The EKF Solution to

Simultaneous Localization and Mapping

José Neira
Universidad de Zaragoza

Outline

1. Basic EKF SLAM
   1. **Introduction: the need for SLAM**
   2. The basic EKF SLAM algorithm
   3. Feature extraction

2. The Data Association Problem
   1. Introduction
   2. Continuous Data Association
   3. The Loop Closing Problem
   4. The Global Localization Problem

3. Advanced EKF SLAM
   1. Computational complexity of EKF SLAM
   2. Consistency of the EKF SLAM
   3. SLAM using local maps
      1. Sequential Map Joining
      2. Divide and Conquer SLAM
      3. Hierarchical SLAM

Simultaneous Localization and Mapping

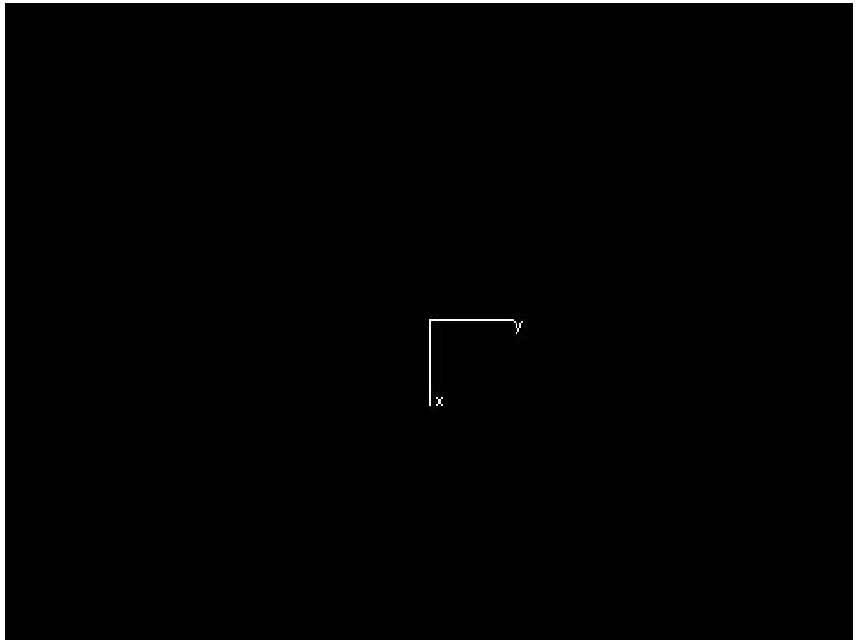
Is it possible to use a vehicle, starting at an

- **unknown initial location**, in an
- **unknown environment**, to
- **incrementally**

build a map of the environment,

- and **at the same time**

use the map to determine the vehicle location?



(image: Paul Newman)

Chicken and egg problem?

Simultaneous Localization and Mapping

The solution to the SLAM problem is, in many respects, a ‘Holy Grail’ of the autonomous vehicle research community, as the ability to build a map and navigate simultaneously would indeed make a robot ‘autonomous’.

(Newman 1999, Leonard 2000, Thrun 2001)

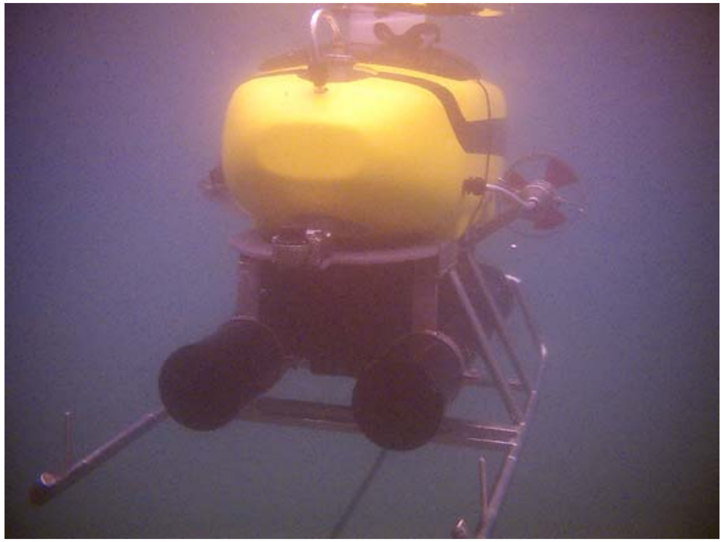
- There is a large amount of potential **applications**
- It gives the vehicle real **autonomy**
- A solution is indeed **possible**

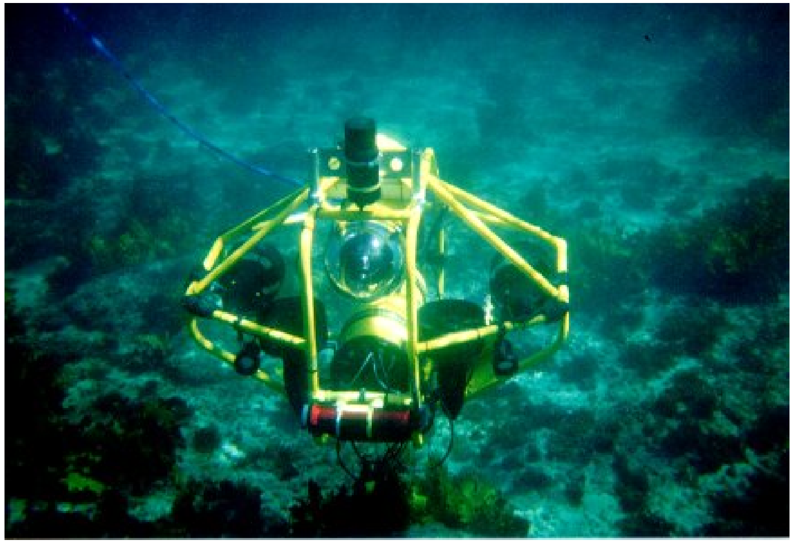
From Wikipedia:

- **Simultaneous localization and mapping (SLAM)** is a technique used by **robots** and **autonomous vehicles** to build up a map within an unknown environment while at the same time keeping track of its current position.
- **This is not as straightforward as it might sound** due to inherent uncertainties in discerning the robot's relative movement from its various **sensors**.
- ... Some of the statistical techniques used in SLAM include **Kalman filters**, **particle filters** and scan matching of range data.
- Pioneering work in this field was conducted by **Hugh F. Durrant-Whyte.**
- Much of the SLAM work is based on concepts imported from **computer vision**...
- **SLAM has not yet been fully perfected**, but it is starting to be employed in unmanned aerial vehicles, autonomous underwater vehicles, planetary rovers and newly emerging domestic robots.

Mobile Robots

Mobile Robots





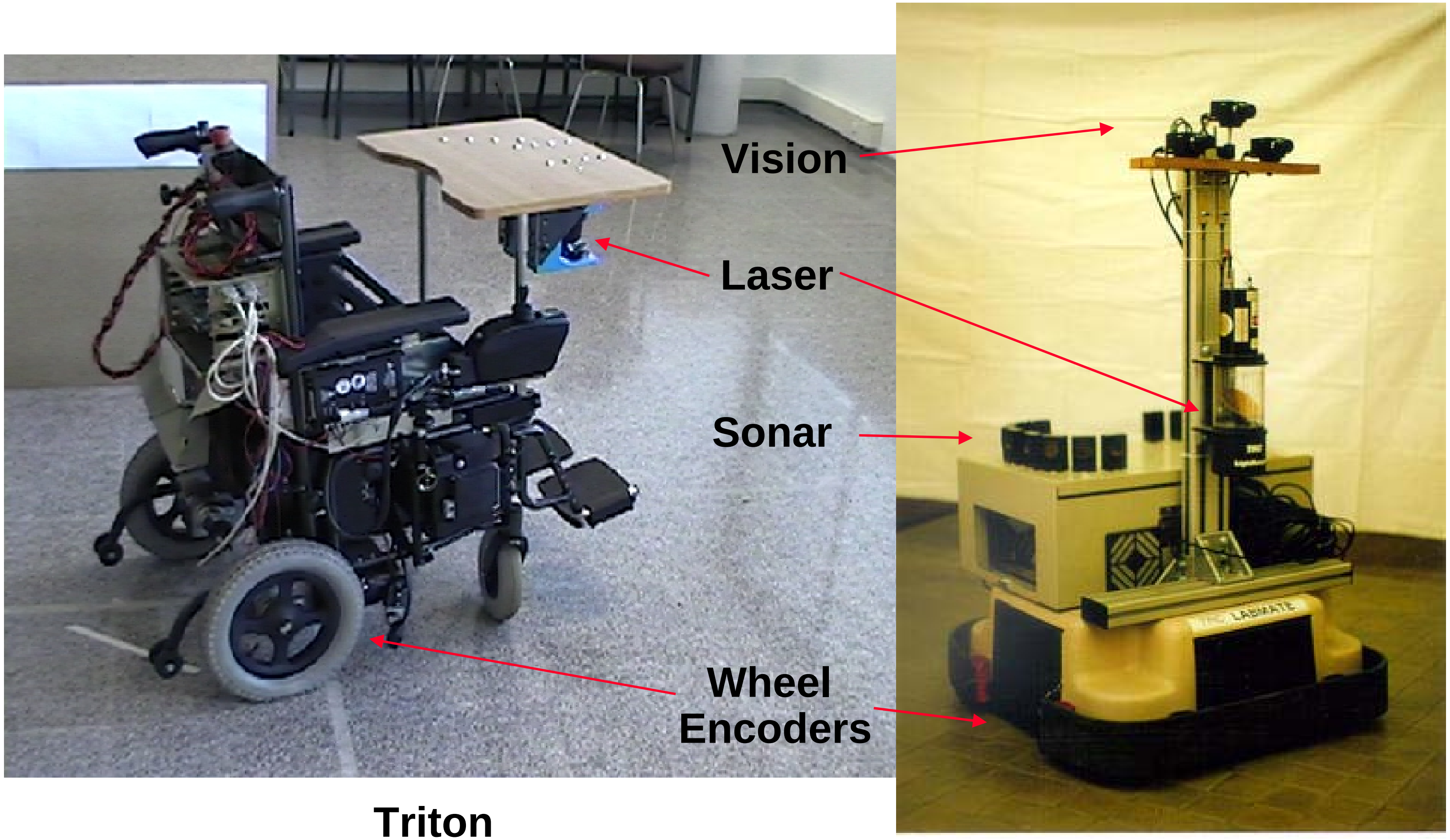
Mobile Robots

ROBOTICS
UNIVERSIDAD DE ZARAGOZA

Mobile Sensors

ROBOTICS
UNIVERSIDAD DE ZARAGOZA

Origins: the localization problem

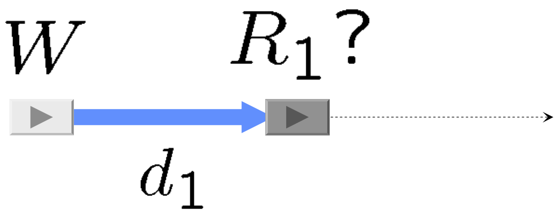


Triton

Otilio

A Simple Example: MonoRob

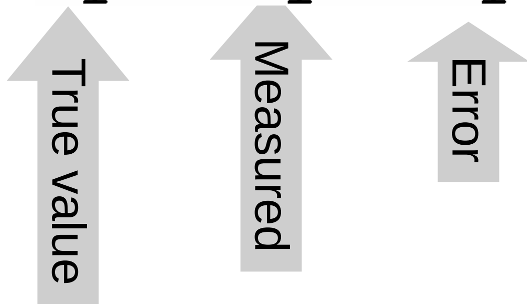
- A robot moves in a mono-dimensional space:



Uncertainty in Robot Position

- Odometry:

$$d_1 = \hat{d}_1 + \tilde{d}_1$$



- Uncertainty model:

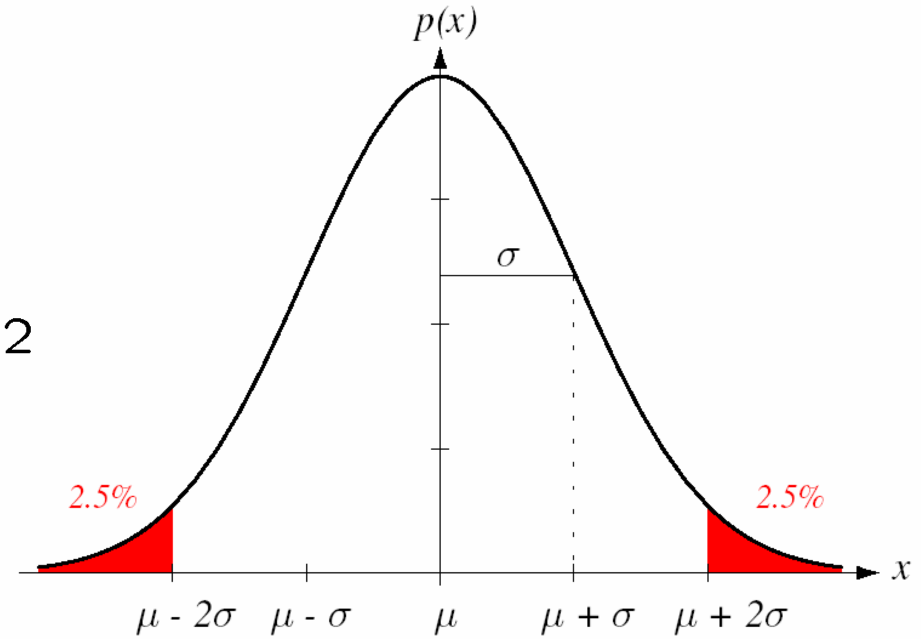
$$\begin{aligned} E\left(\tilde{d}_1\right) &= E\left(\left[d_1 - \hat{d}_1\right]\right) = \mu_1 \\ \mathtt{Var}(\tilde{d}_1) &= E\left(\left[d_1 - \hat{d}_1\right]\left[d_1 - \hat{d}_1\right]^T\right) = \sigma_1^2 \end{aligned}$$

A Simple Example

- Gaussianity assumption:

$$\tilde{x} \sim N(\mu, \sigma^2)$$

$$p(\tilde{x}) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\left(\frac{\tilde{x}-\mu}{\sigma}\right)^2}$$



(image: Duda, Hart, Stork)

$$Pr\{| \tilde{x} - \mu | \leq \sigma\} \simeq 0.68$$
$$Pr\{| \tilde{x} - \mu | \leq 2\sigma\} \simeq 0.95$$
$$Pr\{| \tilde{x} - \mu | \leq 3\sigma\} \simeq 0.997$$

Robot Odometry: Example

- Odometry error model:

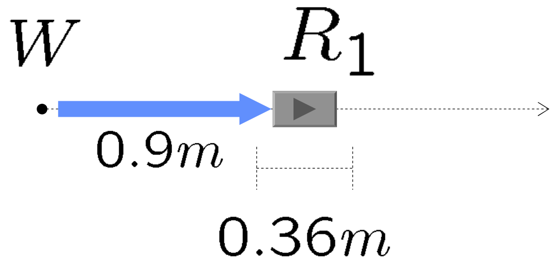
$$d_1 = \hat{d}_1 + \tilde{d}_1$$

$$\tilde{d}_1 \sim N(\mu_1, \sigma_1^2)$$

$$\mu_1 = 0$$

$$\sigma_1 = 0.1 \cdot \hat{d}_1$$

- Example: move 0.9m



$$\hat{x}_{WR_1} = \hat{d}_1$$

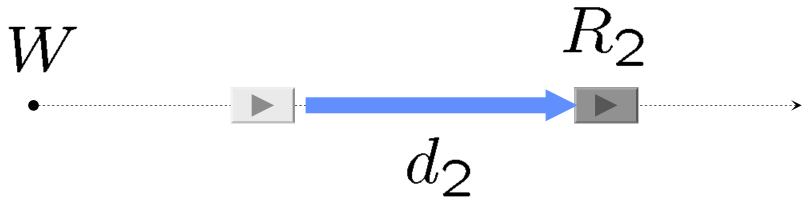
$$= 0.9m$$

$$\sigma_{x_{WR_1}} = 0.09m$$

$$Pr\left\{ | \tilde{d}_1 | \leq 0.18m \right\} \simeq 0.95$$

Robot Odometry: Example

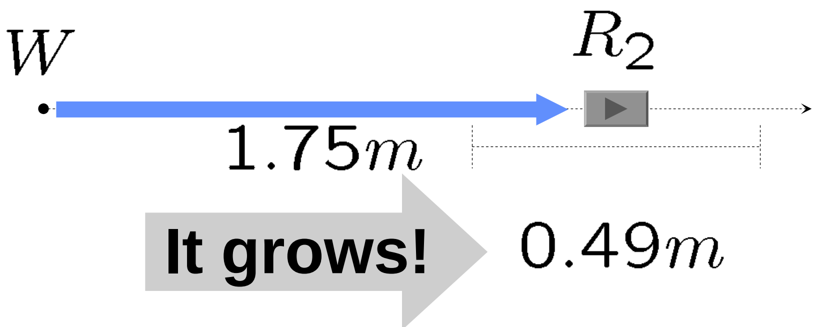
- Robot moves again 0.85m: $\hat{d}_2 = 0.85m$



$$x_{WR_2} = \hat{d}_1 + \tilde{d}_1 + \hat{d}_2 + \tilde{d}_2$$

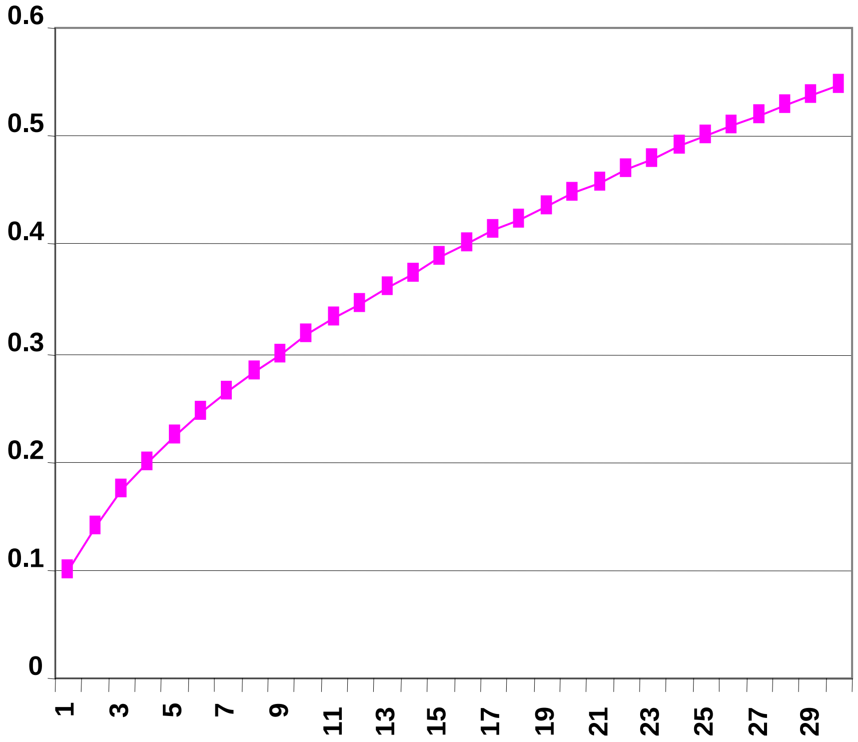
$$\begin{aligned} E(\tilde{d}_1 + \tilde{d}_2) &= \mu_1 + \mu_2 = 0 \\ \mathtt{Var}(\tilde{d}_1 + \tilde{d}_2) &= \sigma_1^2 + \sigma_2^2 \\ &= 0.1^2 \left(\hat{d}_1^2 + \hat{d}_2^2\right) \end{aligned}$$

- New estimation:



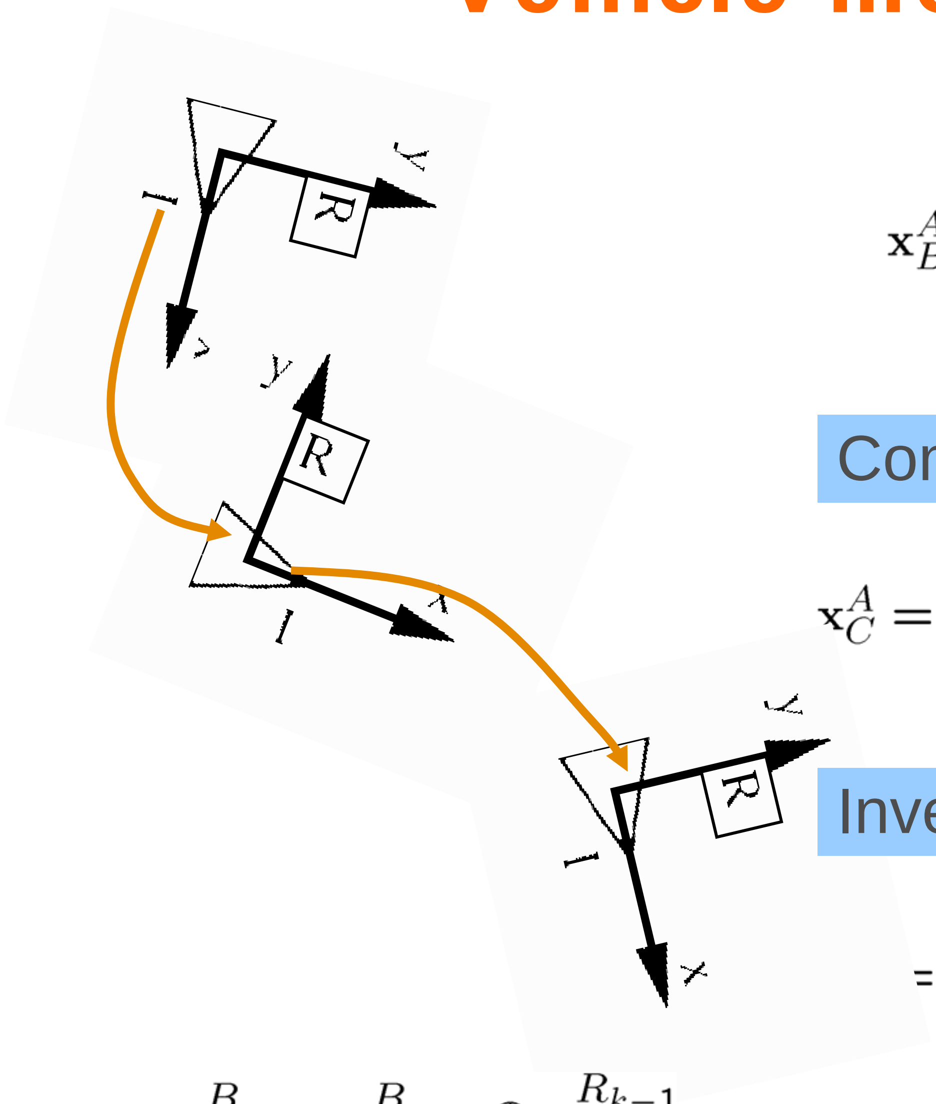
$$\begin{aligned} \hat{x}_{WR_2} &= 1.75m \\ \sigma_{x_{WR_2}} &\simeq 0.12m \end{aligned}$$

Odometry



Odometry error grows unbounded
(with the square root of n)

Vehicle motion in 2D



$$\mathbf{x}_B^A = \begin{bmatrix} x_1 \\ y_1 \\ \phi_1 \end{bmatrix} \qquad \mathbf{x}_C^B = \begin{bmatrix} x_2 \\ y_2 \\ \phi_2 \end{bmatrix}$$

Composition:

$$\mathbf{x}_C^A = \mathbf{x}_B^A \oplus \mathbf{x}_C^B = \begin{bmatrix} x_1 + x_2 \cos\phi_1 - y_2 \sin\phi_1 \\ y_1 + x_2 \sin\phi_1 + y_2 \cos\phi_1 \\ \phi_1 + \phi_2 \end{bmatrix}$$

Inversion:

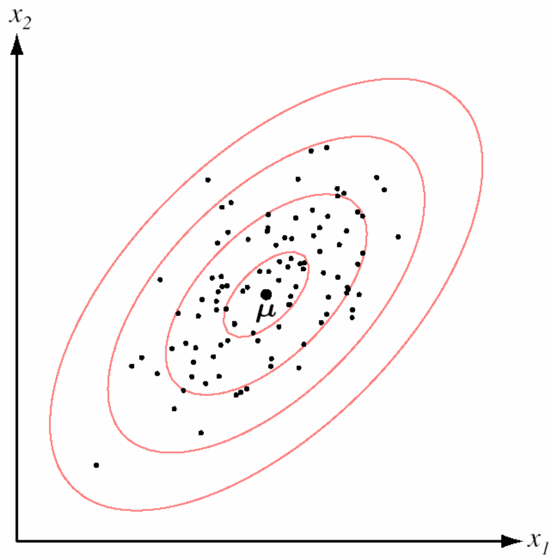
$$= \ominus \mathbf{x}_B^A = \begin{bmatrix} -x_1 \cos\phi_1 - y_1 \sin\phi_1 \\ x_1 \sin\phi_1 - y_1 \cos\phi_1 \\ -\phi_1 \end{bmatrix}$$

$$\mathbf{x}_{R_k}^B = \mathbf{x}_{R_{k-1}}^B \oplus \mathbf{x}_{R_k}^{R_{k-1}}$$

Odometry in 2D

Odometry model:

$$
\begin{aligned}
\mathbf{x}_{R_k}^{R_{k-1}} &= \hat{\mathbf{x}}_{R_k}^{R_{k-1}} + \mathbf{v}_k \\
E[\mathbf{v}_k] &= \mathbf{0} \\
E[\mathbf{v}_k \mathbf{v}_j^T] &= \delta_{kj} \mathbf{Q}_k
\end{aligned}
$$



Composition:

$$
\begin{aligned}
\hat{\mathbf{x}}_{R_k}^{B} &= \hat{\mathbf{x}}_{R_{k-1}}^{B} \oplus \hat{\mathbf{x}}_{R_k}^{R_{k-1}} \\
\mathbf{P}_{R_k} &\simeq J_1 \mathbf{P}_{R_{k-1}} J_1^T + J_2 \mathbf{Q}_k J_2^T
\end{aligned}
$$

$$
J_1 = \left. \frac{\partial \left(\mathbf{x}_{R_{k-1}}^{B} \oplus \mathbf{x}_{R_k}^{R_{k-1}} \right)}{\partial \mathbf{x}_{R_{k-1}}^{B}} \right|_{(\hat{\mathbf{x}}_{R_{k-1}}^{B}, \hat{\mathbf{x}}_{R_k}^{R_{k-1}})}
$$

$$
J_2 = \left. \frac{\partial \left(\mathbf{x}_{R_{k-1}}^{B} \oplus \mathbf{x}_{R_k}^{R_{k-1}} \right)}{\partial \mathbf{x}_{R_k}^{R_{k-1}}} \right|_{(\hat{\mathbf{x}}_{R_{k-1}}^{B}, \hat{\mathbf{x}}_{R_k}^{R_{k-1}})}
$$

Odometry in 2D

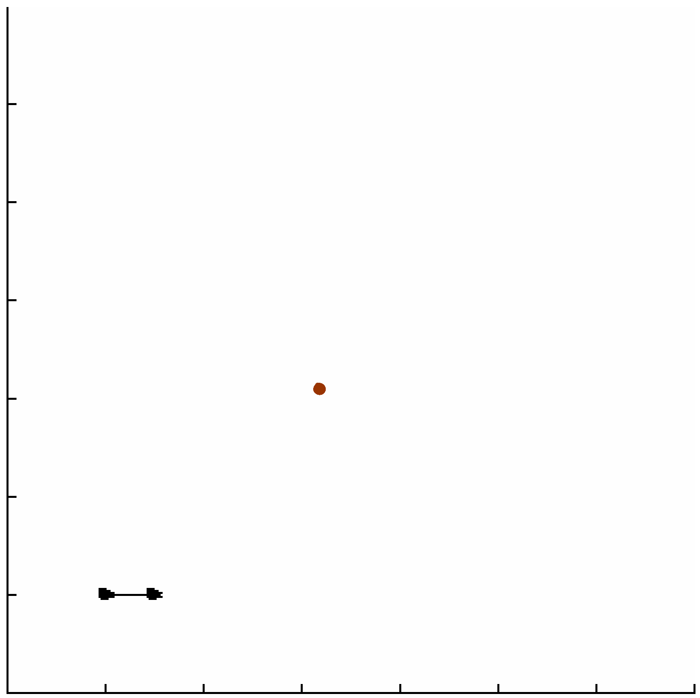
Jacobians:

$$J_{1\oplus}\{\mathbf{x}_B^A, \mathbf{x}_C^B\} = \left.\frac{\partial\left(\mathbf{x}_B^A \oplus \mathbf{x}_C^B\right)}{\partial \mathbf{x}_B^A}\right|_{(\hat{\mathbf{x}}_B^A, \hat{\mathbf{x}}_C^B)} = \begin{bmatrix} 1 & 0 & -x_2 \sin\phi_1 - y_2 \cos\phi_1 \\ 0 & 1 & x_2 \cos\phi_1 - y_2 \sin\phi_1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$J_{2\oplus}\{\mathbf{x}_B^A, \mathbf{x}_C^B\} = \left.\frac{\partial\left(\mathbf{x}_B^A \oplus \mathbf{x}_C^B\right)}{\partial \mathbf{x}_C^B}\right|_{(\hat{\mathbf{x}}_B^A, \hat{\mathbf{x}}_C^B)} = \begin{bmatrix} \cos\phi_1 & -\sin\phi_1 & 0 \\ \sin\phi_1 & \cos\phi_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

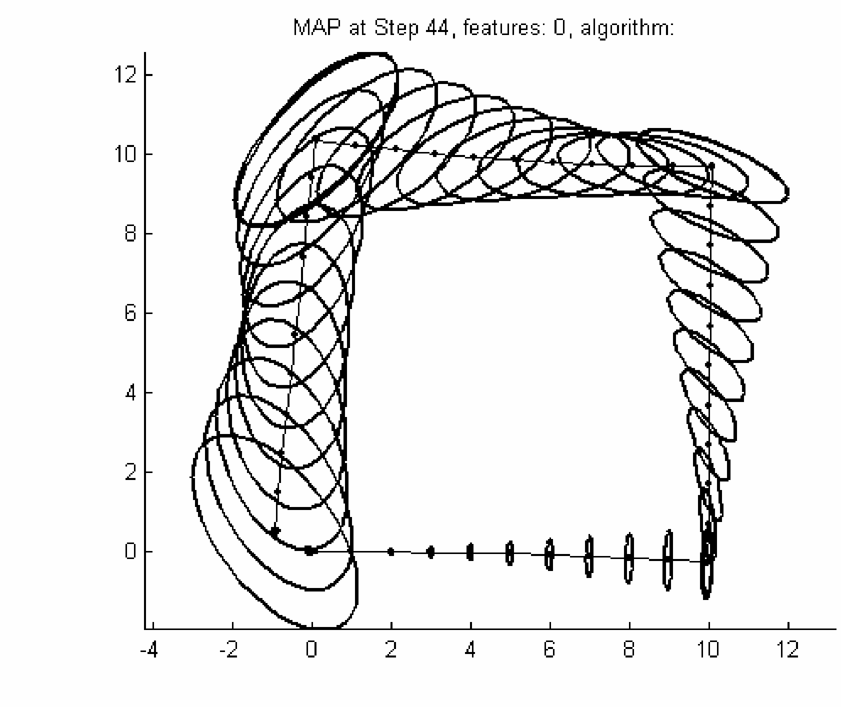
$$J_{\ominus}\{\mathbf{x}_B^A\} = \left.\frac{\partial\left(\ominus \mathbf{x}_B^A\right)}{\partial \mathbf{x}_B^A}\right|_{(\hat{\mathbf{x}}_B^A)} = \begin{bmatrix} -\cos\phi_1 & -\sin\phi_1 & -x_1 \sin\phi_1 - y_1 \cos\phi_1 \\ \sin\phi_1 & -\cos\phi_1 & x_1 \cos\phi_1 + y_1 \sin\phi_1 \\ 0 & 0 & -1 \end{bmatrix}$$

Odometry in 2D

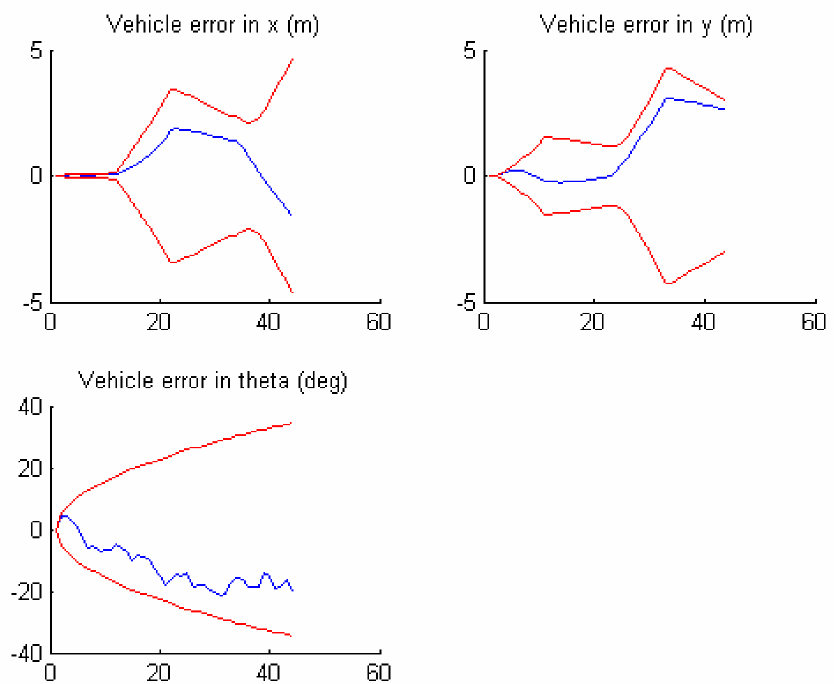


How can we avoid drift?

The need for SLAM



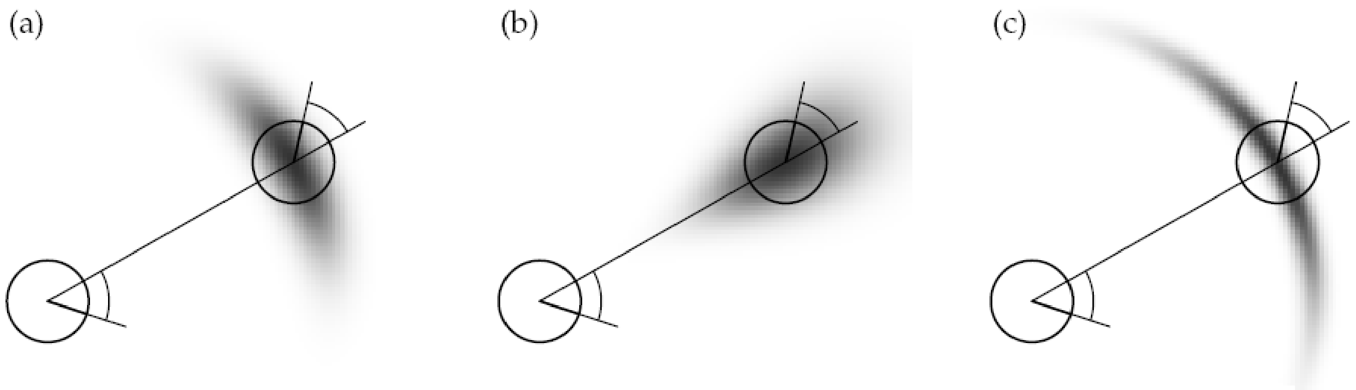
Error $\pm 2\sigma$ (prob. 0.95)



Odometry in 2D

$$\mathbf{x}_{R_k}^{R_{k-1}} = (0, 0, \phi_1)^t \oplus (x_1, 0, 0)^t \oplus (0, 0, \phi_2)^t$$

For different rotation and translational errors

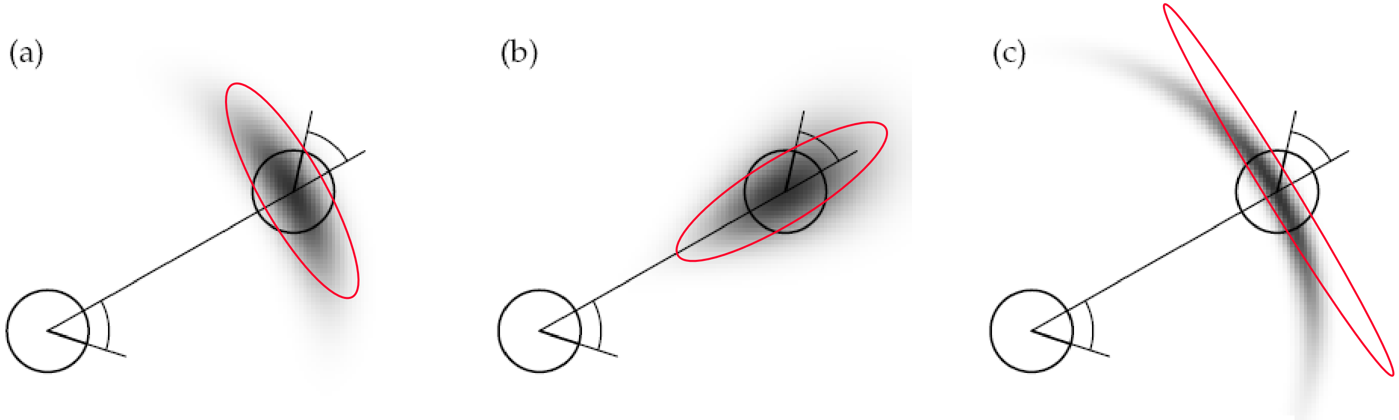


(image: Thrun, Burgard, Fox)

Odometry in 2D

$$\mathbf{x}_{R_k}^{R_{k-1}} = (0, 0, \phi_1)^t \oplus (x_1, 0, 0)^t \oplus (0, 0, \phi_2)^t$$

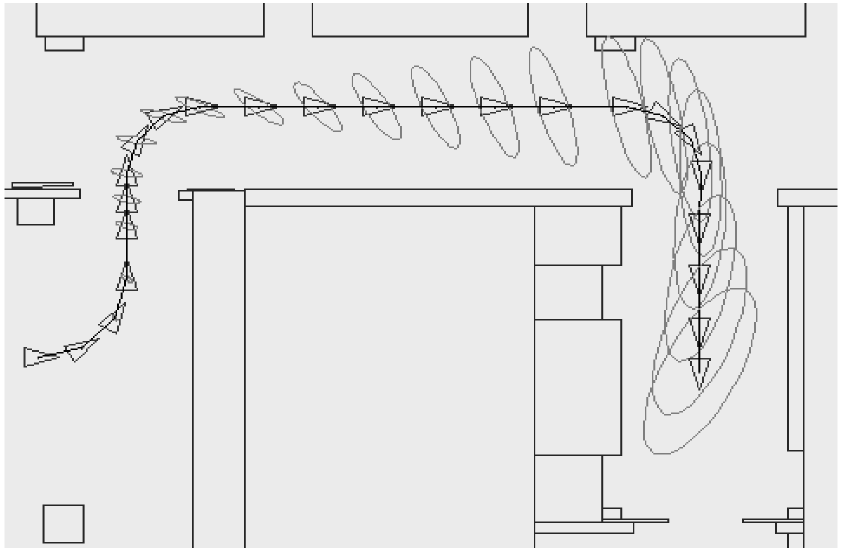
We are linearizing errors!



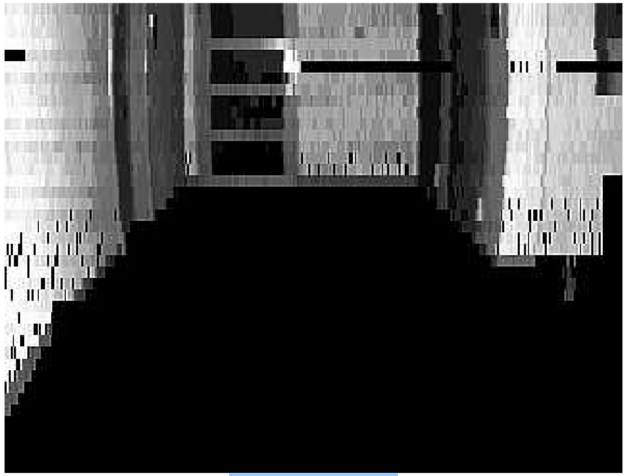
(image: Thrun, Burgard, Fox)

Map-based Localization

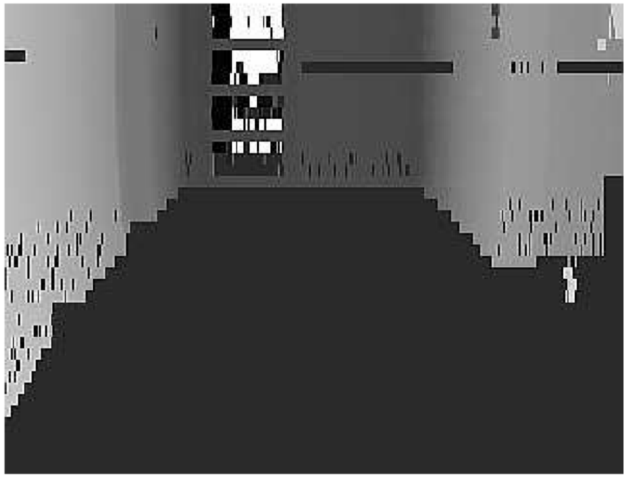
- Cumulative odometry errors



- A priori Map + Perception



Vision



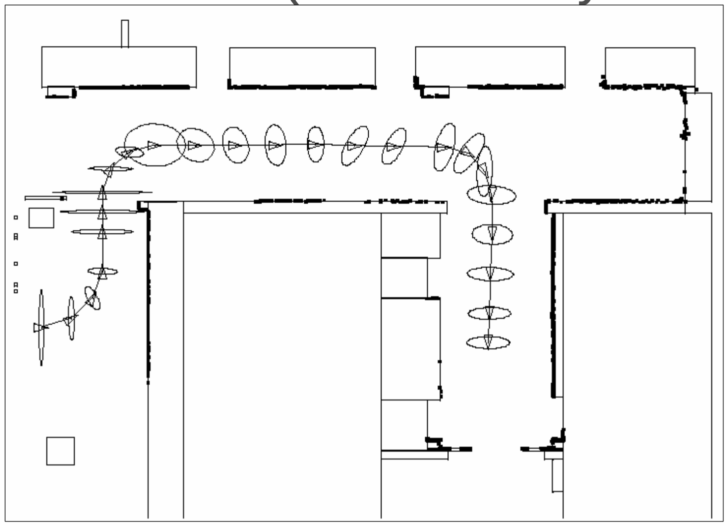
Laser

J. Neira, J.D. Tardós, J. Horn and G. Schmidt: **Fusing Range and Intensity Images for Mobile Robot Localization**, IEEE Trans. Robotics and Automation, Vol. 15, No. 1,Feb 1999, pp 76-84.

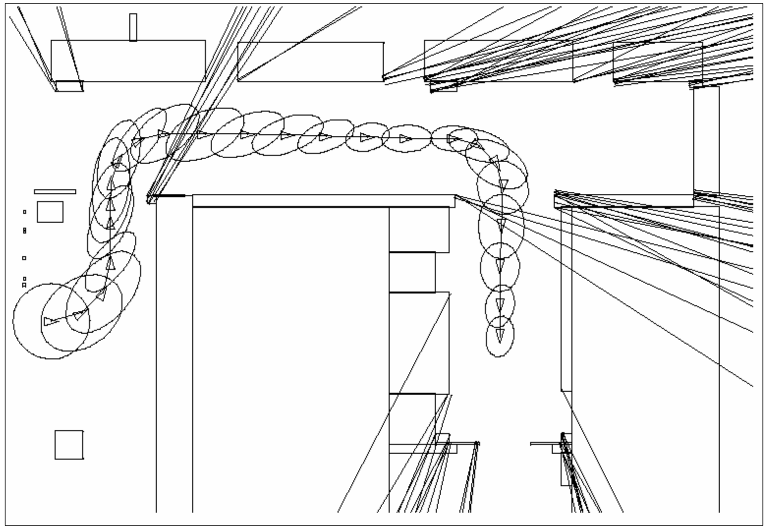
ROBOTICS UNIVERSIDAD DE ZARAGOZA

Multisensor Robot Localization

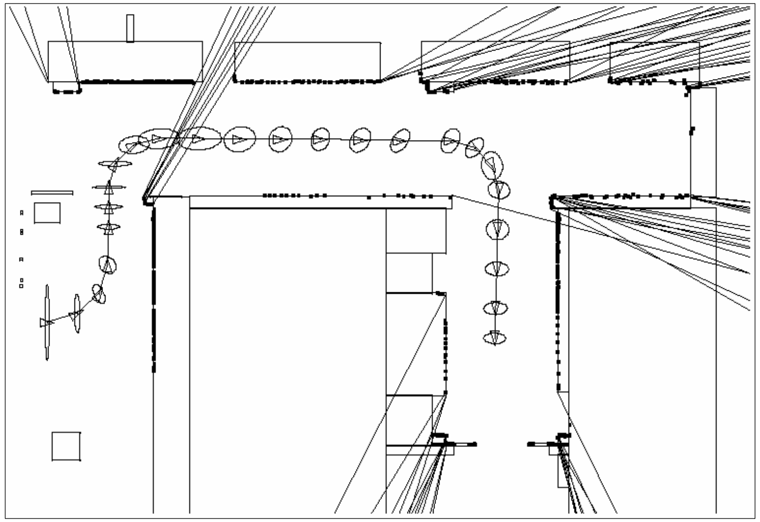
- Results (uncertainty x30)



Laser



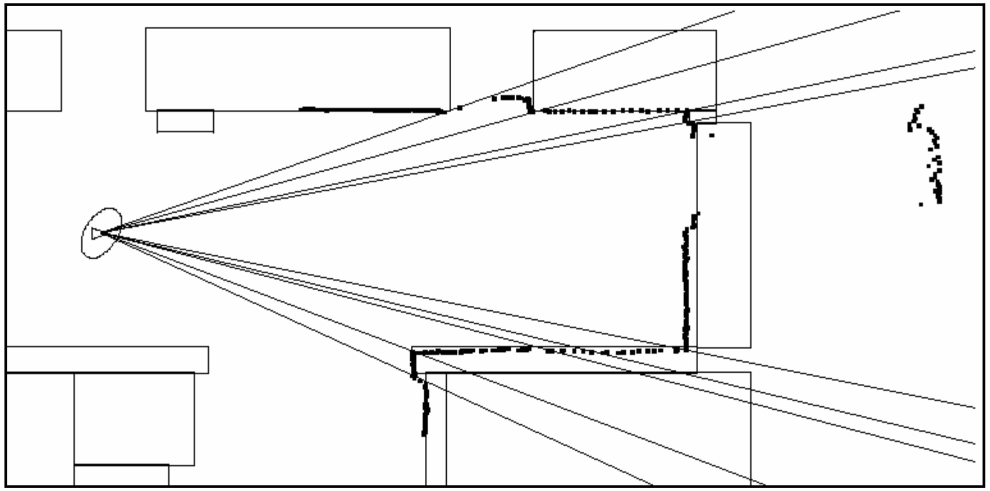
Vision



Laser + vision

The need for SLAM

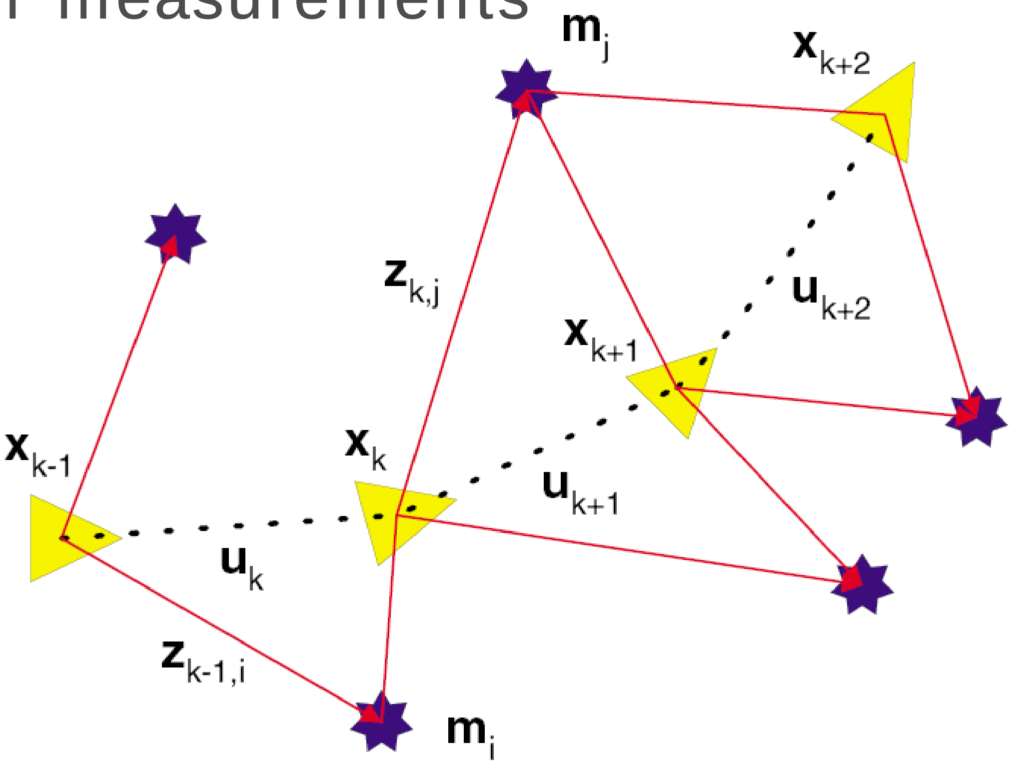
- In many applications the environment is unknown
- A priori maps usually are:
  - Costly to obtain
  - Inaccurate
  - Incomplete
  - Out of date



➔ Map Building

Localization and Mapping elements

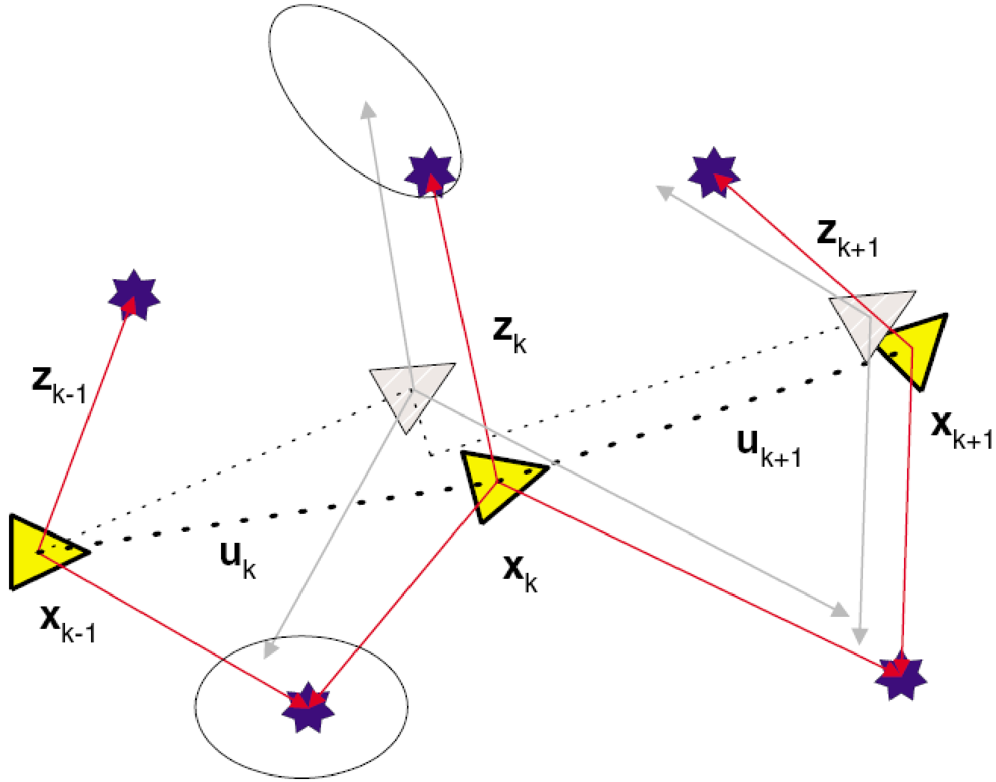
- M: environment features
- U: control inputs
- X: vehicle locations
- Z: sensor measurements



