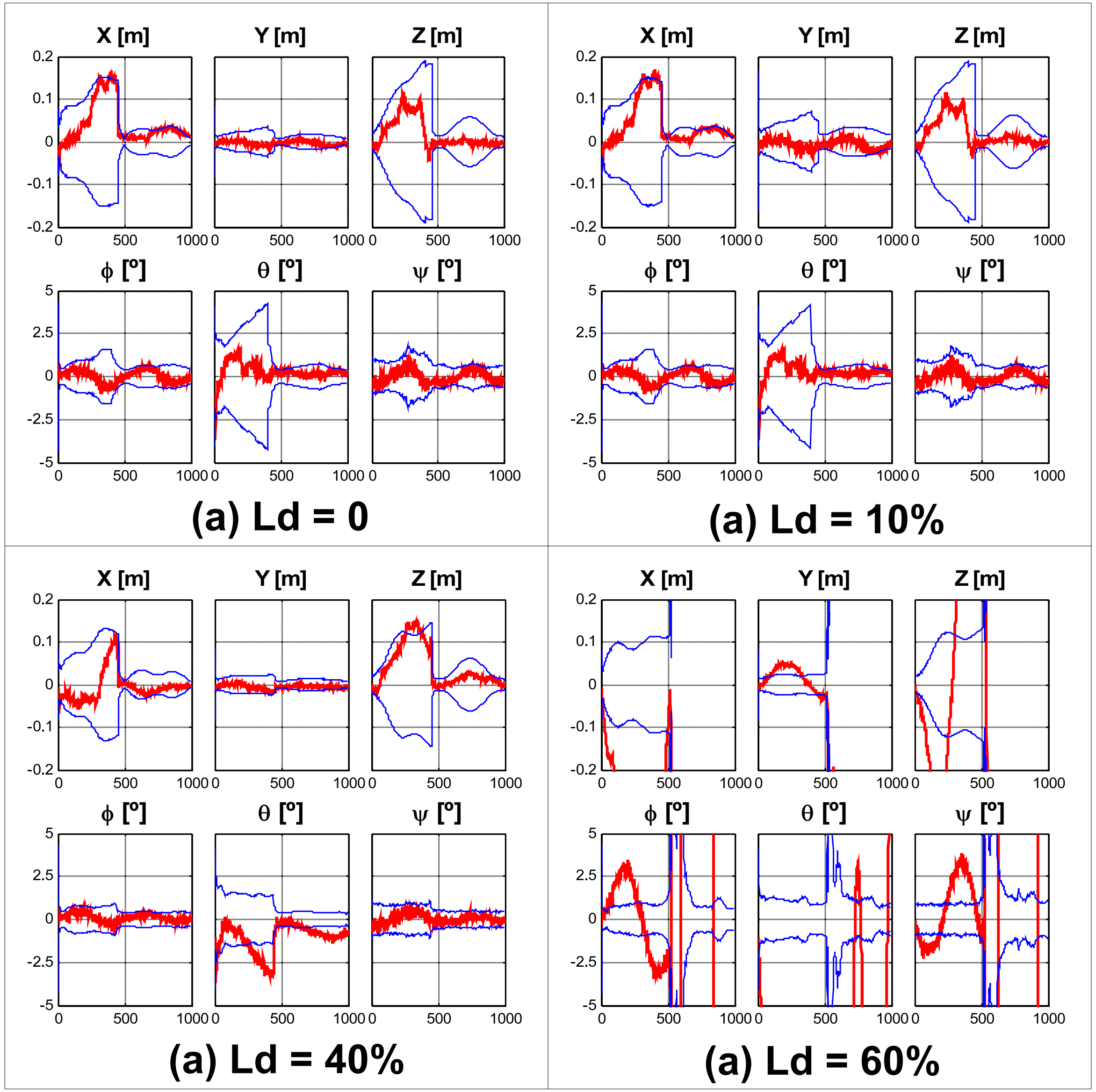
Inverse depth to XYZ conversion threshold