(image: Durrant-Whyte)

Map-based localization

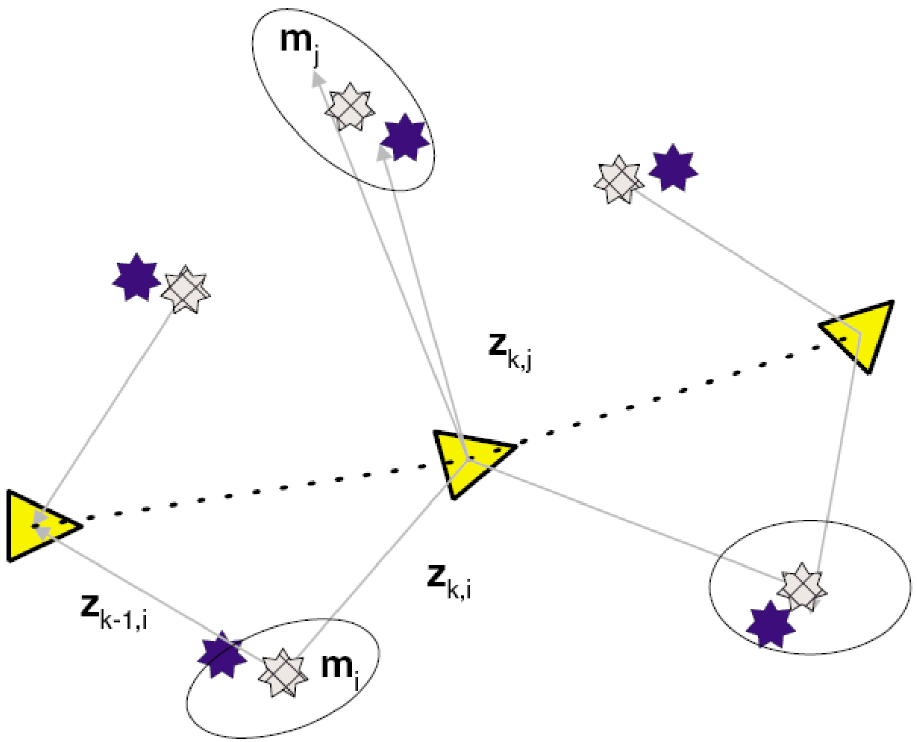
- Given M, U, Z
- Compute X



(image: Durrant-Whyte)

Mapping

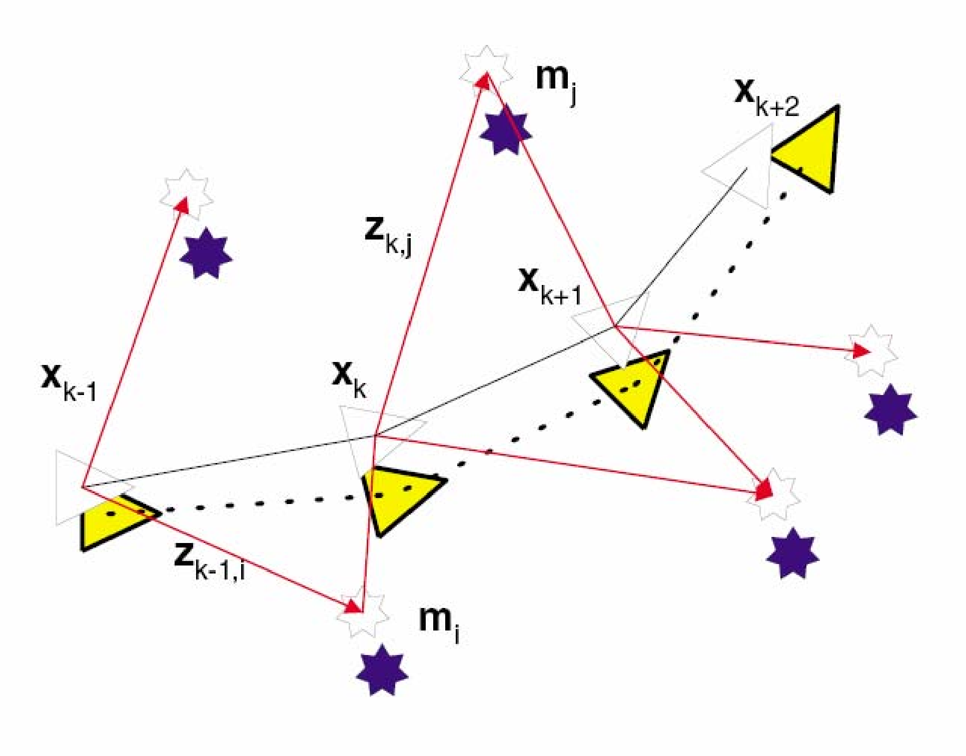
- Given X, Z
- Compute M



(image: Durrant-Whyte)

SLAM

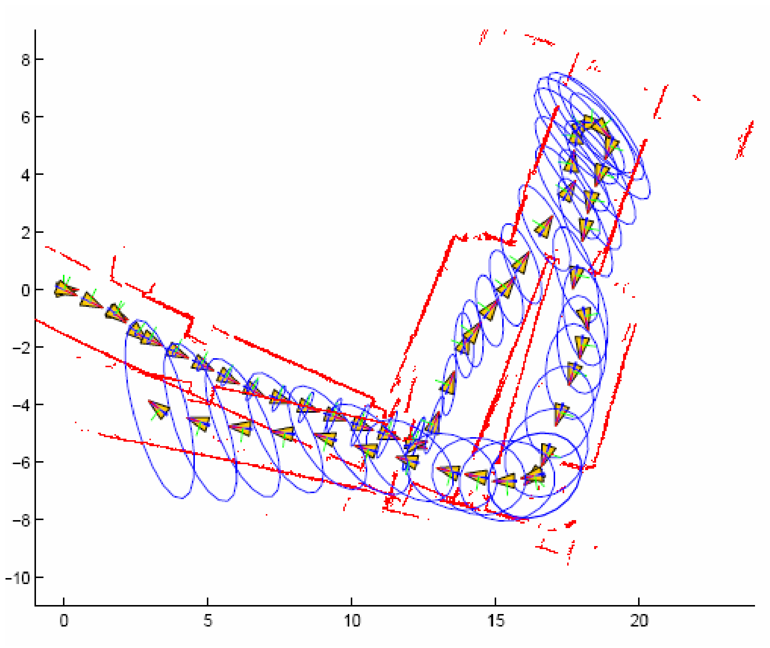
- Given U, Z
- Compute X, M

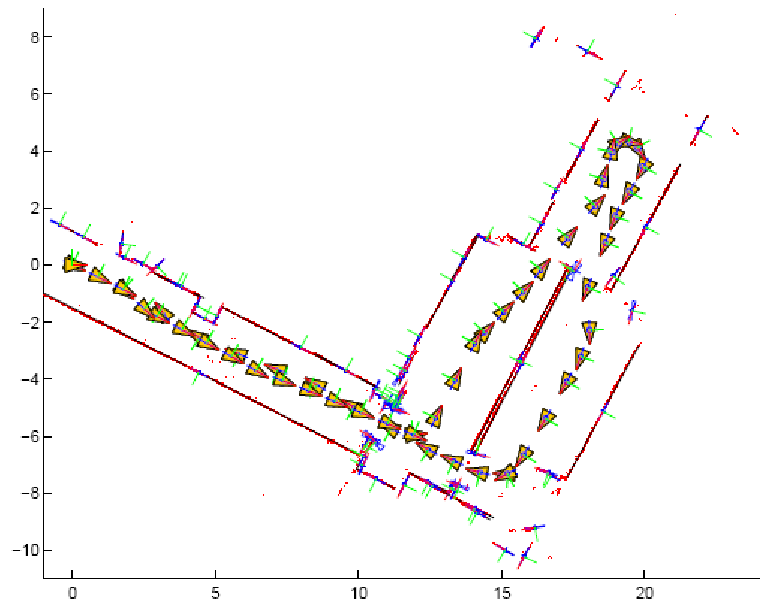


(image: Durrant-Whyte)

The need for SLAM

- Stochastic map + Extended Kalman Filter:





- First proposed by:

R. Smith and P. Cheeseman, “On the Representation and Estimation of Spatial Uncertainty”, Int. J. Robotics Research 5(4), pp. 56-68, 1986.

R. Smith, M. Self and P. Cheeseman, “A Stochastic Map for Uncertain Spatial Relationships”, In O. Faugeras and G. Giralt (eds.), Robotics Research, The Fourth Int. Symp., pp. 467-474. The MIT Press, 1988

ROBOTICS
UNIVERSIDAD DE ZARAGOZA

Outline

1. Basic EKF SLAM
    1. Introduction: the need for SLAM
    2. **The basic EKF SLAM algorithm**
    3. Feature extraction

2. The Data Association Problem
    1. Introduction
    2. Continuous Data Association
    3. The Loop Closing Problem
    4. The Global Localization Problem

3. Advanced EKF SLAM
    1. Computational complexity of EKF SLAM
    2. Consistency of the EKF SLAM
    3. SLAM using local maps
        1. Sequential Map Joining
        2. Divide and Conquer SLAM
        3. Hierarchical SLAM

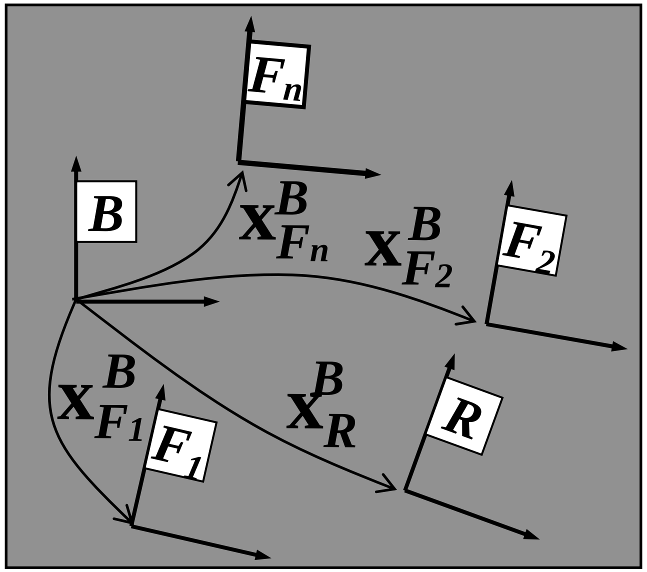
EKF-SLAM: approach

- Environment information related to a set of elements:

$$\mathcal{F} = \{B, R, F_1, \ldots, F_n\}$$

- represented by a **stochastic map**:

$$\mathcal{M}^B = \left(\hat{\mathbf{x}}^B, \mathbf{P}^B\right)$$



$$\hat{\mathbf{x}}^B = \begin{bmatrix} \hat{\mathbf{x}}^B_R \\ \vdots \\ \hat{\mathbf{x}}^B_{F_n} \end{bmatrix}$$

$$\mathbf{P}^B = \begin{bmatrix} \mathbf{P}^B_{RR} & \cdots & \mathbf{P}^B_{RF_n} \\ \vdots & \ddots & \vdots \\ \mathbf{P}^B_{F_nR} & \cdots & \mathbf{P}^B_{F_nF_n} \end{bmatrix}$$

Map Features in 2D

Points:

$$\mathbf{x}_P^B = \begin{bmatrix} x_2 \\ y_2 \end{bmatrix}$$

$$\mathbf{x}_P^A = \mathbf{x}_B^A \oplus \mathbf{x}_P^B = \begin{bmatrix} x_1 + x_2 \cos\phi_1 - y_2 \sin\phi_1 \\ y_1 + x_2 \sin\phi_1 + y_2 \cos\phi_1 \end{bmatrix}$$

$$J_{1\oplus}\{\mathbf{x}_B^A, \mathbf{x}_P^B\} = \begin{bmatrix} 1 & 0 & -x_2 \sin\phi_1 - y_2 \cos\phi_1 \\ 0 & 1 & x_2 \cos\phi_1 - y_2 \sin\phi_1 \end{bmatrix}$$

$$J_{2\oplus}\{\mathbf{x}_B^A, \mathbf{x}_P^B\} = \begin{bmatrix} \cos\phi_1 & -\sin\phi_1 \\ \sin\phi_1 & \cos\phi_1 \end{bmatrix}$$

Lines:

$$\mathbf{x}_L^B = \begin{bmatrix} \rho_2 \\ \theta_2 \end{bmatrix}$$

$$\mathbf{x}_L^A = \mathbf{x}_B^A \oplus \mathbf{x}_L^B = \begin{bmatrix} x_1 \cos(\phi_1 + \theta_2) + y_1 \sin(\phi_1 + \theta_2) + \rho_2 \\ \phi_1 + \theta_2 \end{bmatrix}$$

$$J_{1\oplus}\{\mathbf{x}_B^A, \mathbf{x}_L^B\} = \begin{bmatrix} \cos(\phi_1 + \theta_2) & \sin(\phi_1 + \theta_2) & -x_1 \sin(\phi_1 + \theta_2) + y_1 \cos(\phi_1 + \theta_2) \\ 0 & 0 & 1 \end{bmatrix}$$

$$J_{2\oplus}\{\mathbf{x}_B^A, \mathbf{x}_L^B\} = \begin{bmatrix} 1 & -x_1 \sin(\phi_1 + \theta_2) + y_1 \cos(\phi_1 + \theta_2) \\ 0 & 1 \end{bmatrix}$$

EKF-SLAM

Algorithm 1 SLAM:

$\mathbf{x}_0^B = 0$; $\mathbf{P}_0^B = 0$ {*Map initialization*}

$[\mathbf{z}_0, \mathbf{R}_0]$ = get_measurements

$[\mathbf{x}_0^B, \mathbf{P}_0^B]$ = add_new_features$(\mathbf{x}_0^B, \mathbf{P}_0^B, \mathbf{z}_0, \mathbf{R}_0)$

for $k = 1$ to steps **do**

$[\mathbf{x}_{R_k}^{R_{k-1}}, \mathbf{Q}_k]$ = get_odometry

$[\mathbf{x}_{k|k-1}^B, \mathbf{P}_{k|k-1}^B]$ = compute_motion$(\mathbf{x}_{k-1}^B, \mathbf{P}_{k-1}^B, \mathbf{x}_{R_k}^{R_{k-1}}, \mathbf{Q}_k)$ {*EKF prediction*}

$[\mathbf{z}_k, \mathbf{R}_k]$ = get_measurements

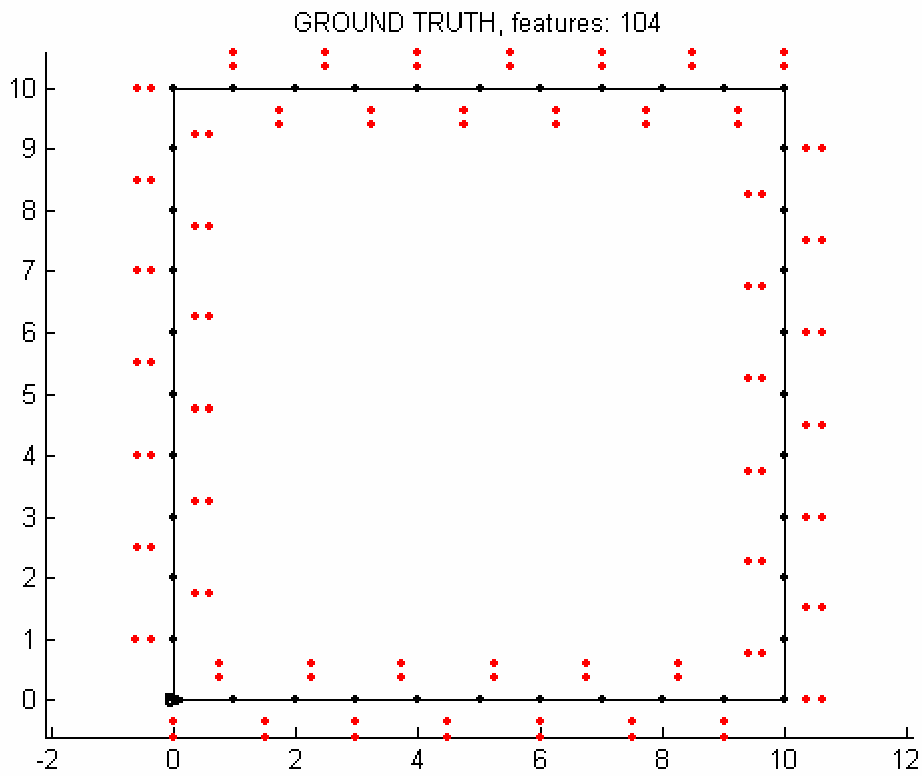
$\mathcal{H}_k$ = data_association$(\mathbf{x}_{k|k-1}^B, \mathbf{P}_{k|k-1}^B, \mathbf{z}_k, \mathbf{R}_k)$

$[\mathbf{x}_k^B, \mathbf{P}_k^B]$ = update_map$(\mathbf{x}_{k|k-1}^B, \mathbf{P}_{k|k-1}^B, \mathbf{z}_k, \mathbf{R}_k, \mathcal{H}_k)$ {*EKF update*}

$[\mathbf{x}_k^B, \mathbf{P}_k^B]$ = add_new_features$(\mathbf{x}_k^B, \mathbf{P}_k^B, \mathbf{z}_k, \mathbf{R}_k, \mathcal{H}_k)$

end for

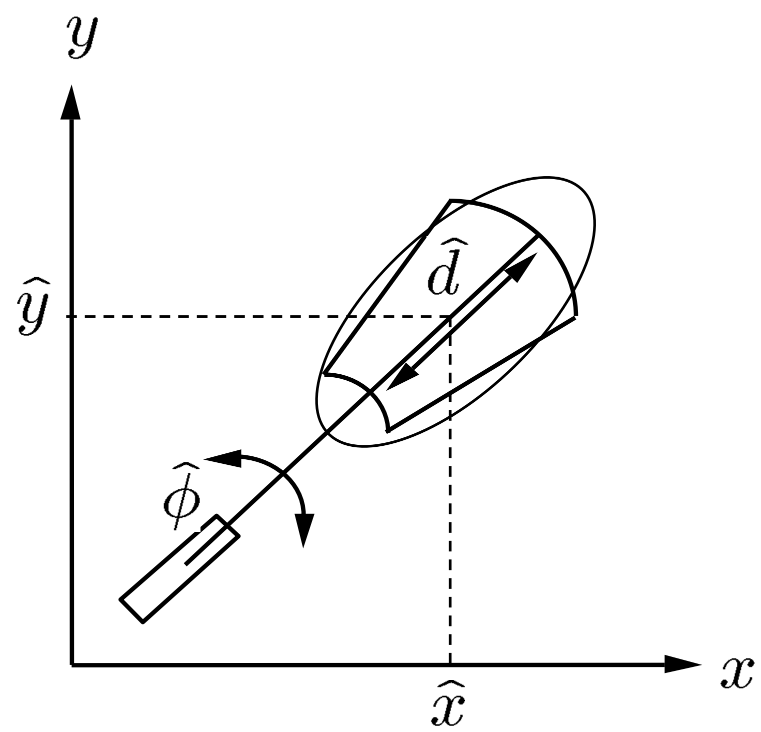
Example: SLAM in a cloister



- Red dots: environment features (columns)
- Black line: robot trajetory

Sensor measurements

- In polar coordinates:



$$\hat{\mathbf{p}} = (\hat{d},\ \hat{\phi})^T$$
$$P_{\mathbf{p}} = \mathrm{diag}(\sigma_d^2,\ \sigma_\phi^2)$$

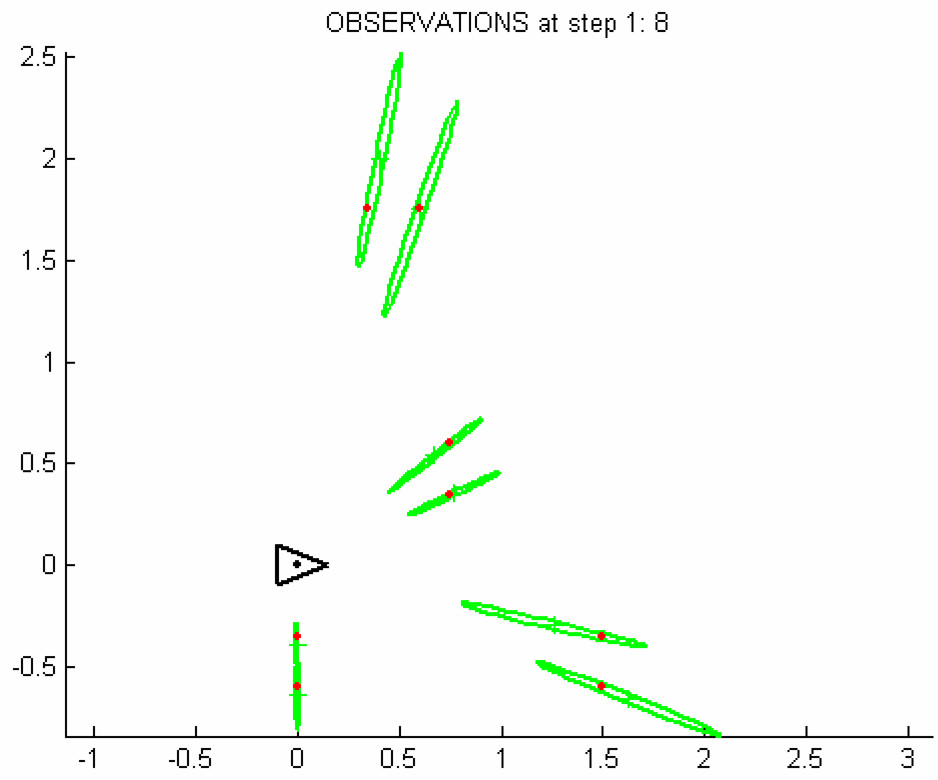
- In Cartesian coordinates:

$$\hat{x} = \hat{d}\cos\hat{\phi}$$
$$\hat{y} = \hat{d}\sin\hat{\phi}$$

$$\mathbf{x} = f(\mathbf{p})$$
$$P_{\mathbf{x}} \simeq J\,P_{\mathbf{p}}\,J^T$$
$$J = \begin{bmatrix} \frac{\partial x}{\partial d} & \frac{\partial x}{\partial \phi} \\ \frac{\partial y}{\partial d} & \frac{\partial y}{\partial \phi} \end{bmatrix}$$

$$\hat{\mathbf{x}} = (\hat{x},\ \hat{y})^T$$
$$P_{\mathbf{x}} = \begin{bmatrix} \sigma_x^2 & \sigma_{xy} \\ \sigma_{xy} & \sigma_y^2 \end{bmatrix}$$

The basic EKF SLAM Algorithms



Sensor measurements

EKF-SLAM: add new features

$$\mathbf{x}_k^B = \begin{pmatrix} \mathbf{x}_{R_k}^B \\ \mathbf{x}_{F_{1,k}}^B \\ \vdots \\ \mathbf{x}_{F_{n,k}}^B \end{pmatrix} \Rightarrow \mathbf{x}_{k+}^B = \begin{pmatrix} \mathbf{x}_{R_k}^B \\ \mathbf{x}_{F_{1,k}}^B \\ \vdots \\ \mathbf{x}_{F_{n,k}}^B \\ \mathbf{x}_{F_{n+1,k}}^B \end{pmatrix} = \begin{pmatrix} \mathbf{x}_{R_k}^B \\ \mathbf{x}_{F_{1,k}}^B \\ \vdots \\ \mathbf{x}_{F_{n,k}}^B \\ \mathbf{x}_{R_k}^B \oplus \mathbf{z}_i \end{pmatrix}$$

Linearization:

$$\mathbf{x}_{k+}^B \simeq \hat{\mathbf{x}}_{k+}^B + \mathbf{F}_k(\mathbf{x}_k^B - \hat{\mathbf{x}}_k^B) + \mathbf{G}_k(\mathbf{z}_i - \hat{\mathbf{z}}_i)$$

$$\mathbf{P}_{k+}^B = \mathbf{F}_k \mathbf{P}_k^B \mathbf{F}_k^T + \mathbf{G}_k \mathbf{R}_k \mathbf{G}_k^T$$

Where:

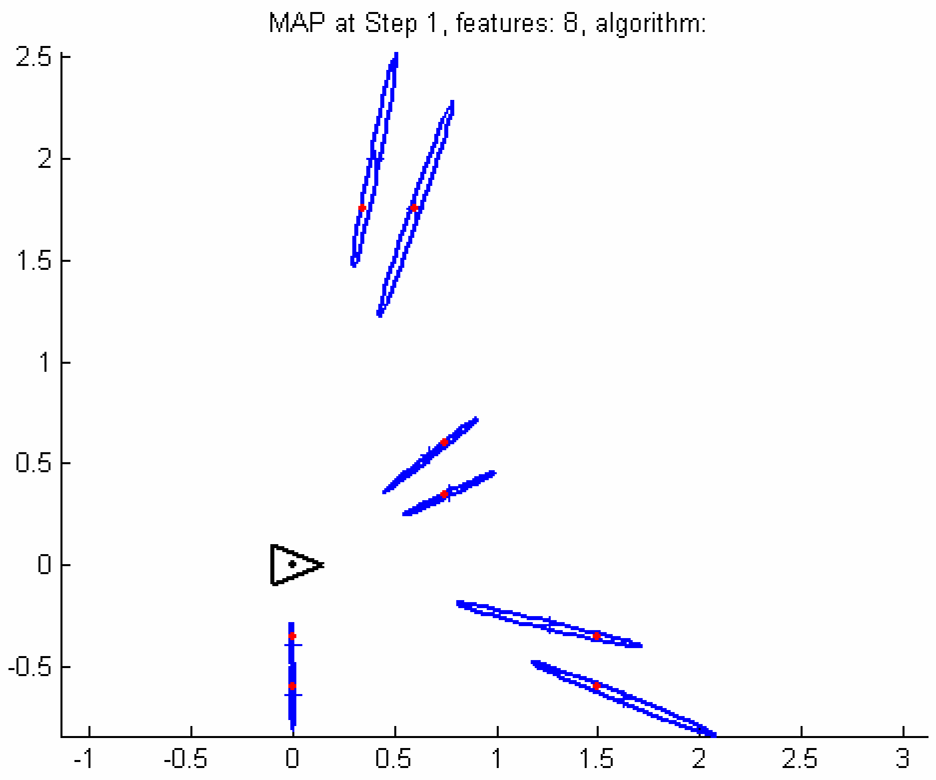
$$\mathbf{F}_k = \frac{\partial \mathbf{x}_{k+}^B}{\partial \mathbf{x}_k^B} = \begin{pmatrix} \mathbf{I} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \ddots & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{I} \\ \mathbf{J}_{1\oplus}\left\{\hat{\mathbf{x}}_{R_k}^B, \hat{\mathbf{z}}_i\right\} & \mathbf{0} & \cdots & \mathbf{0} \end{pmatrix} ; \mathbf{G}_k = \frac{\partial \mathbf{x}_{k+}^B}{\partial \mathbf{z}_i} = \begin{pmatrix} \mathbf{0} \\ \vdots \\ \mathbf{0} \\ \mathbf{J}_{2\oplus}\left\{\hat{\mathbf{x}}_{R_k}^B, \hat{\mathbf{z}}_i\right\} \end{pmatrix}$$

EKF-SLAM: add new features

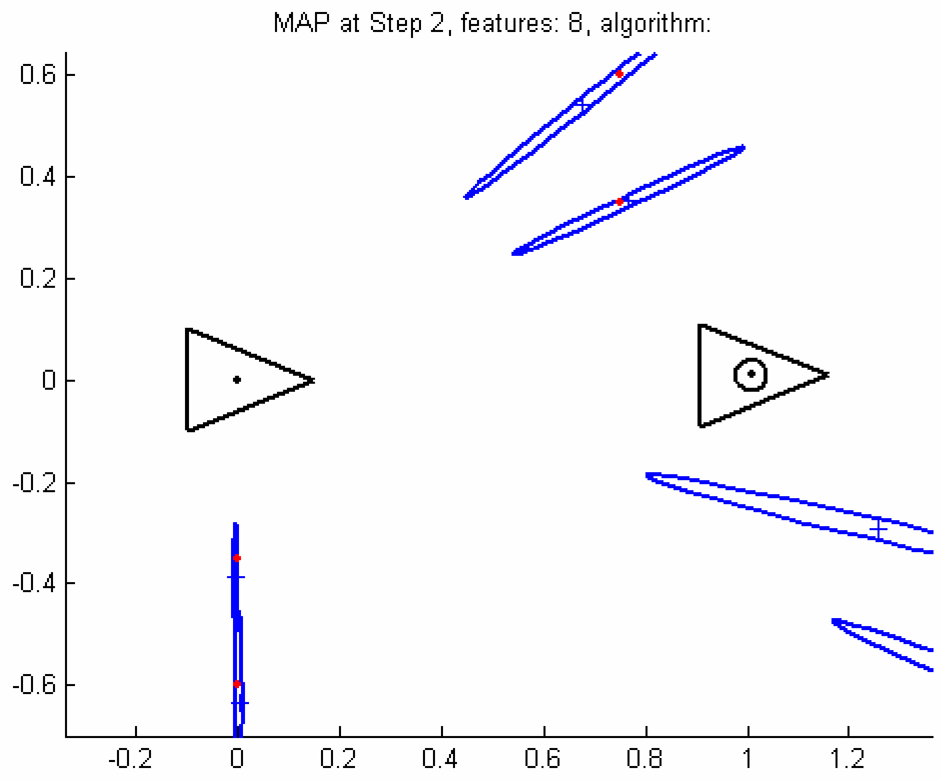
$$\mathbf{P}_k^B = \begin{pmatrix} \mathbf{P}_R & \mathbf{P}_{RF_1} & \cdots & \mathbf{P}_{RF_n} \\ \mathbf{P}_{RF_1}^T & \mathbf{P}_{F_1} & \cdots & \mathbf{P}_{F_1F_n} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{P}_{RF_n}^T & \mathbf{P}_{F_1F_n}^T & \cdots & \mathbf{P}_{F_n} \end{pmatrix}$$

$$\mathbf{P}_{k+}^B = \begin{pmatrix} \mathbf{P}_R & \mathbf{P}_{RF_1} & \cdots & \mathbf{P}_{RF_n} & \mathbf{P}_R\mathbf{J}_{1\oplus}^T \\ \mathbf{P}_{RF_1}^T & \mathbf{P}_{F_1} & \cdots & \mathbf{P}_{F_1F_n} & \mathbf{P}_{RF_1}^T\mathbf{J}_{1\oplus}^T \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \mathbf{P}_{RF_n}^T & \mathbf{P}_{F_1F_n}^T & \cdots & \mathbf{P}_{F_n} & \mathbf{P}_{RF_n}^T\mathbf{J}_{1\oplus}^T \\ \mathbf{J}_{1\oplus}\mathbf{P}_R & \mathbf{J}_{1\oplus}\mathbf{P}_{RF_1} & \cdots & \mathbf{J}_{1\oplus}\mathbf{P}_{RF_n} & \mathbf{J}_{1\oplus}\mathbf{P}_R\mathbf{J}_{1\oplus}^T + \mathbf{J}_{2\oplus}\mathbf{R}_k\mathbf{J}_{2\oplus}^T \end{pmatrix}$$

EKF-SLAM: add new features



EKF-SLAM: compute robot motion



EKF-SLAM: compute robot motion

$$\mathbf{x}^B_{R_k} = \mathbf{x}^B_{R_{k-1}} \oplus \mathbf{x}^{R_{k-1}}_{R_k}$$

Odometry model (white noise):

$$\begin{aligned} \mathbf{x}^{R_{k-1}}_{R_k} &= \hat{\mathbf{x}}^{R_{k-1}}_{R_k} + \mathbf{v}_k \\ E[\mathbf{v}_k] &= \mathbf{0} \\ E[\mathbf{v}_k \mathbf{v}_j^T] &= \delta_{kj}\mathbf{Q}_k \end{aligned}$$

EKF prediction:

$$\hat{\mathbf{x}}^B_{k|k-1} = \begin{bmatrix} \hat{\mathbf{x}}^B_{R_{k-1}} \oplus \hat{\mathbf{x}}^{R_{k-1}}_{R_k} \\ \hat{\mathbf{x}}^B_{F_{1,k-1}} \\ \vdots \\ \hat{\mathbf{x}}^B_{F_{m,k-1}} \end{bmatrix}$$

$$\mathbf{P}^B_{k|k-1} = \mathbf{F}_k \mathbf{P}^B_{k-1} \mathbf{F}_k^T + \mathbf{G}_k \mathbf{Q}_k \mathbf{G}_k^T$$

$$\mathbf{F}_k = \begin{bmatrix} \mathbf{J}_{1\oplus}\left\{\hat{\mathbf{x}}^B_{R_{k-1}}, \hat{\mathbf{x}}^{R_{k-1}}_{R_k}\right\} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{I} & & \vdots \\ \vdots & & \ddots & \\ \mathbf{0} & \cdots & & \mathbf{I} \end{bmatrix}$$

$$\mathbf{G}_k = \begin{bmatrix} \mathbf{J}_{2\oplus}\left\{\hat{\mathbf{x}}^B_{R_{k-1}}, \hat{\mathbf{x}}^{R_{k-1}}_{R_k}\right\} \\ \mathbf{0} \\ \vdots \\ \mathbf{0} \end{bmatrix}$$

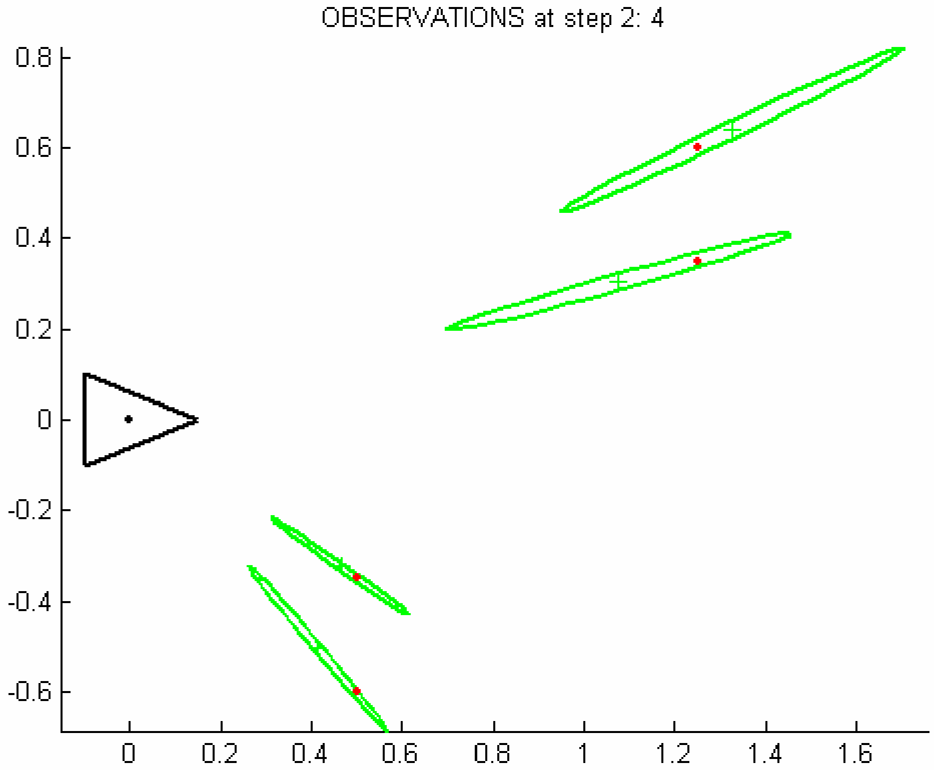
ROBOTICS
UNIVERSIDAD DE ZARAGOZA

EKF-SLAM: compute robot motion

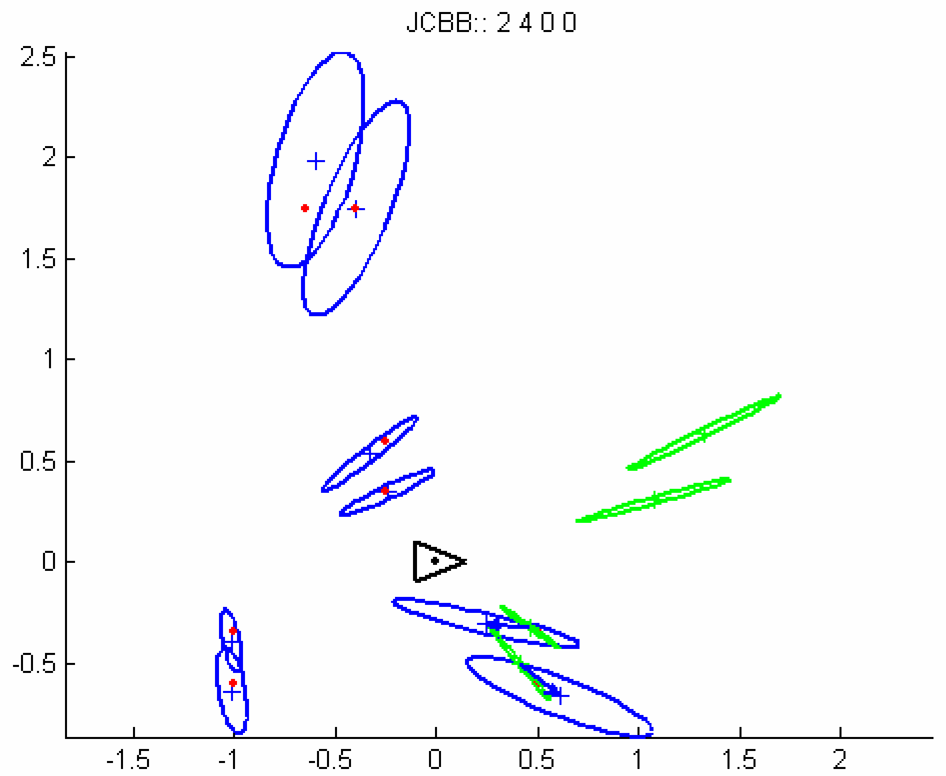
$$\mathbf{P}^B_{k-1|k-1} = \begin{pmatrix} \mathbf{P}_R & \mathbf{P}_{RF_1} & \cdots & \mathbf{P}_{RF_n} \\ \mathbf{P}^T_{RF_1} & \mathbf{P}_{F_1} & \cdots & \mathbf{P}_{F_1F_n} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{P}^T_{RF_n} & \mathbf{P}^T_{F_1F_n} & \cdots & \mathbf{P}_{F_n} \end{pmatrix}$$

$$\mathbf{P}^B_{k|k-1} = \begin{pmatrix} \mathbf{J}_{1\oplus}\mathbf{P}_R\mathbf{J}^T_{1\oplus} + \mathbf{J}_{2\oplus}\mathbf{Q}_k\mathbf{J}^T_{2\oplus} & \mathbf{J}_{1\oplus}\mathbf{P}_{RF_1} & \cdots & \mathbf{J}_{1\oplus}\mathbf{P}_{RF_n} \\ \mathbf{J}^T_{1\oplus}\mathbf{P}^T_{RF_1} & \mathbf{P}_{F_1} & \cdots & \mathbf{P}_{F_1F_n} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{J}^T_{1\oplus}\mathbf{P}^T_{RF_n} & \mathbf{P}^T_{F_1F_n} & \cdots & \mathbf{P}_{F_n} \end{pmatrix}$$

EKF-SLAM: Observations



EKF-SLAM: Data association



Predicted map .vs. measurements

EKF-SLAM: Observations

Observations at instant k:

$$\mathbf{z}_{k,i} \quad \text{with } i = 1 \ldots s$$

Association Hypothesis (obs. i with map feature j_i) :

$$\mathcal{H}_k = [j_1, j_2, \cdots, j_s]$$

Measurement equation:

$$\begin{aligned} \mathbf{z}_k &= \mathbf{h}_k(\mathbf{x}_k^B) + \mathbf{w}_k \\ \mathbf{h}_k &= \begin{bmatrix} \mathbf{h}_{1j_1} \\ \mathbf{h}_{2j_2} \\ \vdots \\ \mathbf{h}_{sj_s} \end{bmatrix} \end{aligned}$$

Sensor model (white noise):

$$\begin{aligned} E[\mathbf{w}_k] &= 0 \\ E[\mathbf{w}_k \mathbf{w}_j^T] &= \delta_{kj} \mathbf{R}_k \\ E[\mathbf{w}_k \mathbf{v}_j^T] &= 0 \end{aligned}$$

EKF-SLAM: Observations

Linearization:

$$
\begin{aligned}
\mathbf{z}_k &\simeq \mathbf{h}_k(\hat{\mathbf{x}}^B_{k|k-1}) + \mathbf{H}_k(\mathbf{x}^B_k - \hat{\mathbf{x}}^B_{k|k-1}) \\
\mathbf{H}_k &= \left.\frac{\partial \mathbf{h}_k}{\partial \mathbf{x}^B_k}\right|_{(\hat{\mathbf{x}}^B_{k|k-1})} = \begin{pmatrix} \mathbf{H}_R & \mathbf{0} & \cdots & \mathbf{H}_F \cdots & \mathbf{0} \end{pmatrix} \\
\mathbf{H}_R &= \left.\frac{\partial \mathbf{h}_k}{\partial \mathbf{x}^B_{R_k}}\right|_{(\hat{\mathbf{x}}^B_{k|k-1})} \; ; \; \mathbf{H}_F = \left.\frac{\partial \mathbf{h}_k}{\partial \mathbf{x}^B_{F_k}}\right|_{(\hat{\mathbf{x}}^B_{k|k-1})}
\end{aligned}
$$

Data association

Innovation:

$$\begin{aligned} \nu_k &= \mathbf{z}_k - \hat{\mathbf{z}}_k \\ \mathrm{Cov}(\nu_k) &= \mathbf{H}_k \mathbf{P}_k^B \mathbf{H}_k^T + \mathbf{R}_k^T \end{aligned}$$

Mahalanobis distance:

$$D^2 = \nu_k^T \mathrm{Cov}(\nu_k)^{-1} \nu_k \sim \chi_r^2$$

where $r =$dim(ν_k)

Hypothesis test:

$D^2 \leq \chi^2_{r,\alpha} \;\Rightarrow\; \mathbf{z}_k$ **compatible with** $\hat{\mathbf{z}}_k$

where $\alpha = 0.05$ (common)

EKF-SLAM: map update

State update:

$$\hat{\mathbf{x}}_k^B = \hat{\mathbf{x}}_{k|k-1}^B + \mathbf{K}_k(\mathbf{z}_k - \mathbf{h}_k(\hat{\mathbf{x}}_{k|k-1}^B))$$

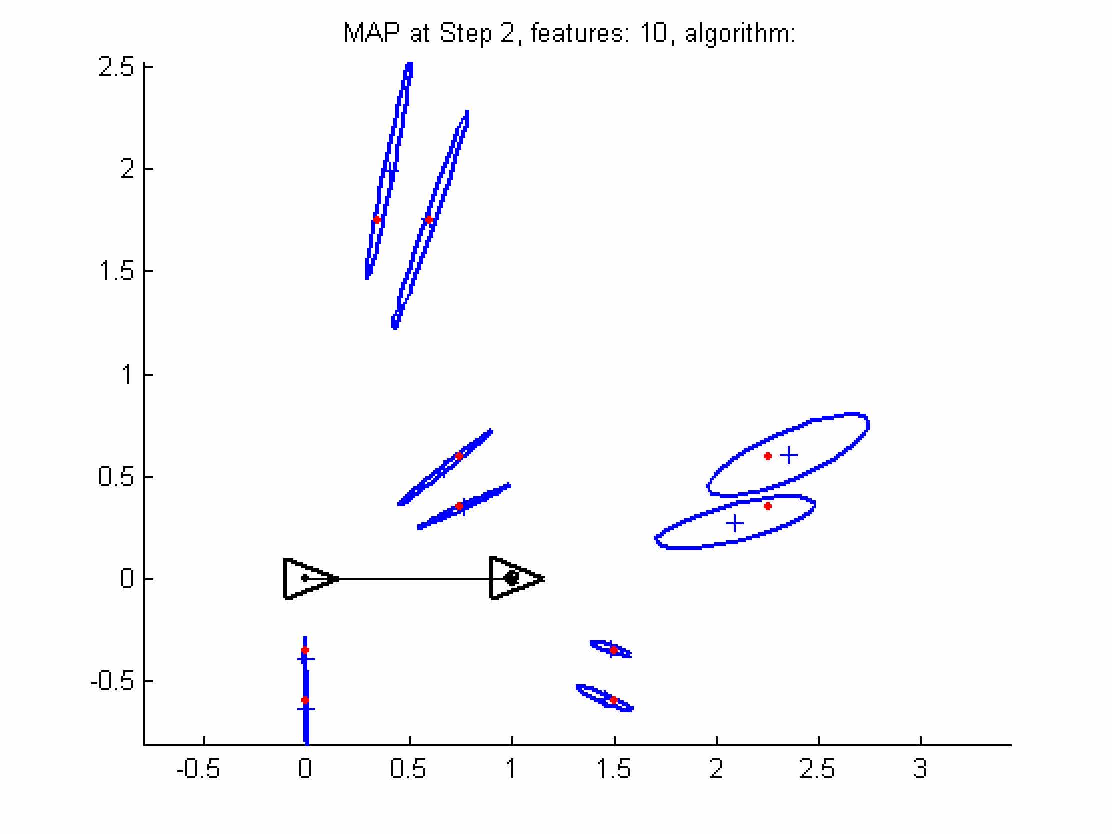
Covariance update:

$$\mathbf{P}_k^B = (\mathbf{I} - \mathbf{K}_k \mathbf{H}_k)\mathbf{P}_{k|k-1}^B$$

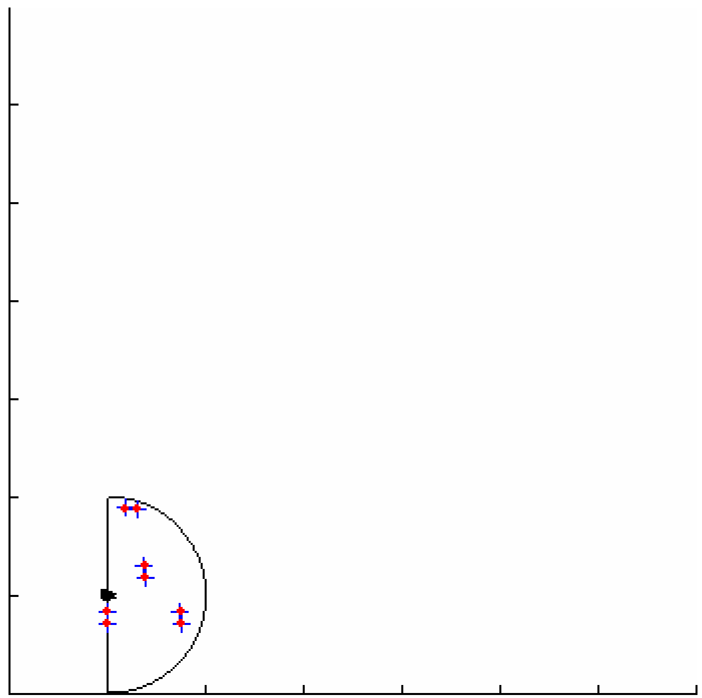
Filter gain:

$$\mathbf{K}_k = \mathbf{P}_{k|k-1}^B \mathbf{H}_k^T (\mathbf{H}_k \mathbf{P}_{k|k-1}^B \mathbf{H}_k^T + \mathbf{R}_k)^{-1}$$

EKF-SLAM: map update

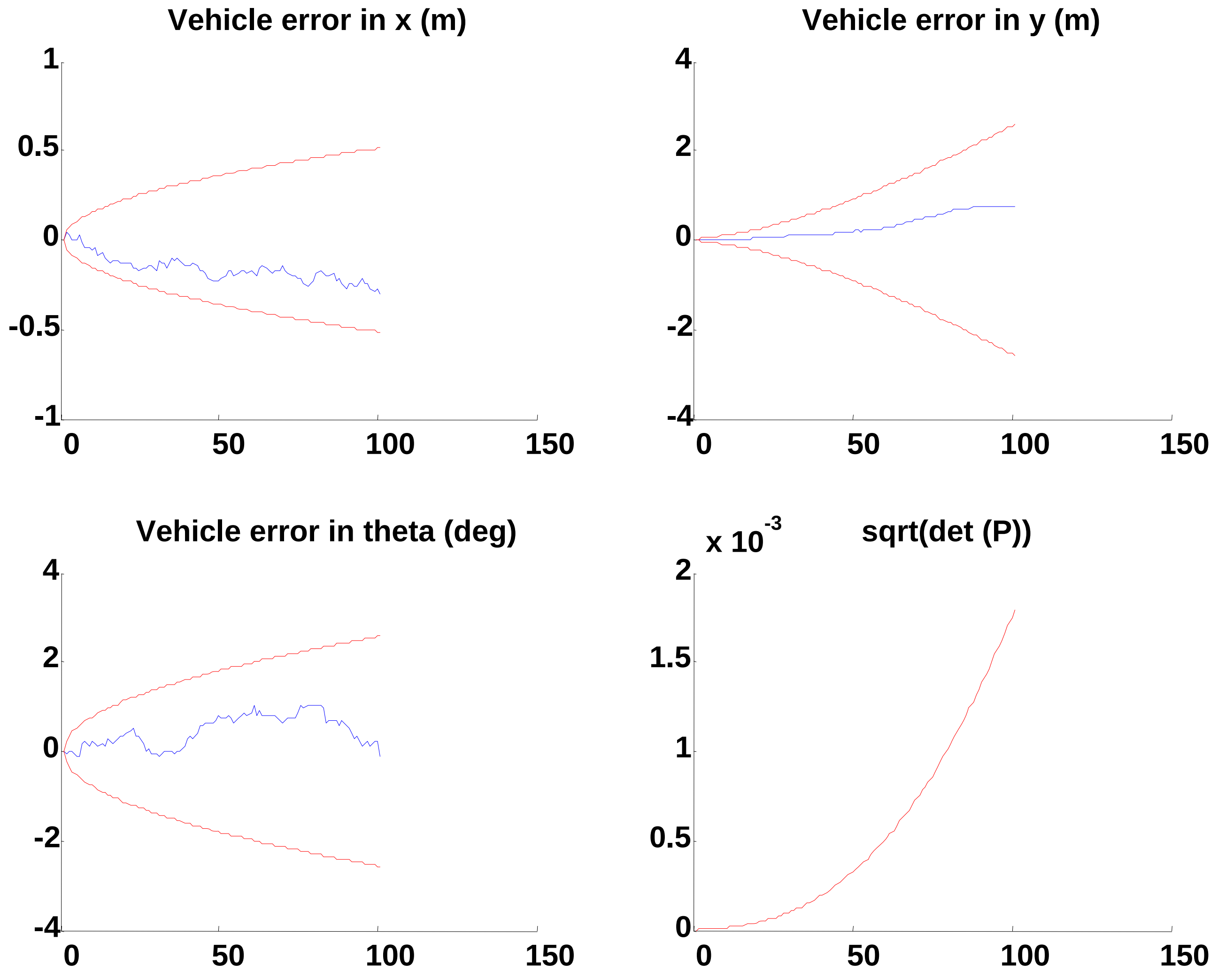


Why we do SLAM

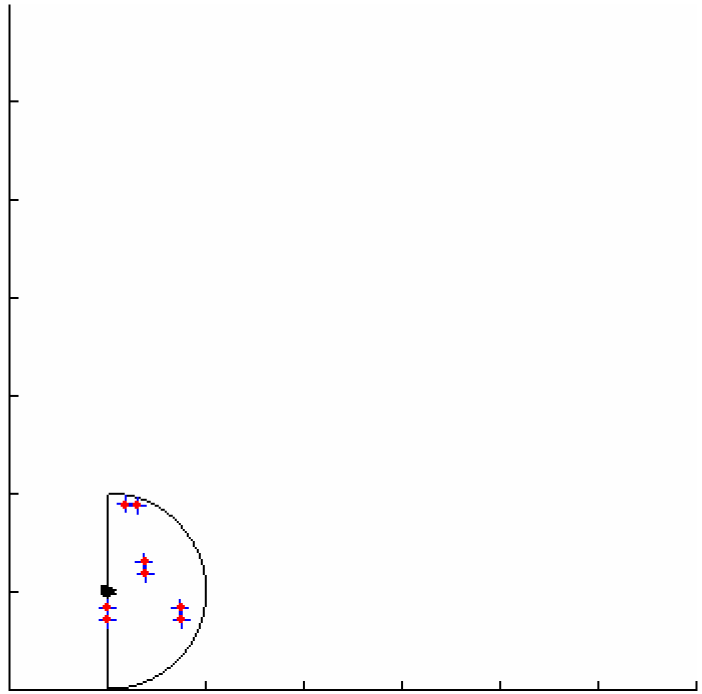


Uncertainty still grows!

Dead-reckoning, moving forward

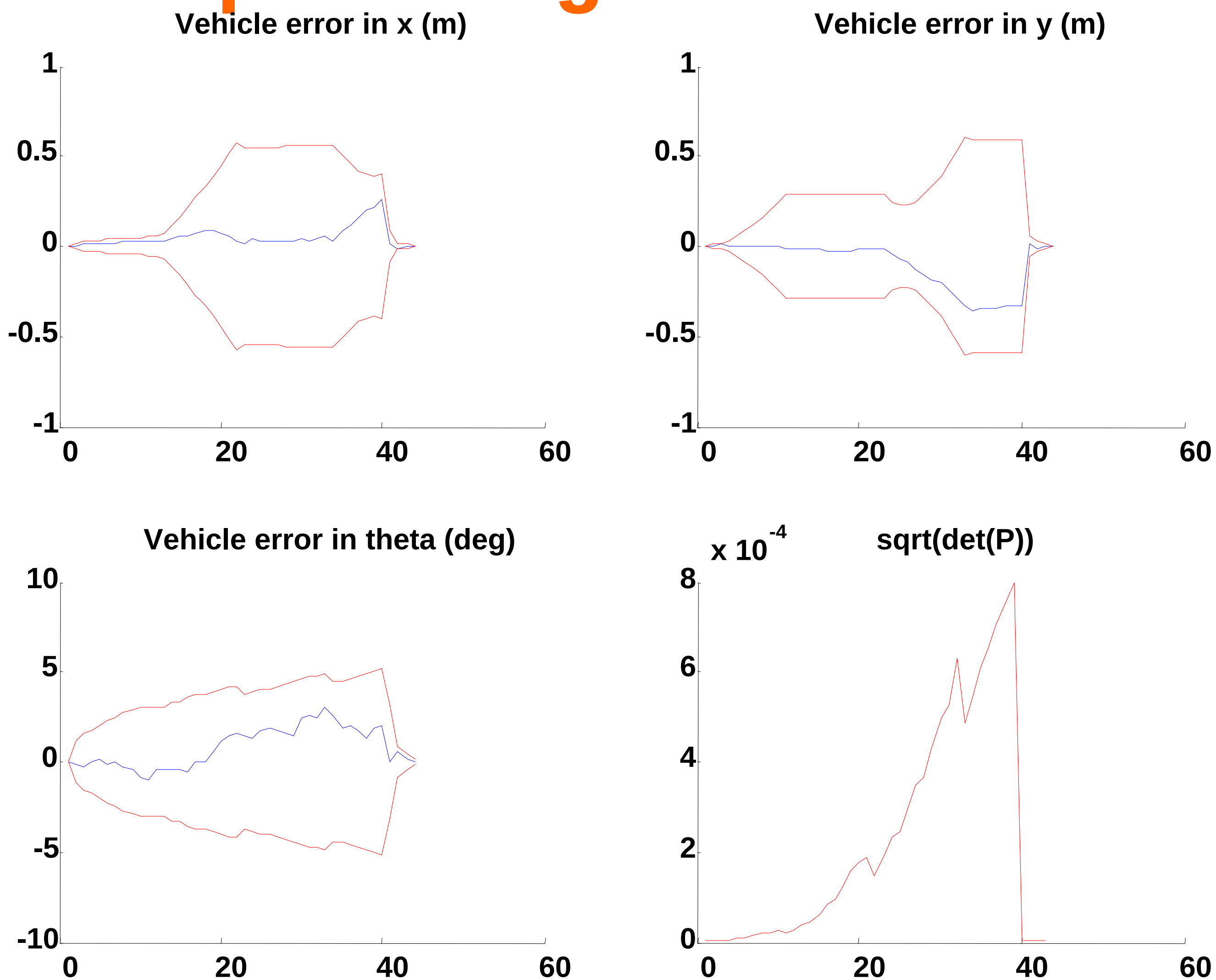


Good news!



Loop closing reduces uncertainty!

Loop closing in EKF-SLAM

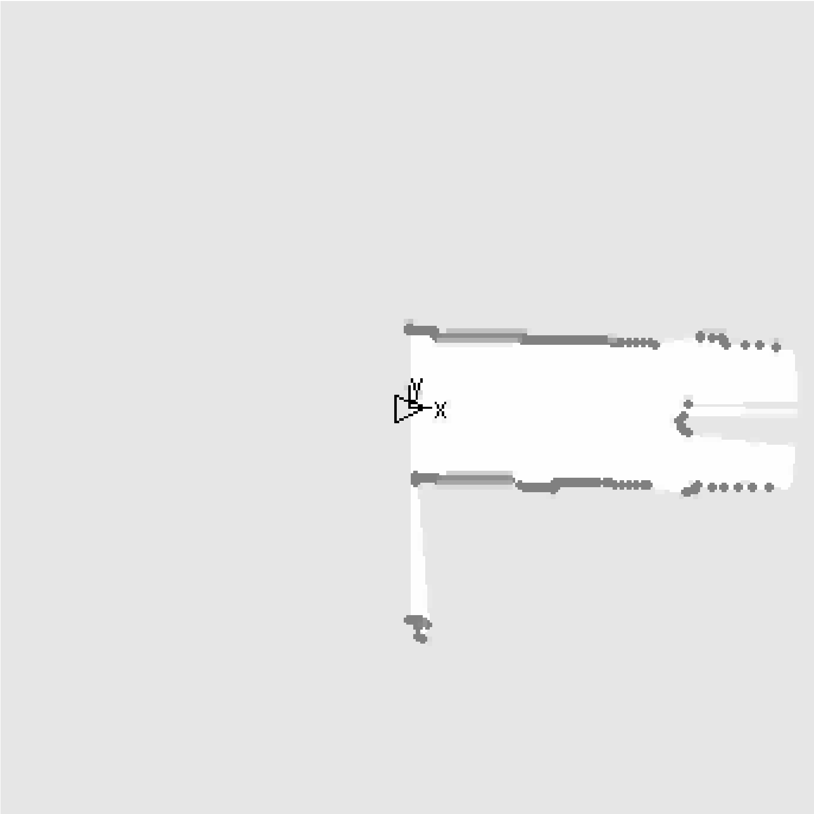


Loop closing reduces uncertainty!

Outline

1. Basic EKF SLAM
    1. Introduction: the need for SLAM
    2. The basic EKF SLAM algorithm
    3. **Feature extraction**
2. The Data Association Problem
    1. Introduction
    2. Continuous Data Association
    3. The Loop Closing Problem
    4. The Global Localization Problem
3. Advanced EKF SLAM
    1. Computational complexity of EKF SLAM
    2. Consistency of the EKF SLAM
    3. SLAM using local maps
        1. Sequential Map Joining
        2. Divide and Conquer SLAM
        3. Hierarchical SLAM

Feature extraction: Laser

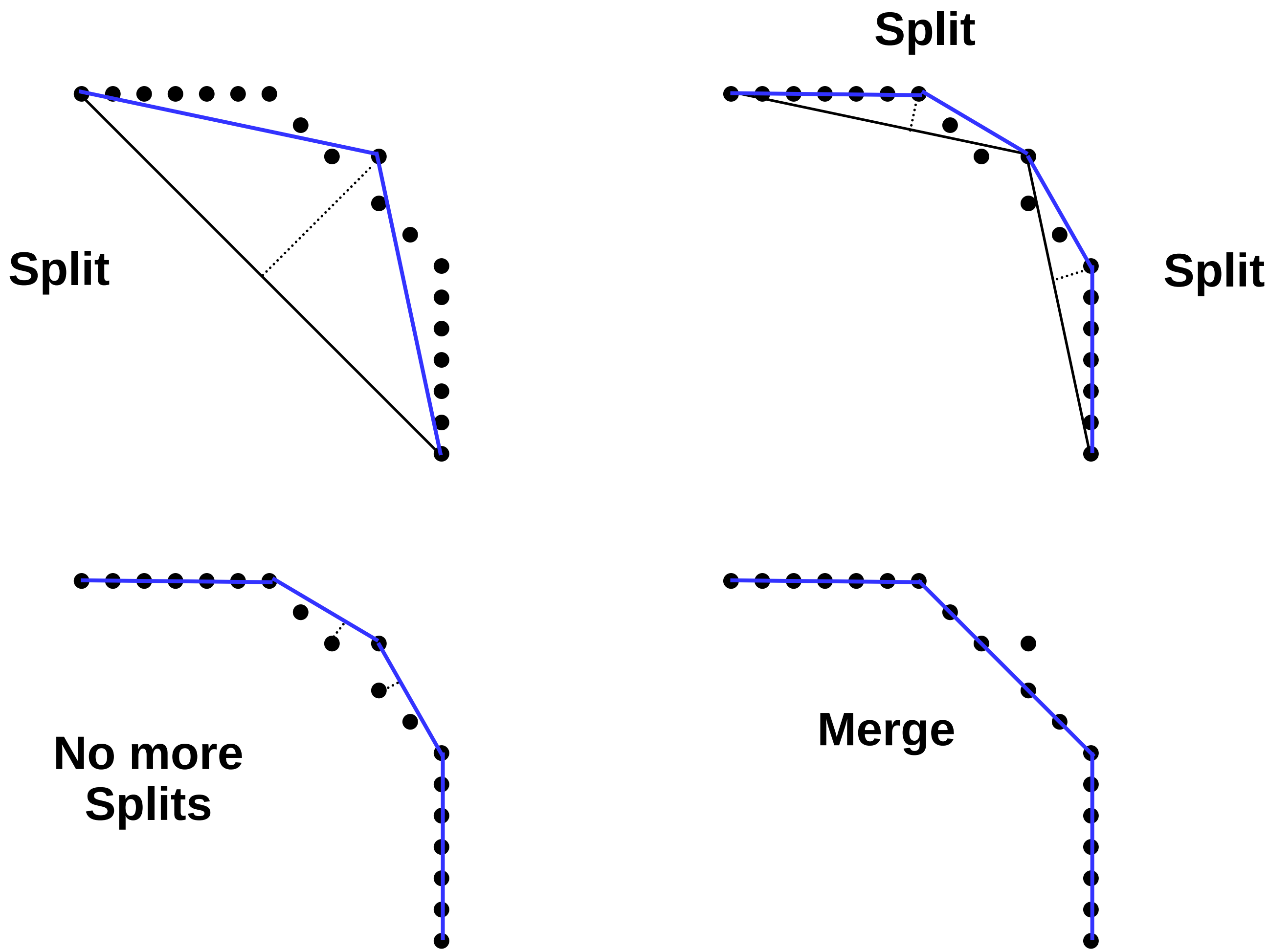


- Obtain line segments from a laser scan:
  - Segmentation
  - Line estimation

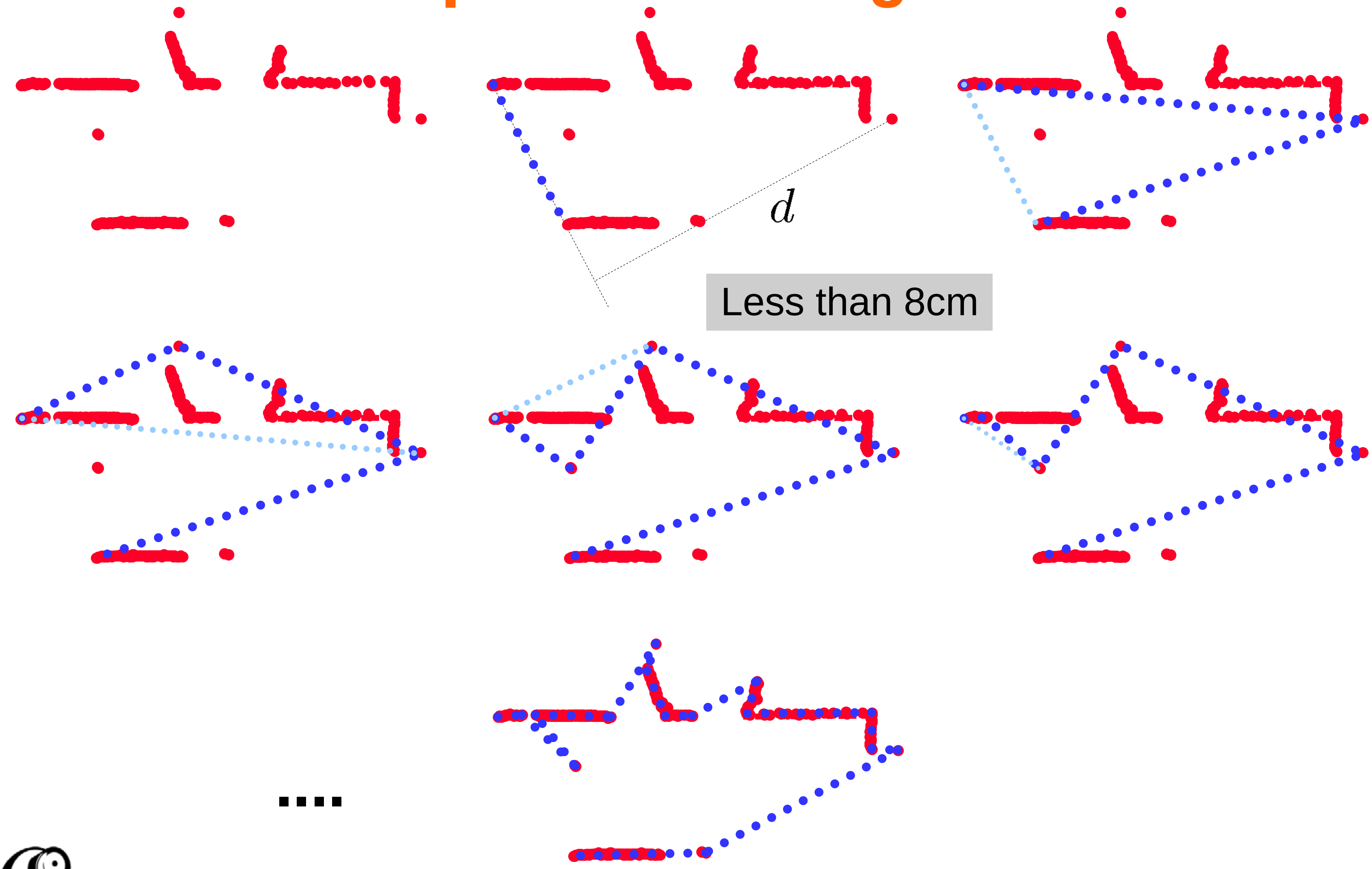
Split and merge:

1. **Recursive Split:**
   1. Obtain the line passing by the two extreme points
   2. Obtain the point more distant to the line
   3. If distance > error_max, split and repeat with the left and right sub-scan
2. **Merge:**
   1. If two consecutive segments are close enough, obtain the common line and the more distant point
   2. If distance <= error_max, merge both segments
3. **Prune short** segments
4. **Estimate line equation**

Split and Merge



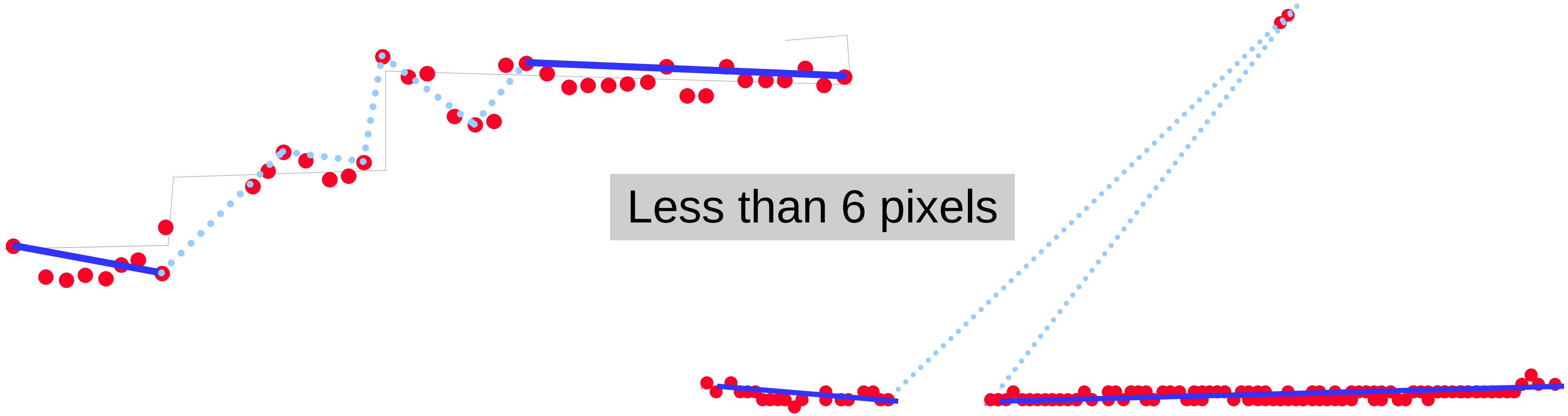
Split and Merge



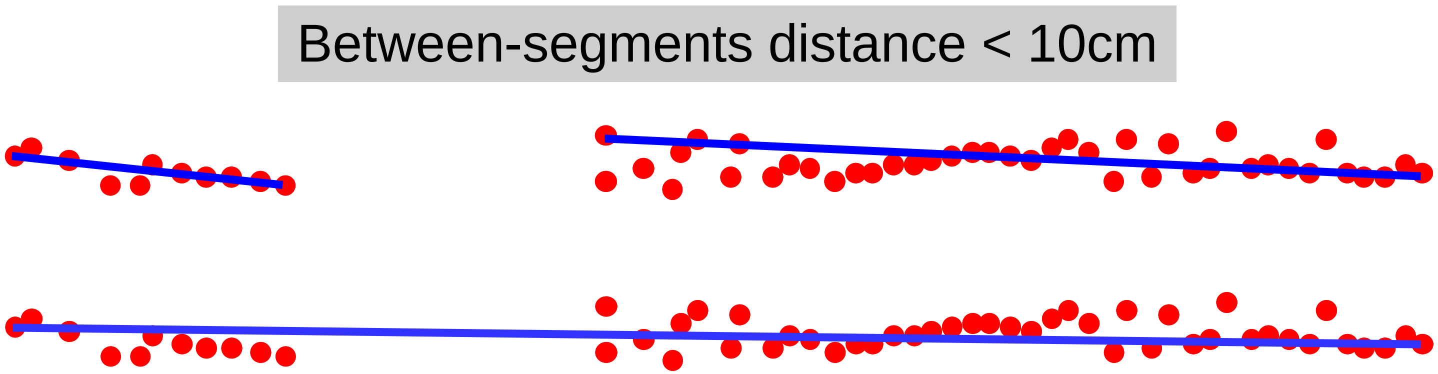
....

Split and Merge

- Elimination of small segments:

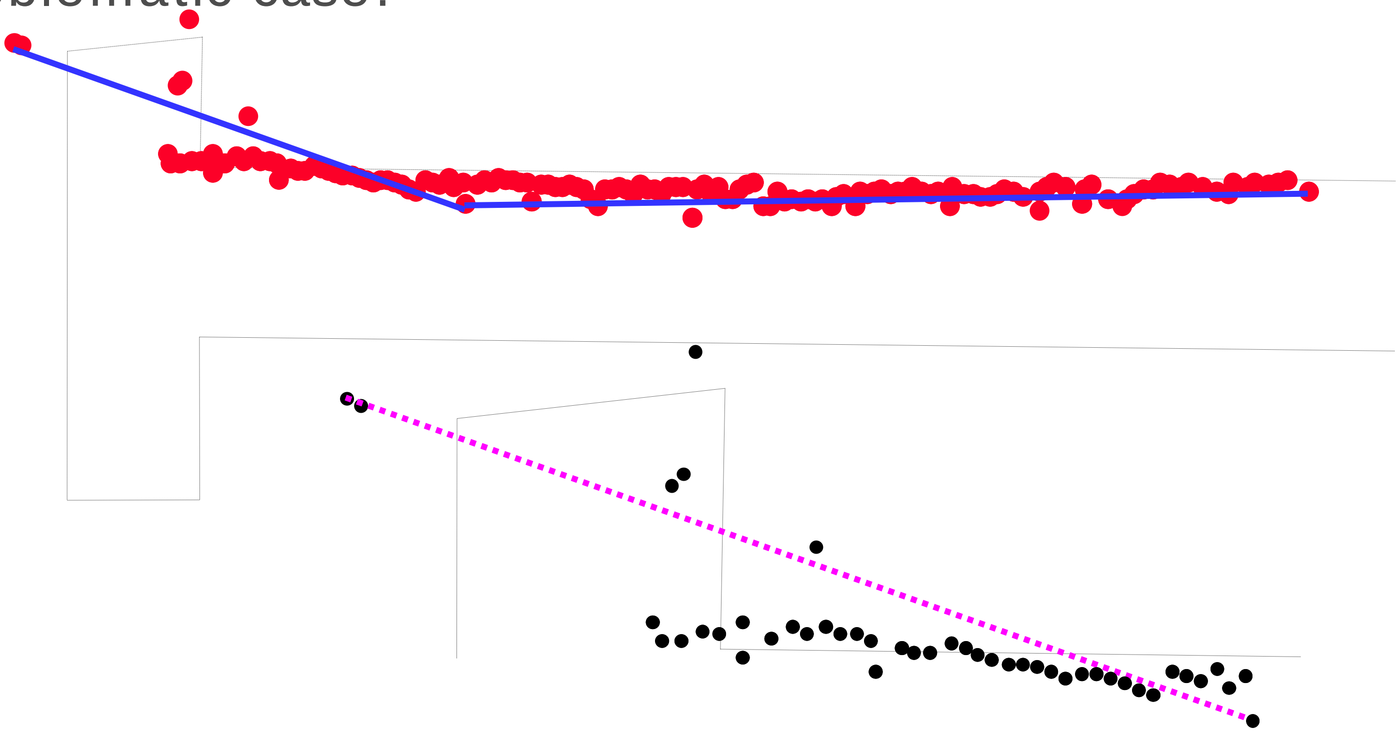


- Segment fusion:



Split and Merge

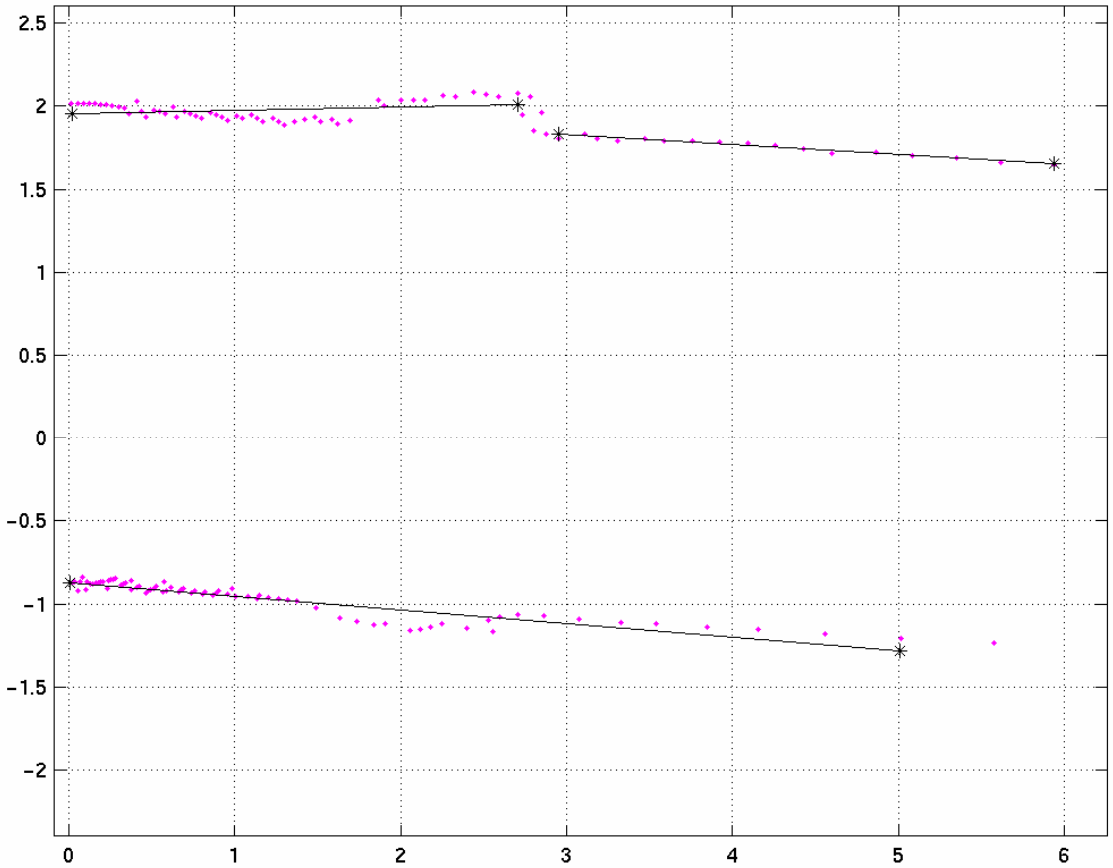
- Problematic case:



- **Split and merge:** only the extreme points are used
- **Alternatives:**
  - Linear regression
  - RANSAC
  - Hough transform

ROBOTICS UNIVERSIDAD DE ZARAGOZA

Split and Merge

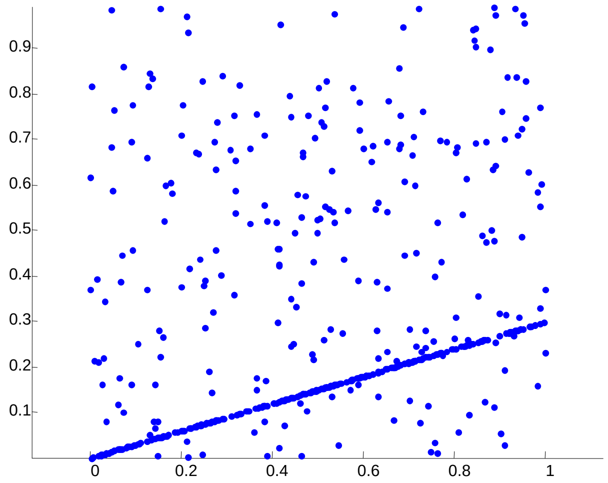


Not robust to complex and/or spurious data

RANSAC

- Given a model that requires n data points to compute a solution and a set of data points P, with #(P) > n :
  - Randomly select a subset S1 of n data points and compute the model M1
  - Determine the **consensus** set S1* of points is P compatible with M1 (within some error tolerance)
  - If #(S1*) > th, use S1* to compute (maybe using least squares) a new model M1*
  - If #(S1*) < th, randomly select another subset S2 and repeat
  - If, after t trials there is no consensus set with th points, return with failure

RANSAC



z acceptable probability of failure

t tries ¿?

$$(1 - w^n)^t = z$$

$$t = \left\lceil \frac{\log z}{\log(1 - w^n)} \right\rceil$$

w	0,5
n	2
z	0,05
t	11

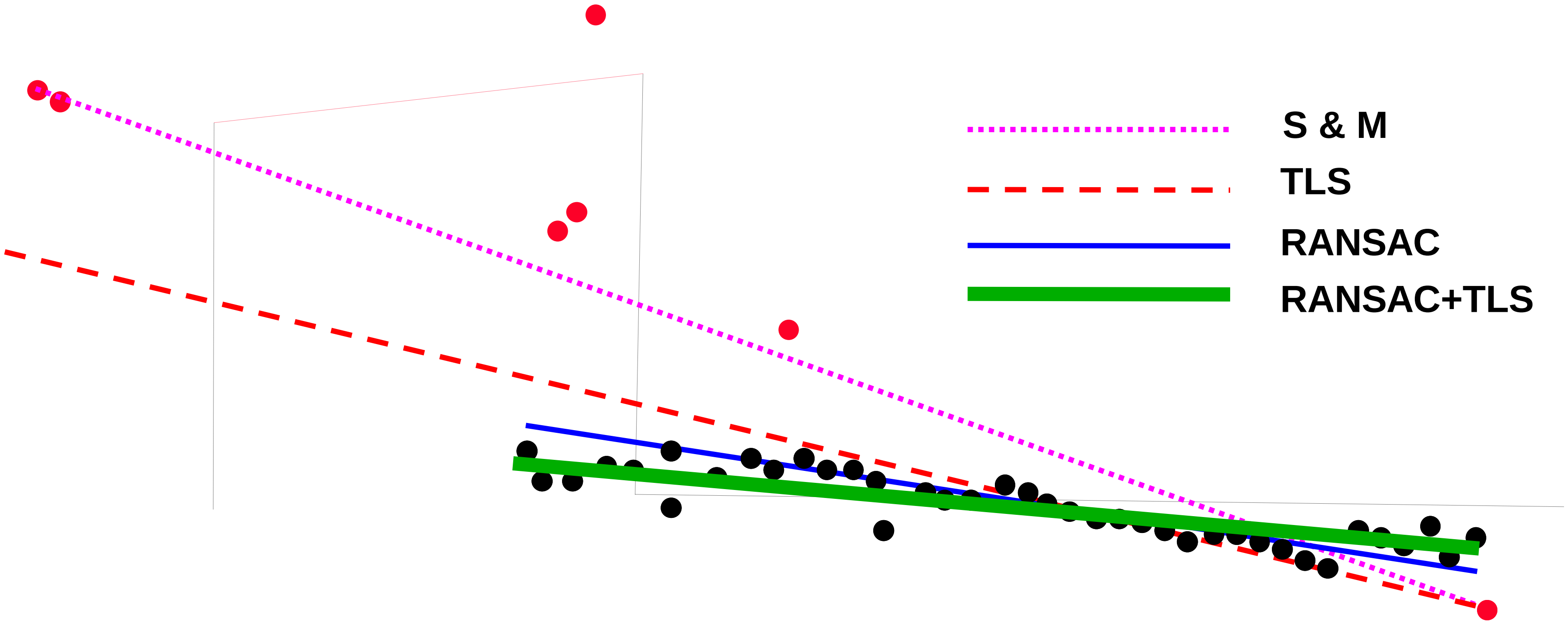
p no. of points

n points to build model

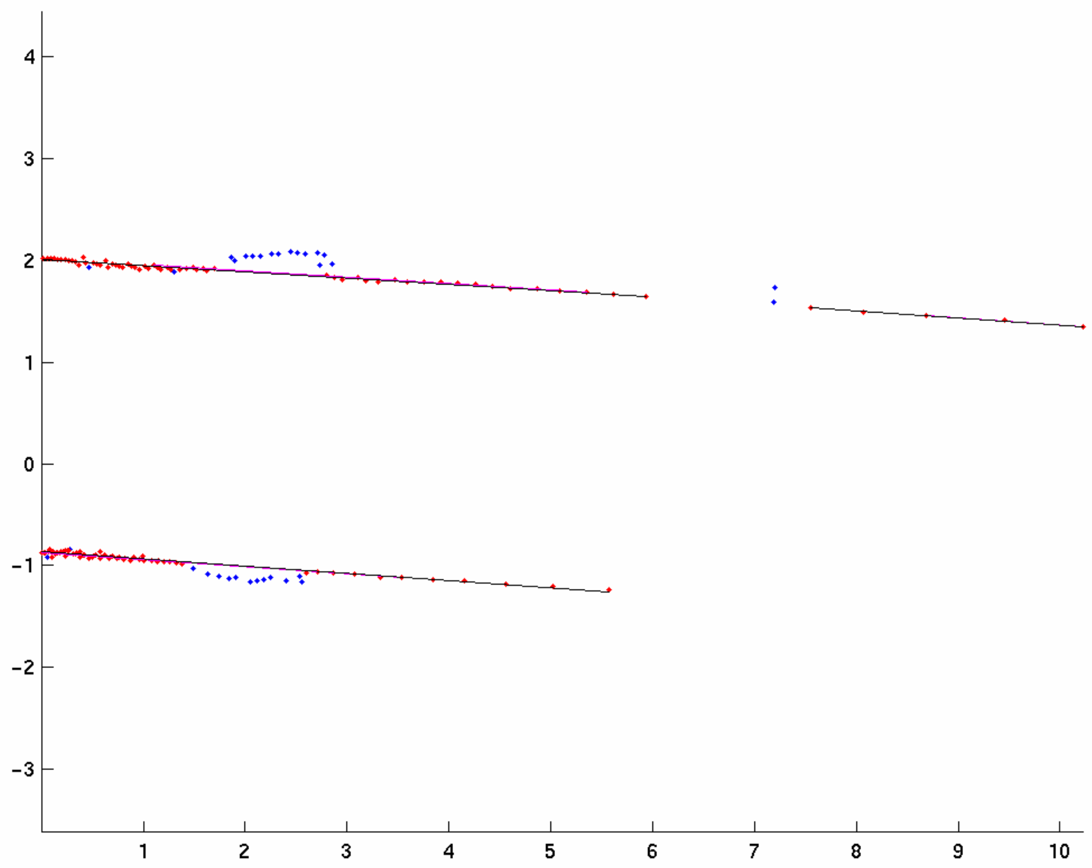
w probability that a point is good

$O(p^n)$_possible_models

RANSAC

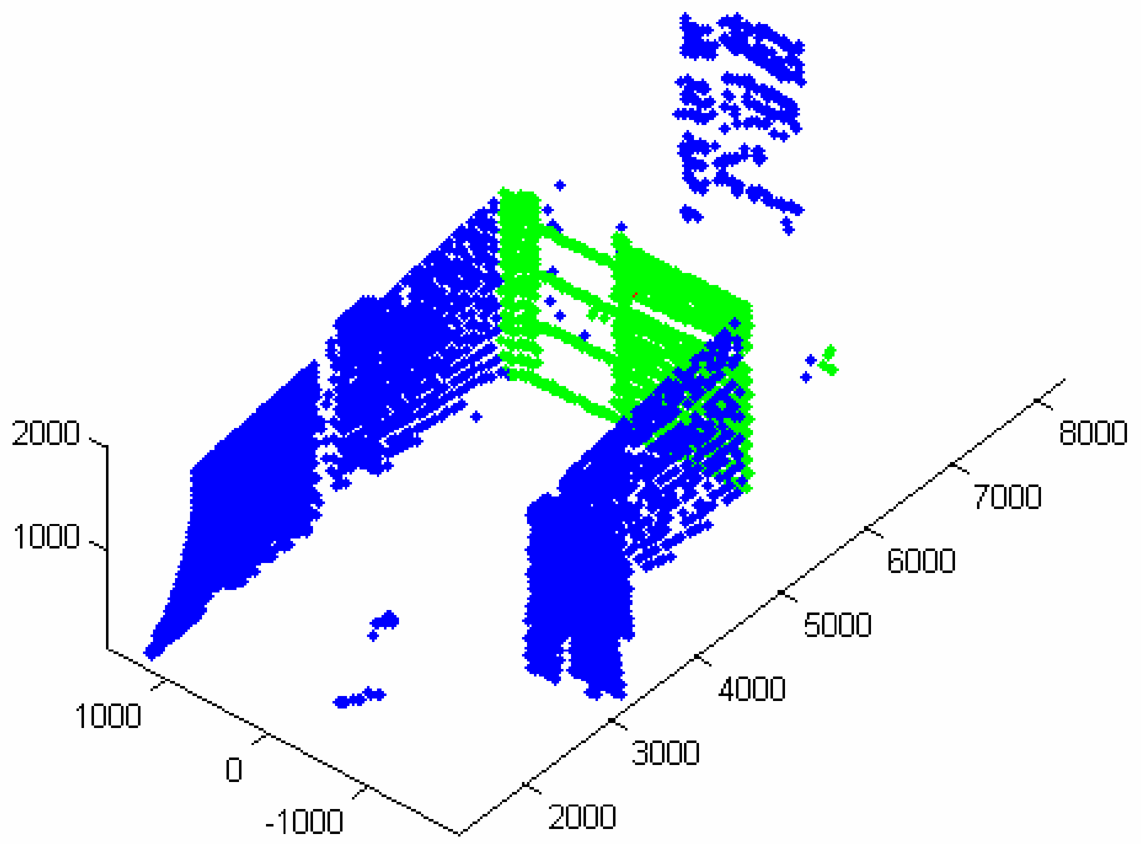


RANSAC



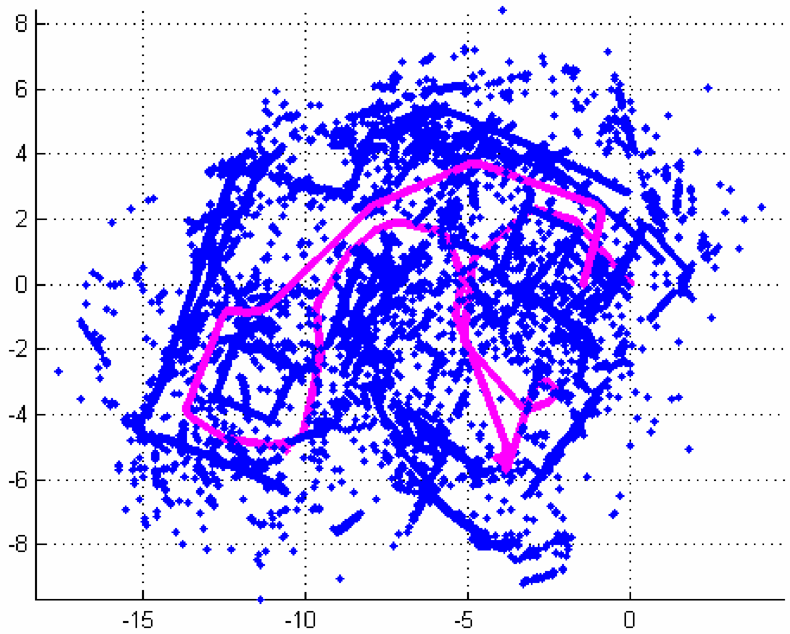
Robust statistics deal with spuriousness

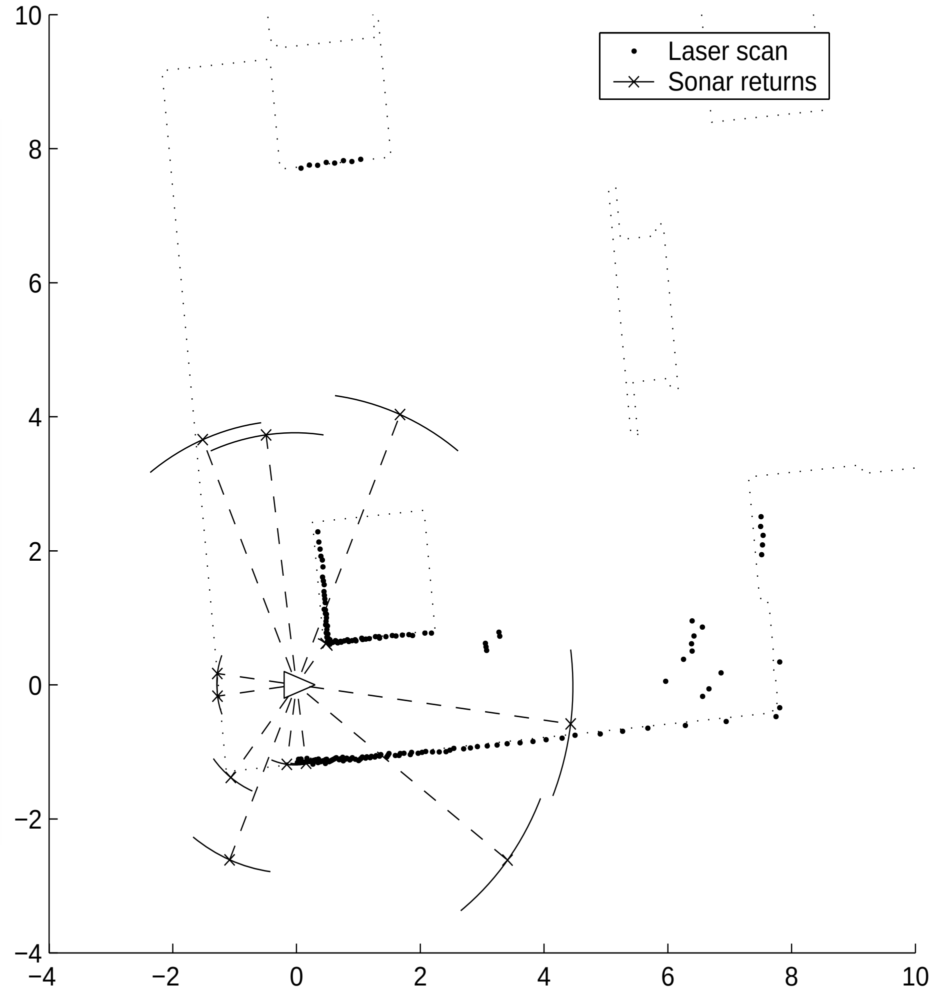
RANSAC for 3D planes



P.M. Newman, J.J. Leonard, J. Neira and J.D. Tardós: **Explore and Return: Experimental Validation of Real Time Concurrent Mapping and Localization**. IEEE Int. Conf. Robotics and Automation, May, 2002

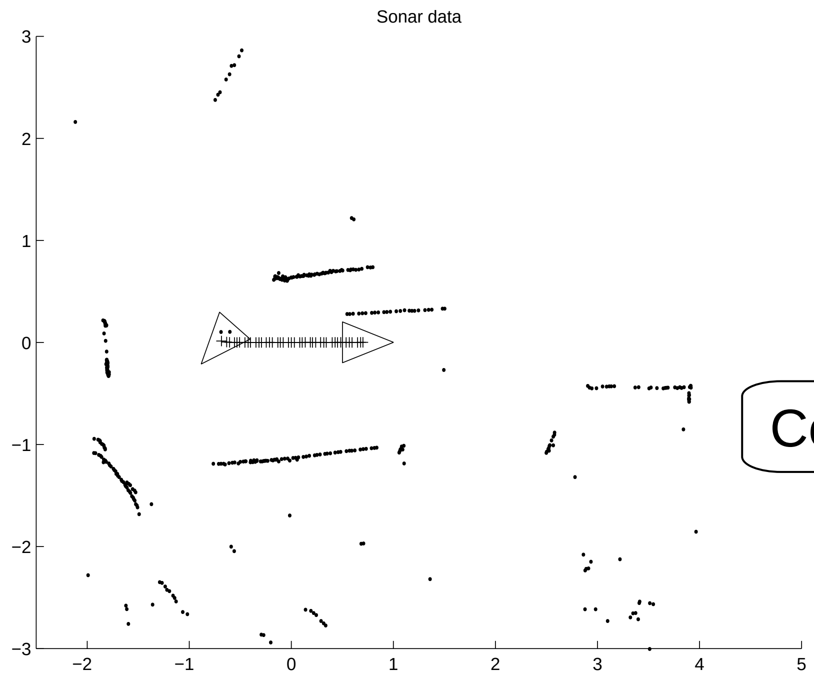
Sonar

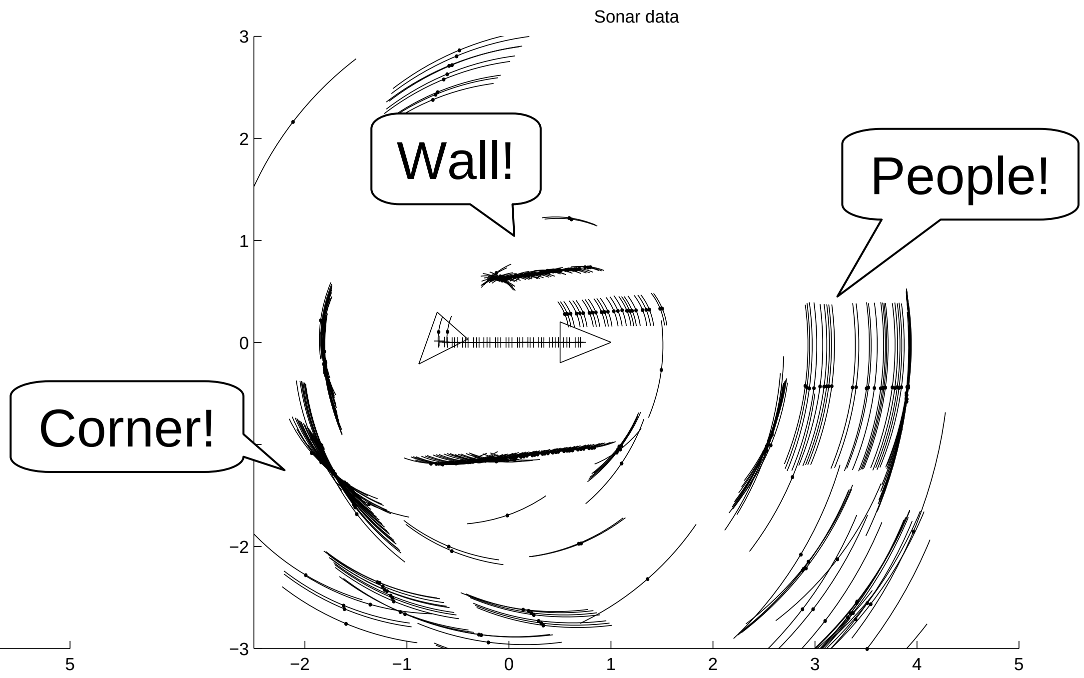




Very sparse and noisy data

Move and build a local map

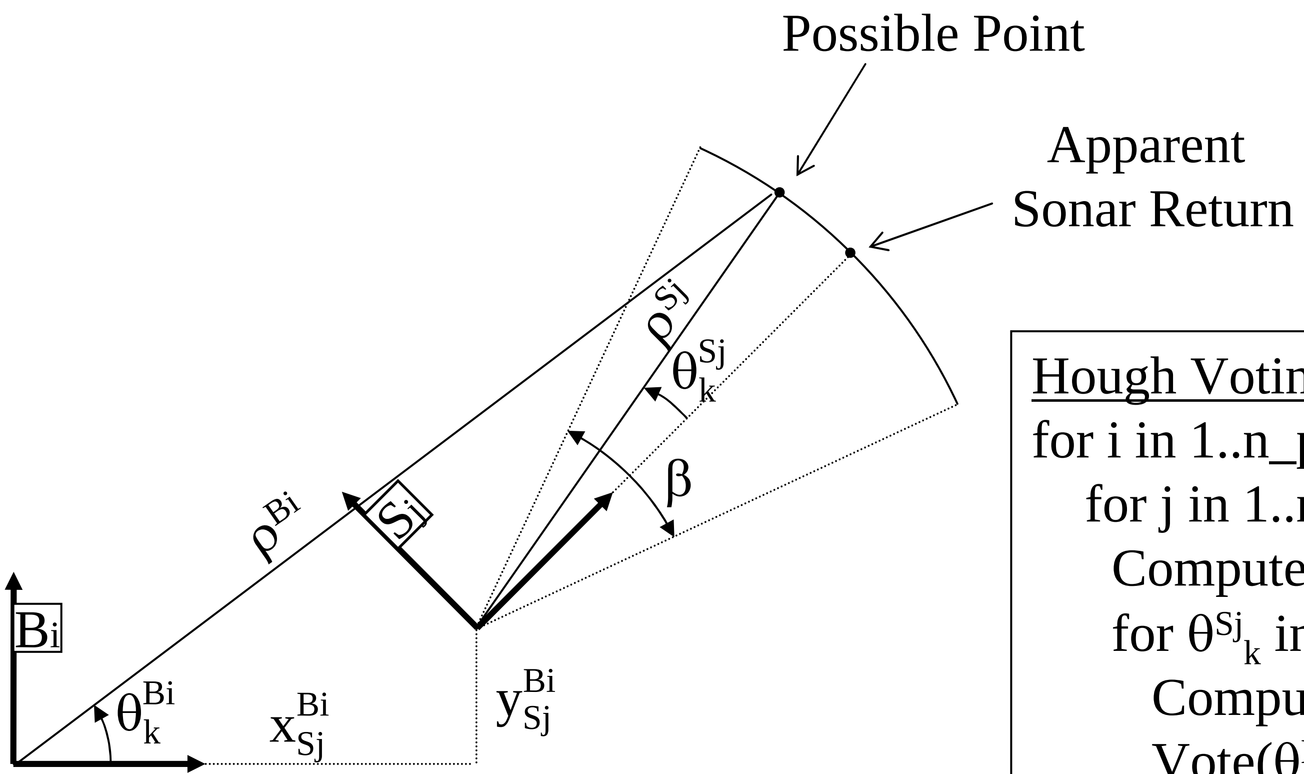




Exploit redundancy

Use a good sensor model

Sonar Model for Points

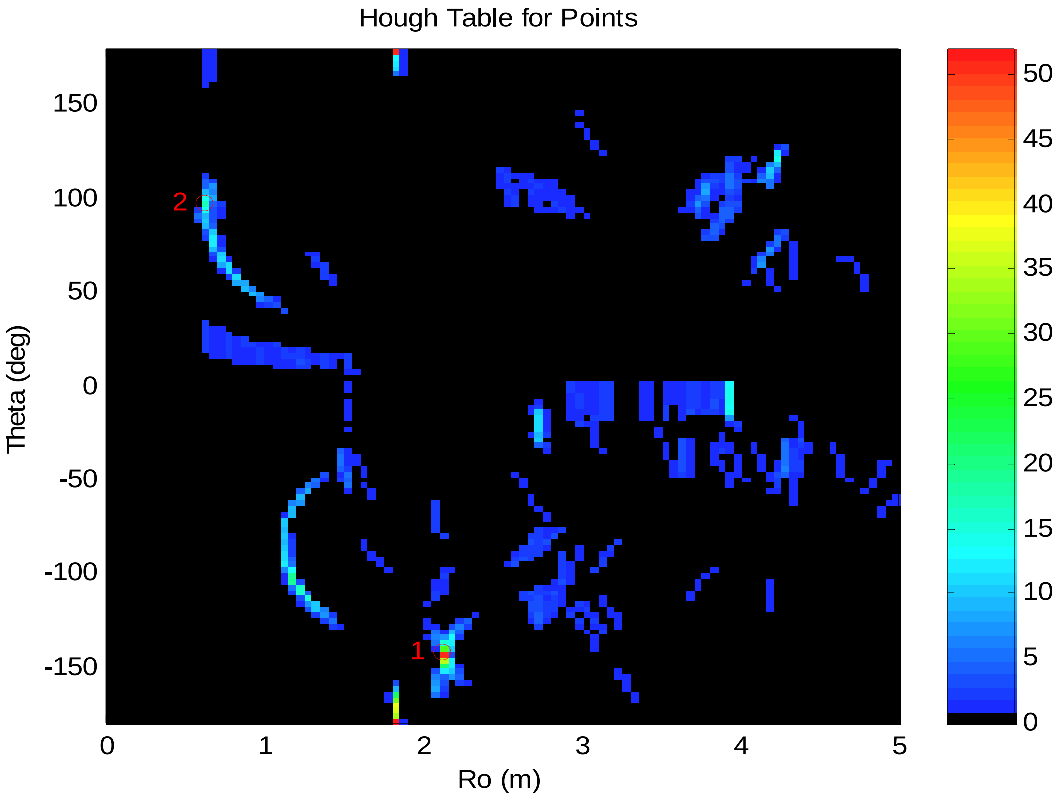


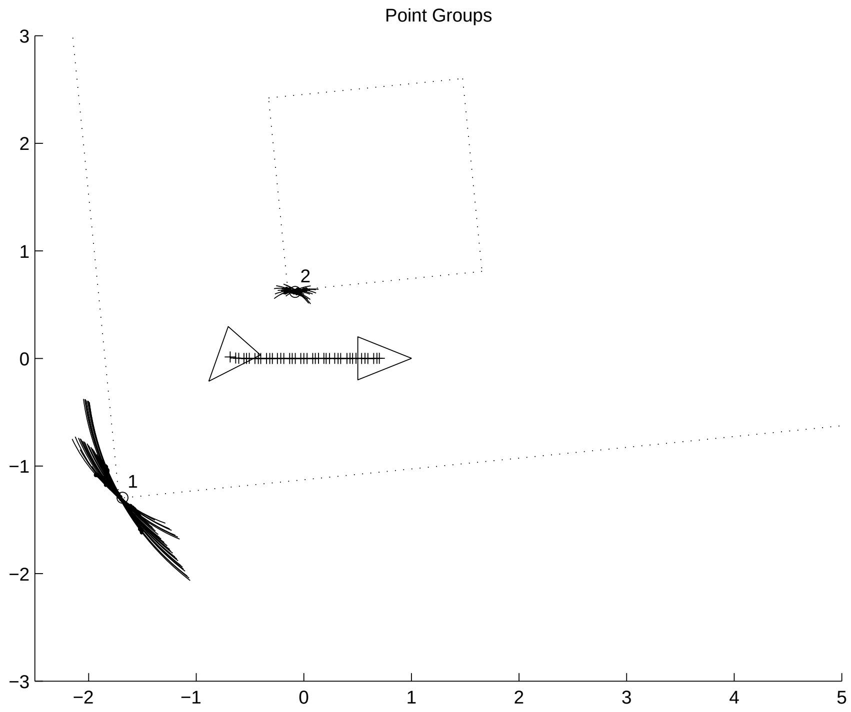
<u>Hough Voting</u>

for i in 1..n_positions
    for j in 1..n_sensors
        Compute $\mathbf{x}^{Bi}{}_{Sj}$
        for $\theta^{Sj}{}_k$ in $-\beta/2..\beta/2$ step δ
            Compute $\theta^{Bj}{}_k$ $\rho^{Bj}{}_k$
            Vote($\theta^{Bj}{}_k$, $\rho^{Bj}{}_k$)
        end
    end
end