Camera location error   95% acceptance error

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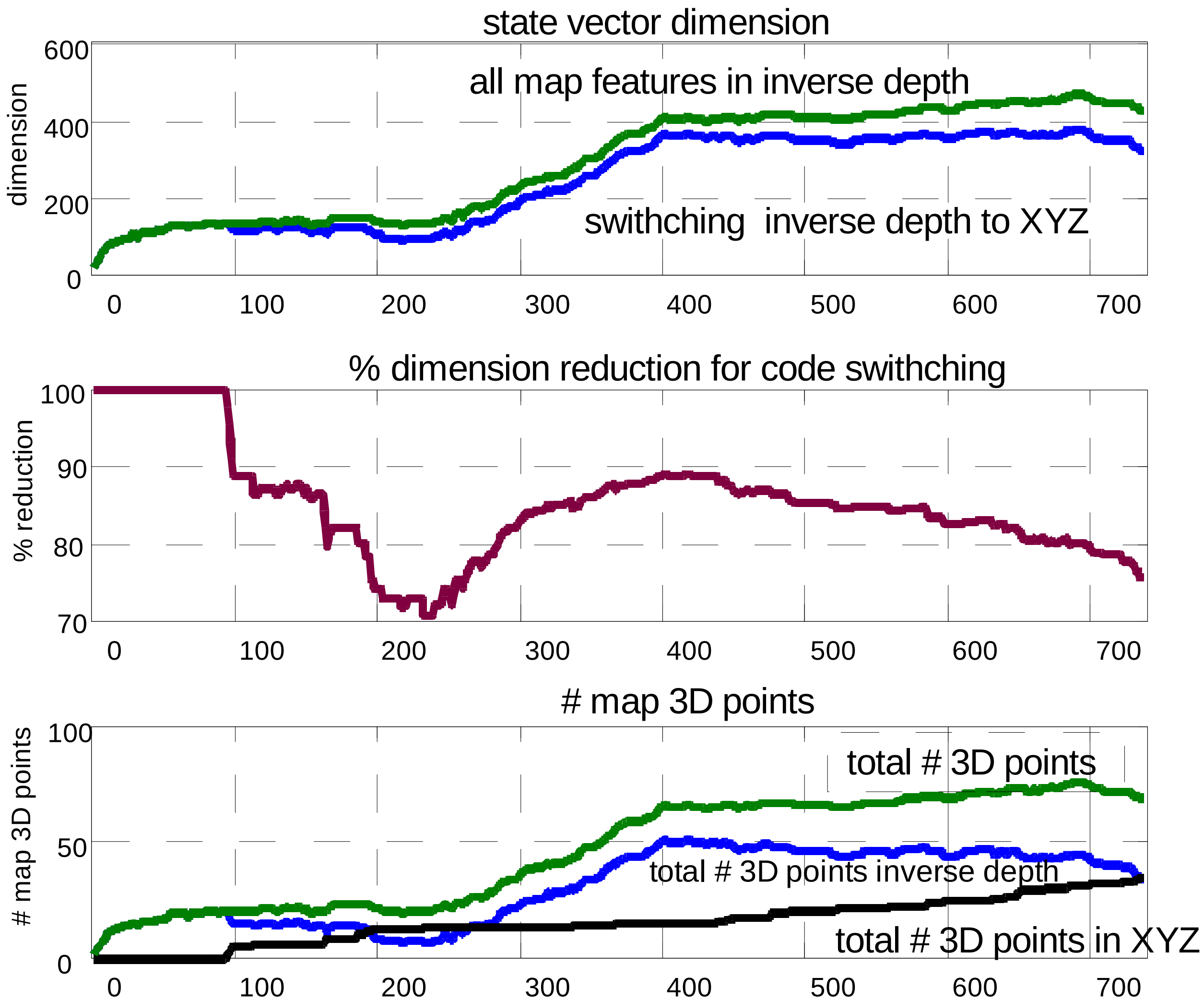
Switching evolution



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Switching evolution

☆ *Inverse depth coding*

△ *Depth coding*

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