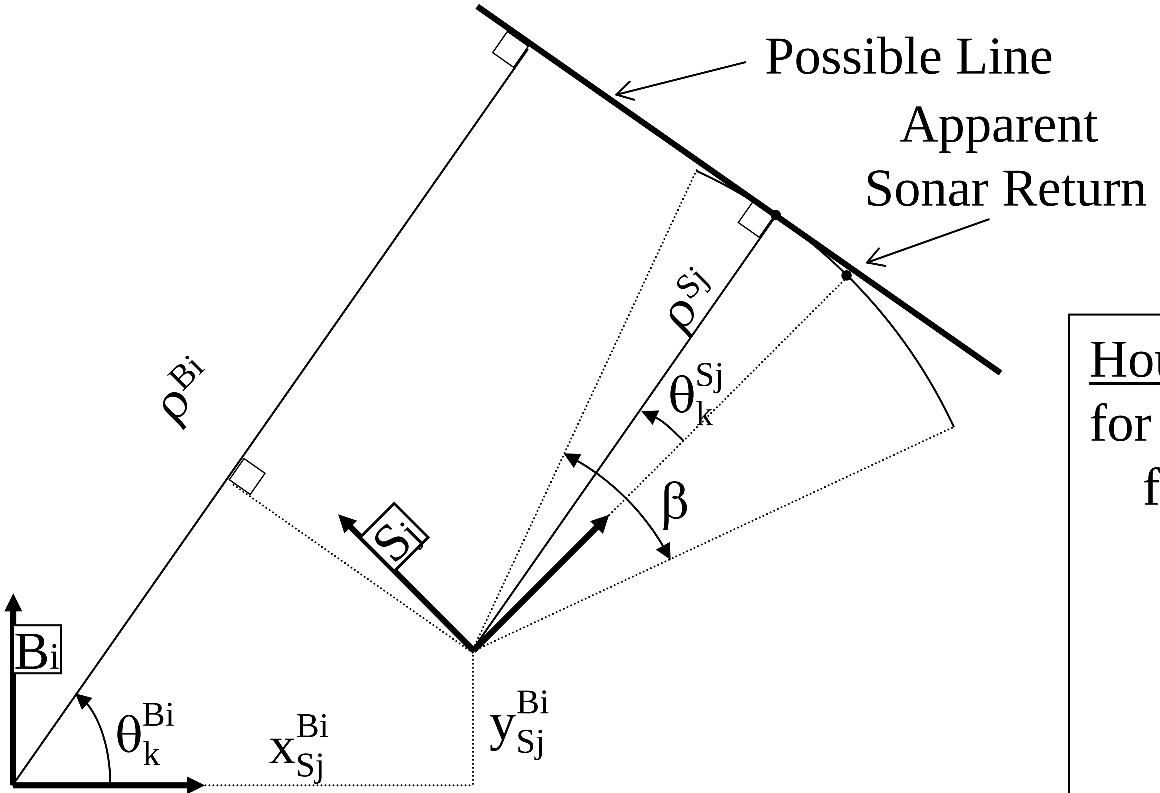
Hough Transform: Corners

- Sonar returns **vote** for points
- Look for local maxima





Sonar Model for Lines

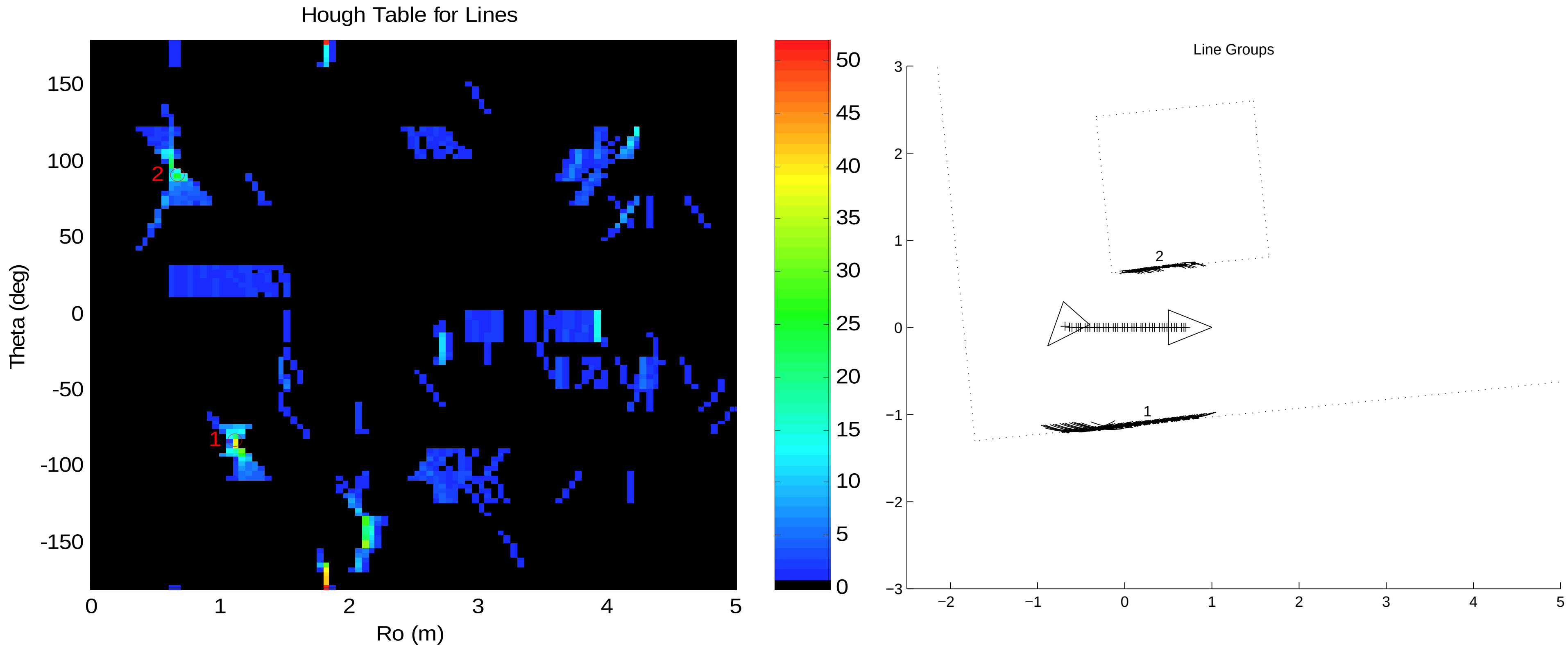


<u>Hough Voting</u>

for i in 1..n_positions
  for j in 1..n_sensors
    Compute $\mathbf{x}^{Bi}_{Sj}$
    for θ^{Sj}_k in -β/2..β/2 step δ
      Compute θ^{Bj}_k ρ^{Bj}_k
      Vote(θ^{Bj}_k, ρ^{Bj}_k)
    end
  end
end

Hough Transform: Lines

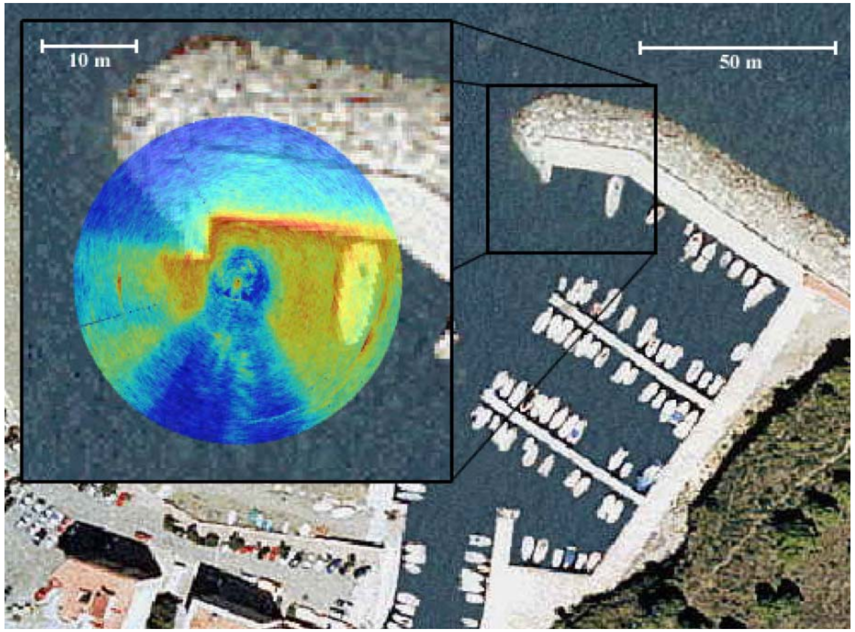
- Sonar returns **vote** for lines
- Look for local maxima

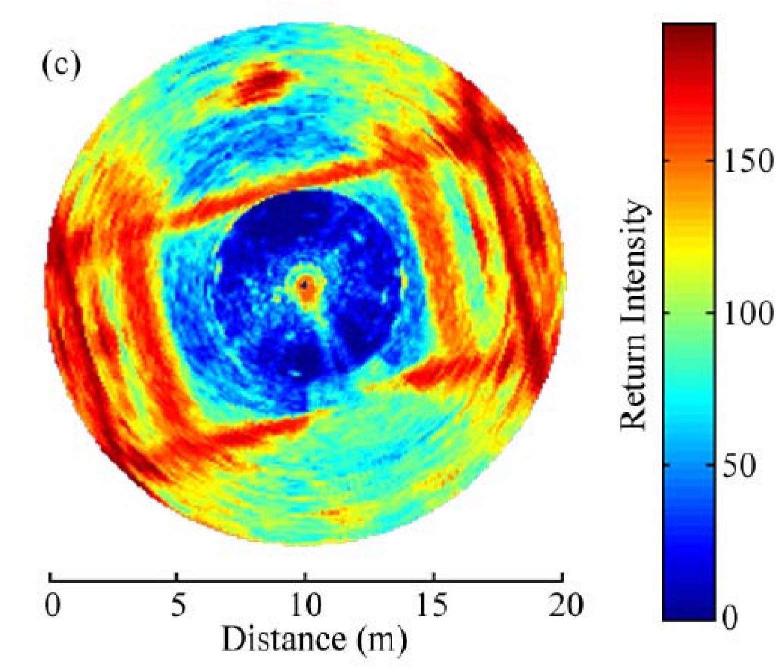


The Hough gives robust **local** data associations

Hough Transform

- Imaging sonar





Hough Transform

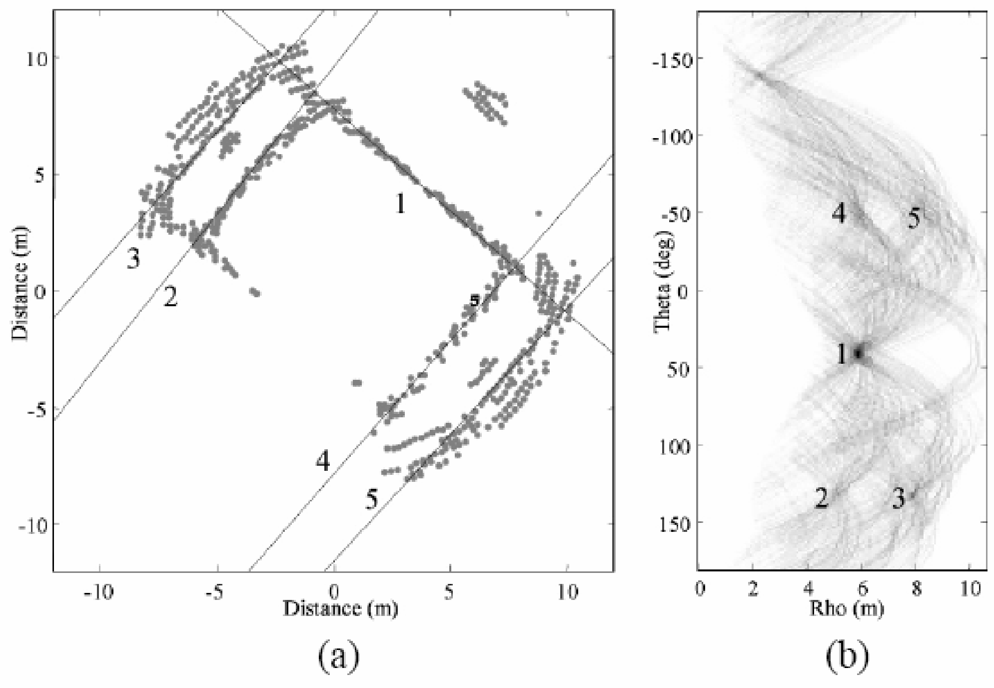
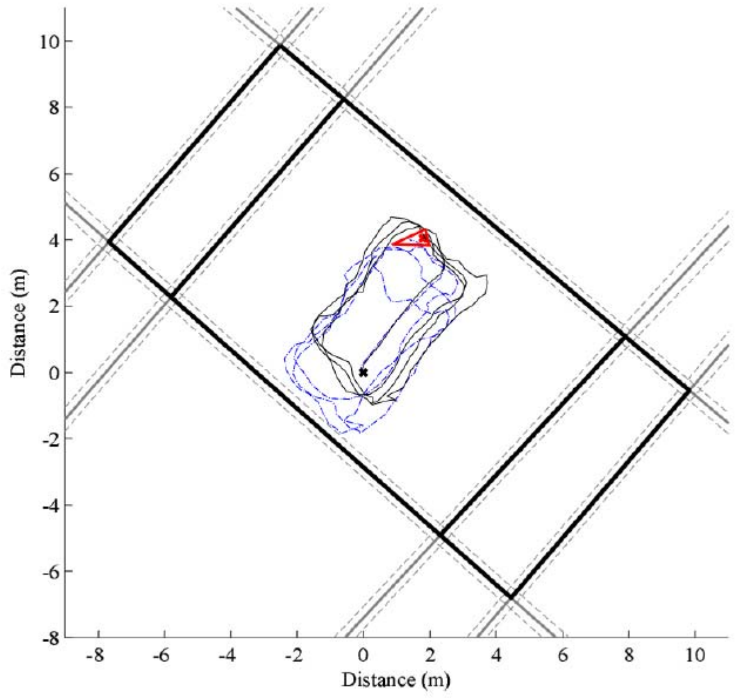


Fig. 3. Hough transform for line detection. (a) High echo-amplitude returns and the winning lines. (b)The obtained Hough voting space



D. Ribas, P. Ridao, J. Neira, J.D. Tardós, **SLAM using an Imaging Sonar for Partially Structured Underwater Environments**, The 2006 IEEE/RSJ International Conference on Intelligent Robots and Systems.

Outline

1. Basic EKF SLAM
    1. Introduction: the need for SLAM
    2. The basic EKF SLAM algorithm
    3. Feature extraction

2. The Data Association Problem
    1. **Introduction**
    2. Continuous Data Association
    3. The Loop Closing Problem
    4. The Global Localization Problem

3. Advanced EKF SLAM
    1. Computational complexity of EKF SLAM
    2. Consistency of the EKF SLAM
    3. SLAM using local maps
        1. Sequential Map Joining
        2. Divide and Conquer SLAM
        3. Hierarchical SLAM

EKF-SLAM

Algorithm 1 SLAM:

$\mathbf{x}_0^B = 0$; $\mathbf{P}_0^B = 0$ {*Map initialization*}

$[\mathbf{z}_0, \mathbf{R}_0]$ = get_measurements

$[\mathbf{x}_0^B, \mathbf{P}_0^B]$ = add_new_features$(\mathbf{x}_0^B, \mathbf{P}_0^B, \mathbf{z}_0, \mathbf{R}_0)$

for $k = 1$ to steps **do**

$[\mathbf{x}_{R_k}^{R_{k-1}}, \mathbf{Q}_k]$ = get_odometry

$[\mathbf{x}_{k|k-1}^B, \mathbf{P}_{k|k-1}^B]$ = compute_motion$(\mathbf{x}_{k-1}^B, \mathbf{P}_{k-1}^B, \mathbf{x}_{R_k}^{R_{k-1}}, \mathbf{Q}_k)$ {*EKF prediction*}

$[\mathbf{z}_k, \mathbf{R}_k]$ = get_measurements

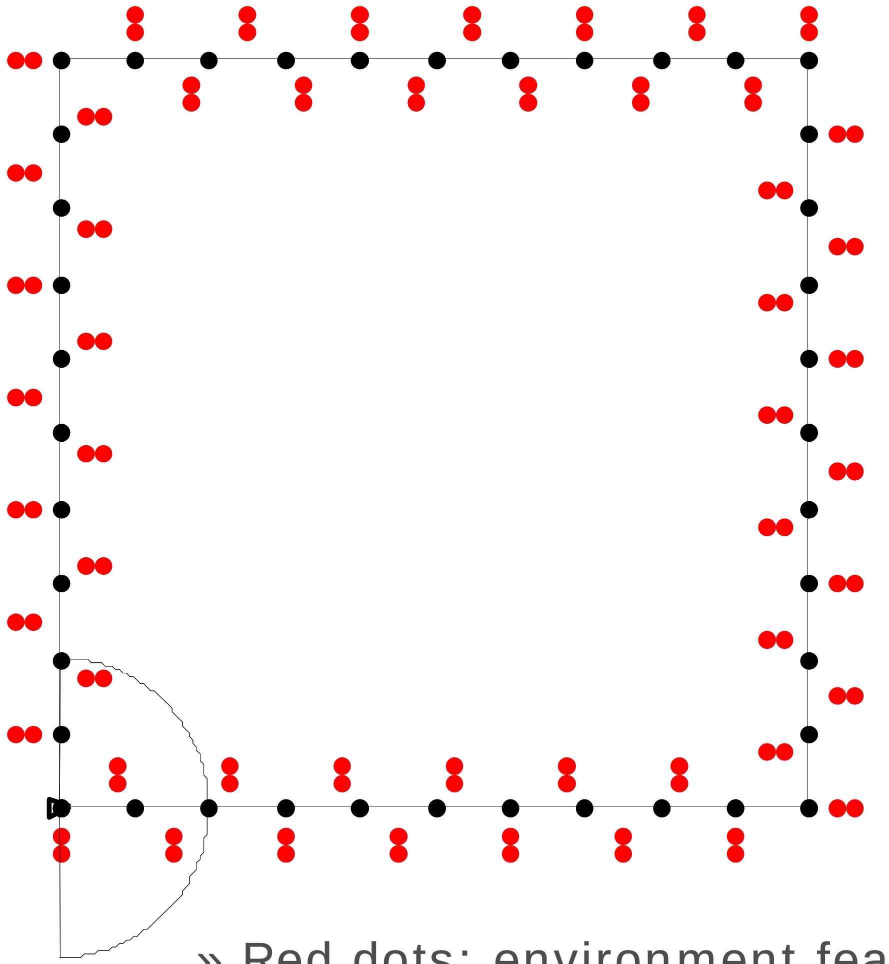
$\mathcal{H}_k$ = data_association$(\mathbf{x}_{k|k-1}^B, \mathbf{P}_{k|k-1}^B, \mathbf{z}_k, \mathbf{R}_k)$

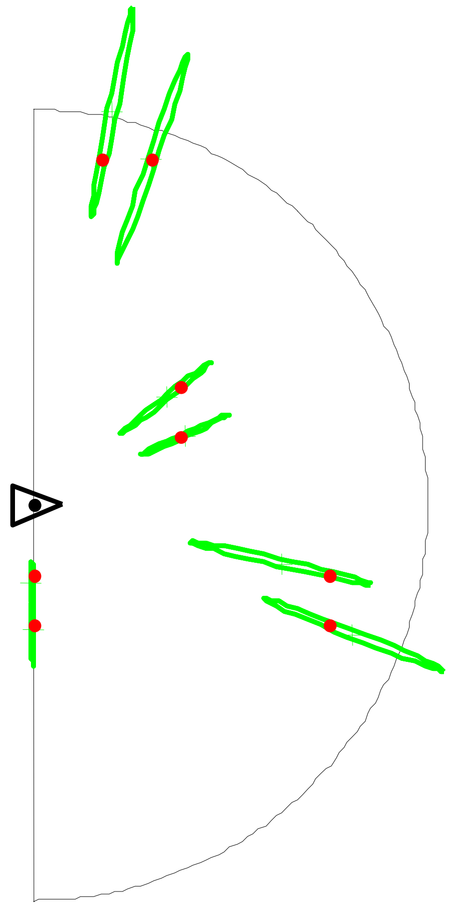
$[\mathbf{x}_k^B, \mathbf{P}_k^B]$ = update_map$(\mathbf{x}_{k|k-1}^B, \mathbf{P}_{k|k-1}^B, \mathbf{z}_k, \mathbf{R}_k, \mathcal{H}_k)$ {*EKF update*}

$[\mathbf{x}_k^B, \mathbf{P}_k^B]$ = add_new_features$(\mathbf{x}_k^B, \mathbf{P}_k^B, \mathbf{z}_k, \mathbf{R}_k, \mathcal{H}_k)$

end for

Example: SLAM in a cloister





» Red dots: environment features (columns)
» Black line: robot trajectory
» Black semicircle: sensor range

The Data Association Problem

- ***n*** map features: $\mathcal{F} = \{F_1 \dots F_n\}$

- ***m*** sensor measurements: $\mathcal{E} = \{E_1 \dots E_m\}$

- Data association should return a hypothesis that associates each observation E_i with a feature F_{j_i}

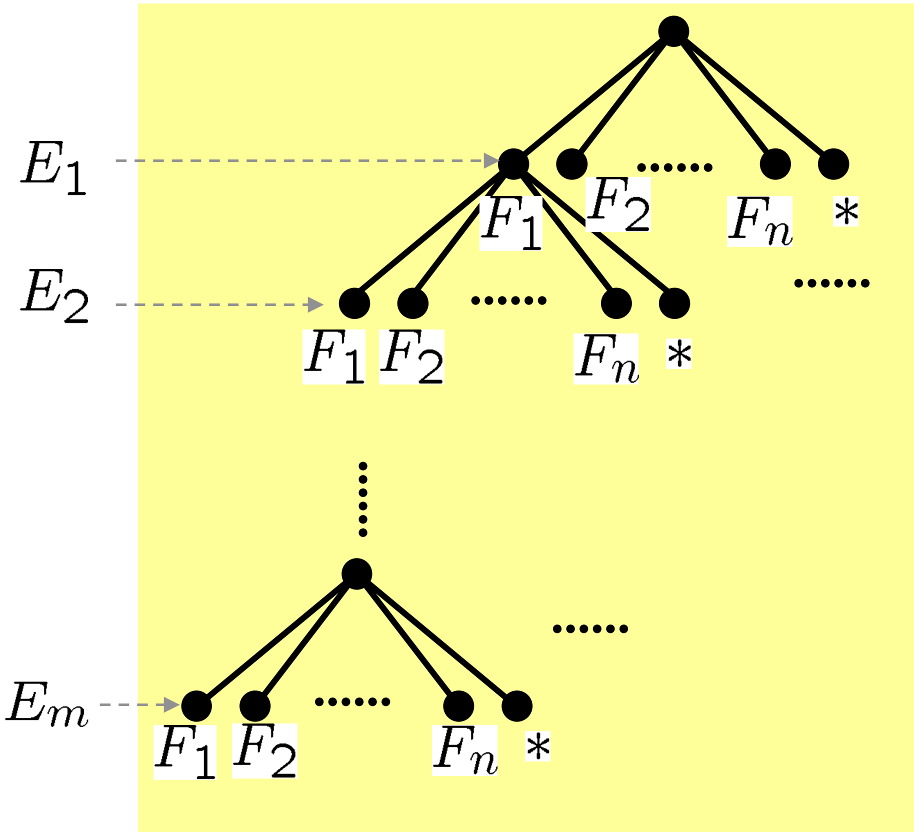
$$\mathcal{H}_m = [j_1 \dots j_i \dots j_m]$$

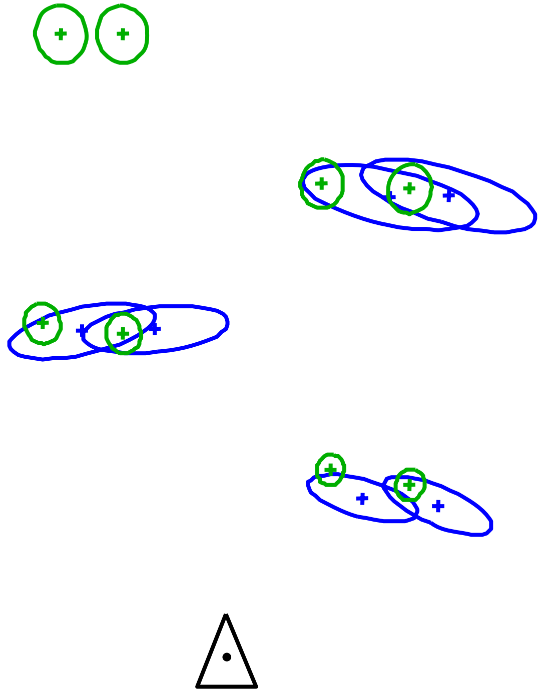
$$E_i \leftarrow j_i \rightarrow F_{j_i}$$

- Non matched observations: $j_i = 0$

The Correspondence Space

Interpretation tree
(Grimson et al. 87):



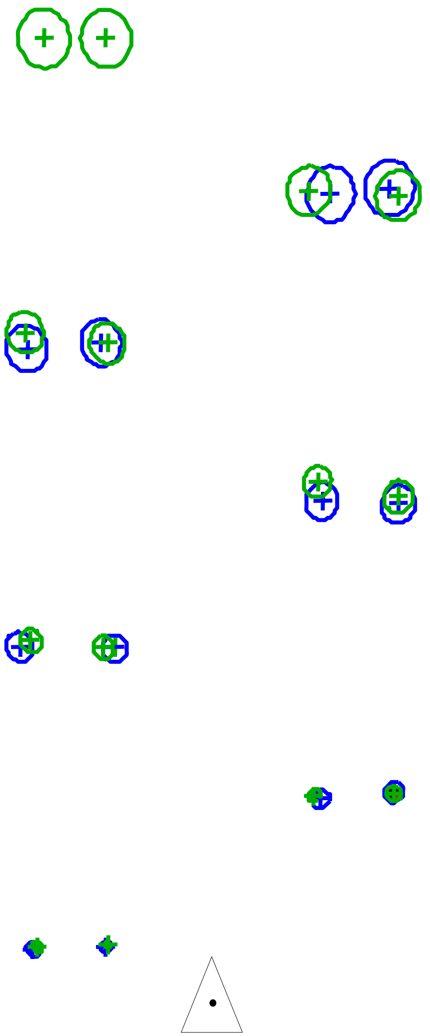


Green points: measurements
Blue Points: predicted features

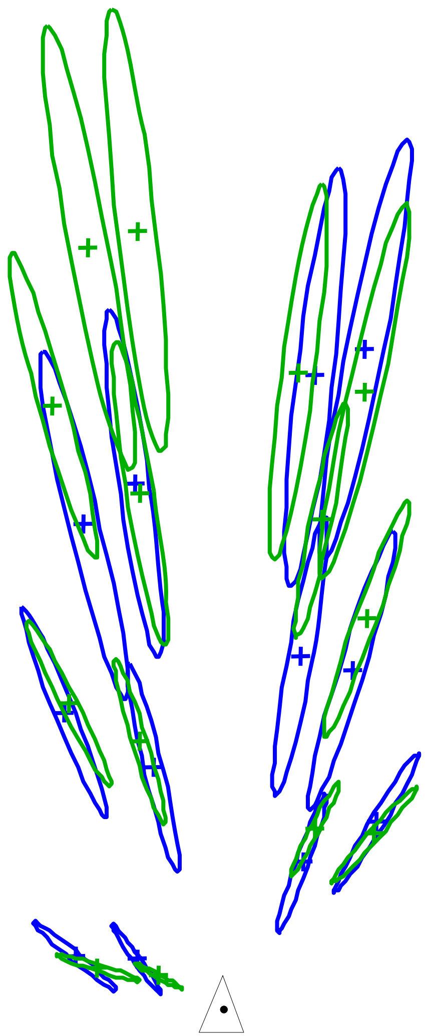
$(n+1)^m$ possible hypotheses

When data association is difficult

- Low sensor error

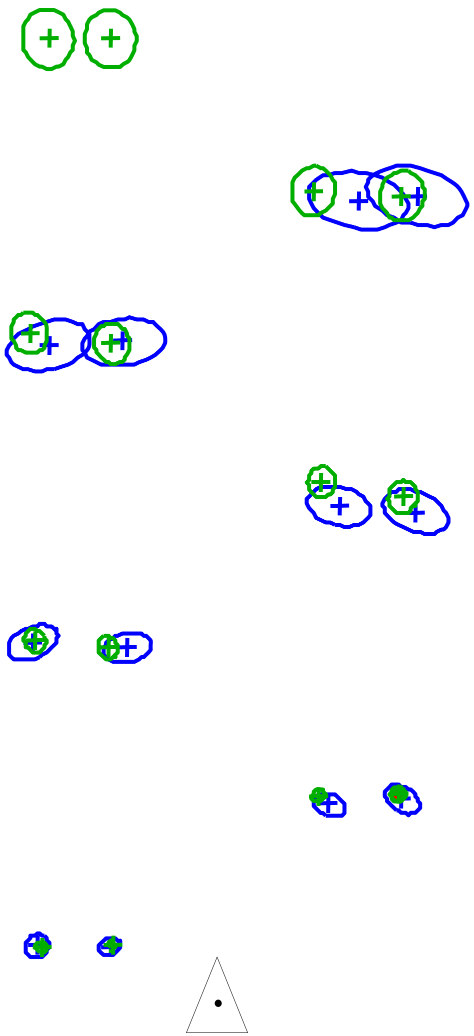


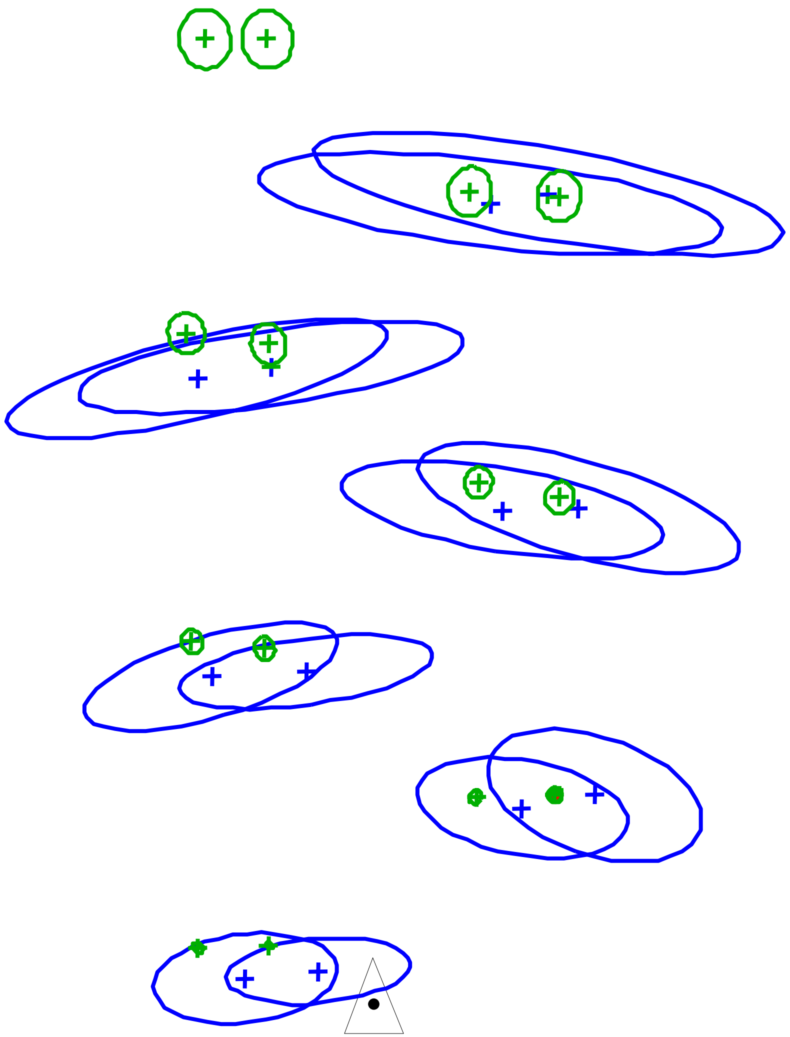
- High sensor error



When data association is difficult

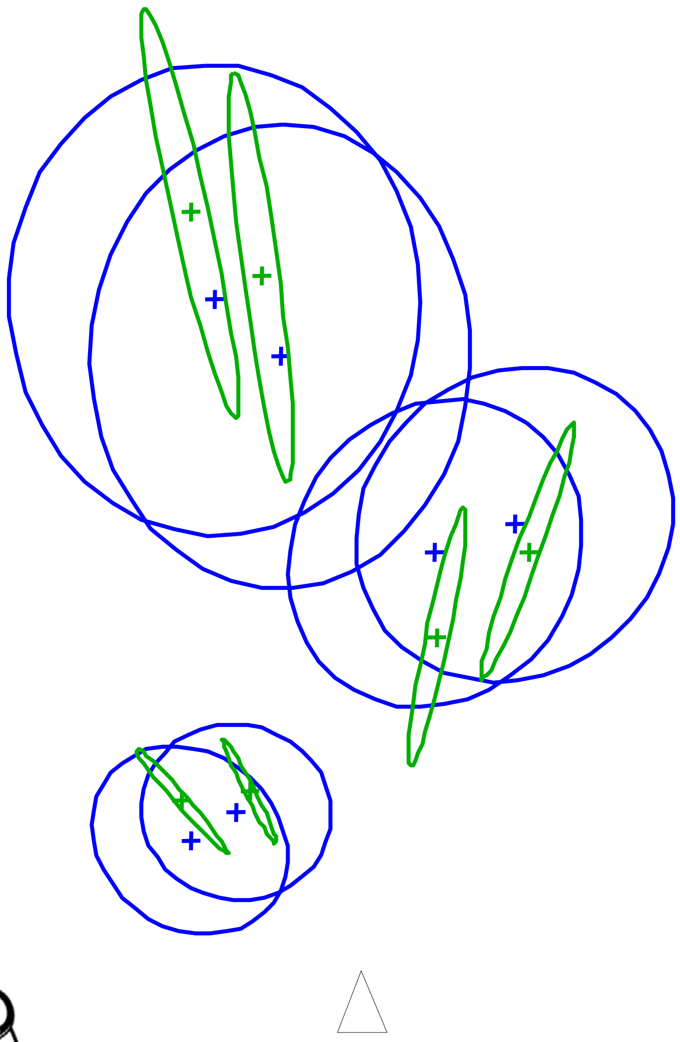
- Low odometry error
- High odometry error



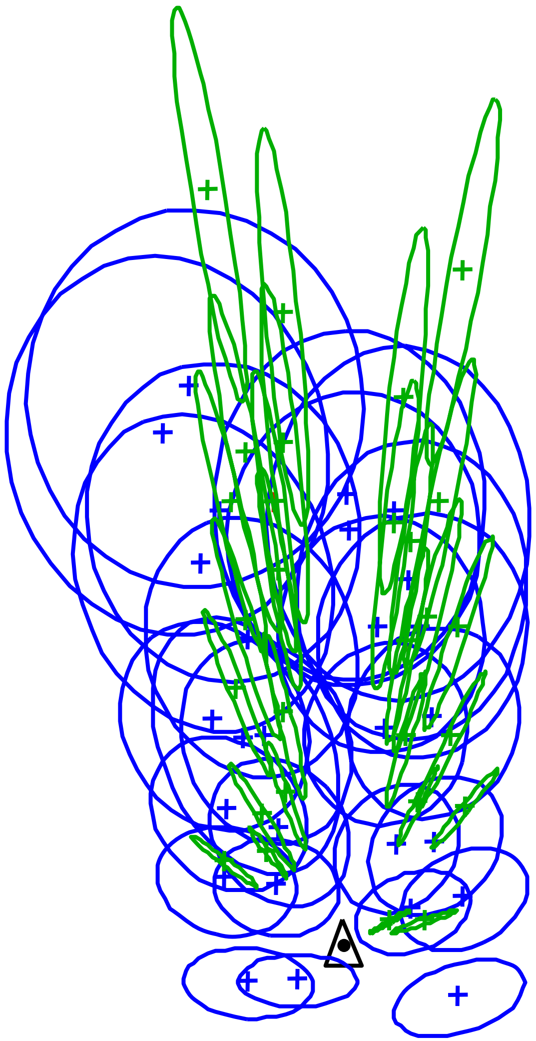


When data association is difficult

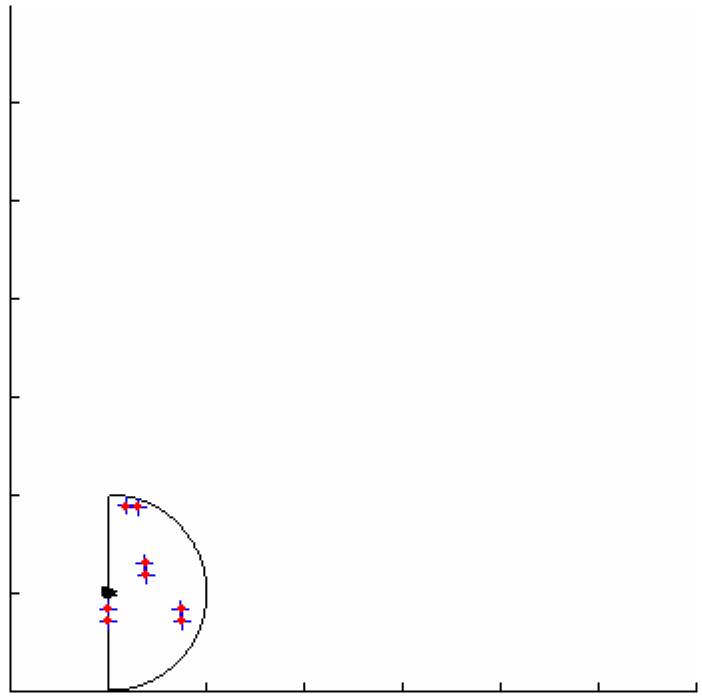
- Low feature density



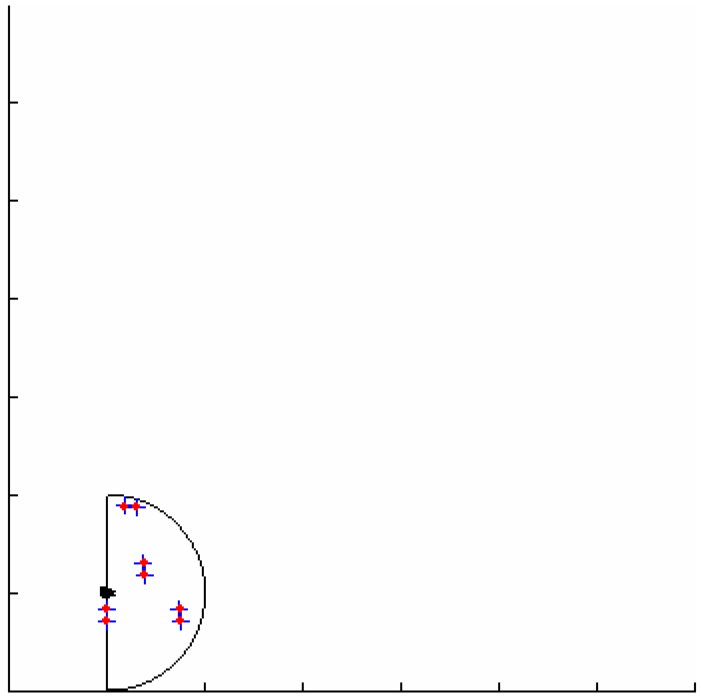
- High feature density



How important is data association?

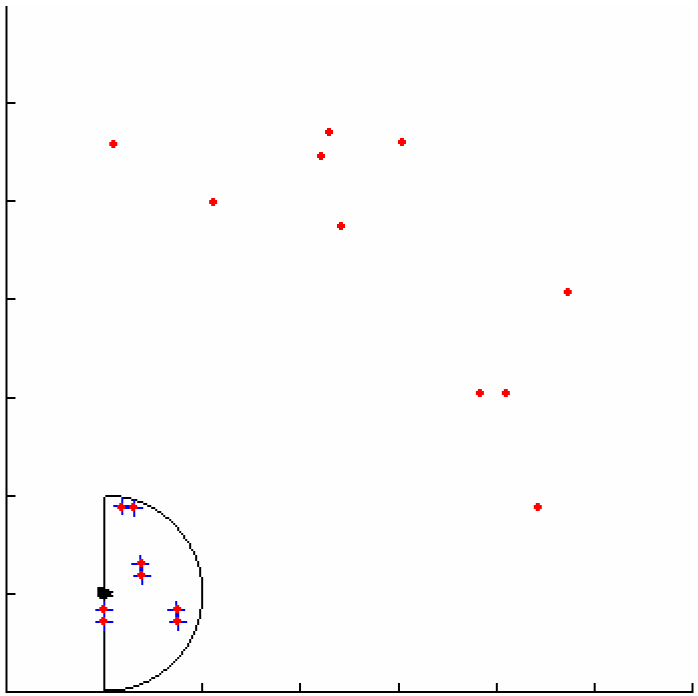


A good algorithm

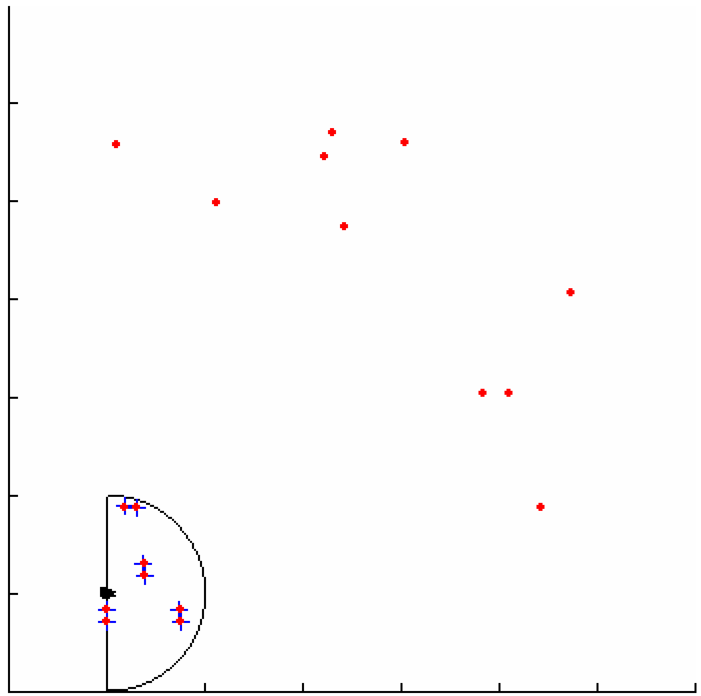


A **bad** algorithm

Why it's difficult?



A good algorithm



A **bad** algorithm

Importance of Data Association

- EKF update:

Values that depend on $\mathcal{H}_m$

$$\hat{\mathbf{x}}_k^B = \hat{\mathbf{x}}_{k|k-1}^B + \mathbf{K}_k \nu_k$$

$$\mathbf{P}_k^B = (\mathbf{I} - \mathbf{K}_k \mathbf{H}_k)\mathbf{P}_{k|k-1}^B$$

$$\mathbf{K}_k = \mathbf{P}_{k|k-1}^B \mathbf{H}_k^T (\mathbf{H}_k \mathbf{P}_{k|k-1}^B \mathbf{H}_k^T + \mathbf{R}_k)^{-1}$$

- If the association of $\boldsymbol{E_i}$ with feature $\boldsymbol{F_j}$ is.....

		correct:	spurious:
error:	$\mathbf{x} - \hat{\mathbf{x}}$	↓	↑
covariance:	P	↓	↓
		Consistency	Divergence!

Outline

1. Basic EKF SLAM
   1. Introduction: the need for SLAM
   2. The basic EKF SLAM algorithm
   3. Feature extraction

2. The Data Association Problem
   1. Introduction
   2. **Continuous Data Association**
   3. The Loop Closing Problem
   4. The Global Localization Problem

3. Advanced EKF SLAM
   1. Computational complexity of EKF SLAM
   2. Consistency of the EKF SLAM
   3. SLAM using local maps
      1. Sequential Map Joining
      2. Divide and Conquer SLAM
      3. Hierarchical SLAM

Individual Compatibility

- Measurement equation for observation E_i and feature F_j

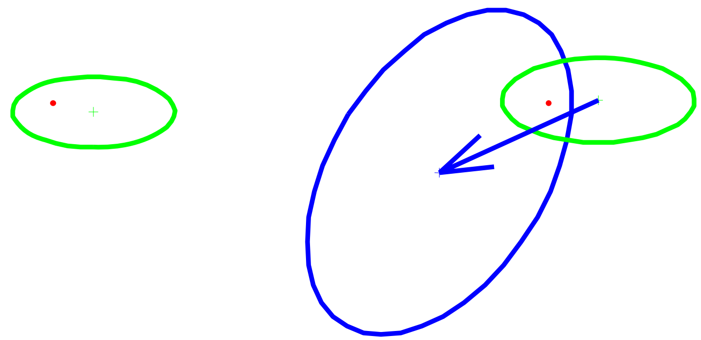
$$\mathbf{z}_i = \mathbf{h}_{ij}(\mathbf{x}^B) + \mathbf{w}_i$$
$$\mathbf{z}_i \simeq \mathbf{h}_{ij}(\hat{\mathbf{x}}^B) + \mathbf{H}_{ij}(\mathbf{x}^B - \hat{\mathbf{x}}^B)$$

$$E[\mathbf{w}_i \mathbf{w}_i^T] = \mathbf{R}_i$$
$$\mathbf{H}_{ij} = \left.\frac{\partial \mathbf{h}_{ij}}{\partial \mathbf{x}^B}\right|_{(\hat{\mathbf{x}}^B)}$$

- E_i and F_j are compatible if:

$$D_{ij}^2 = (\mathbf{z}_i - \mathbf{h}_{ij}(\hat{\mathbf{x}}^B))^T \mathbf{P}_{ij}^{-1} (\mathbf{z}_i - \mathbf{h}_{ij}(\hat{\mathbf{x}}^B)) < \chi^2_{d,\alpha}$$
$$\mathbf{P}_{ij} = \mathbf{H}_{ij} \mathbf{P}^B \mathbf{H}_{ij}^T + \mathbf{R}_i$$

d = length(z_i)

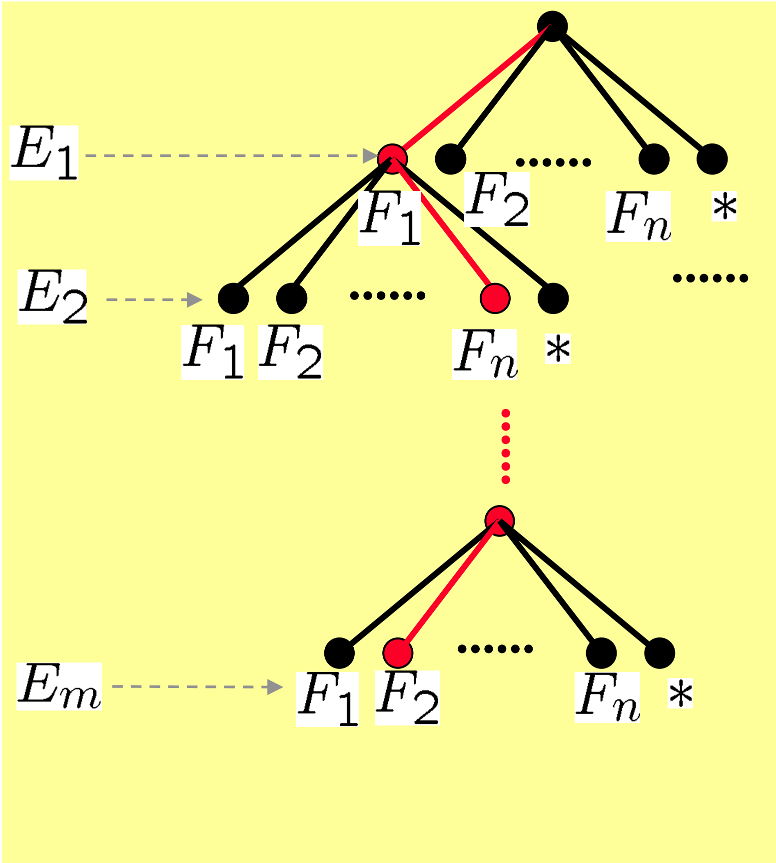


Nearest Neighbor

Algorithm 2 Individual Compatibility Nearest Neighbor ICNN $(E_{1\cdots m}, F_{1\cdots n})$

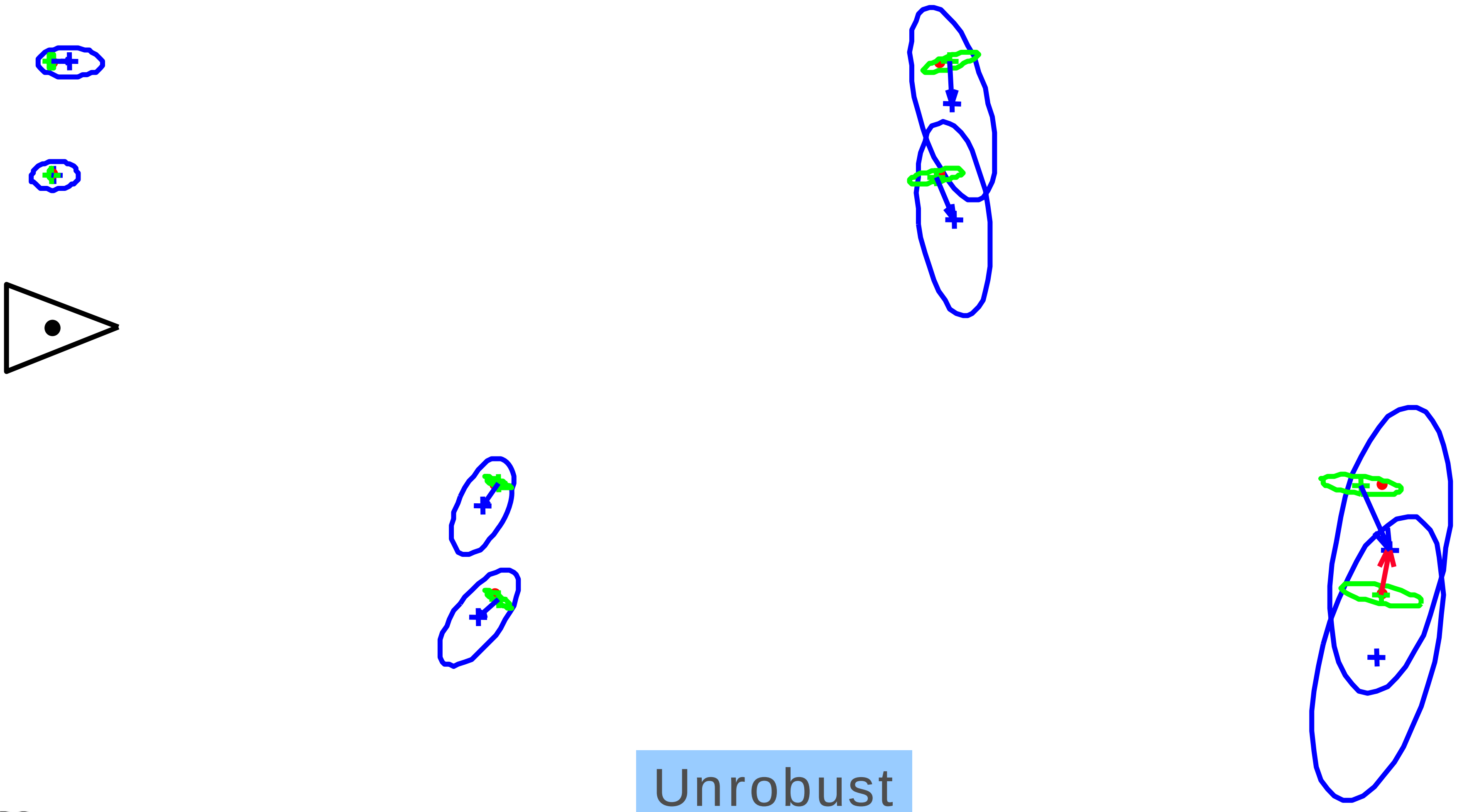
```
for i = 1 to m do {measurement E_i}
  D²_min ← mahalanobis2(E_i, F_1)
  nearest ← 1
  for j = 2 to n do {feature F_j}
    D²_ij ← mahalanobis2(E_i, F_j)
    if D²_ij < D²_min then
      nearest ← j
      D²_min ← D²_ij
    end if
  end for
  if D²_min ≤ χ²_{d_i,1−α} then
    H_i ← nearest
  else
    H_i ← 0
  end if
end for
return H
```

Greedy algorithm: *O(mn)*



ROBOTICS UNIVERSIDAD DE ZARAGOZA

The Fallacy of the Nearest Neighbor



Unrobust

Joint Compatibility

- Given a hypothesis $\mathcal{H} = [j_1, j_2, \cdots, j_s]$
- Joint measurement equation

$$\mathbf{z}_{\mathcal{H}} = \mathbf{h}_{\mathcal{H}}(\mathbf{x}^B) + \mathbf{w}_{\mathcal{H}}$$

$$\mathbf{h}_{\mathcal{H}} = \begin{bmatrix} \mathbf{h}_{1j_1} \\ \mathbf{h}_{2j_2} \\ \vdots \\ \mathbf{h}_{sj_s} \end{bmatrix}$$

- The joint hypothesis is compatible if:

$$D^2_{\mathcal{H}} = (\mathbf{z}_{\mathcal{H}} - \mathbf{h}_{\mathcal{H}}(\hat{\mathbf{x}}^B))^T C_{\mathcal{H}}^{-1} (\mathbf{z}_{\mathcal{H}} - \mathbf{h}_{\mathcal{H}}(\hat{\mathbf{x}}^B)) < \chi^2_{d,\alpha}$$

$$C_{\mathcal{H}} = \mathbf{H}_{\mathcal{H}} \mathbf{P}^B \mathbf{H}_{\mathcal{H}}^T + \mathbf{R}_{\mathcal{H}}$$

d = length(z)

Joint Compatibility Branch and Bound

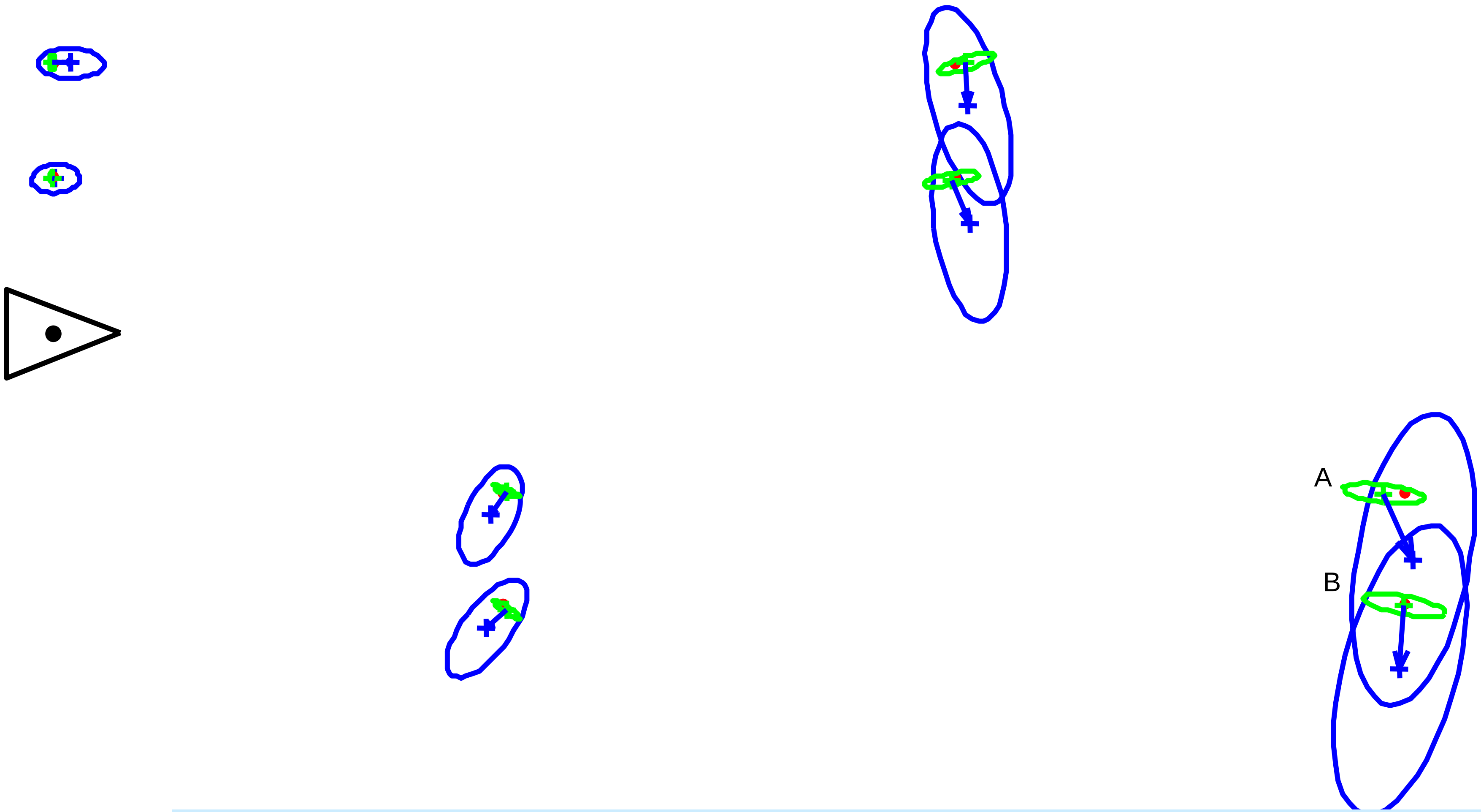
- Find the largest hypothesis with **jointly consistent** pairings

```
procedure JCBB (H, i):   -- find pairings for observation E_i

if i > m -- leaf node?
   if pairings(H) > pairings(Best)
      Best = H
   fi
else
   for j in {1...n}
      if individual_compatibility(i, j) and then
         joint_compatibility(H, i, j)
         JCBB([H j], i + 1) -- pairing (E_i, F_j) accepted
      fi
   rof
   if pairings(H) + m - i> pairings(Best) -- can do better?
      JCBB([H 0], i + 1) -- star node, E_i not paired
   fi
fi
```

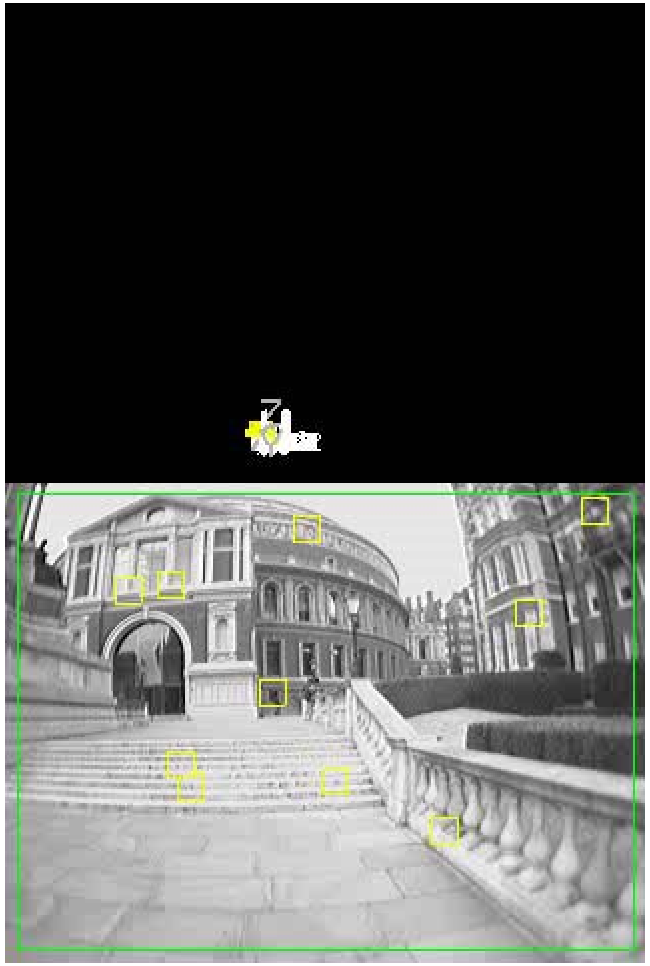
Selects the largest set of pairings where there is **consensus**

The Fallacy of the Nearest Neighbor

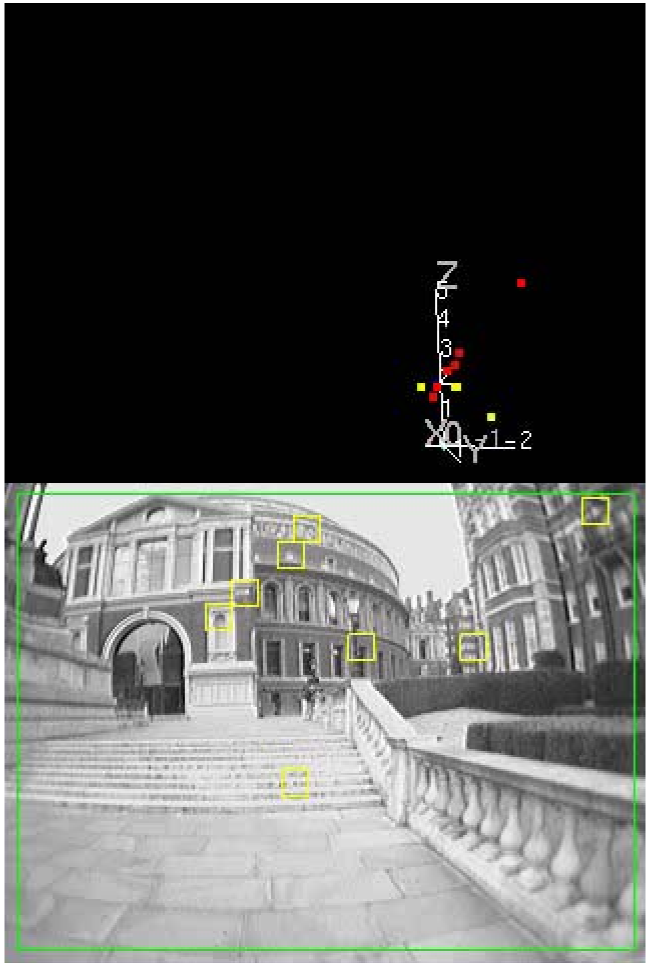


J. Neira, J.D. Tardós. **Data Association in Stochastic Mapping using the Joint Compatibility Test** IEEE Trans. Robotics and Automation, Vol. 17, No. 6, Dec 2001, pp 890 –897

Nearest neighbor .vs. Joint Compatibility



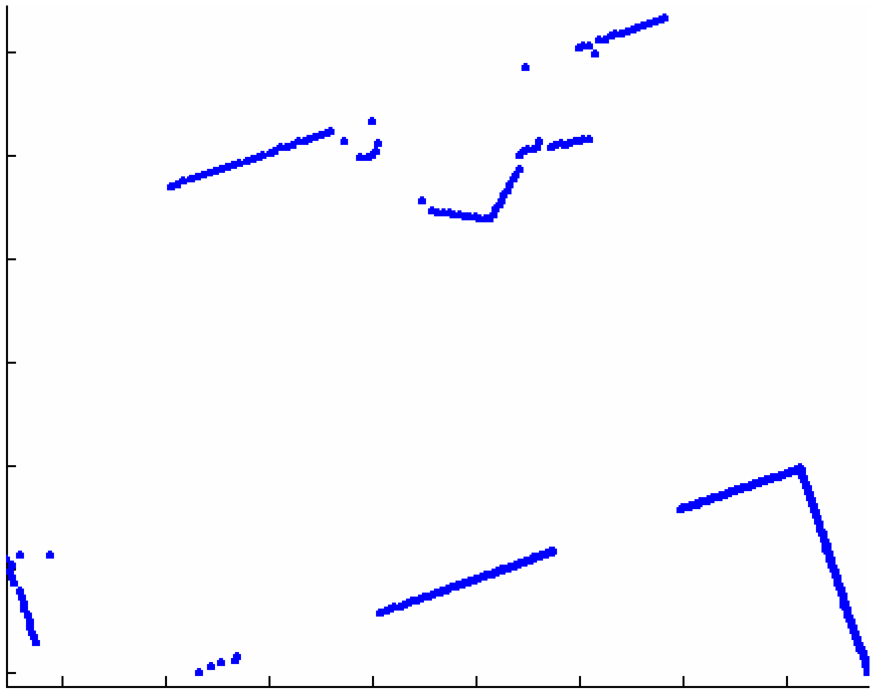
NN

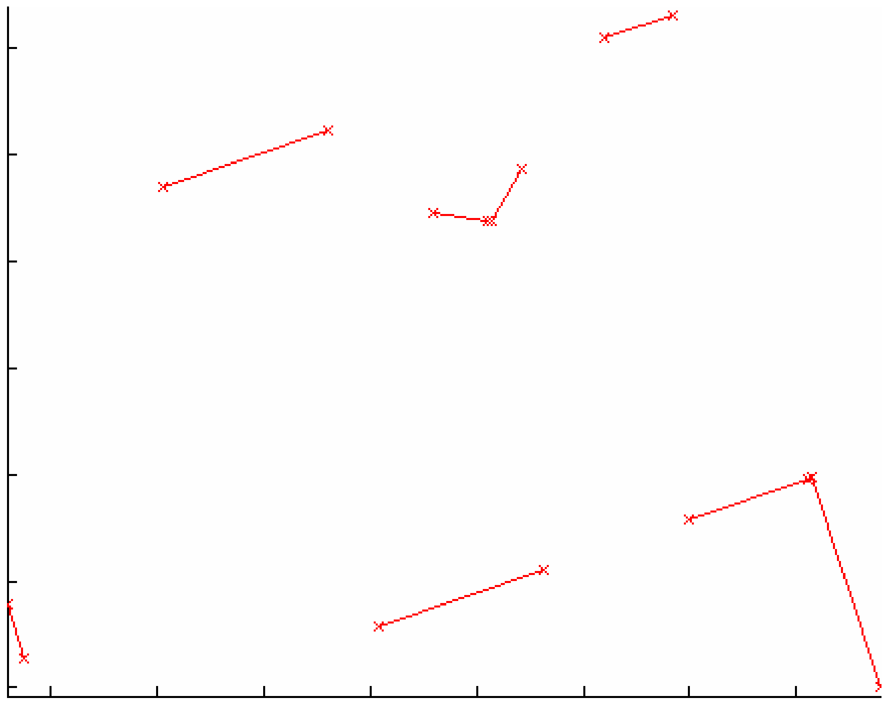


JCBB

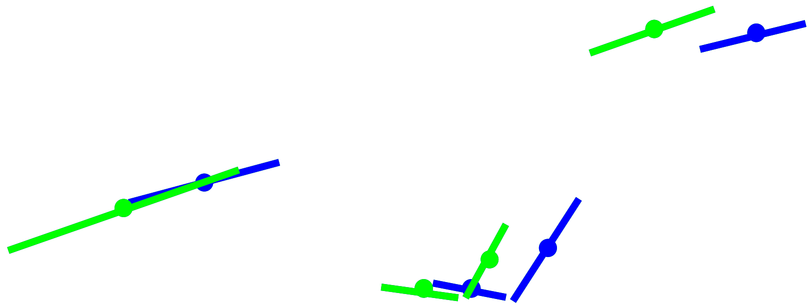
SLAM without odometry

- No estimation of the vehicle motion
- Segments in the environment

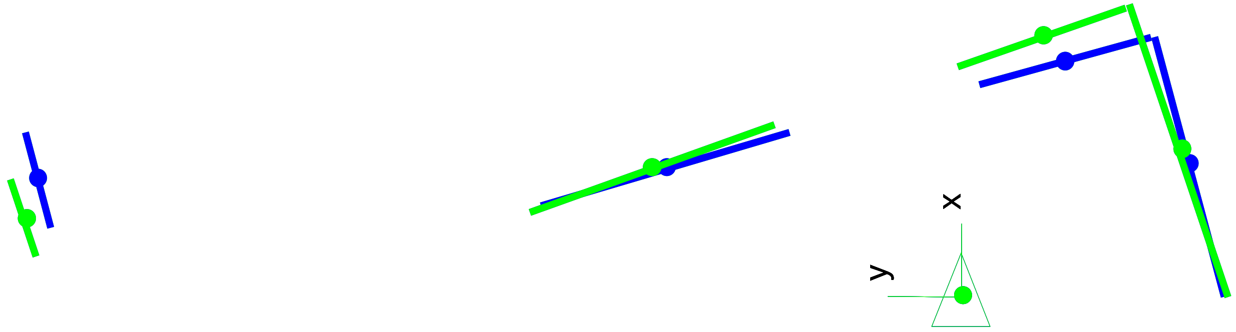




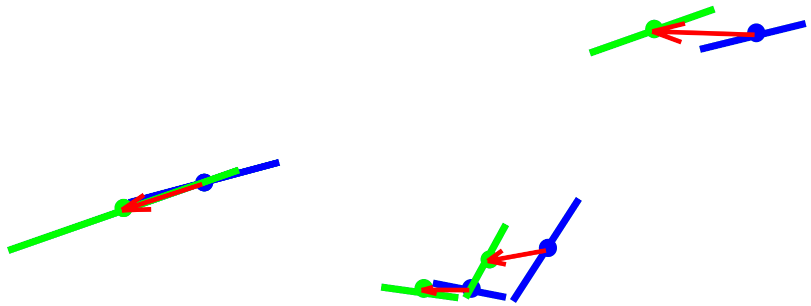
SLAM without odometry



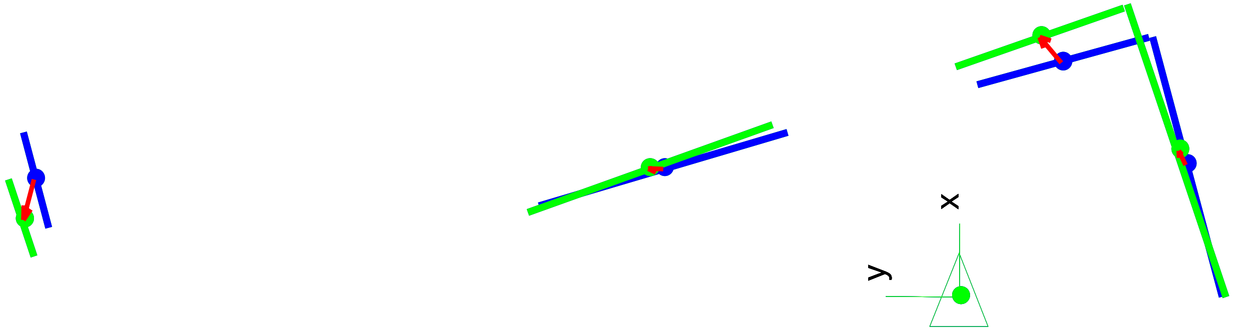
Assuming small motions



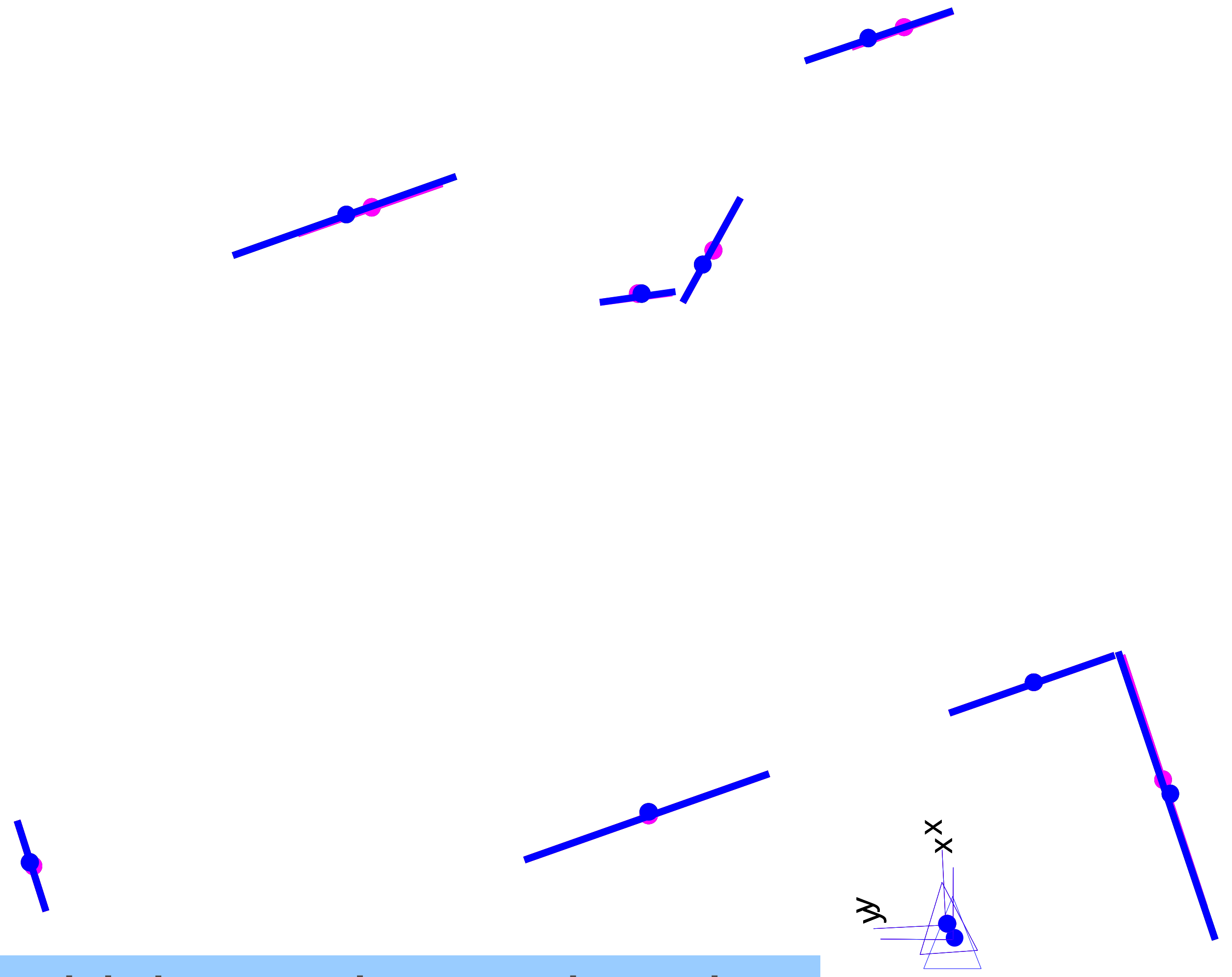
SLAM without odometry



Data association using Joint Compatibility

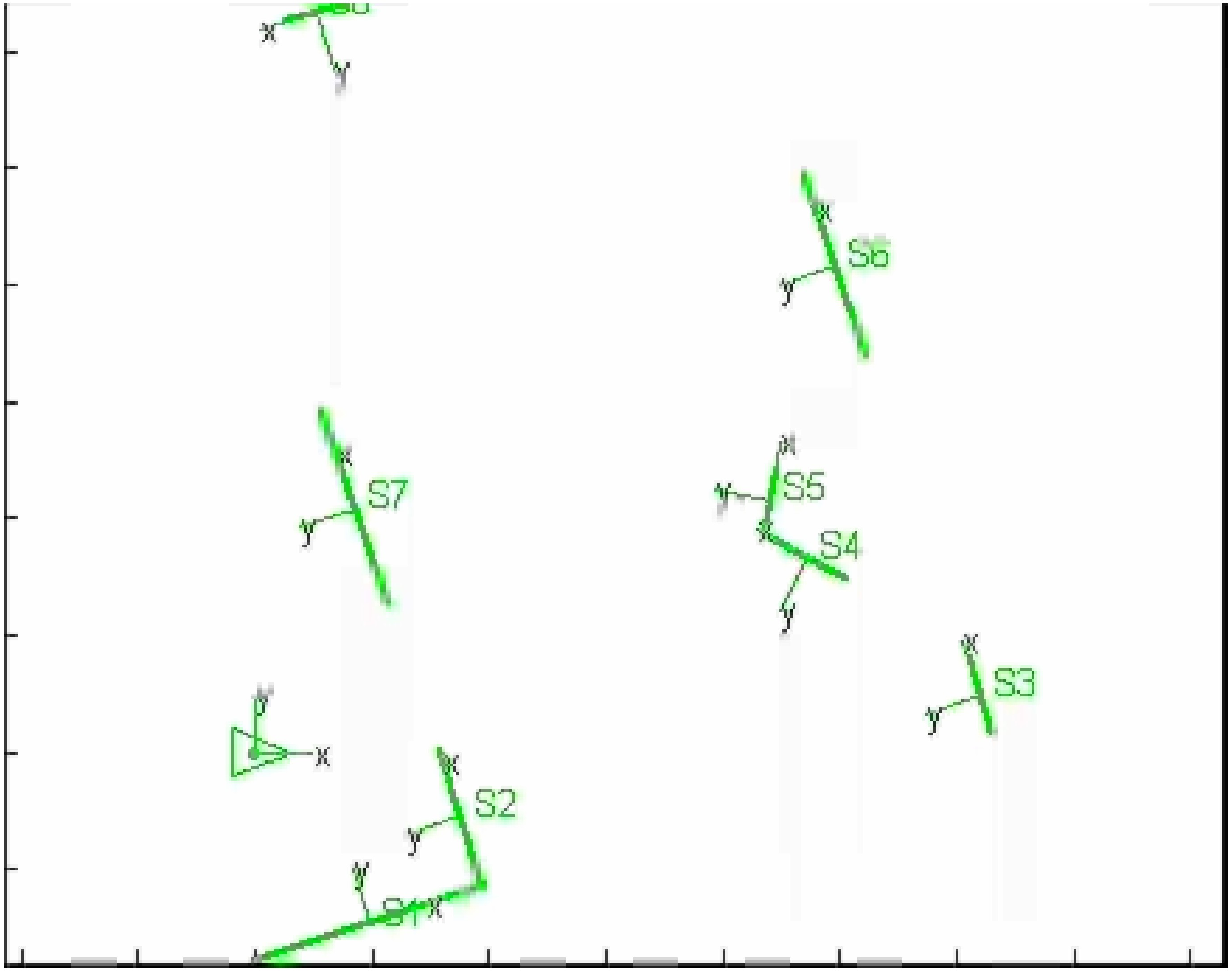


SLAM without odometry



Vehicle motion estimation

SLAM without odometry



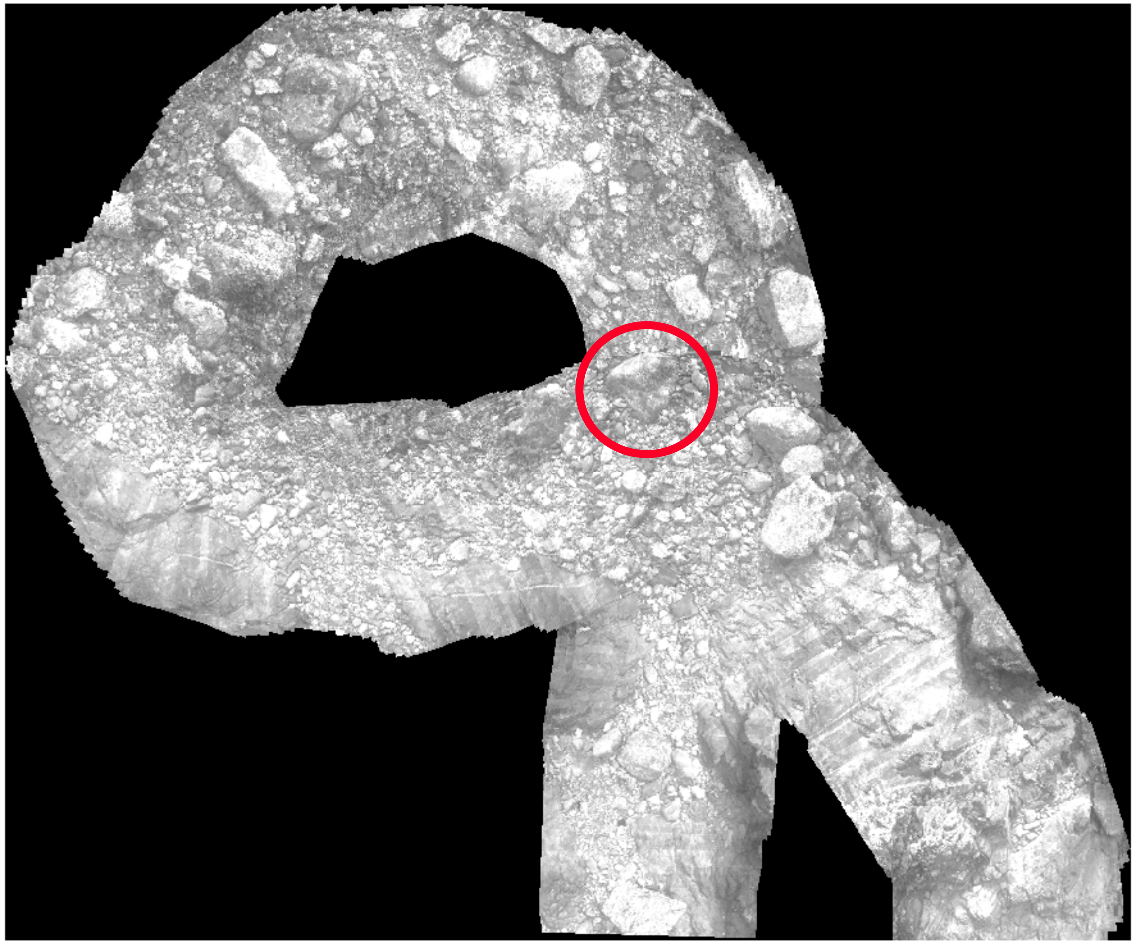
Outline

1. Basic EKF SLAM
   1. Introduction: the need for SLAM
   2. The basic EKF SLAM algorithm
   3. Feature extraction

2. The Data Association Problem
   1. Introduction
   2. Continuous Data Association
   3. **The Loop Closing Problem**
   4. The Global Localization Problem

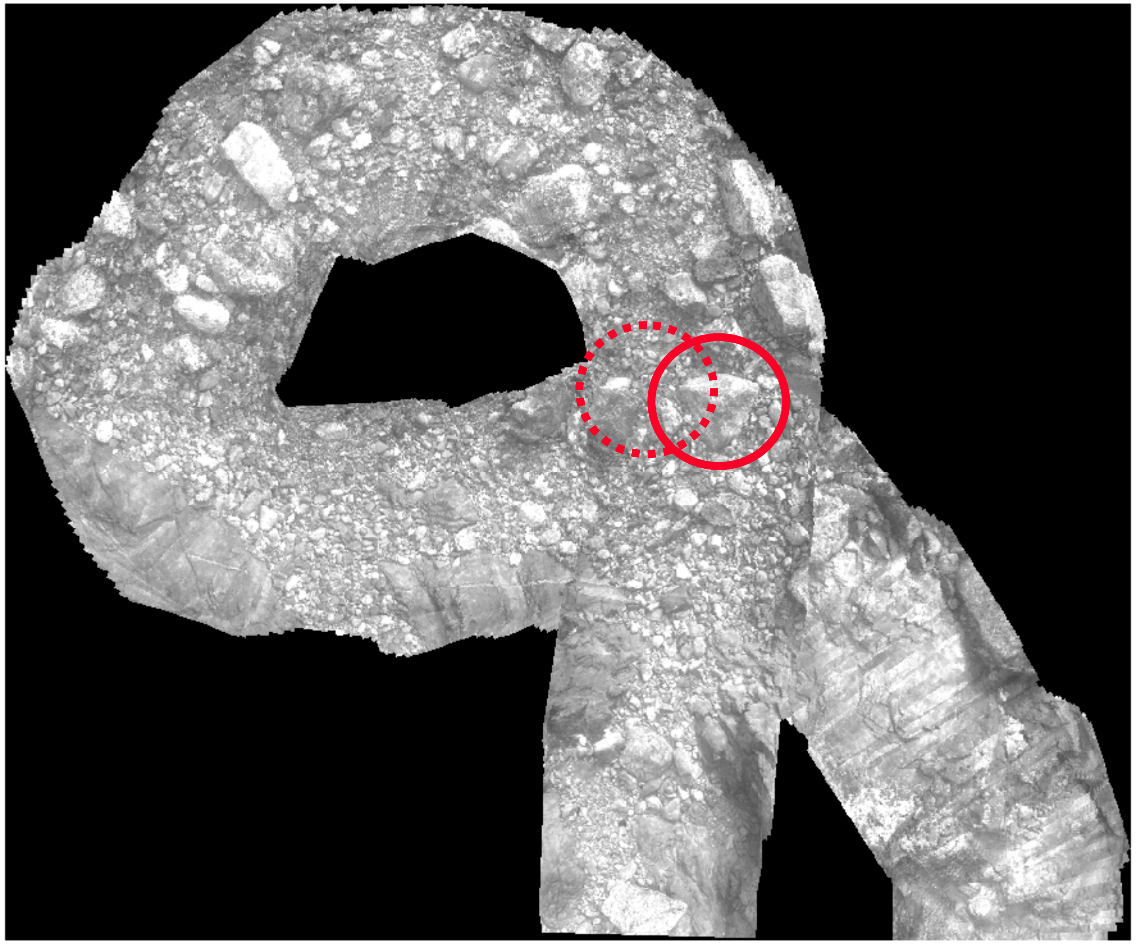
3. Advanced EKF SLAM
   1. Computational complexity of EKF SLAM
   2. Consistency of the EKF SLAM
   3. SLAM using local maps
      1. Sequential Map Joining
      2. Divide and Conquer SLAM
      3. Hierarchical SLAM

Loop closing in mosaicing
use first



Joint work with R. García, University of Girona

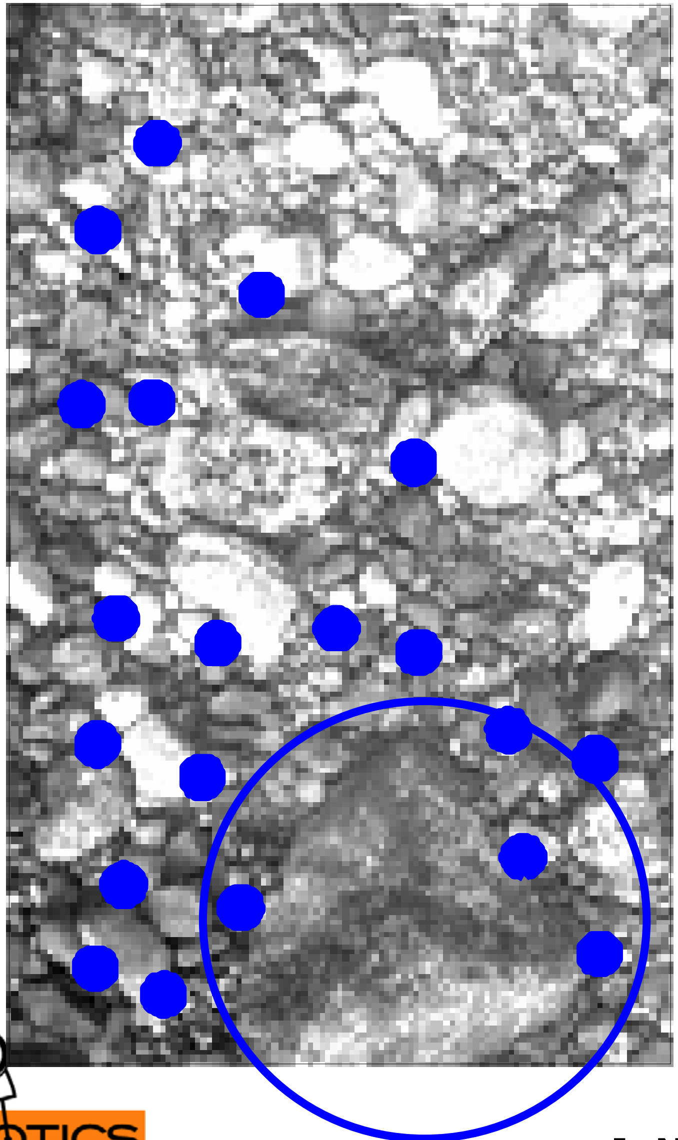
Loop closing in mosaicing: use Last



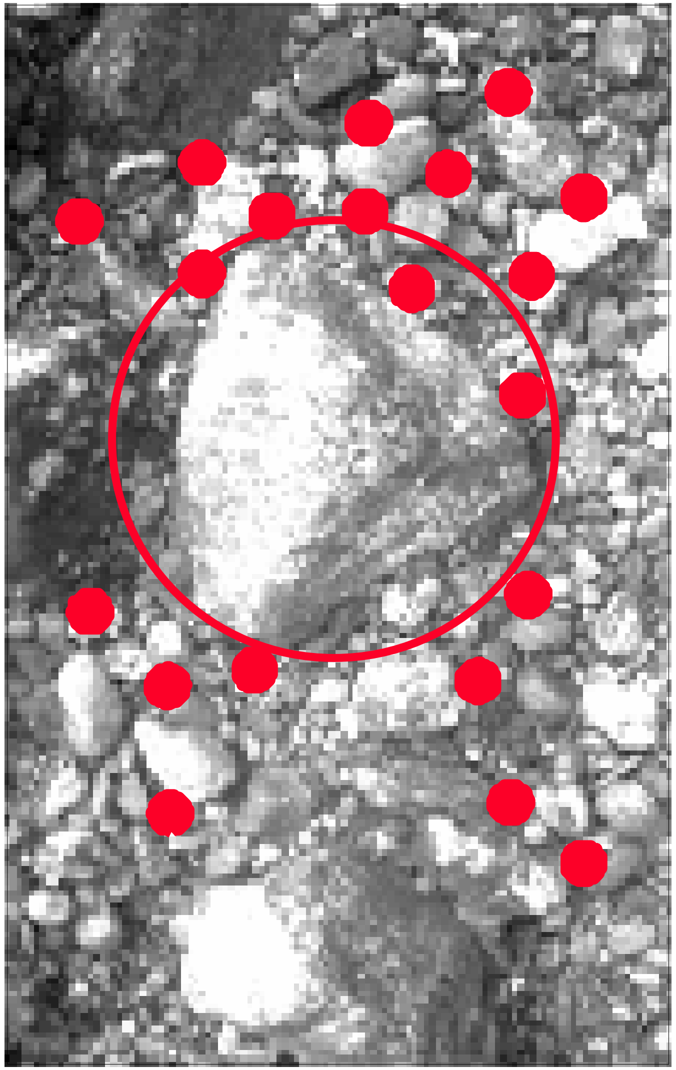
Sequential mosaicing is a form of odometry

The loop closing problem

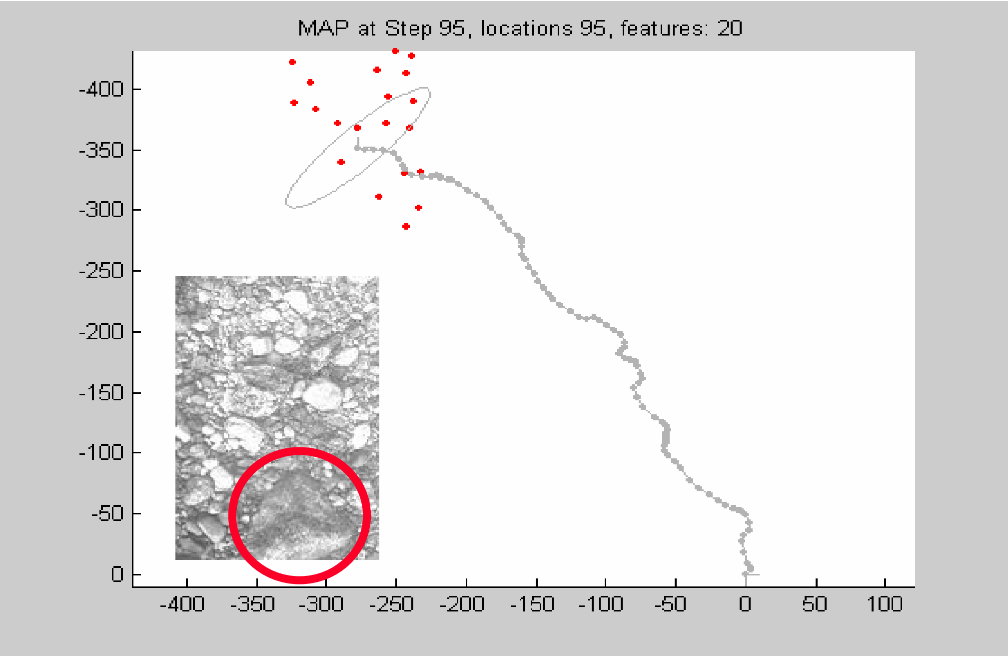
- Loop beginning

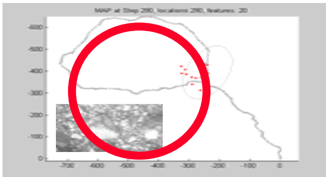


- Loop end

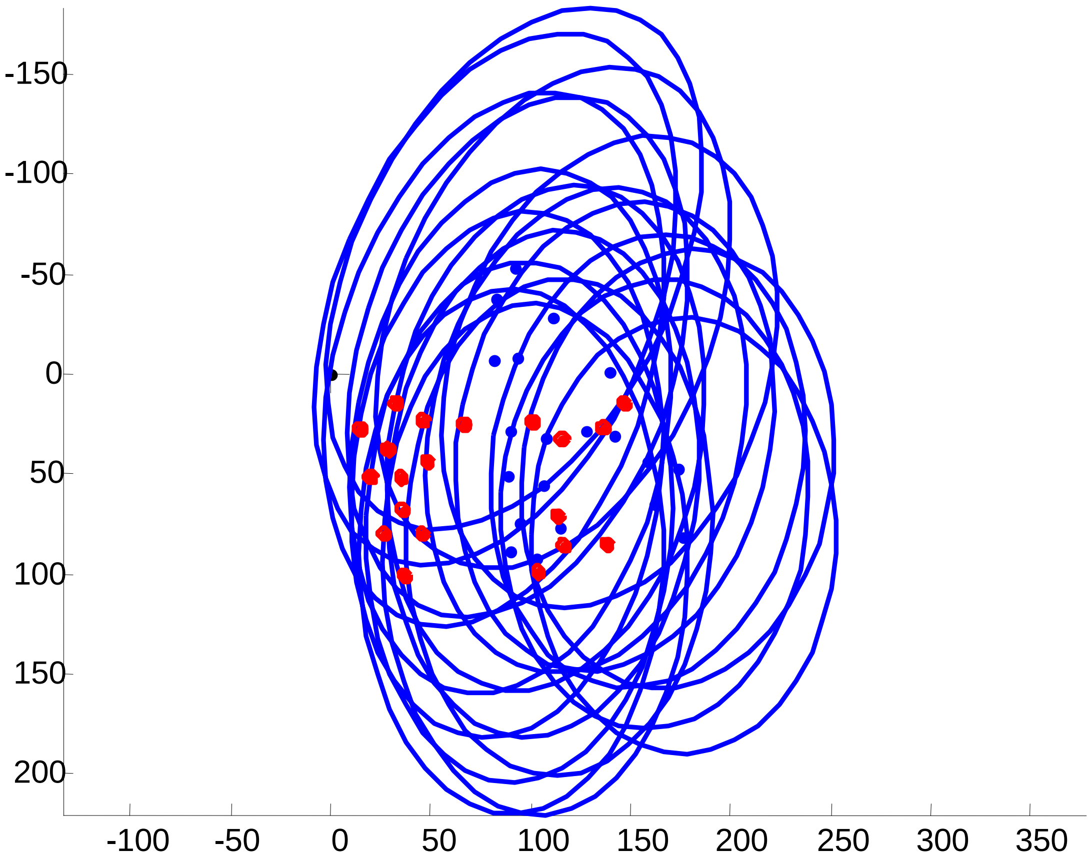


The loop closing problem





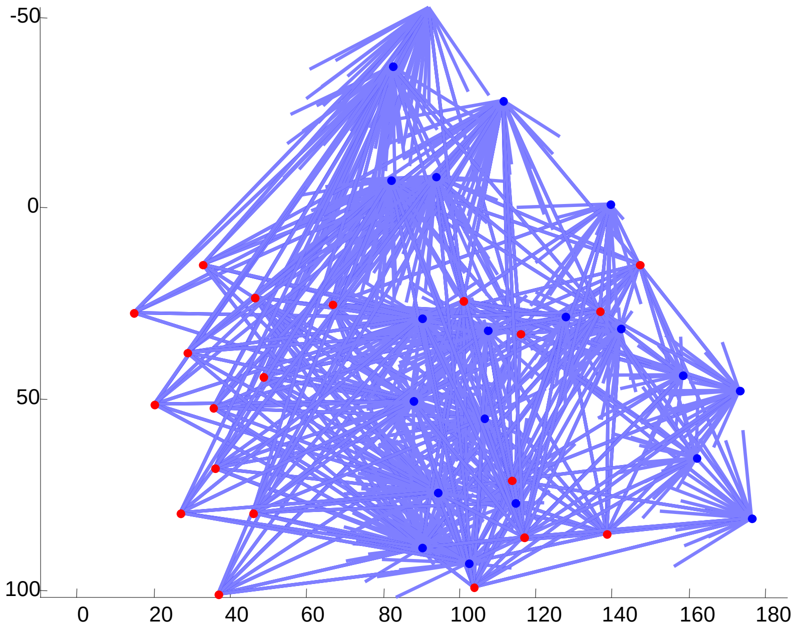
The loop closing problem



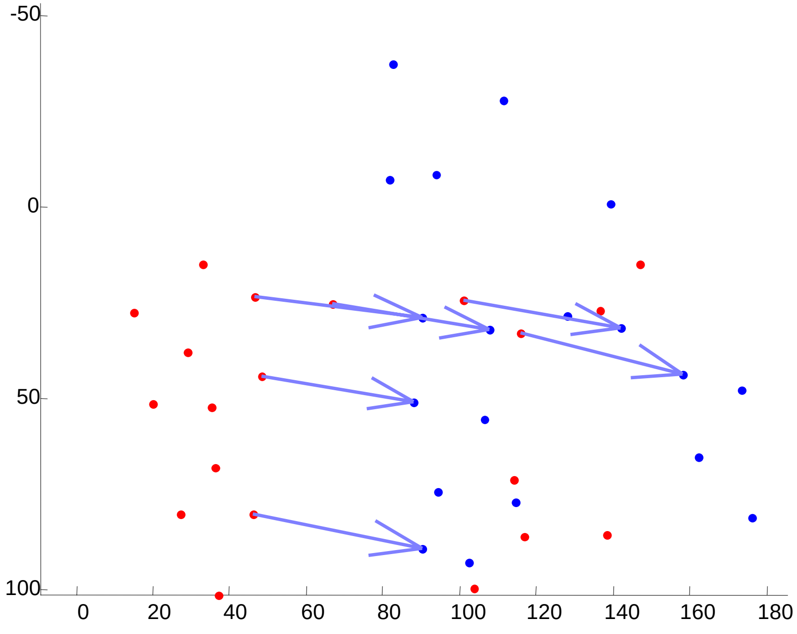
Measurements (red) and predicted features (blue)

The loop closing problem

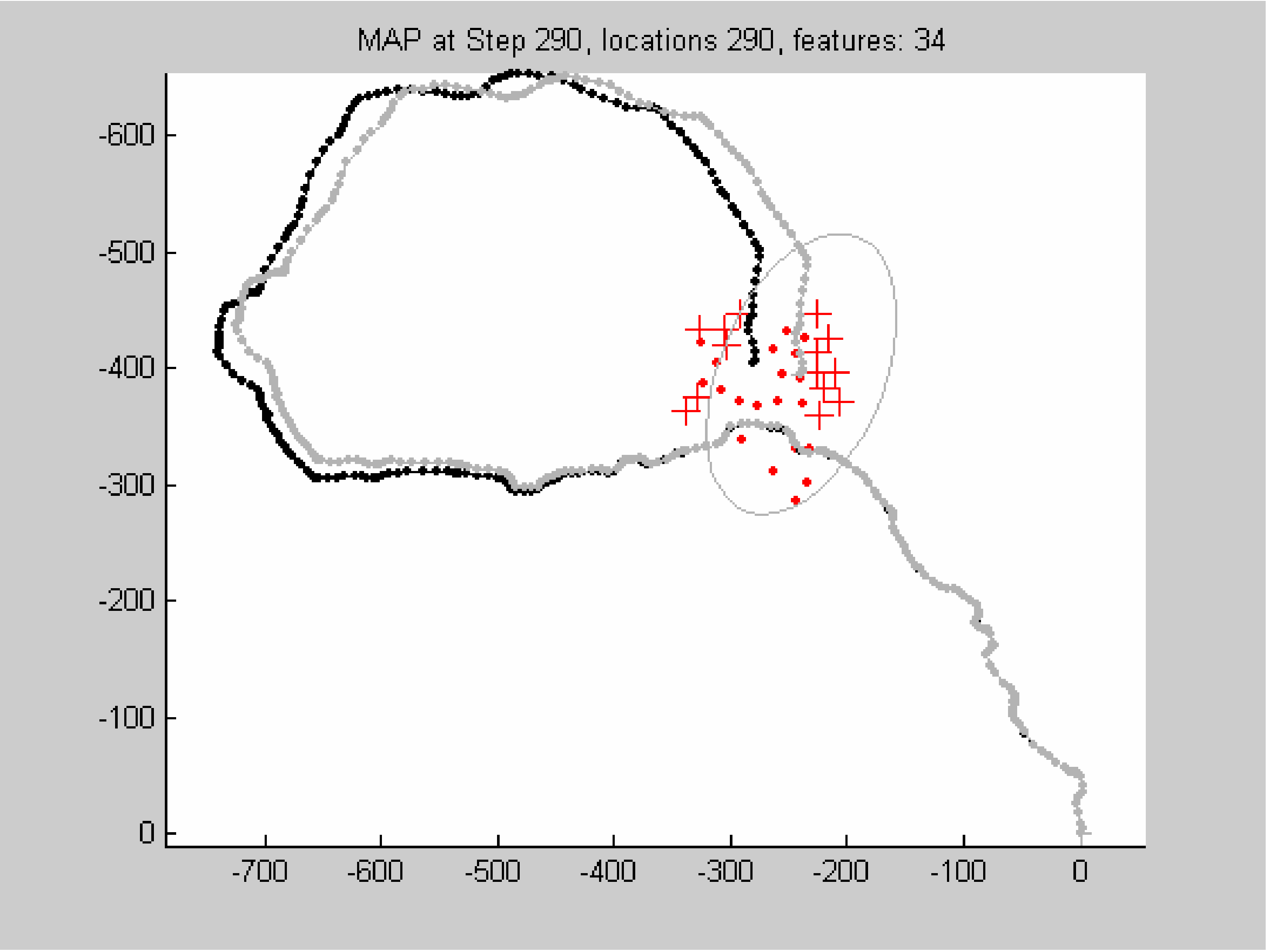
- Individual compatibility



- Joint Compatibility



The loop closing problem



Outline

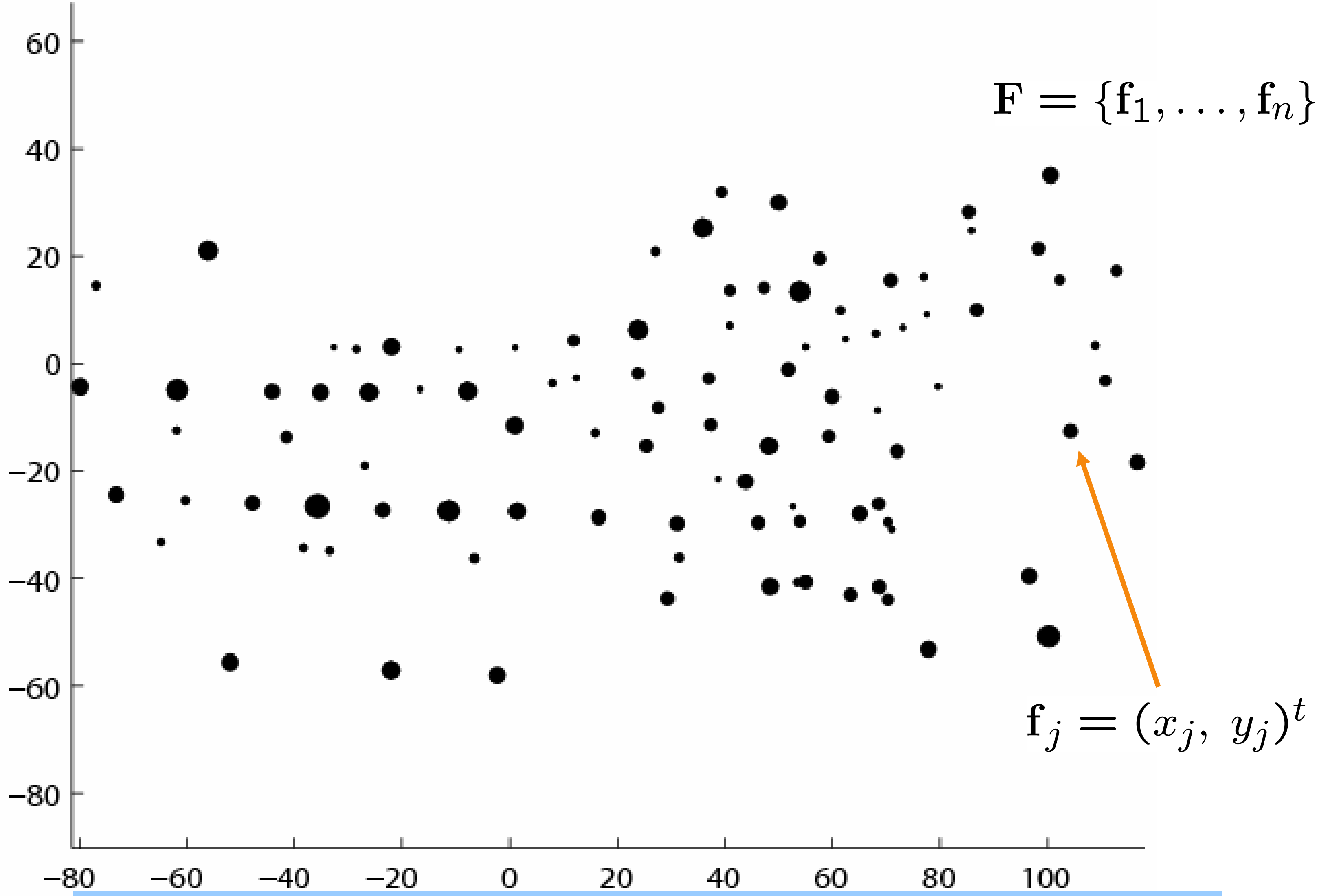
1. Basic EKF SLAM
   1. Introduction: the need for SLAM
   2. The basic EKF SLAM algorithm
   3. Feature extraction

2. The Data Association Problem
   1. Introduction
   2. Continuous Data Association
   3. The Loop Closing Problem
   4. **The Global Localization Problem**

3. Advanced EKF SLAM
   1. Computational complexity of EKF SLAM
   2. Consistency of the EKF SLAM
   3. SLAM using local maps
      1. Sequential Map Joining
      2. Divide and Conquer SLAM
      3. Hierarchical SLAM

Global Localization

- Robot placed in a previously mapped environment

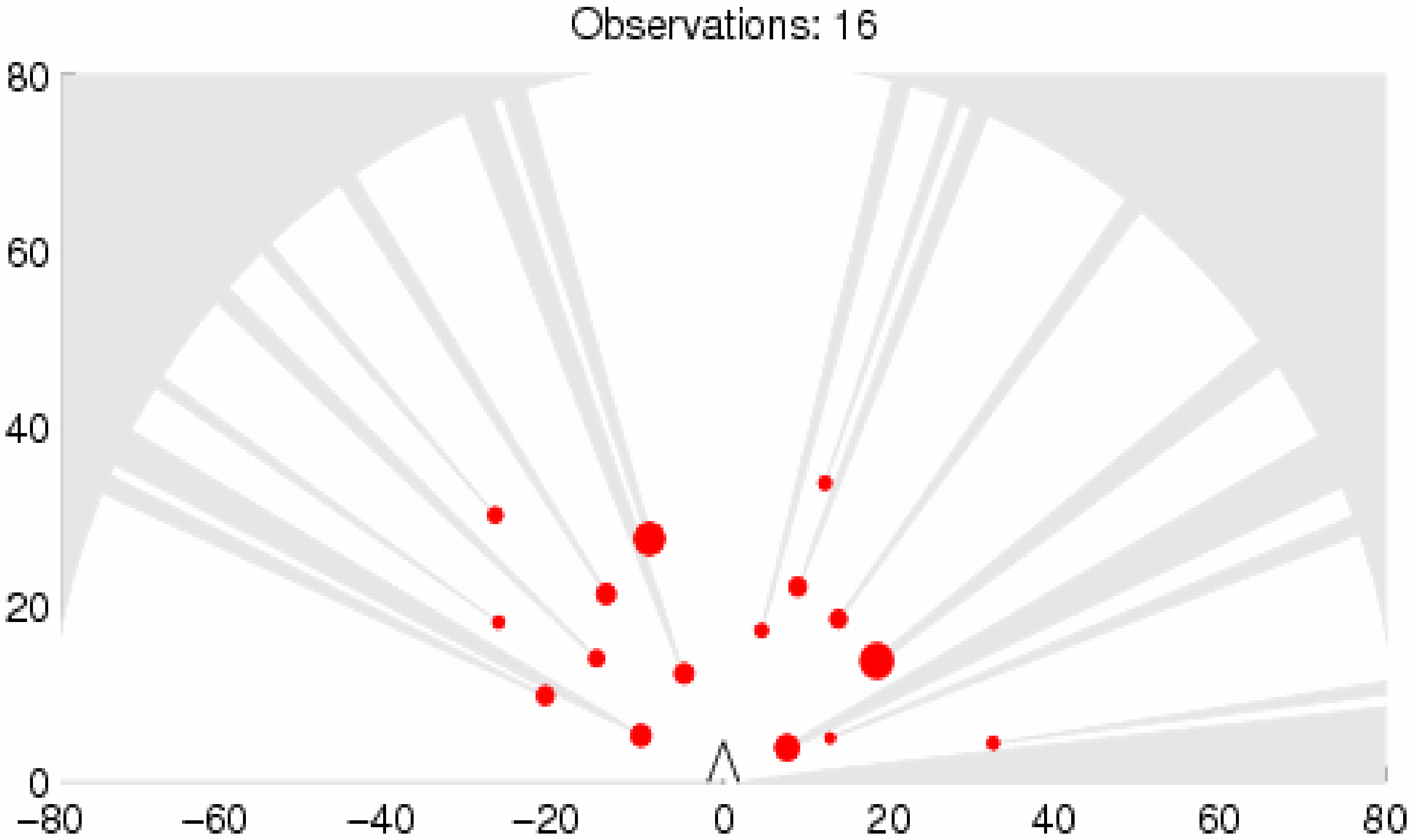


http://www.acfr.usyd.edu.au/homepages/academic/enebot/victoria_park.htm

Problem Definition

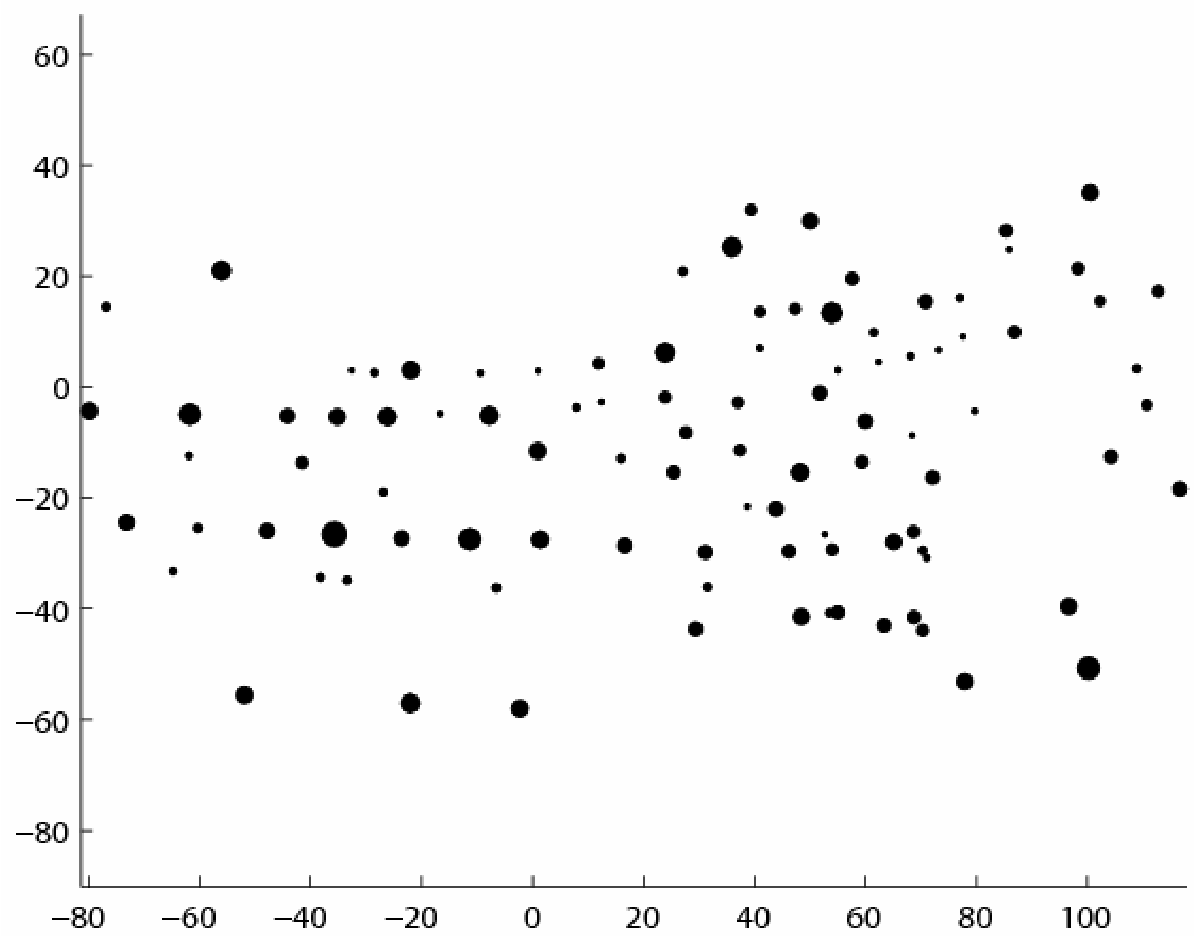
- On-board sensor obtains *m* measurements:

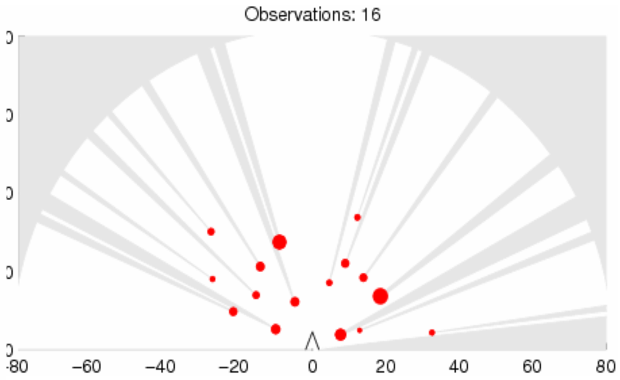
$$\mathbf{Z} = \{\mathbf{z}_1, \ldots, \mathbf{z}_m\}$$



http://www.acfr.usyd.edu.au/homepages/academic/enebot/victoria_park.htm

Problem Definition





- Twofold question:
  - Is the vehicle in the map?
  - If so, where?

http://www.acfr.usyd.edu.au/homepages/academic/enebot/victoria_park.htm

Global Localization algorithms

- Correspondence space
  - Consider consistent combinations of measurement-feature pairings.
    - » Branch and Bound (Grimson, 1990)
    - » Maximum Clique (Bailey et. al. 2000)
    - » Random Sampling (Neira et. al. 2003)

- Configuration space
  - Consider different vehicle location hypotheses.
    - » Monte Carlo Localization (Fox et. al. 1999)
    - » Markov Localization (Fox et. al. 1998)

In correspondence space

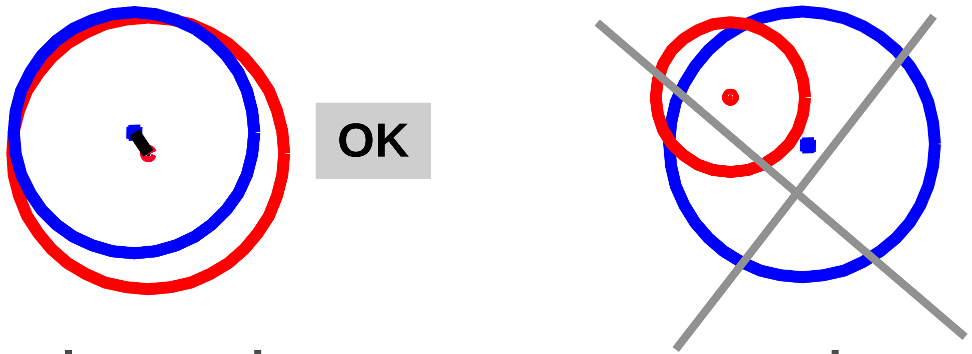
No vehicle location

- **Unary constraints:**

$$p_{ij} = (E_i, F_j)?$$

depend on a single matching (size, color,...)

– Trees: trunk diameter



– Walls: length, corners: angle....

61 714 354 176 000 valid hypotheses

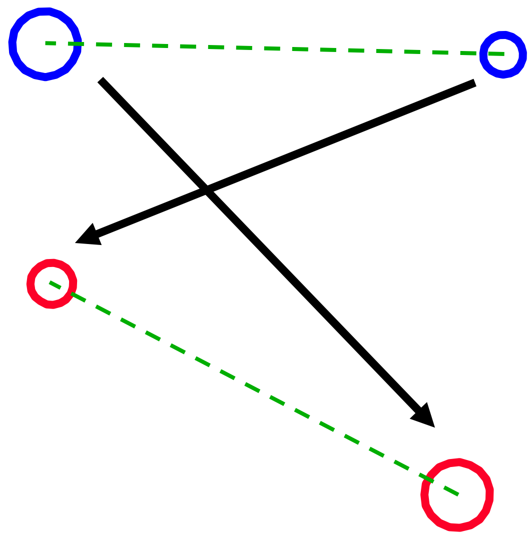
m constraints

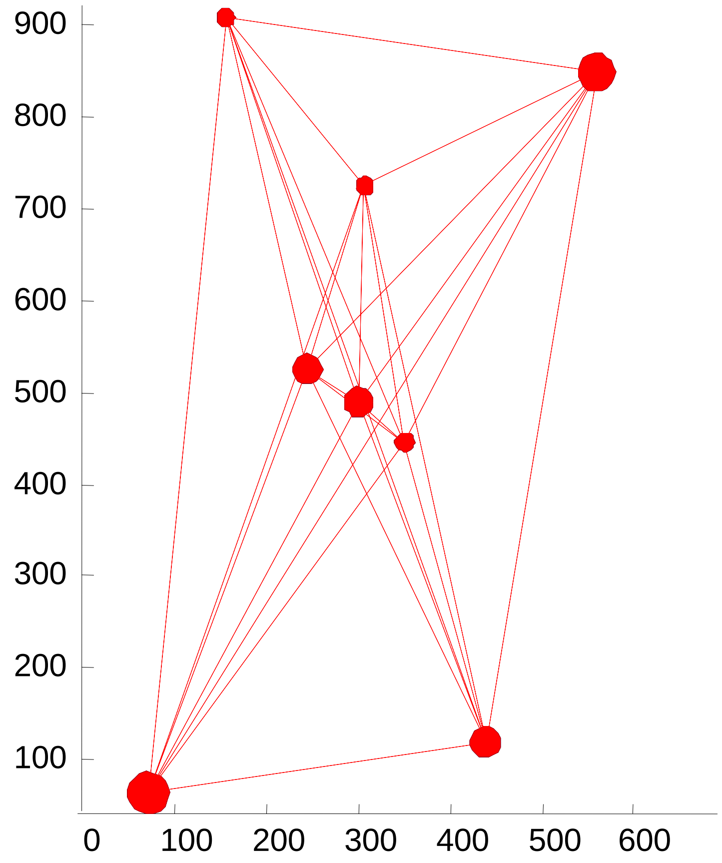
No vehicle location

- **Binary constraints:**

$$p_{ij} = (E_i, F_j)?$$
$$p_{kl} = (E_k, F_l)?$$

distances between points:

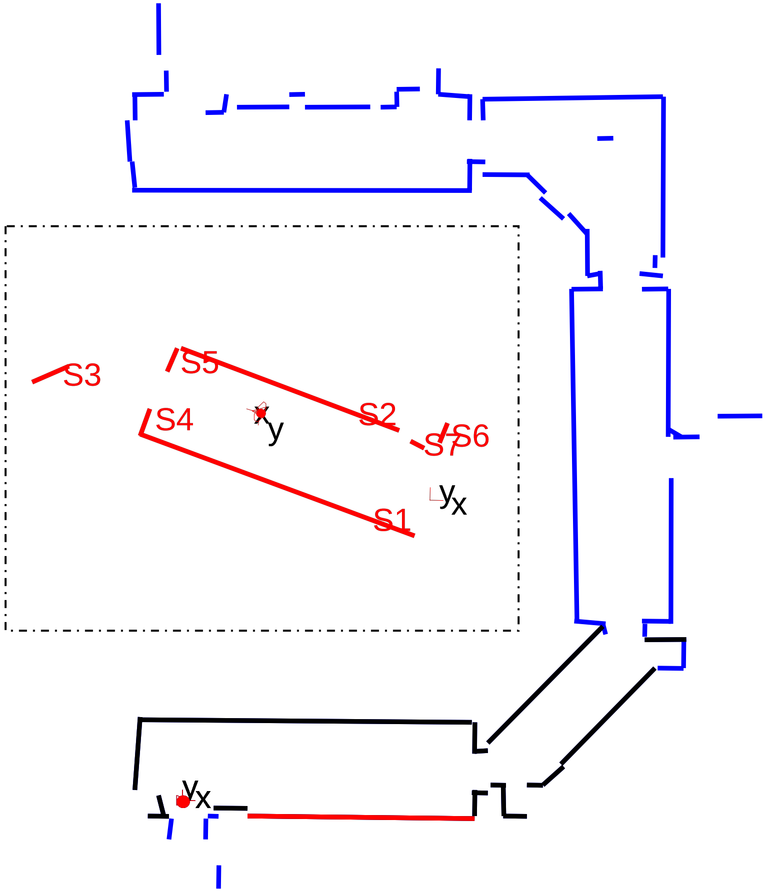


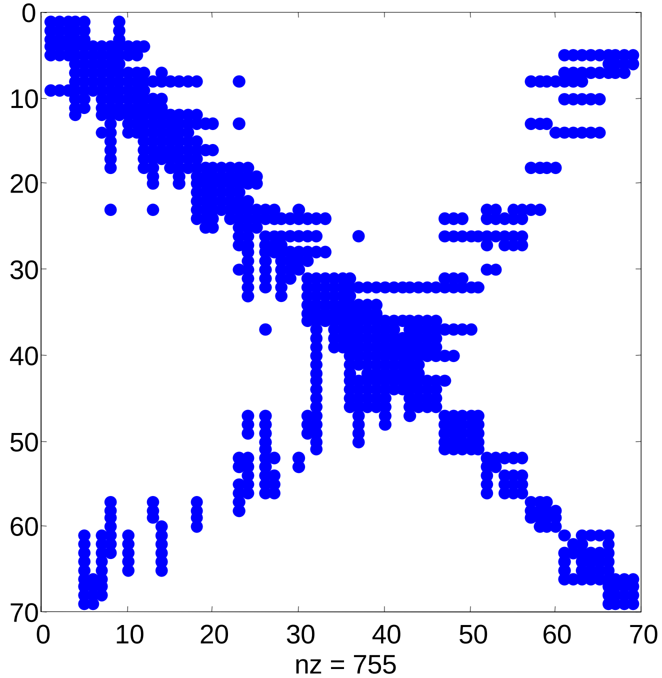


$$\frac{m\,(m-1)}{2}\text{constraints}$$

Locality

- Features F_i and F_j may belong to the same hypotesis iff they are 'close enough'.



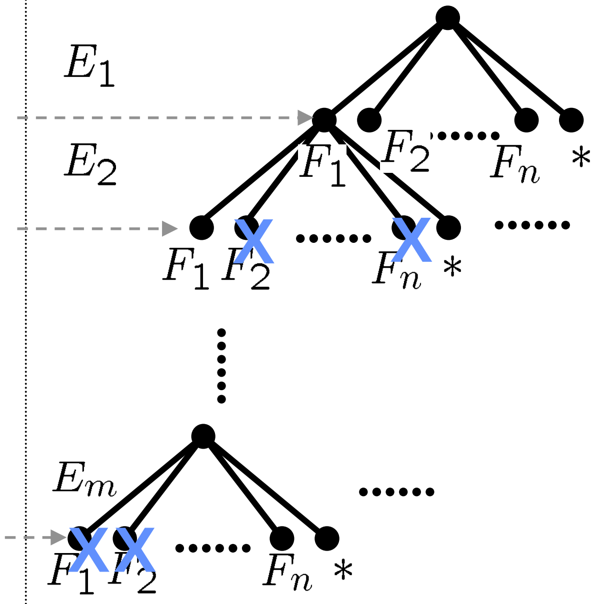


Covisibility matrix

Limit search in the map to subsets of **covisible** features
Locality makes search **linear** with the global map size

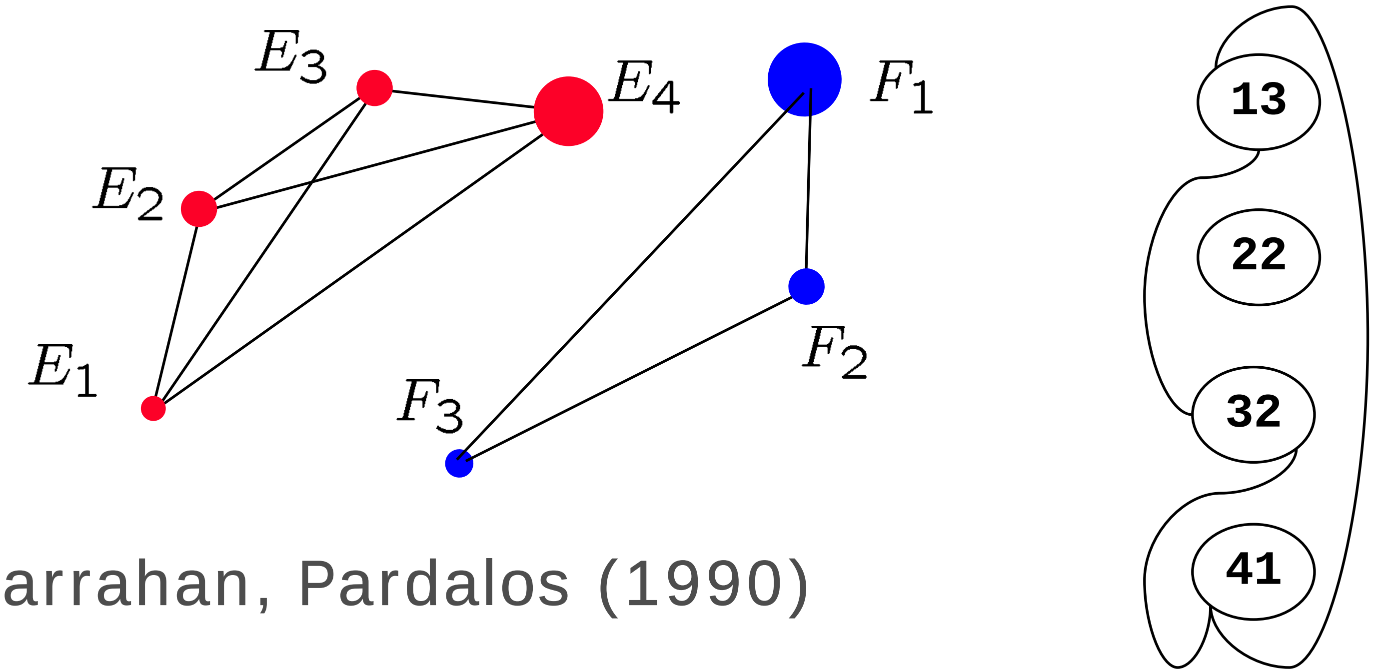
Algorithm 1: Geometric Constraints Branch and Bound (Grimson, 1990)

```
procedure GCBB (H, i):
if i > m -- leaf node?
   if pairings(H) > pairings(Best) -- did better?
      estimate_location(H)
      if joint_compatibility(H)
         Best = H
      fi
   fi
else
   for j in {1...n}
      if unary(i, j) ∧ binary(i, j, H)
         GCBB([H j], i + 1) -- (E_i, F_j) accepted
      fi
   rof
   if pairings(H) + m - i > pairings(Best)
      GCBB([H 0], i + 1) -- try star node
   fi
fi
```



Algorithm 2: Maximum Clique

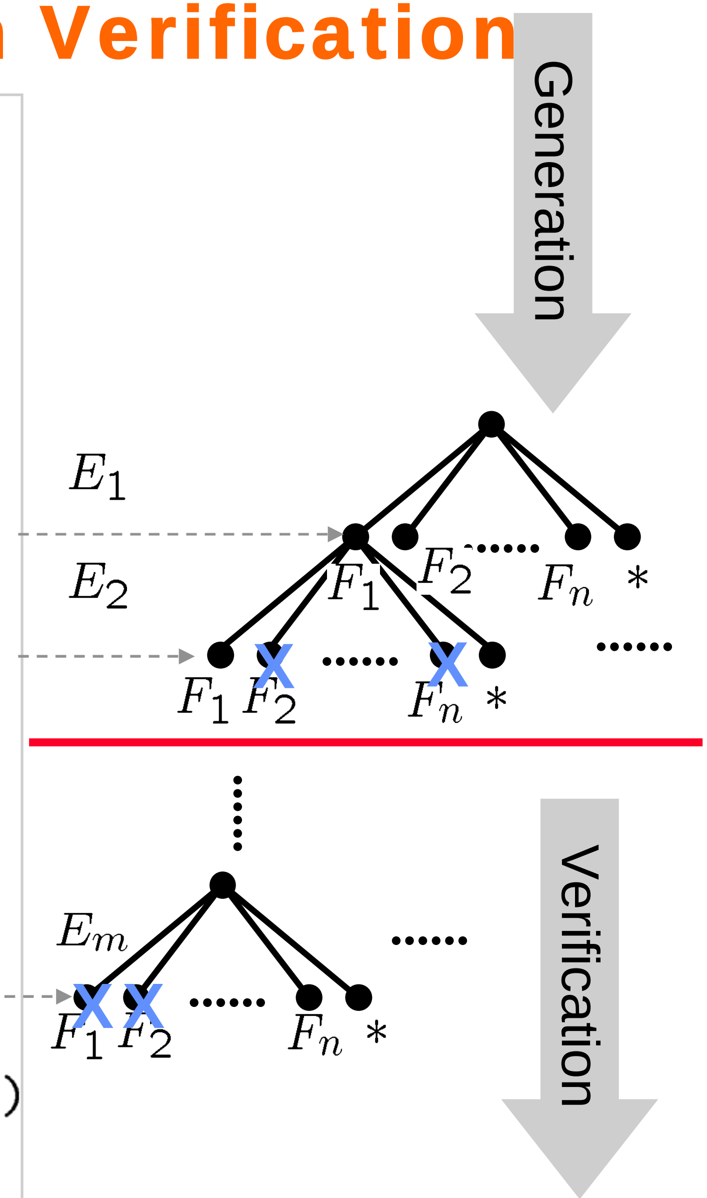
- All unary and binary constraints can be precomputed
- Build a compatibility graph where:
  - Nodes represent unary compatible pairings
  - Arcs represent pairs of binary compatible pairings



- Carrahan, Pardalos (1990)

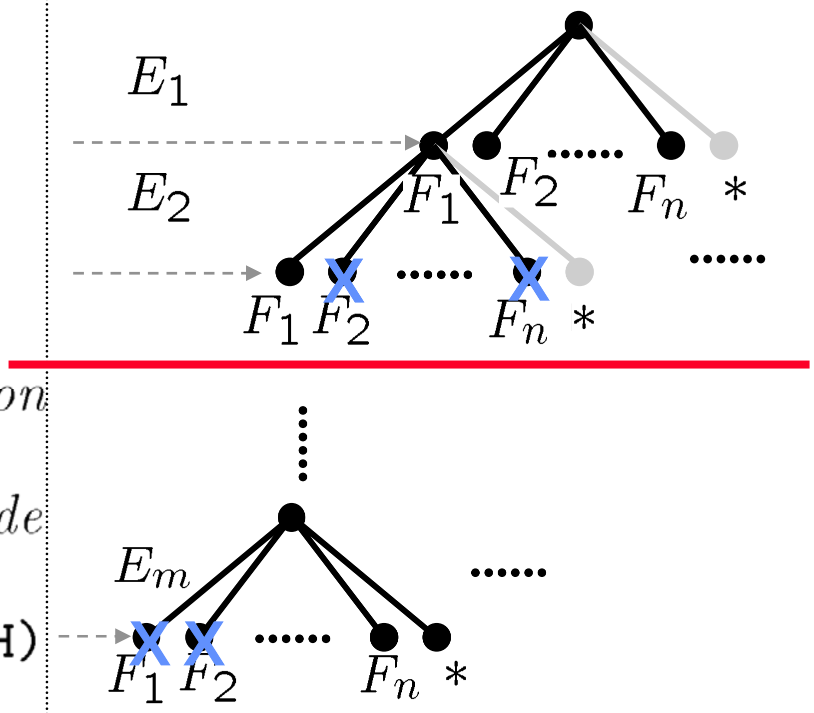
Algorithm 3: Generation Verification

```
procedure GV (H, i):
if i > m
   if pairings(H) > pairings(Best)
      Best = H
   fi
elseif pairings(H) == 3
   estimate_location_(H)
   if joint_compatibility(H)
      JCBB(H, i) -- hypohtesis verification
   fi
else
   for j in {1...n}
      if unary(i, j) ∧ binary(i, j, H)
         GV([H j], i + 1)
      fi
   rof
   if pairings(H) + m - i > pairings(Best)
      GV([H 0], i + 1)
   fi
fi
```



Algorithm 4: RANSAC

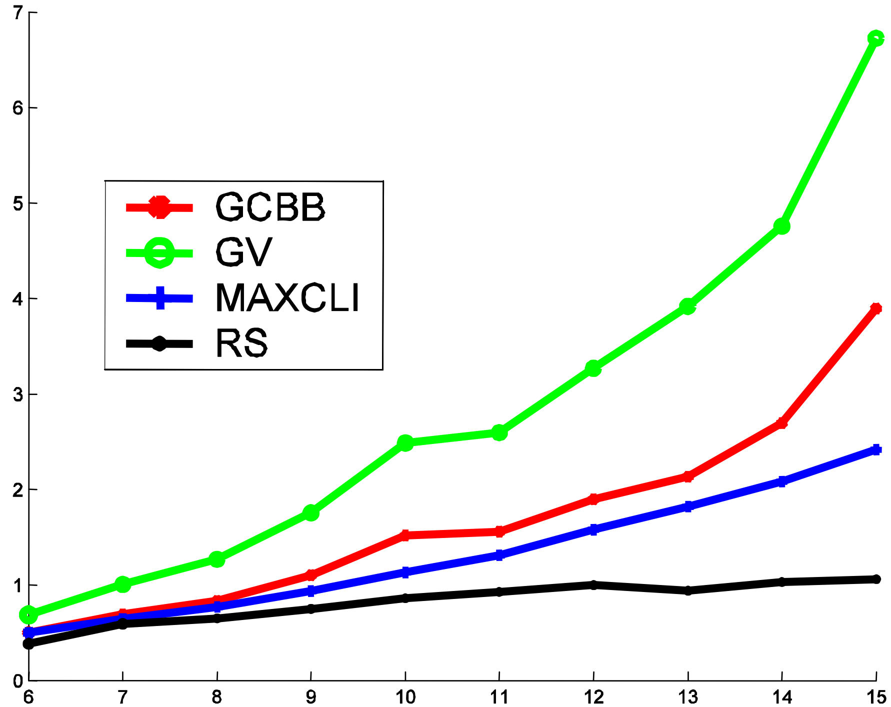
```
procedure RS (H, i):
if i > m
   if pairings(H) > pairings(Best)
      Best = H
   fi
elseif pairings(H) == 3
   estimate_location_(H)
   if joint_compatibility(H)
      JCBB(H, i) -- hypohtesis verification
   fi
else -- branch and bound without star node
   for j in {1 ... n}
      if unary(i, j) ∧ binary(i, j, H)
         RS([H j], i + 1)
      fi
   rof
fi
```



Experiments

1. No significant difference in **effectiveness** of the considered algorithms
2. All algorithms are made **linear** with the size of the map
3. **Efficiency** when the vehicle is in the map

mean time .vs. m

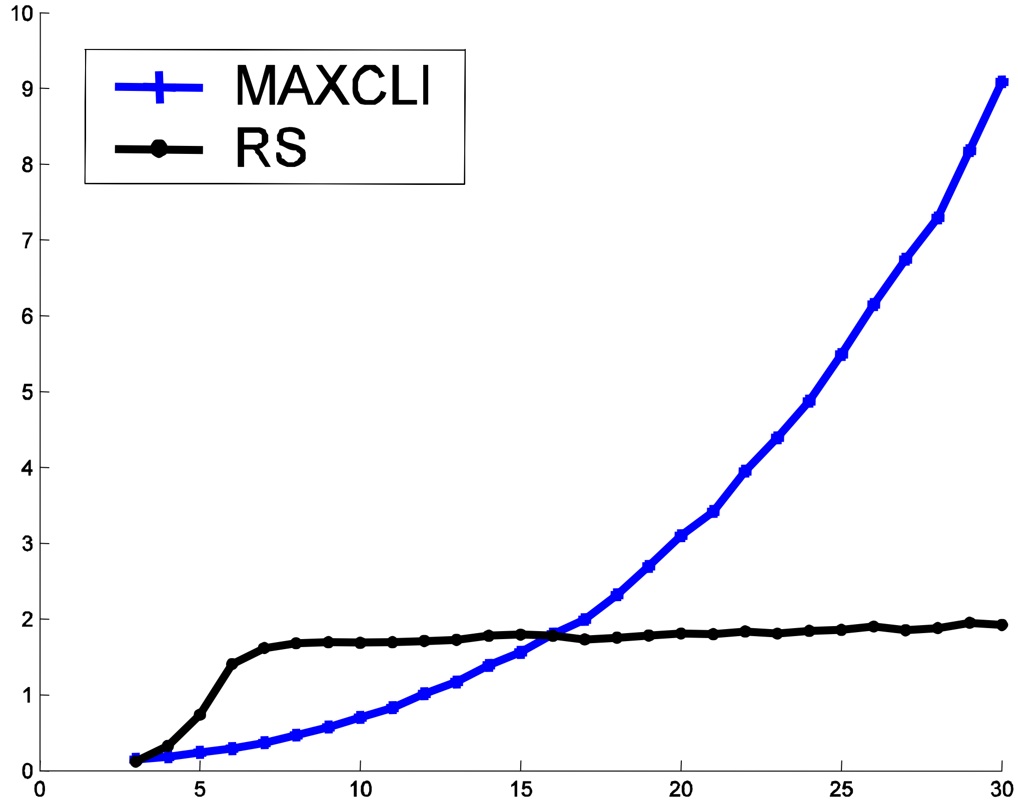


Experiments

4. When the vehicle is **NOT** in the map:

random measurements, 100 for each m=3..30

mean time .vs. m



J. Neira, J.D. Tardós, J.A. Castellanos, **Linear time vehicle relocation in SLAM.** IEEE Int. Conf. Robotics and Automation, Taipei, Taiwan, May, 2003

ROBOTICS UNIVERSIDAD DE ZARAGOZA

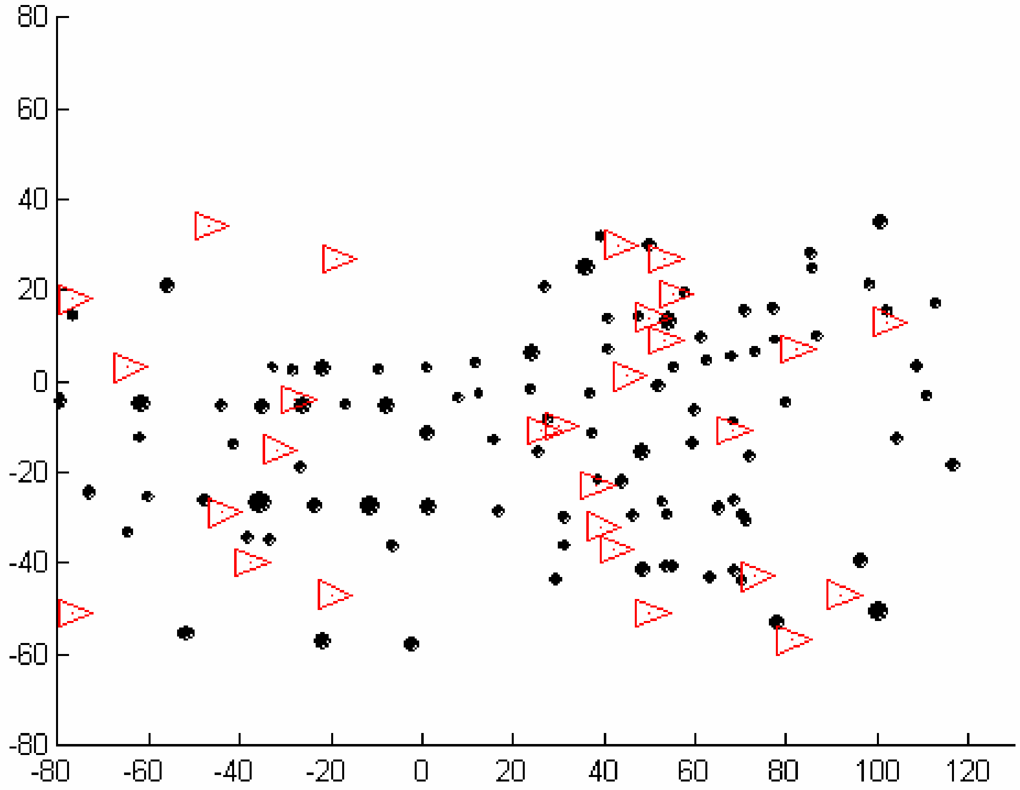
In configuration space

In Configuration Space: RANDOM sampling

- Consider *s* randomly chosen vehicle locations hypotheses:

$$\mathbf{x} = (x, y, \phi)^t$$

$$x \in [x_{min}, x_{max}]$$
$$y \in [y_{min}, y_{max}]$$
$$\phi \in [\phi_{min}, \phi_{max}]$$



- Monte Carlo Localization

Alternative 1: location-driven

- Consider each alternative **location hypothesis** in turn

Algorithm 1 Loc_driven:

$votes = 0$

for each hypothesis $\mathbf{x} \in \mathbf{X}$ **do**

  for each measurement $\mathbf{z}_i \in \mathbf{Z}$ **do**

    $\mathbf{F}_i = \texttt{predict\_features}(\mathbf{x}, \mathbf{z}_i)$

    if $\texttt{any\_compatible\_feature}(\mathbf{F}_i, \mathbf{F})$ **then**

      $votes(\mathbf{x}) = votes(\mathbf{x}) + 1$

    end if

  end for

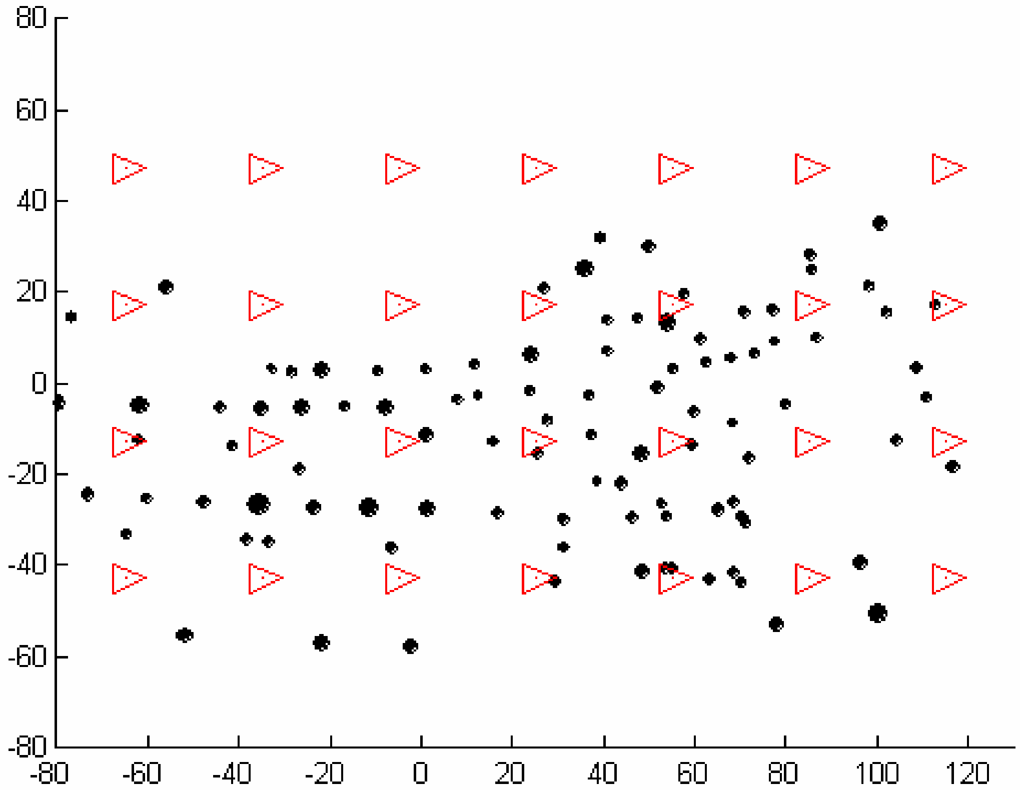
end for

In Configuration Space: GRID sampling

- Uniformly tessellate the space in $s = n_x \, n_y \, n_\phi$ grid cells:

$$\mathbf{x} = (x, y, \phi)^t$$

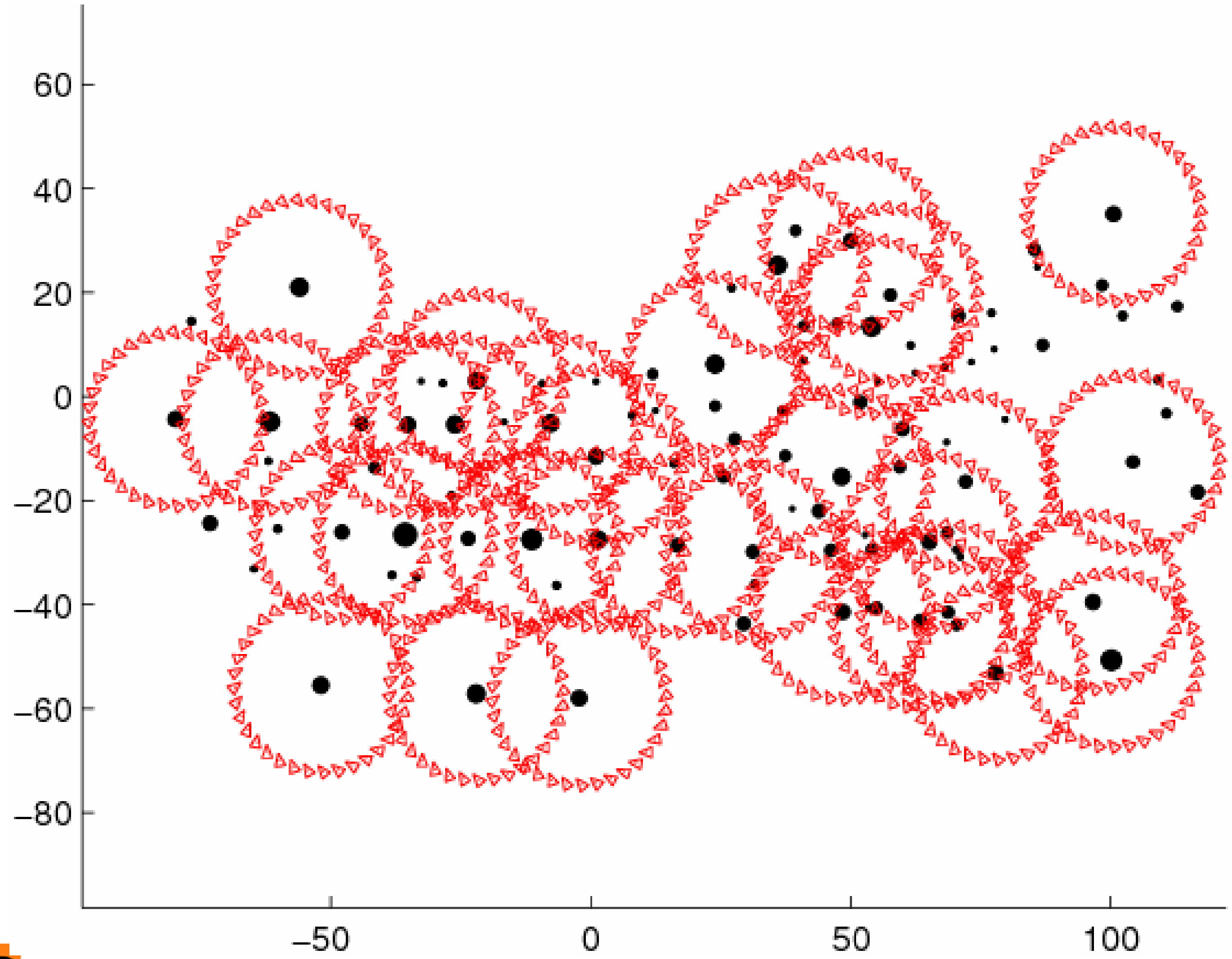
$$x \in [x_{min}, x_{max}]$$
$$y \in [y_{min}, y_{max}]$$
$$\phi \in [\phi_{min}, \phi_{max}]$$



- Markov Localization

Voting in configuration space

- Each measurement-feature pairing constrains the set of possible vehicle locations:



Alternative 2: pairing-driven

- Consider each **measurement-feature pairing** in turn

Algorithm 2 Pair_driven:

$votes = 0$

for each measurement $\mathbf{z}_i \in \mathbf{Z}$ **do**

  for each feature $\mathbf{f}_j \in \mathbf{F}$ **do**

    $\mathbf{X}_{ij} = \texttt{hypothesize\_locations}(\mathbf{f}_j, \mathbf{z}_i)$

    $\mathbf{X}_v = \texttt{compute\_compatible\_locations}(\mathbf{X}_{ij}, \mathbf{X})$

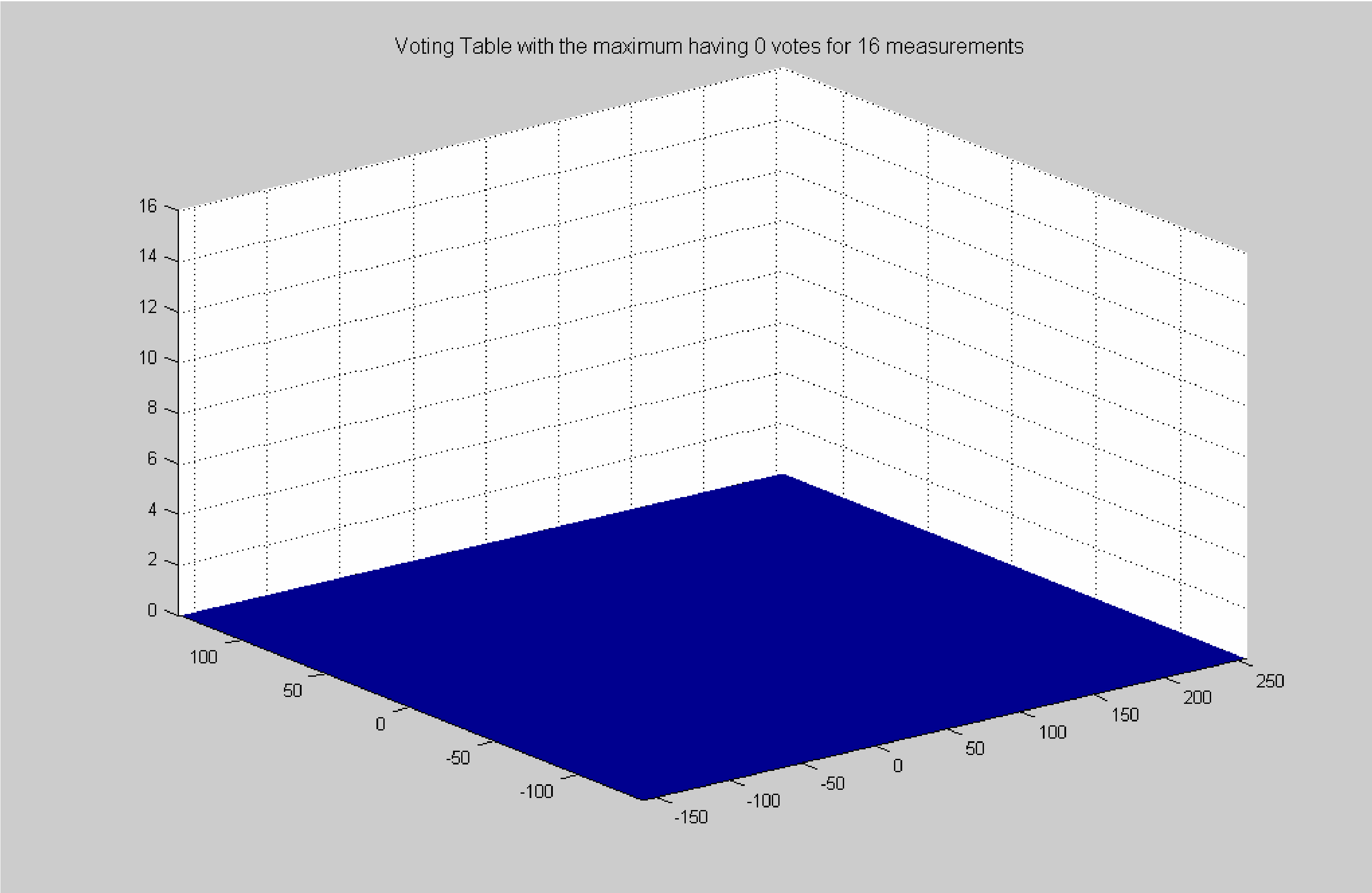
    $votes(\mathbf{X}_v) = votes(\mathbf{X}_v) + 1$

  end for

end for

Results

– Resolution: 1.5m for x and y, and 1º for theta



Computational Complexity

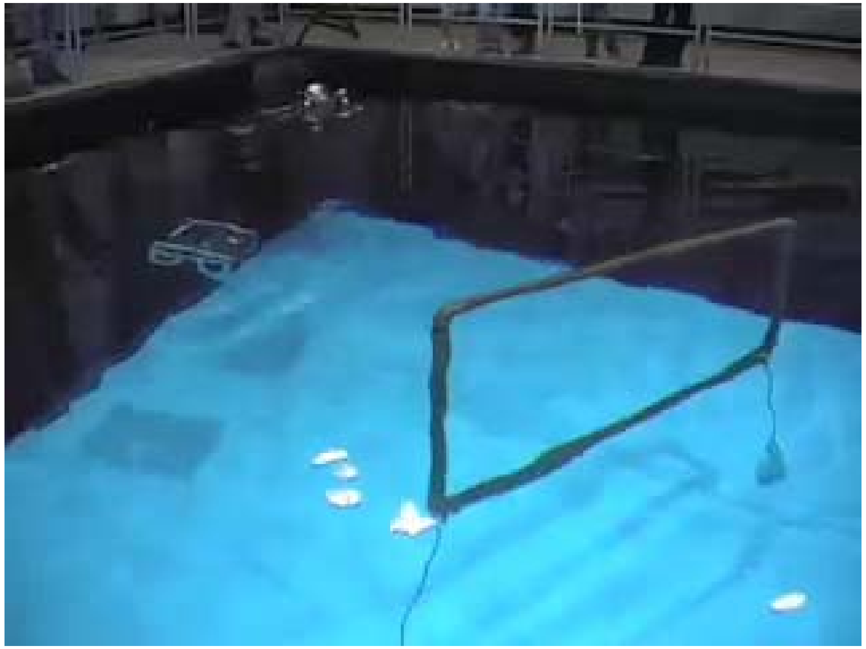
Sensor	Loc_driven	Pair_driven
Range and bearing	$O(n_x \cdot n_y \cdot n_\phi \cdot m)$	$O(n \cdot m \cdot n_\phi)$
Range-only	$O(n_x \cdot n_y \cdot n_\phi \cdot m \cdot n_\theta)$	$O(n \cdot m \cdot n_\phi \cdot n_\theta)$
Bearing-only	$O(n_x \cdot n_y \cdot n_\phi \cdot m \cdot n_r)$	$O(n \cdot m \cdot n_\phi \cdot n_r)$

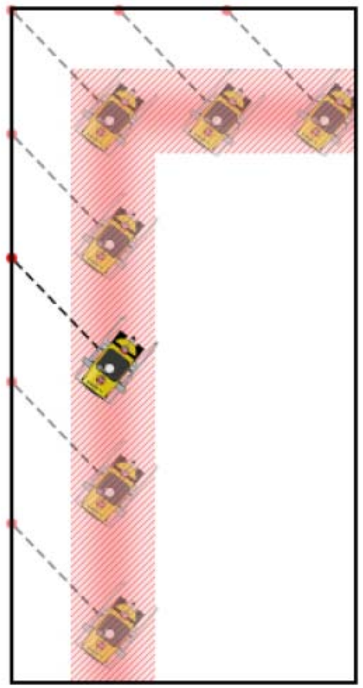
- How do they compare?

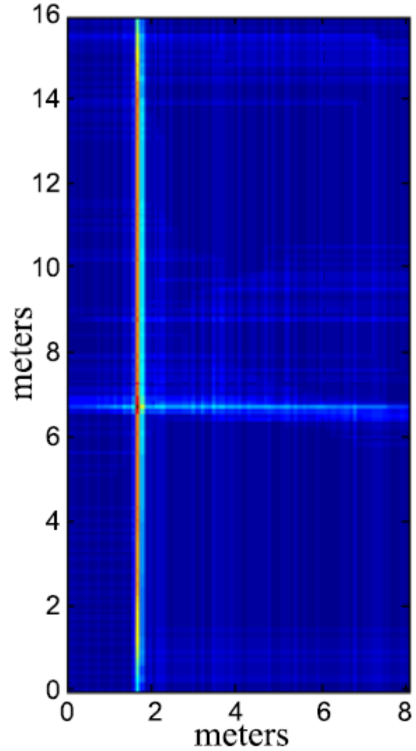
$$\frac{\text{Pair\_driven}}{\text{Loc\_driven}} = \frac{n}{n_x \cdot n_y} = \rho$$

- Pair_driven is better in proportion to the **density of features** in the environment.
- Victoria Park: 18321 sq. m.,sample every 1.5m, expect pair_driven to be **82** times faster.

Voting for map based localization







Ribas D., Palomeras, N. Hernandez, E., Ridao, P., Carreras, M.
ICTINEU AUV Wins the First SAUC-E Competition
2007 IEEE International Conference on Robotics and Automation, Rome, Italy.

ROBOTICS UNIVERSIDAD DE ZARAGOZA

Conclusions

- A solution to SLAM is indeed possible
- Data association: algorithms based on some form of **consensus** provide the best results

    » Joint Compatibility

    » RANSAC

    » Hough Transform

But…

- What happens at a large scale?

Outline

1. Basic EKF SLAM
    1. Introduction: the need for SLAM
    2. The basic EKF SLAM algorithm
    3. Feature extraction

2. The Data Association Problem
    1. Introduction
    2. Continuous Data Association
    3. The Loop Closing Problem
    4. The Global Localization Problem

3. Advanced EKF SLAM
    1. **Computational complexity of EKF SLAM**
    2. Consistency of the EKF SLAM
    3. SLAM using local maps
        1. Sequential Map Joining
        2. Divide and Conquer SLAM
        3. Hierarchical SLAM

The EKF SLAM algorithm

Algorithm 1 SLAM:

$\mathbf{x}_0^B = \mathbf{0};\ \mathbf{P}_0^B = \mathbf{0}$ {*Map initialization*}

$[\mathbf{z}_0, \mathbf{R}_0] = \text{get\_measurements}$

$[\mathbf{x}_0^B, \mathbf{P}_0^B] = \text{add\_new\_features}(\mathbf{x}_0^B, \mathbf{P}_0^B, \mathbf{z}_0, \mathbf{R}_0)$

for $k = 1$ to steps **do**

$[\mathbf{x}_{R_k}^{R_{k-1}}, \mathbf{Q}_k] = \text{get\_odometry}$

$[\mathbf{x}_{k|k-1}^B, \mathbf{P}_{k|k-1}^B] = \text{EKF\_prediction}(\mathbf{x}_{k-1}^B, \mathbf{P}_{k-1}^B, \mathbf{x}_{R_k}^{R_{k-1}}, \mathbf{Q}_k)$

$[\mathbf{z}_k, \mathbf{R}_k] = \text{get\_measurements}$

$\mathcal{H}_k = \text{data\_association}(\mathbf{x}_{k|k-1}^B, \mathbf{P}_{k|k-1}^B, \mathbf{z}_k, \mathbf{R}_k)$

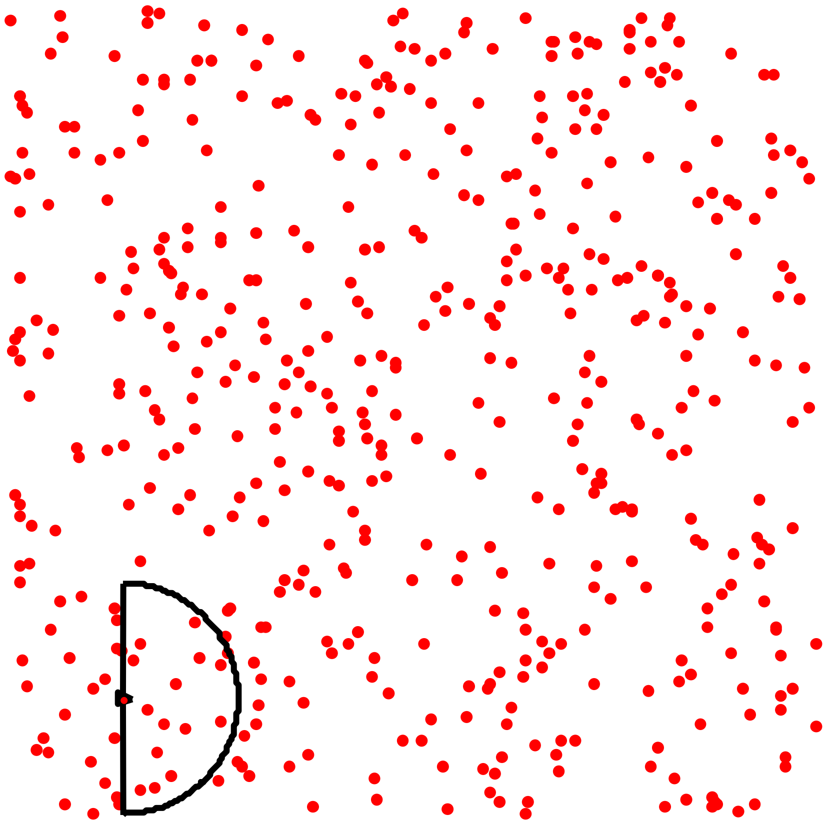
$[\mathbf{x}_k^B, \mathbf{P}_k^B] = \text{EKF\_update}(\mathbf{x}_{k|k-1}^B, \mathbf{P}_{k|k-1}^B, \mathbf{z}_k, \mathbf{R}_k, \mathcal{H}_k)$

$[\mathbf{x}_k^B, \mathbf{P}_k^B] = \text{add\_new\_features}(\mathbf{x}_k^B, \mathbf{P}_k^B, \mathbf{z}_k, \mathbf{R}_k, \mathcal{H}_k)$

end for

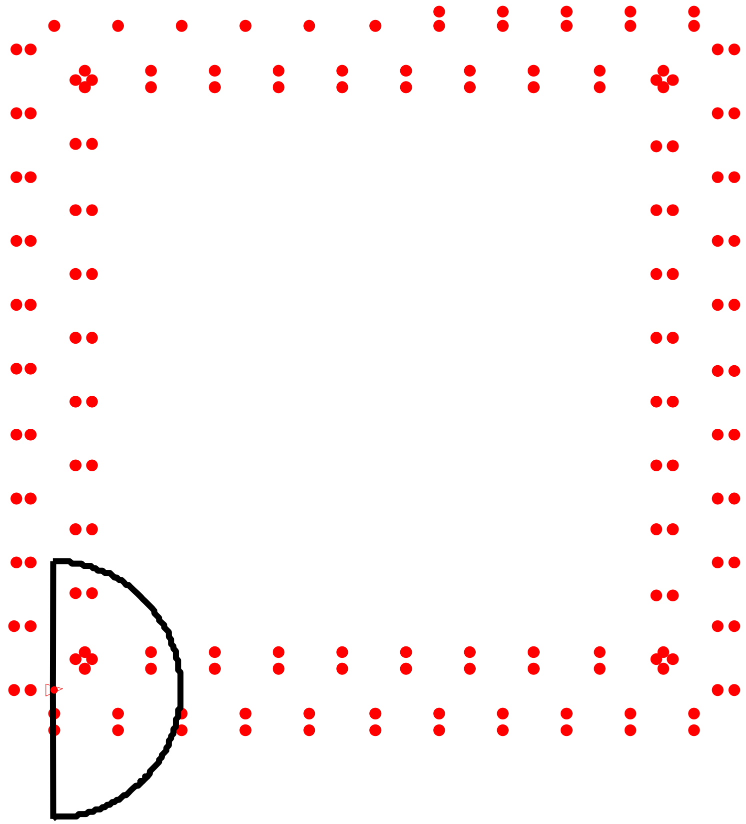
Without loss of generality...

- Environment to be mapped has more or less uniform density of features



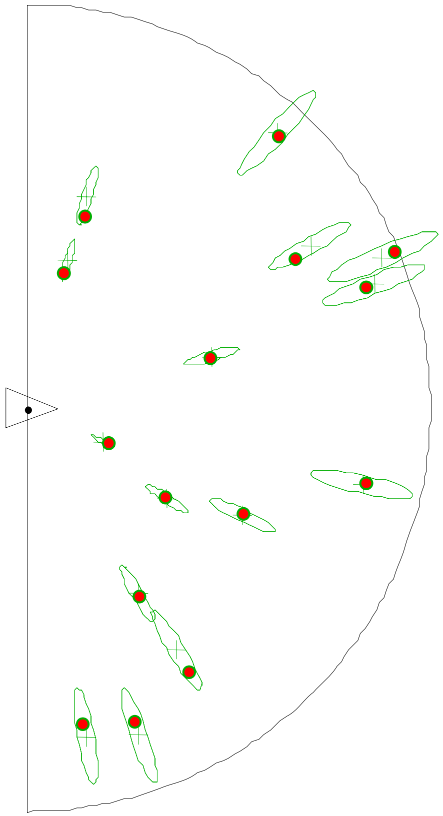
Without loss of generality...

- Enviroment to be mapped has more or less uniform density of features



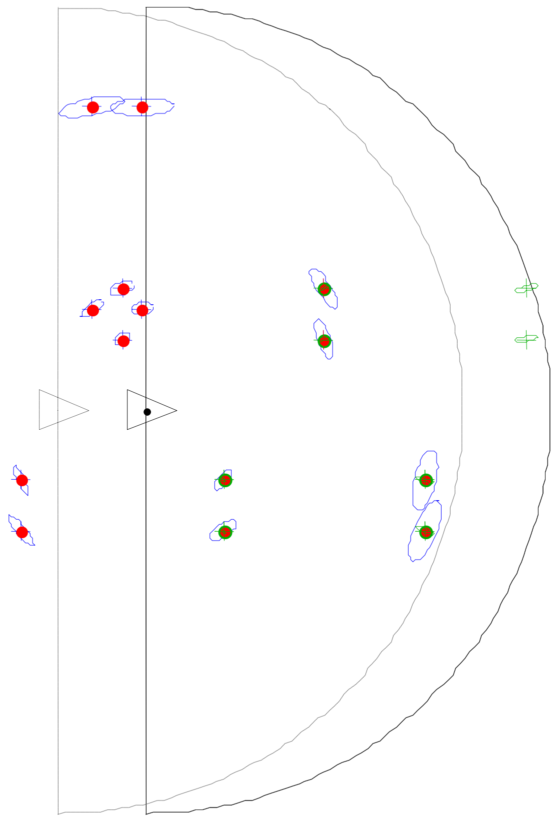
Without loss of generality...

- Onboard range and bearing sensor obtains ***m*** measurements



Without loss of generality...

- Vehicle performs an exploratory trajectory, re-observing ***r*** features, and seeing ***s = m – r*** new features.



The prediction step

EKF SLAM prediction

$$\hat{\mathbf{x}}^B_{k|k-1} = \begin{bmatrix} \hat{\mathbf{x}}^B_{R_{k-1}} \oplus \hat{\mathbf{x}}^{R_{k-1}}_{R_k} \\ \hat{\mathbf{x}}^B_{F_{1,k-1}} \\ \vdots \\ \hat{\mathbf{x}}^B_{F_{m,k-1}} \end{bmatrix}$$

$$\mathbf{P}^B_{k|k-1} \simeq \mathbf{F}_k \mathbf{P}^B_{k-1} \mathbf{F}^T_k + \mathbf{G}_k \mathbf{Q}_k \mathbf{G}^T_k$$

$$\mathbf{F}_k = \begin{bmatrix} \mathbf{J}_{1\oplus}\left\{\hat{\mathbf{x}}^B_{R_{k-1}}, \hat{\mathbf{x}}^{R_{k-1}}_{R_k}\right\} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{I} & & \vdots \\ \vdots & & \ddots & \\ \mathbf{0} & \cdots & & \mathbf{I} \end{bmatrix} \; ; \; \mathbf{G}_k = \begin{bmatrix} \mathbf{J}_{2\oplus}\left\{\hat{\mathbf{x}}^B_{R_{k-1}}, \hat{\mathbf{x}}^{R_{k-1}}_{R_k}\right\} \\ \mathbf{0} \\ \vdots \\ \mathbf{0} \end{bmatrix}$$

EKF SLAM: prediction

$$\mathbf{P}_{k-1|k-1} = \begin{pmatrix} \mathbf{P}_R & \mathbf{P}_{RF_1} & \cdots & \mathbf{P}_{RF_n} \\ \mathbf{P}_{RF_1}^T & \mathbf{P}_{F_1} & \cdots & \mathbf{P}_{F_1F_n} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{P}_{RF_n}^T & \mathbf{P}_{F_1F_n}^T & \cdots & \mathbf{P}_{F_n} \end{pmatrix}$$

O(n)

O(1)

$$\mathbf{P}_{k|k-1} = \begin{pmatrix} \mathbf{J}_{1\oplus}\mathbf{P}_R\mathbf{J}_{1\oplus}^T + \mathbf{J}_{2\oplus}\mathbf{Q}_k\mathbf{J}_{2\oplus}^T & \mathbf{J}_{1\oplus}\mathbf{P}_{RF_1} & \cdots & \mathbf{J}_{1\oplus}\mathbf{P}_{RF_n} \\ \mathbf{J}_{1\oplus}^T\mathbf{P}_{RF_1}^T & \mathbf{P}_{F_1} & \cdots & \mathbf{P}_{F_1F_n} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{J}_{1\oplus}^T\mathbf{P}_{RF_n}^T & \mathbf{P}_{F_1F_n}^T & \cdots & \mathbf{P}_{F_n} \end{pmatrix}$$

EKF prediction is *O(n)*

Adding new features

EKF SLAM: add new features

$$\mathbf{x}_k^B = \begin{pmatrix} \mathbf{x}_{R_k}^B \\ \mathbf{x}_{F_{1,k}}^B \\ \vdots \\ \mathbf{x}_{F_{n,k}}^B \end{pmatrix} \Rightarrow \mathbf{x}_{k+}^B = \begin{pmatrix} \mathbf{x}_{R_k}^B \\ \mathbf{x}_{F_{1,k}}^B \\ \vdots \\ \mathbf{x}_{F_{n,k}}^B \\ \mathbf{x}_{F_{n+1,k}}^B \end{pmatrix} = \begin{pmatrix} \mathbf{x}_{R_k}^B \\ \mathbf{x}_{F_{1,k}}^B \\ \vdots \\ \mathbf{x}_{F_{n,k}}^B \\ \mathbf{x}_{R_k}^B \oplus \mathbf{z}_i \end{pmatrix}$$

Linearization:

$$\mathbf{x}_{k+}^B \simeq \hat{\mathbf{x}}_{k+}^B + \mathbf{F}_k(\mathbf{x}_k^B - \hat{\mathbf{x}}_k^B) + \mathbf{G}_k(\mathbf{z}_i - \hat{\mathbf{z}}_i)$$

$$\mathbf{P}_{k+}^B = \mathbf{F}_k \mathbf{P}_k^B \mathbf{F}_k^T + \mathbf{G}_k \mathbf{R}_k \mathbf{G}_k^T$$

Where:

$$\mathbf{F}_k = \frac{\partial \mathbf{x}_{k+}^B}{\partial \mathbf{x}_k^B} = \begin{pmatrix} \mathbf{I} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \ddots & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{I} \\ \mathbf{J}_{1\oplus}\left\{\hat{\mathbf{x}}_{R_k}^B, \hat{\mathbf{z}}_i\right\} & \mathbf{0} & \cdots & \mathbf{0} \end{pmatrix}; \mathbf{G}_k = \frac{\partial \mathbf{x}_{k+}^B}{\partial \mathbf{z}_i} = \begin{pmatrix} \mathbf{0} \\ \vdots \\ \mathbf{0} \\ \mathbf{J}_{2\oplus}\left\{\hat{\mathbf{x}}_{R_k}^B, \hat{\mathbf{z}}_i\right\} \end{pmatrix}$$

EKF SLAM: add new features

$$\mathbf{P}_k = \begin{pmatrix} \mathbf{P}_R & \mathbf{P}_{RF_1} & \cdots & \mathbf{P}_{RF_n} \\ \mathbf{P}_{RF_1}^T & \mathbf{P}_{F_1} & \cdots & \mathbf{P}_{F_1F_n} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{P}_{RF_n}^T & \mathbf{P}_{F_1F_n}^T & \cdots & \mathbf{P}_{F_n} \end{pmatrix}$$

O(n)

$$\mathbf{P}_{k+} = \begin{pmatrix} \mathbf{P}_R & \mathbf{P}_{RF_1} & \cdots & \mathbf{P}_{RF_n} & \mathbf{P}_R\mathbf{J}_{1\oplus}^T \\ \mathbf{P}_{RF_1}^T & \mathbf{P}_{F_1} & \cdots & \mathbf{P}_{F_1F_n} & \mathbf{P}_{RF_1}^T\mathbf{J}_{1\oplus}^T \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \mathbf{P}_{RF_n}^T & \mathbf{P}_{F_1F_n}^T & \cdots & \mathbf{P}_{F_n} & \mathbf{P}_{RF_n}^T\mathbf{J}_{1\oplus}^T \\ \mathbf{J}_{1\oplus}\mathbf{P}_R & \mathbf{J}_{1\oplus}\mathbf{P}_{RF_1} & \cdots & \mathbf{J}_{1\oplus}\mathbf{P}_{RF_n} & \mathbf{J}_{1\oplus}\mathbf{P}_R\mathbf{J}_{1\oplus}^T + \mathbf{J}_{2\oplus}\mathbf{R}_k\mathbf{J}_{2\oplus}^T \end{pmatrix}$$

O(s)

Adding new features is *O(n)*

The update step

EKF SLAM: map update

m **observations:**

$$\mathbf{z}_k = \mathbf{h}_k(\mathbf{x}^B_{\mathcal{F}_k}) + \mathbf{w}_k$$

$$\mathbf{z}_k \simeq \mathbf{h}_k(\hat{\mathbf{x}}^B_{\mathcal{F}_{k|k-1}}) + \mathbf{H}_k(\mathbf{x}^B_{\mathcal{F}_k} - \hat{\mathbf{x}}^B_{\mathcal{F}_{k|k-1}})$$

$$\mathbf{H}_k = \left.\frac{\partial \mathbf{h}_k}{\partial \mathbf{x}^B_{\mathcal{F}_k}}\right|_{(\hat{\mathbf{x}}^B_{\mathcal{F}_{k|k-1}})}$$

Innovation Matrix: $\mathbf{S}_k = \mathbf{H}_k \mathbf{P}_{k|k-1} \mathbf{H}_k^T + \mathbf{R}_k$

Filter gain: $\mathbf{K}_k = \mathbf{P}_{k|k-1} \mathbf{H}_k^T (\mathbf{S}_k)^{-1}$

State update: $\hat{\mathbf{x}}_k = \hat{\mathbf{x}}_{k|k-1} + \mathbf{K}_k(\mathbf{z}_k - \mathbf{h}_k(\hat{\mathbf{x}}_{k|k-1}))$

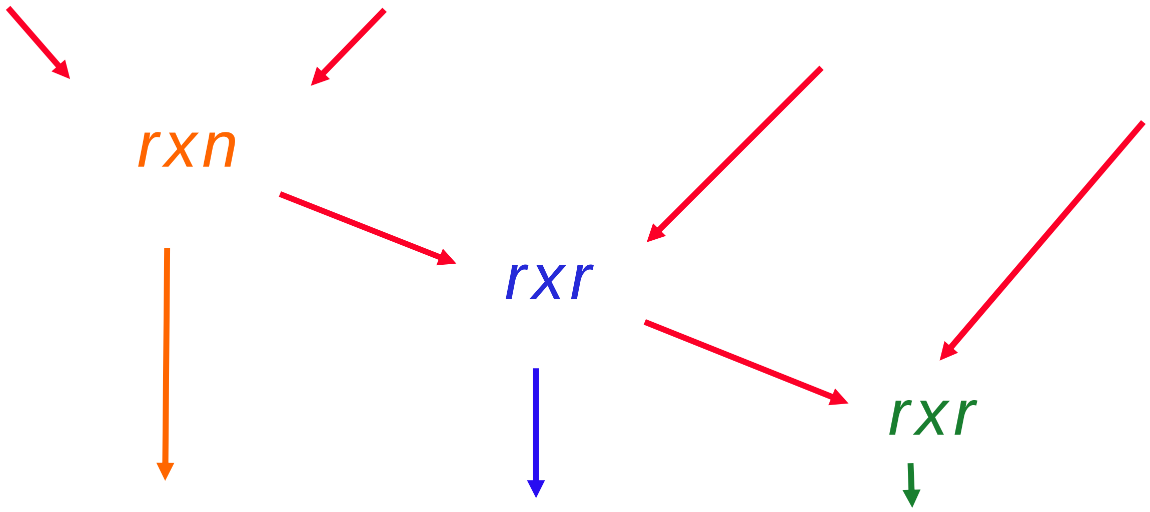
Covariance update: $\mathbf{P}_k = (\mathbf{I} - \mathbf{K}_k \mathbf{H}_k)\mathbf{P}_{k|k-1}$

ROBOTICS UNIVERSIDAD DE ZARAGOZA

The innovation matrix

$$\mathbf{S}_k = (\mathbf{H}_k \mathbf{P}_{k|k-1})\mathbf{H}_k^T + \mathbf{R}_k$$

$r x r$ = $r x n$ · $n x n$ · $n x r$ + $r x r$



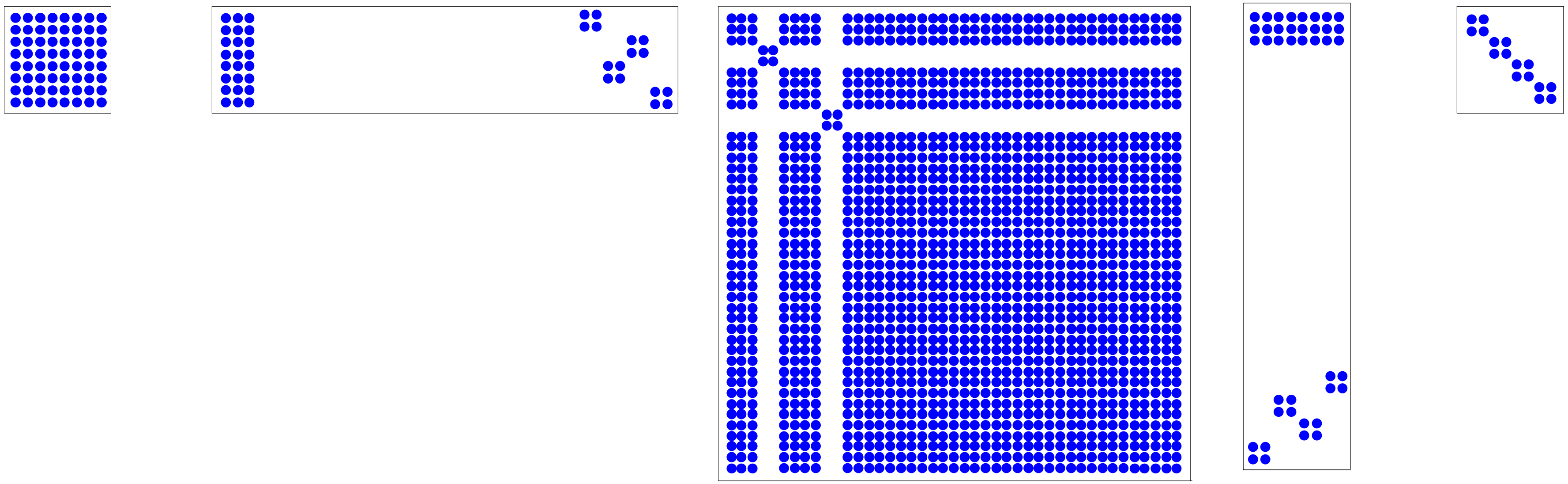
$rn^2 + r^2n + r^2$

O(n²) ?

The innovation matrix

$$\mathbf{S}_k = (\mathbf{H}_k \mathbf{P}_{k|k-1})\mathbf{H}_k^T + \mathbf{R}_k$$

rxr *rxc* *nxn* *cxr* *rxr*

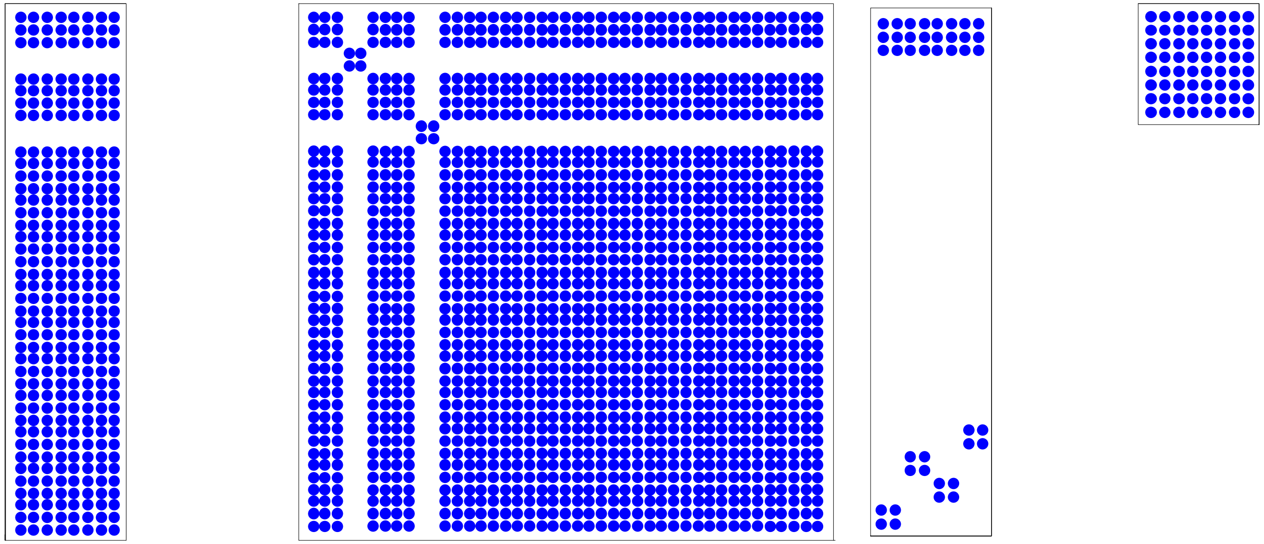


O(rn)

The Kalman gain matrix

$$\mathbf{K}_k = \mathbf{P}_{k|k-1} \mathbf{H}_k^T (\mathbf{S}_k)^{-1}$$

nxr *nxn* *nxc* *rxr*



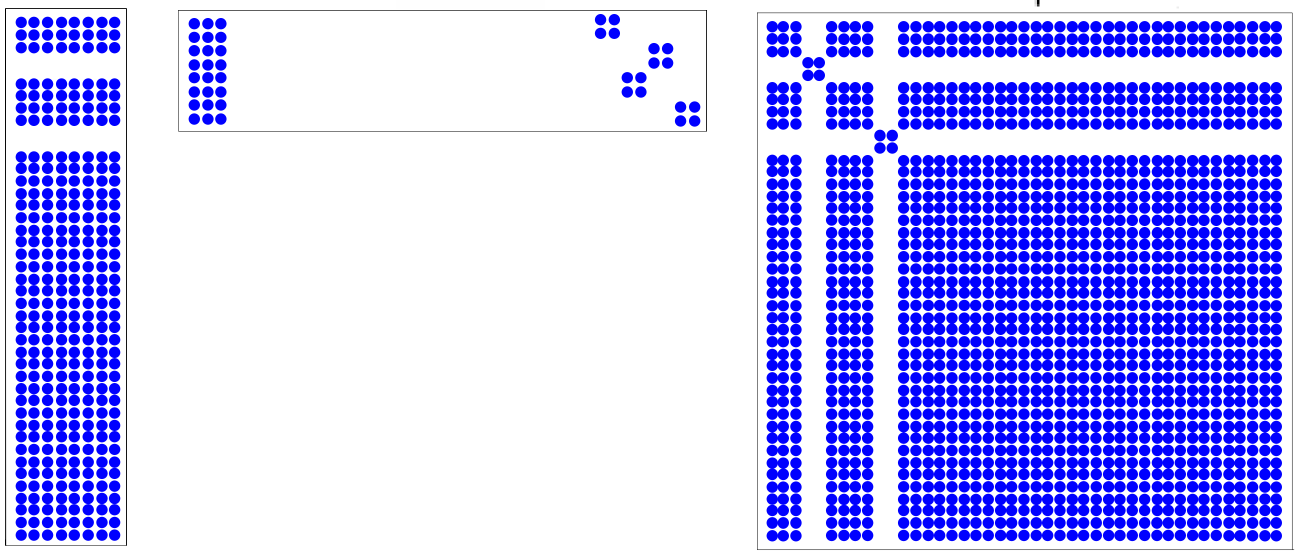
O(r²n)

The covariance matrix

$$\mathbf{P}_k = (\mathbf{I} - \mathbf{K}_k \mathbf{H}_k)\mathbf{P}_{k|k-1}$$

..... = ... $\mathbf{K}_k$ $\mathbf{H}_k$ $\mathbf{P}_{k|k-1}$

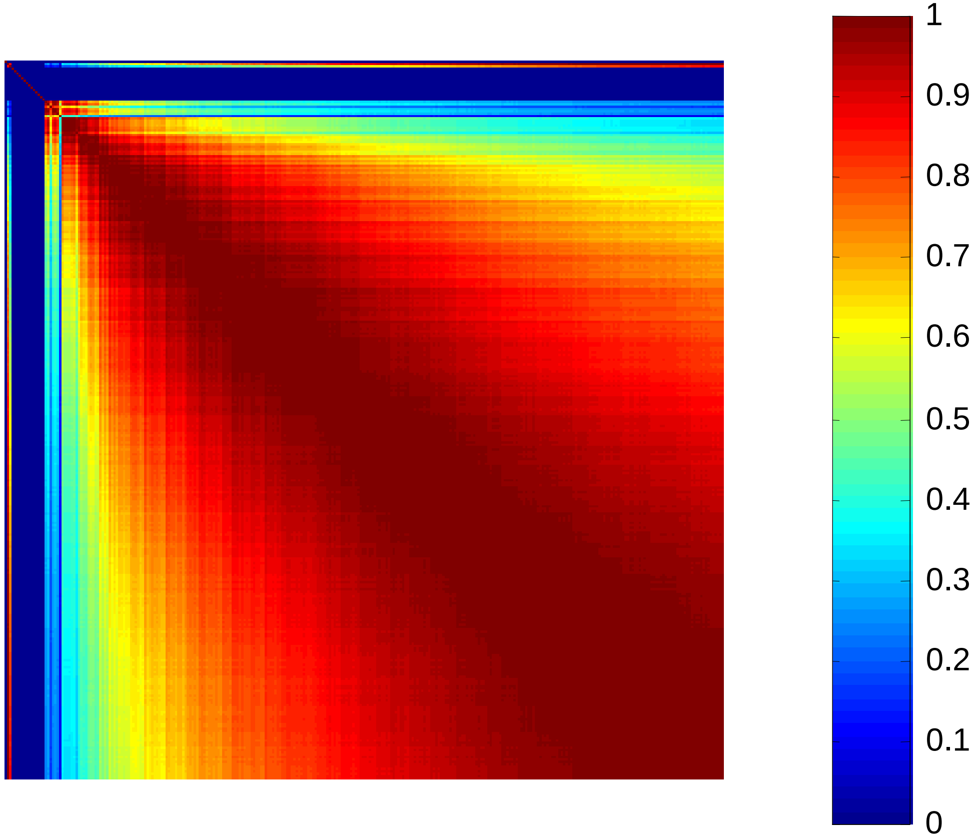
....



nxr *cxn* *nxn*

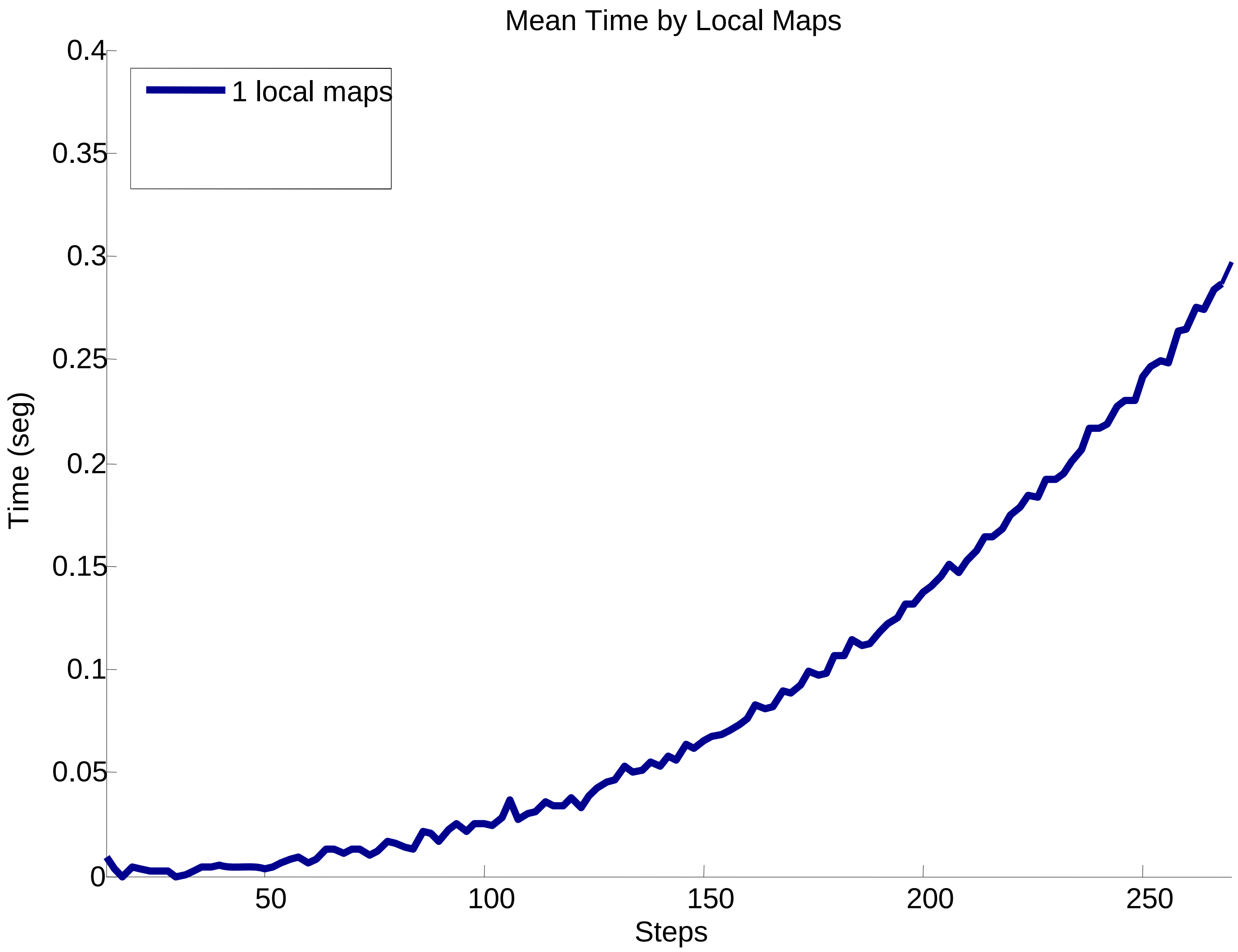
EKF update step is $O(n^2)$

The mixed blessing of Covariance



- Covariance provides data association
- But the covariance matrix is full

EKF-SLAM updates are $O(n^2)$



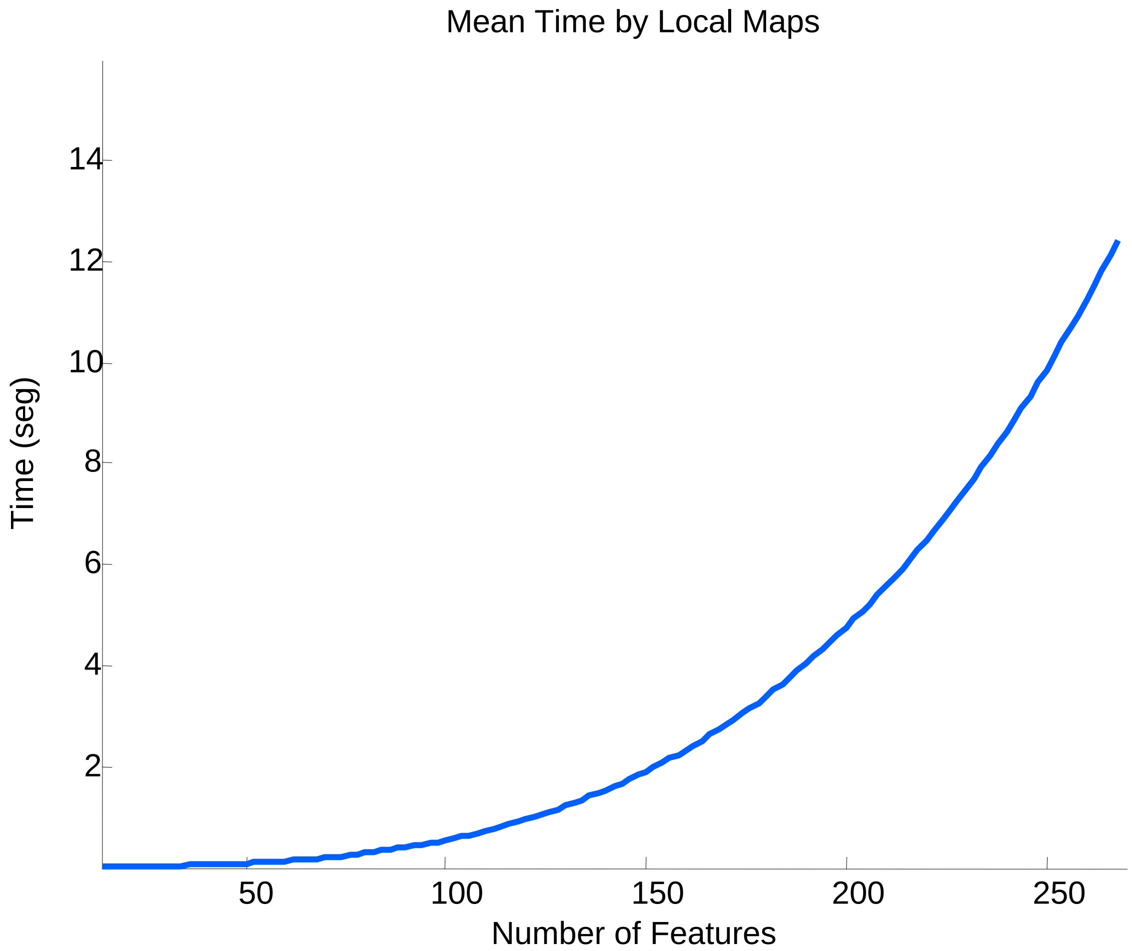
Total EKF SLAM is *O(n³)*

- The total cost of computing a map os size ***n***, with ***m*** observations per step, with ***s=m-r*** new features per step:

$$
\begin{aligned}
O\left(\sum_{k=1}^{n/s}(ks)^2\right) &= O\left(s^2\sum_{k=1}^{n/s}k^2\right) \\
&= O\left(s^2\frac{(n/s)(n/s+1)(2n/s+1)}{6}\right) \\
&= O\left(\frac{1}{6}2n^3/s+3n^2+ns\right) \\
&= O(n^3)
\end{aligned}
$$

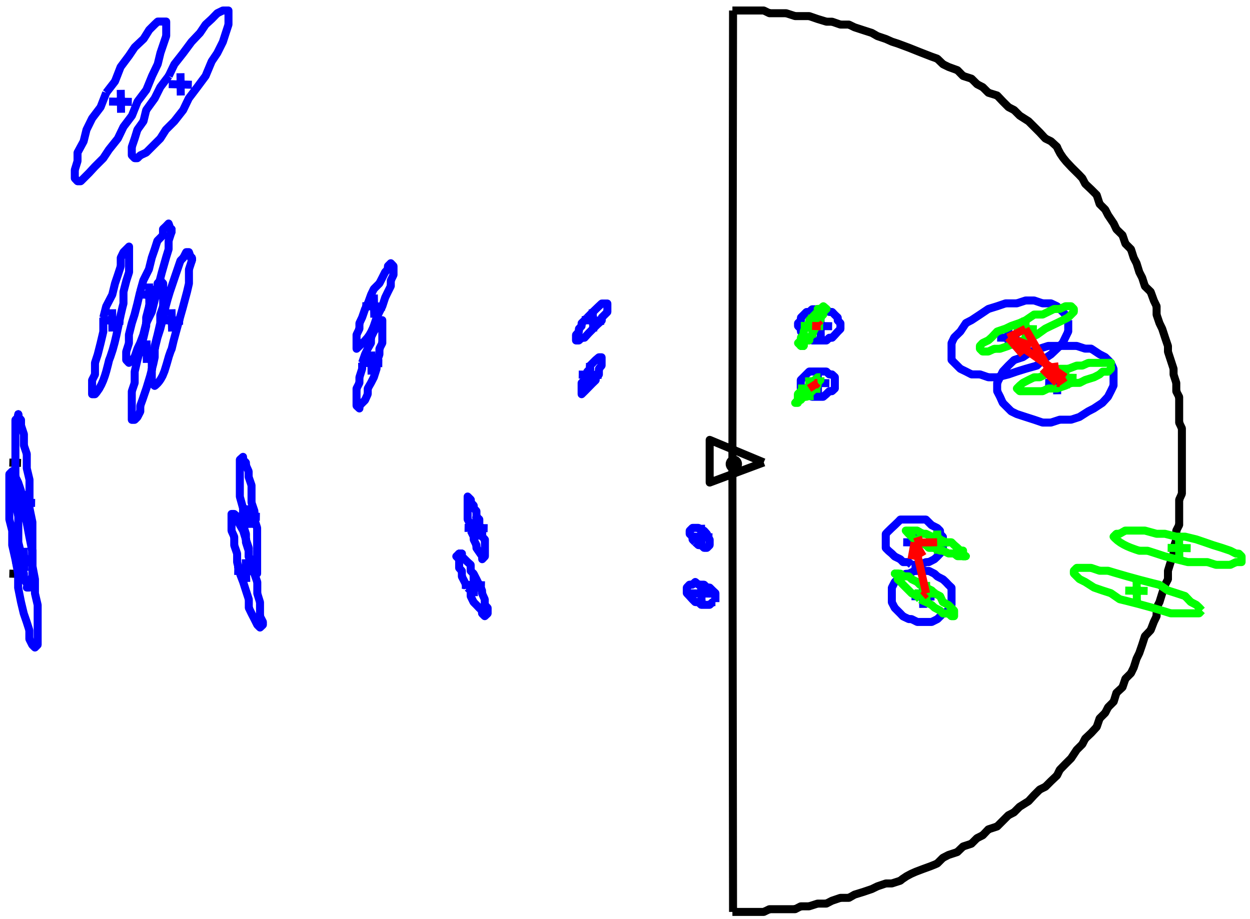
ROBOTICS
UNIVERSIDAD DE ZARAGOZA

Total cost of EKF-SLAM



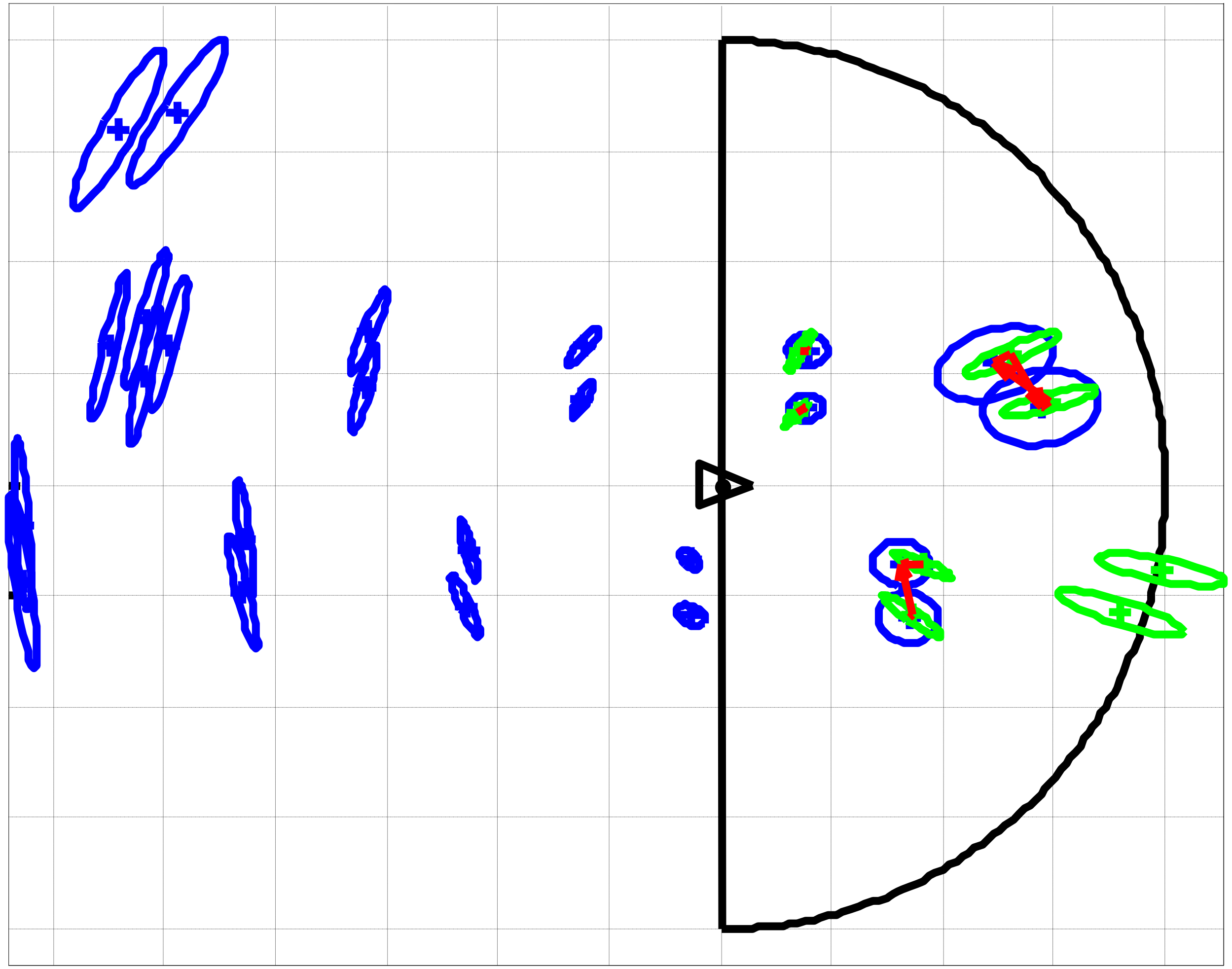
Total SLAM is $O(n^3)$

Continuous data association



Individual compatibility is $O(nm) = O(n)$

Map tessellation



Individual compatibility can be *O(1)*

Global localization: Computational Complexity

Sensor	Pair_driven
Range and bearing	$O(n \cdot m \cdot n_\phi)$
Range-only	$O(n \cdot m \cdot n_\phi \cdot n_\theta)$
Bearing-only	$O(n \cdot m \cdot n_\phi \cdot n_r)$

Global localization is *O(nm)* = *O(n)*

Outline

1. Basic EKF SLAM
   1. Introduction: the need for SLAM
   2. The basic EKF SLAM algorithm
   3. Feature extraction

2. The Data Association Problem
   1. Introduction
   2. Continuous Data Association
   3. The Loop Closing Problem
   4. The Global Localization Problem

3. Advanced EKF SLAM
   1. Computational complexity of EKF SLAM
   2. **Consistency of the EKF SLAM**
   3. SLAM using local maps
      1. Sequential Map Joining
      2. Divide and Conquer SLAM
      3. Hierarchical SLAM

ROBOTICS UNIVERSIDAD DE ZARAGOZA

Consistency of EKF-SLAM

- Nice **"convergence"** properties of $\mathbf{P}_k^W$ (Dissanayake et al. 2001):
  - Landmark covariance decreases monotonically
  - In the limit, landmarks become fully correlated
  - In the limit, landmark covariance reaches a lower bound related to the initial vehicle covariance

- But SLAM is a non-linear problem
  - The inherent approximations due to linearizations can lead to **divergence (inconsistency)** of the EKF
    - » see for example (Jazwinski, 1970)

EKF-SLAM: Robot Motion

$$\mathbf{x}^B_{R_k} = \mathbf{x}^B_{R_{k-1}} \oplus \mathbf{x}^{R_{k-1}}_{R_k}$$

Odometry model (white noise):

$$\begin{aligned} \mathbf{x}^{R_{k-1}}_{R_k} &= \hat{\mathbf{x}}^{R_{k-1}}_{R_k} + \mathbf{v}_k \\ E[\mathbf{v}_k] &= \mathbf{0} \\ E[\mathbf{v}_k \mathbf{v}_j^T] &= \delta_{kj} \mathbf{Q}_k \end{aligned}$$

EKF prediction:

$$\hat{\mathbf{x}}^B_{k|k-1} = \begin{bmatrix} \hat{\mathbf{x}}^B_{R_{k-1}} \oplus \hat{\mathbf{x}}^{R_{k-1}}_{R_k} \\ \hat{\mathbf{x}}^B_{F_{1,k-1}} \\ \vdots \\ \hat{\mathbf{x}}^B_{F_{m,k-1}} \end{bmatrix}$$

$$\mathbf{P}^B_{\mathcal{F}_{k|k-1}} = \mathbf{F}_k \mathbf{P}^B_{k-1} \mathbf{F}_k^T + \mathbf{G}_k \mathbf{Q}_k \mathbf{G}_k^T$$

Linearization

$$\mathbf{F}_k = \begin{bmatrix} \mathbf{J}_{1\oplus}\left\{\hat{\mathbf{x}}^B_{R_{k-1}}, \hat{\mathbf{x}}^{R_{k-1}}_{R_k}\right\} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{I} & & \vdots \\ \vdots & & \ddots & \\ \mathbf{0} & \cdots & & \mathbf{I} \end{bmatrix}$$

$$\mathbf{G}_k = \begin{bmatrix} \mathbf{J}_{2\oplus}\left\{\hat{\mathbf{x}}^B_{R_{k-1}}, \hat{\mathbf{x}}^{R_{k-1}}_{R_k}\right\} \\ \mathbf{0} \\ \vdots \\ \mathbf{0} \end{bmatrix}$$

EKF-SLAM: Map Update

Feature observations:

$$\mathbf{z}_k = \mathbf{h}_k(\mathbf{x}_k^B) + \mathbf{w}_k$$

$$\mathbf{z}_k \simeq \mathbf{h}_k(\hat{\mathbf{x}}_{k|k-1}^B) + \mathbf{H}_k(\mathbf{x}_k^B - \hat{\mathbf{x}}_{k|k-1}^B)$$

$$\mathbf{H}_k = \left.\frac{\partial \mathbf{h}_k}{\partial \mathbf{x}_k^B}\right|_{(\hat{\mathbf{x}}_{k|k-1}^B)}$$

Linearization

EKF map update:

$$\hat{\mathbf{x}}_k^B = \hat{\mathbf{x}}_{k|k-1}^B + \mathbf{K}_k(\mathbf{z}_k - \mathbf{h}_k(\hat{\mathbf{x}}_{k|k-1}^B))$$

$$\mathbf{P}_k^B = (\mathbf{I} - \mathbf{K}_k\mathbf{H}_k)\mathbf{P}_{k|k-1}^B$$

$$\mathbf{K}_k = \mathbf{P}_{k|k-1}^B\mathbf{H}_k^T(\mathbf{H}_k\mathbf{P}_{k|k-1}^B\mathbf{H}_k^T + \mathbf{R}_k)^{-1}$$

The Consistency Problem

True map value

$$\mathbf{x}_k^W$$

EKF-SLAM estimation

$$\hat{\mathbf{x}}_k^W$$

$$\mathbf{P}_k^W$$

- An estimator is **consistent** if:

$$E\left[\mathbf{x}_k^W - \hat{\mathbf{x}}_k^W\right] = 0$$

$$E\left[\left(\mathbf{x}_k^W - \hat{\mathbf{x}}_k^W\right)\left(\mathbf{x}_k^W - \hat{\mathbf{x}}_k^W\right)^T\right] = \mathbf{P}_k^W$$

Unbiased

The Mean Square Error matches the filter computed Covariance

  - Pessimistic covariance is OK (but not too pessimistic)
  - Optimistic covariance = Inconsistency = Filter divergence

ROBOTICS UNIVERSIDAD DE ZARAGOZA

Consistency Testing

1. Normalized Estimation Error Squared NEES

$$D^2 = \left(\mathbf{x}_k^W - \hat{\mathbf{x}}_k^W\right)^T \left(\mathbf{P}_k^W\right)^{-1} \left(\mathbf{x}_k^W - \hat{\mathbf{x}}_k^W\right)$$

$$D^2 \leq \chi^2_{r,1-\alpha}$$

True map required → Simulations

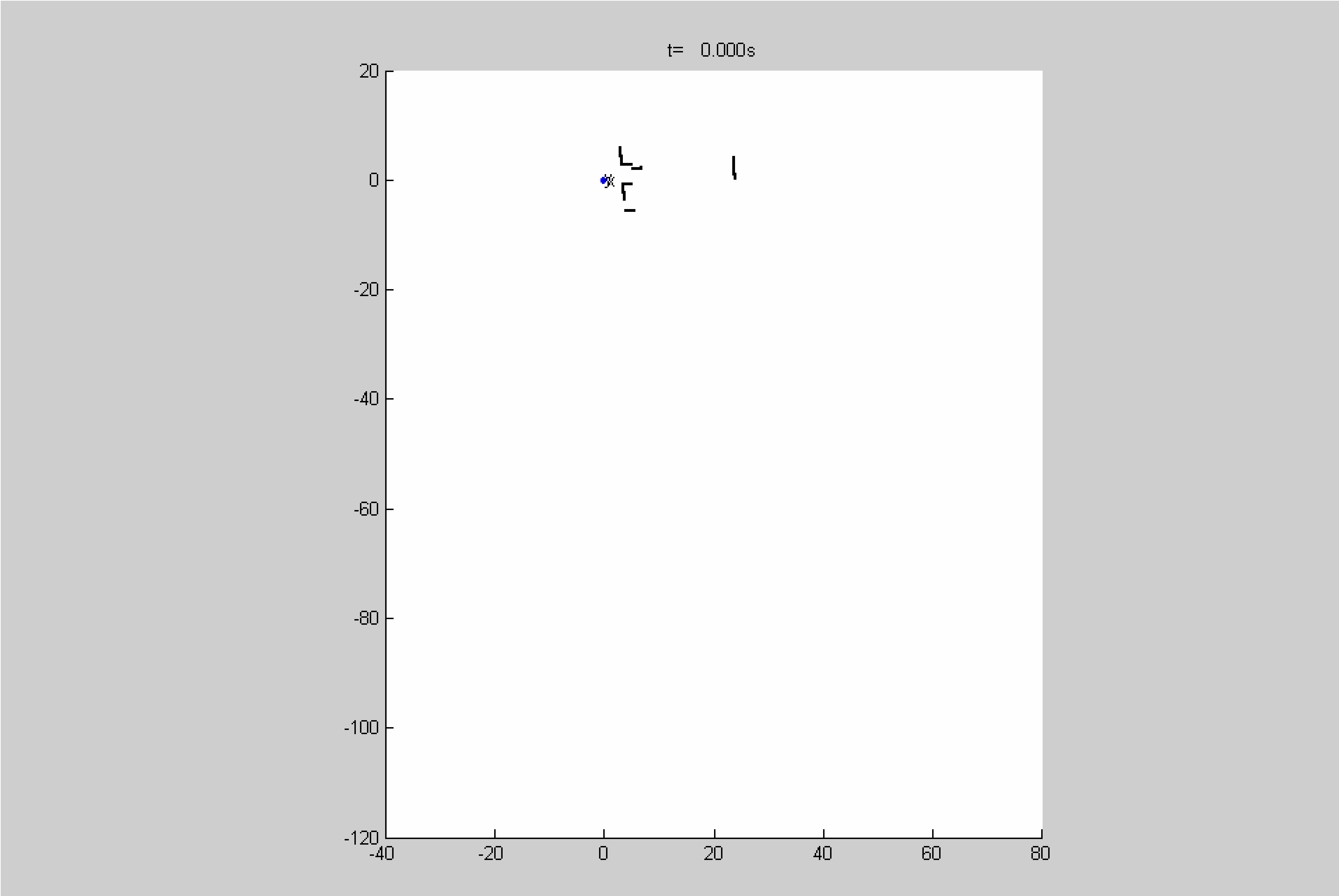
2. Innovation test (observation *i* → map feature *j*)

$$D_{ij}^2 = \left(\mathbf{z}_i - \mathbf{h}_j(\hat{\mathbf{x}}_k^W)\right)^T \left(\mathbf{H}_j \mathbf{P}_k^W \mathbf{H}_j^T + \mathbf{R}_i\right)^{-1} \left(\mathbf{z}_i - \mathbf{h}_j(\hat{\mathbf{x}}_k^W)\right)$$

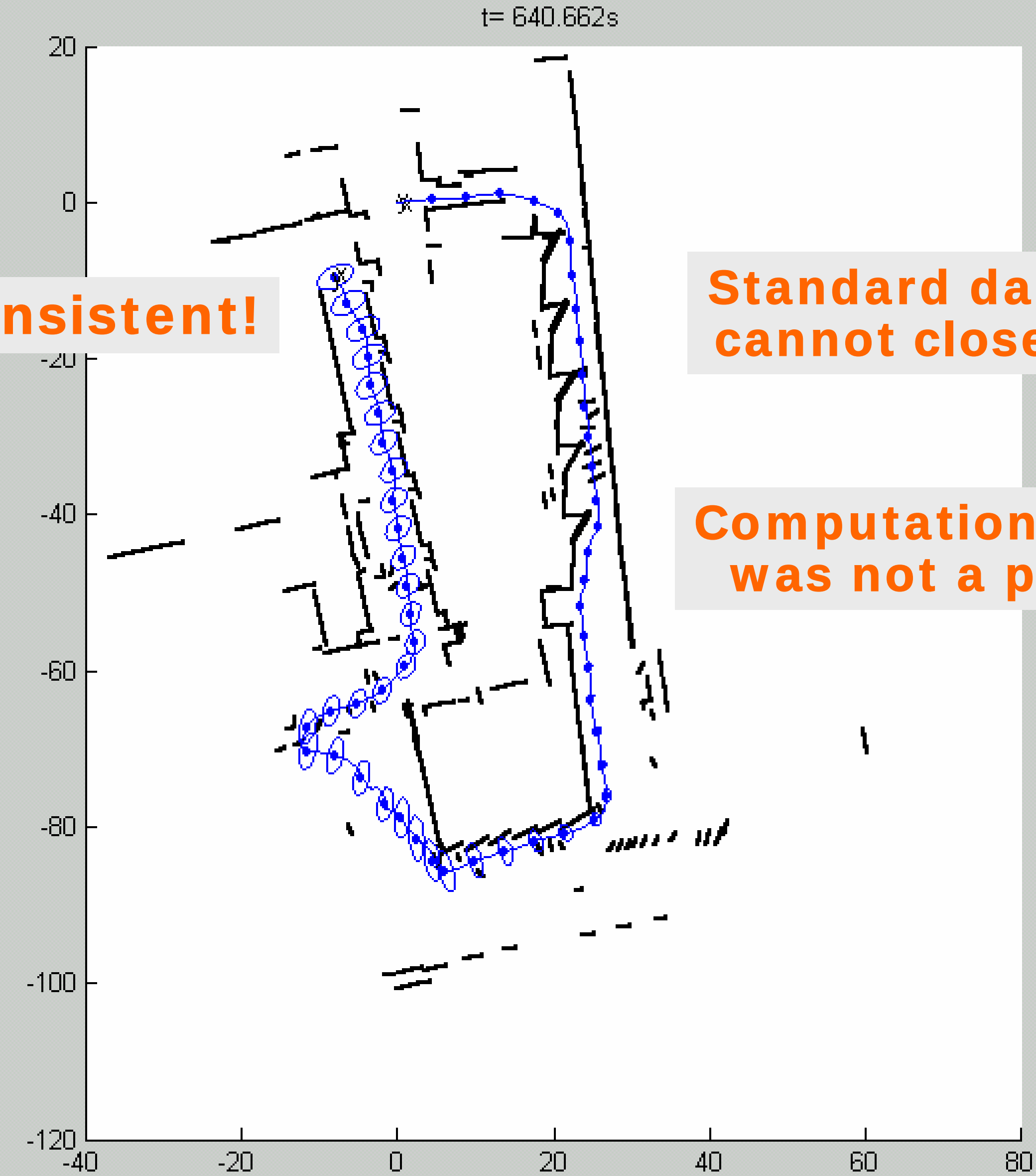
$$D_{ij}^2 \leq \chi^2_{d,1-\alpha}$$

Critical when closing big loops

EKF-SLAM: Real Example



EKF-SLAM: Real Example



Inconsistent!

Standard data association cannot close a 250m loop

Computational complexity was not a problem here!

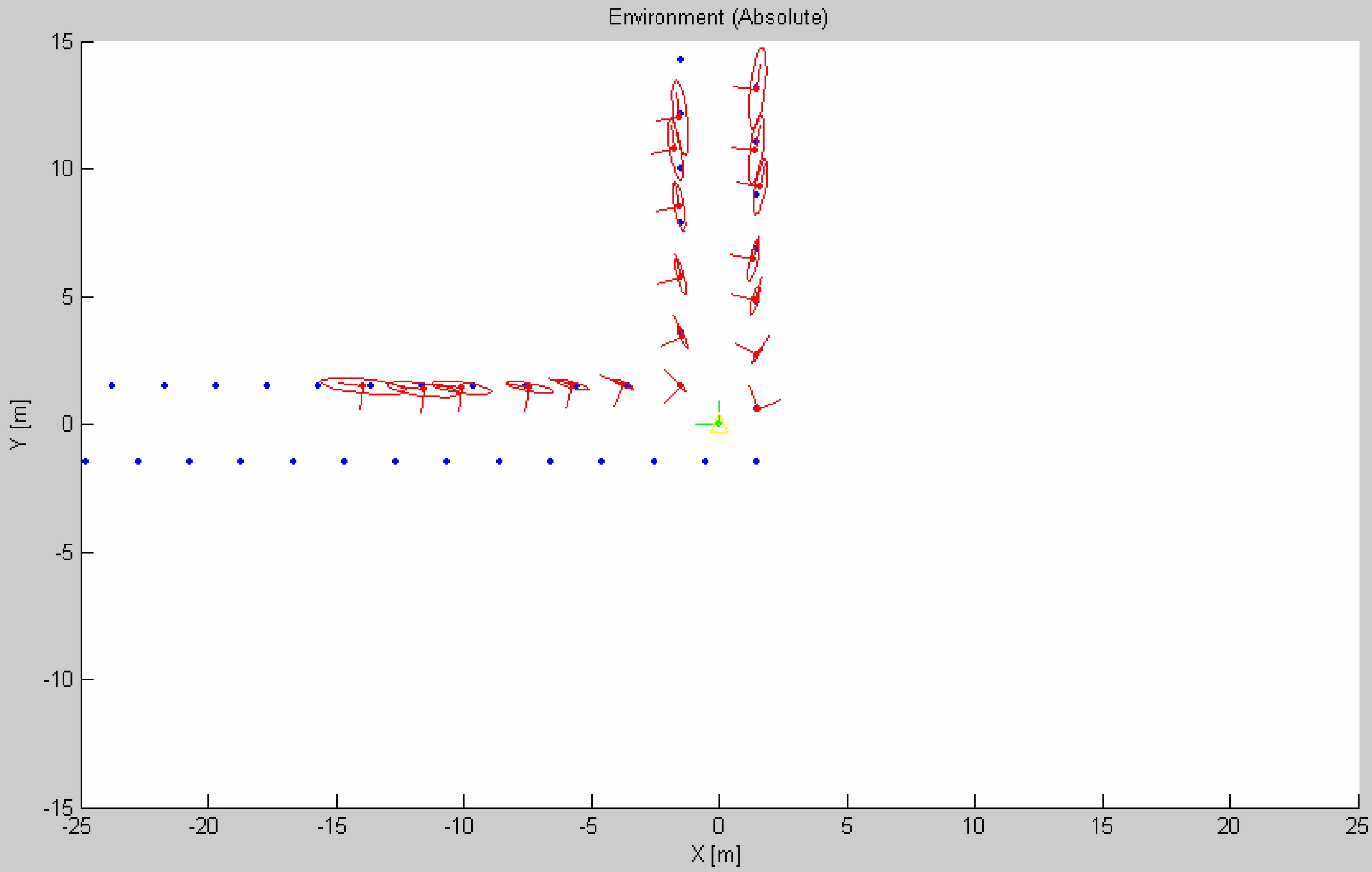
EKF-SLAM: Simulation

- Simulation conditions
  - Perfect data association
  - Ideal odometry and measurement noise
    - » white, Gaussian, known covariance

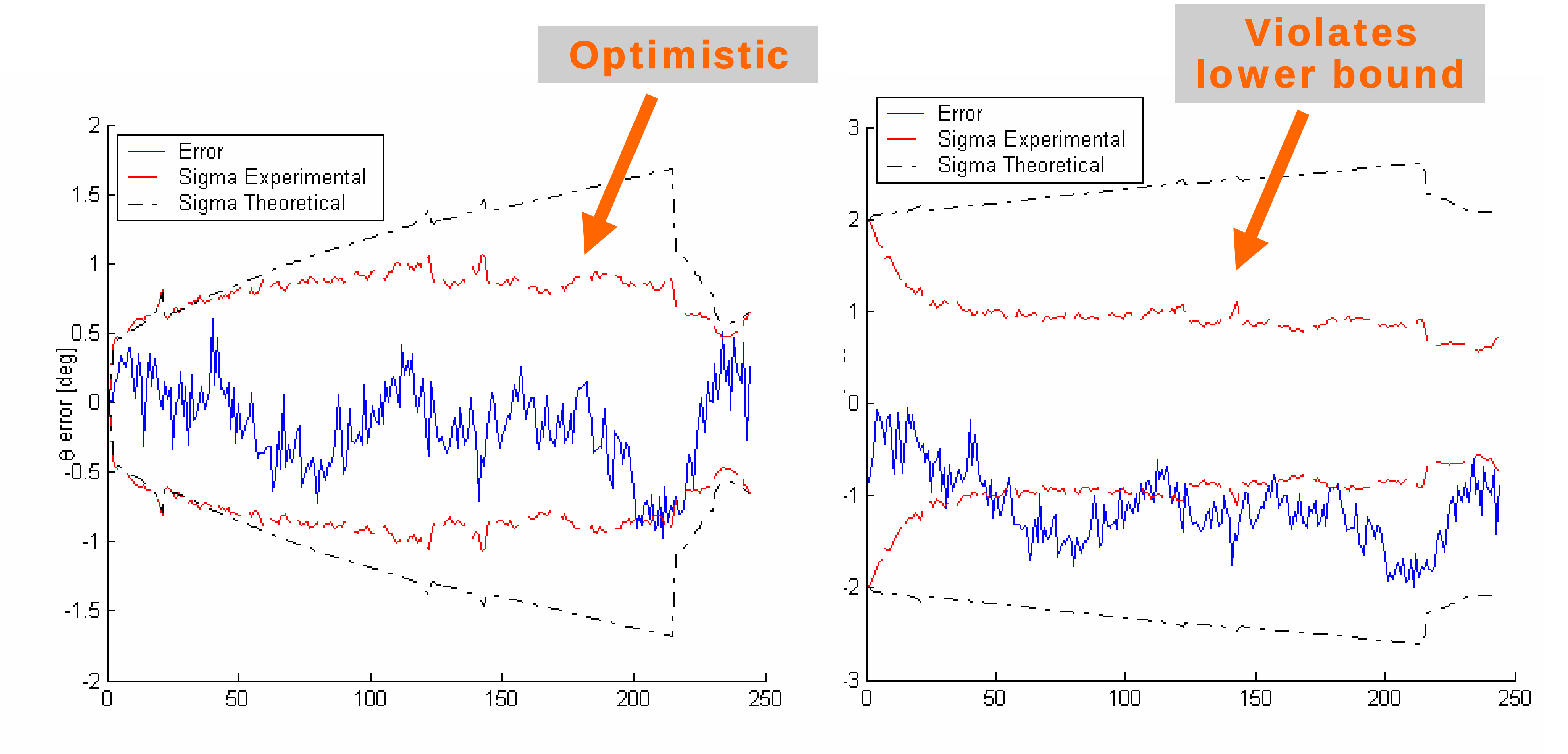
- Advantages of simulation:
  - Consistency can be tested against the true map
  - A simulation with noise=0 gives the theoretical map covariance (without linearization errors)

EKF-SLAM: Simulat

Perfect data association and noise model



EKF-SLAM: Covariance



Initial uncertainty = 0 **Initial uncertainty > 0**

J.A. Castellanos, J. Neira, J.D. Tardós, **Limits to the Consistency of EKF-based SLAM**, 5th IFAC Symposium on Intelligent Autonomous Vehicles, Lisbon, July 2004

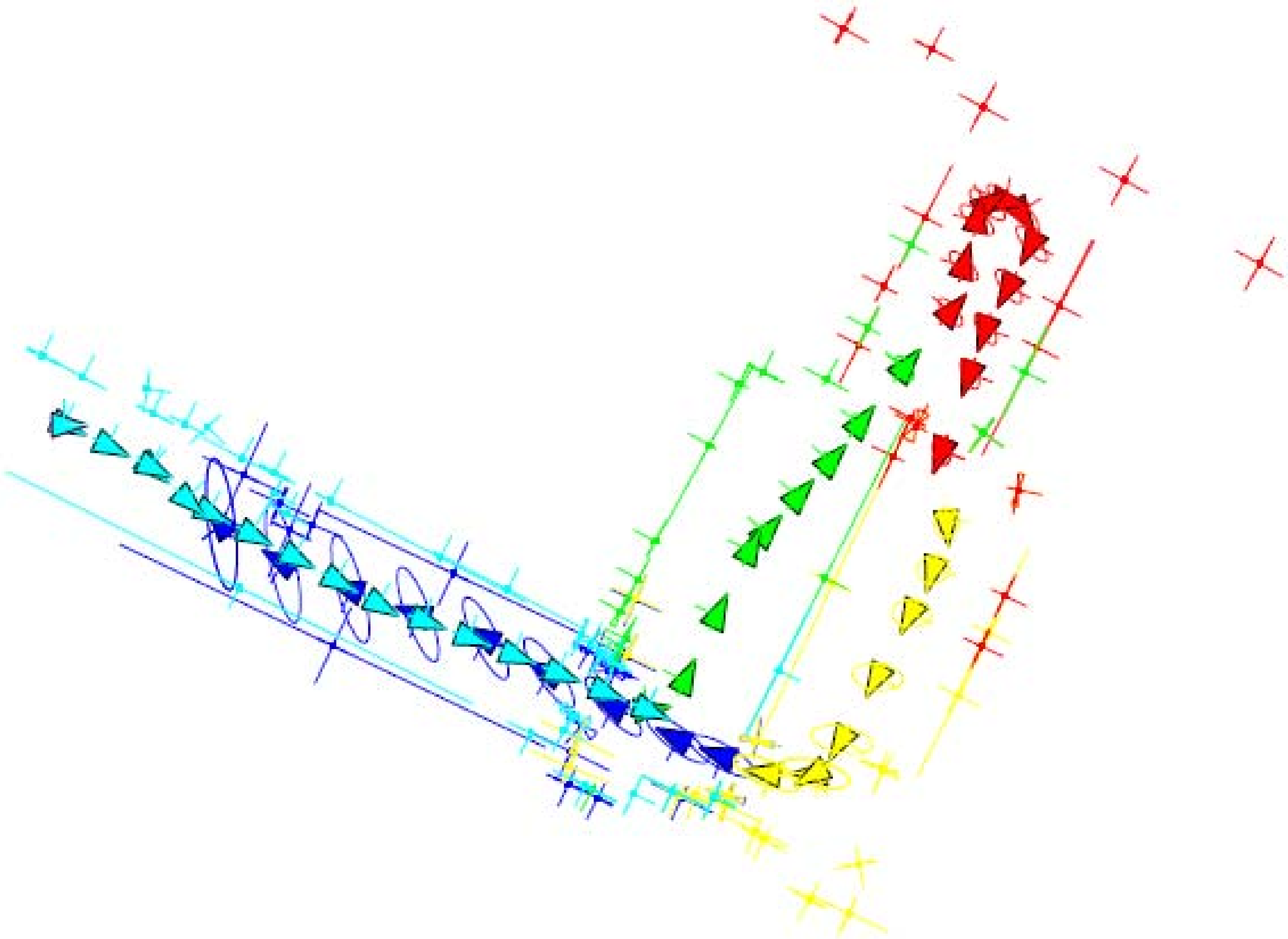
Outline

1. Basic EKF SLAM
    1. Introduction: the need for SLAM
    2. The basic EKF SLAM algorithm
    3. Feature extraction

2. The Data Association Problem
    1. Introduction
    2. Continuous Data Association
    3. The Loop Closing Problem
    4. The Global Localization Problem

3. Advanced EKF SLAM
    1. Computational complexity of EKF SLAM
    2. Consistency of the EKF SLAM
    3. **SLAM using local maps**
        1. Sequential Map Joining
        2. Divide and Conquer SLAM
        3. Hierarchical SLAM

Local submaps



Local map building

- Periodically, the robot starts a new map, relative to its current location:

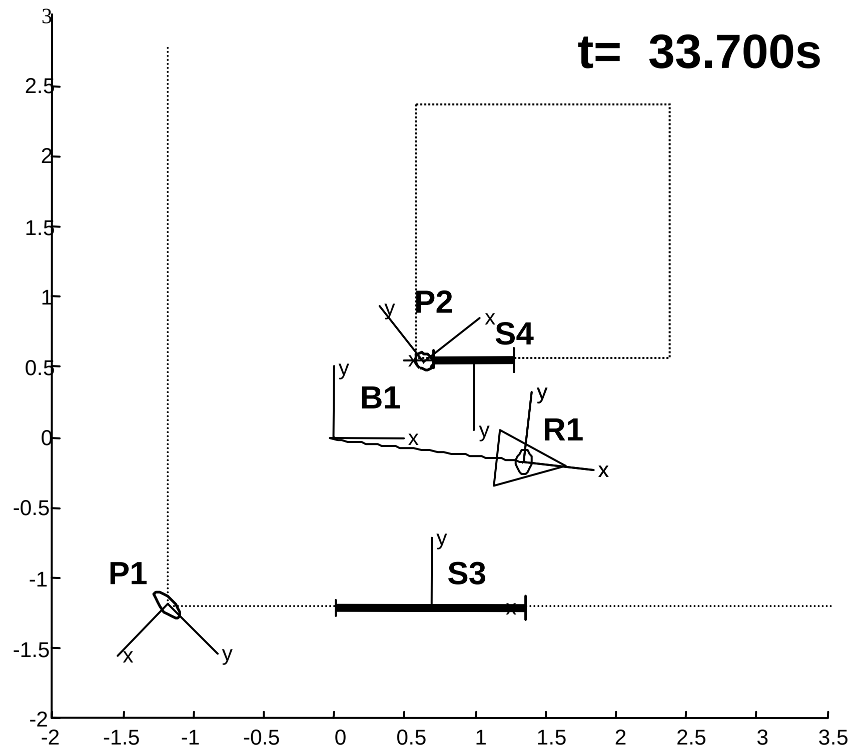
$$\hat{\mathbf{x}}^{B}_{R_0} = 0$$
$$\mathbf{P}^{B}_{R_0} = 0$$

- Given measurements:

$$D^{1\ldots k_1} = \left\{\mathbf{u}_1\,\mathbf{z}_1 \ldots \mathbf{u}_{k_1}\,\mathbf{z}_{k_1}\right\}$$
$$\mathbf{u}_k = \hat{\mathbf{x}}^{R_{k-1}}_{R_k}$$

- EKF approximates the conditional mean:

$$\hat{\mathbf{x}}^{B_1}_{\mathcal{F}_1} \simeq E\left[\mathbf{x}^{B_1}_{\mathcal{F}_1} \mid D^{1\ldots k_1},\ \mathcal{H}^{1\ldots k_1}\right]$$



Local map building

- Second map: $D^{k_1+1\ldots k_2} = \left\{ \mathbf{u}_{k_1+1}\, \mathbf{z}_{k_1+1} \ldots \mathbf{u}_{k_2}\, \mathbf{z}_{k_2} \right\}$

$$\hat{\mathbf{x}}^{B_2}_{\mathcal{F}_2} \simeq E\left[\mathbf{x}^{B_2}_{\mathcal{F}_2} \mid D^{k_1+1\ldots k_2},\ \mathcal{H}^{k_1+1\ldots k_2}\right]$$

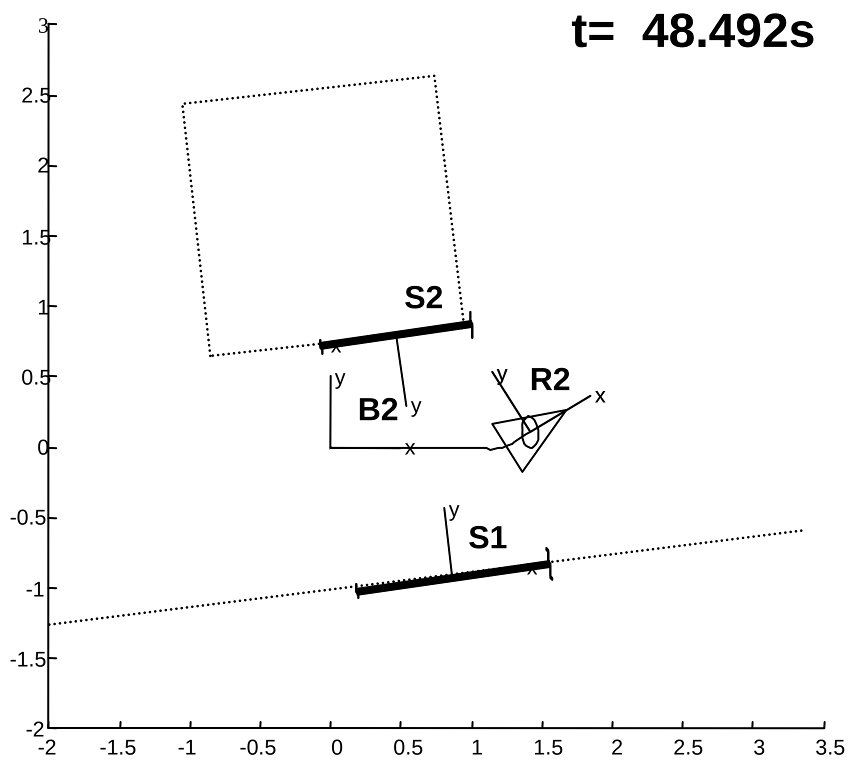
- No information is shared:

$$D^{1\ldots k_1} \cap D^{k_1+1\ldots k_2} = \emptyset$$

Maps are uncorrelated

- Common reference:

$$B_2 = R_1$$



ROBOTICS UNIVERSIDAD DE ZARAGOZA

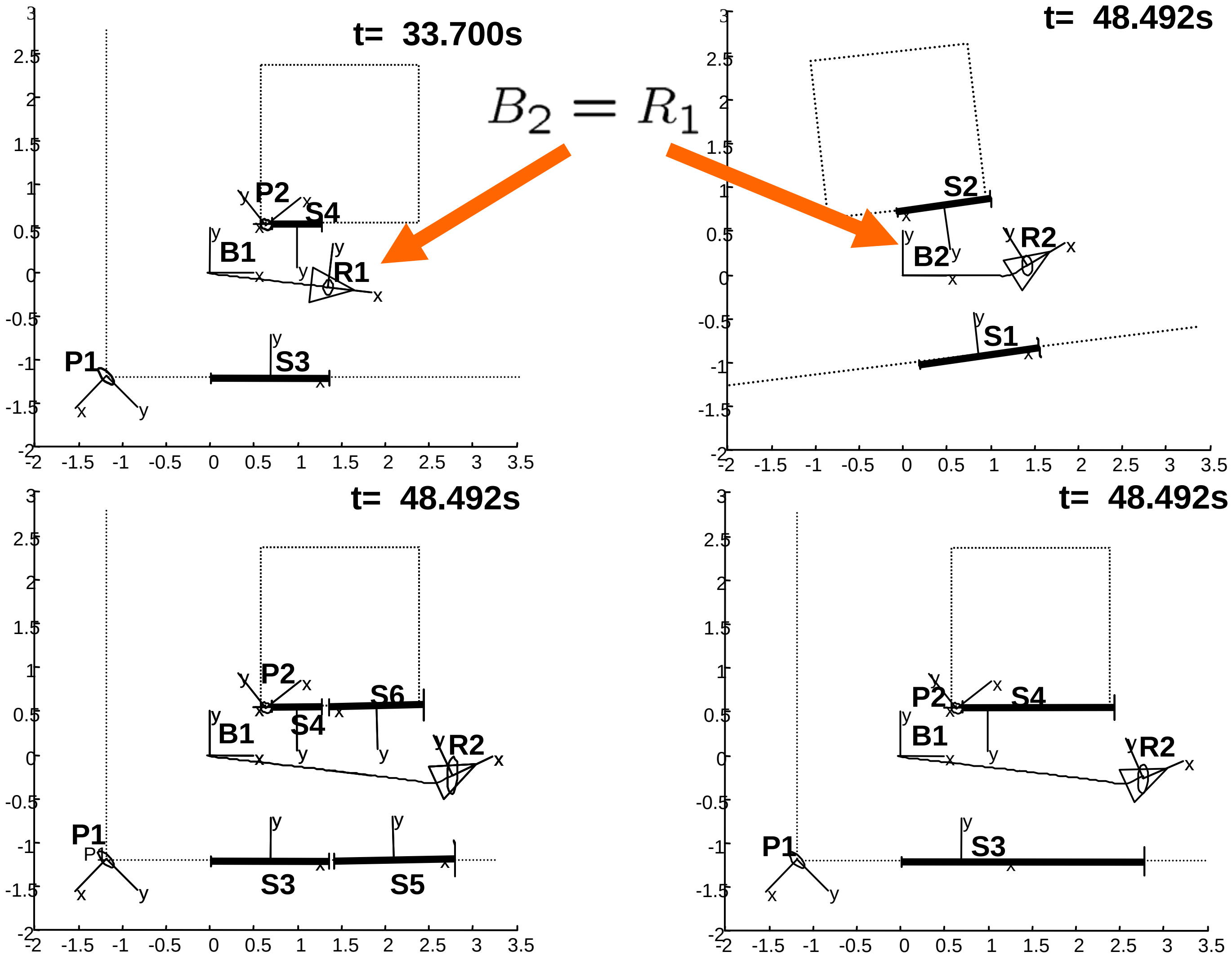
Outline

1. Basic EKF SLAM
    1. Introduction: the need for SLAM
    2. The basic EKF SLAM algorithm
    3. Feature extraction

2. The Data Association Problem
    1. Introduction
    2. Continuous Data Association
    3. The Loop Closing Problem
    4. The Global Localization Problem

3. Advanced EKF SLAM
    1. Computational complexity of EKF SLAM
    2. Consistency of the EKF SLAM
    3. SLAM using local maps
        1. **Sequential Map Joining**
        2. Divide and Conquer SLAM
        3. Hierarchical SLAM

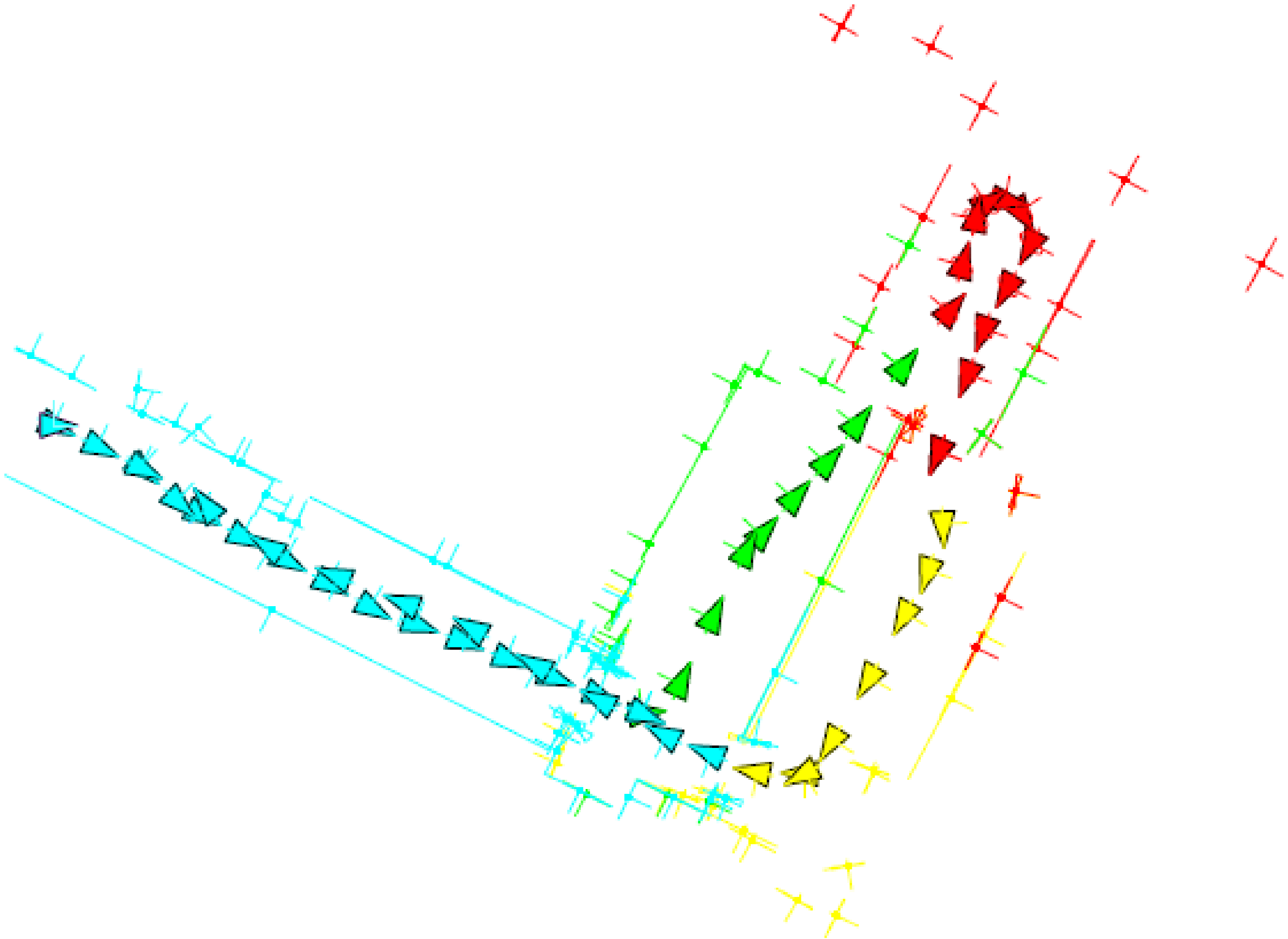
Map Joining: Example



Joined map   After matching and fusion

J.D. Tardós, J. Neira, P.M. Newman and J.J. Leonard, Robust Mapping and Localization in Indoor Environments using Sonar Data, The Int. Journal of Robotics Research, Vol. 21, No. 4, April, 2002, pp 311 –330

Map Joining



Map Joining Step

- New state vector:

$$\hat{\mathbf{x}}_{\mathcal{A}+\mathcal{B}} = \begin{bmatrix} \hat{\mathbf{x}}_{\mathcal{A}} \\ \hat{\mathbf{x}}_{\mathcal{B}} \end{bmatrix}$$

- New covariance matrix:

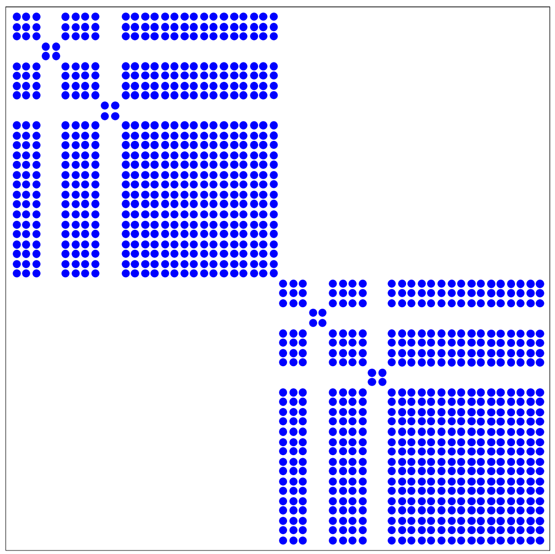
$$\mathbf{P}^{-}_{\mathcal{A}+\mathcal{B}} = \begin{bmatrix} \mathbf{P}_{\mathcal{A}} & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & \mathbf{P}_{\mathcal{B}} \end{bmatrix}$$

$$\mathbf{P}^{-}_{\mathcal{A}+\mathcal{B}} = \quad \mathbf{P}_{\mathcal{A}}$$

nxn

n1xn1

n = n1+n2



$\mathbf{P}_{\mathcal{B}}$

n2xn2

Map joining is $O(n^2{}_2)$ on the maps

Matching and Fusion Step

- Matching function:

$$\mathbf{f}_{ij_i}(\mathbf{x}) = \mathbf{0}$$

- Joint matching function for the hypothesis:

$$\mathbf{f}_{\mathcal{H}}(\mathbf{x}) = \begin{bmatrix} \mathbf{f}_{1j_1}(\mathbf{x}) \\ \vdots \\ \mathbf{f}_{mj_m}(\mathbf{x}) \end{bmatrix} \simeq \mathbf{h}_{\mathcal{H}} + \mathbf{H}_{\mathcal{H}}(\mathbf{x} - \hat{\mathbf{x}}) = \mathbf{0}$$

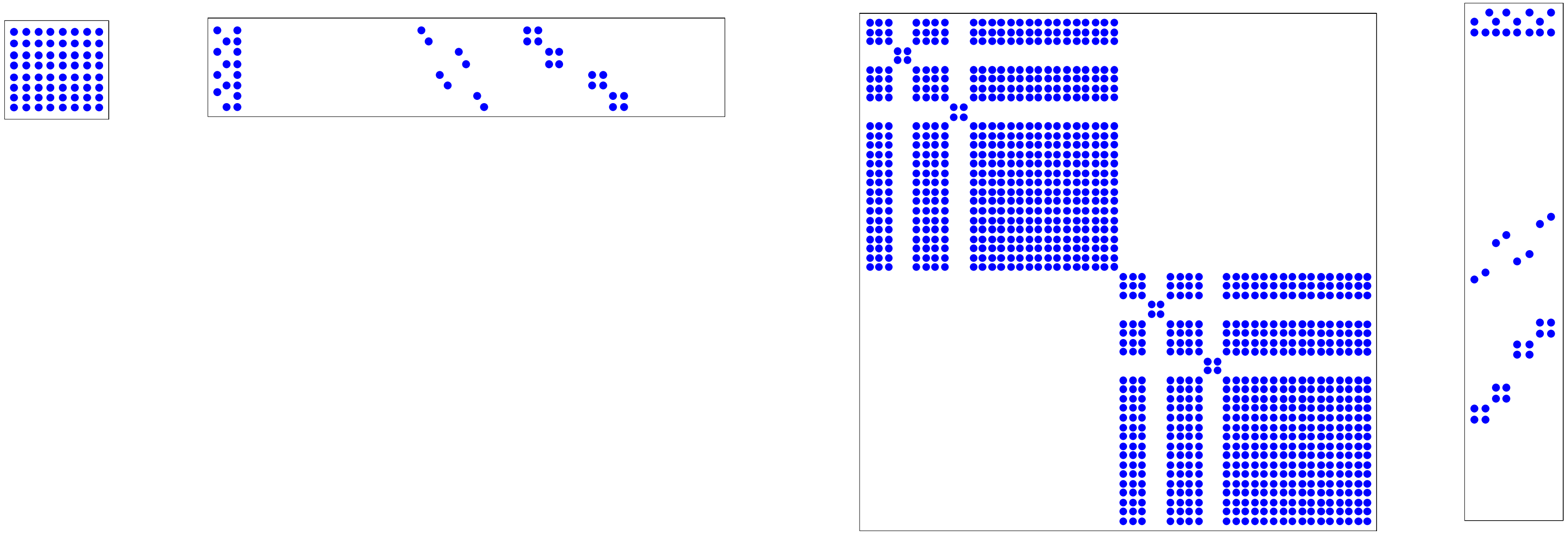
- Map update using EKF:

$$\begin{aligned} \mathbf{S}_{\mathcal{H}} &= \mathbf{H}_{\mathcal{H}} \mathbf{P}^{-}_{\mathcal{A}+\mathcal{B}} \mathbf{H}^{T}_{\mathcal{H}} \\ \mathbf{K}_{\mathcal{H}} &= \mathbf{P}^{-}_{\mathcal{A}+\mathcal{B}} \mathbf{H}^{T}_{\mathcal{H}} \left(\mathbf{S}_{\mathcal{H}}\right)^{-1} \\ \mathbf{P}_{\mathcal{A}+\mathcal{B}} &= \left(\mathbf{I} - \mathbf{K}_{\mathcal{H}} \mathbf{H}_{\mathcal{H}}\right) \mathbf{P}^{-}_{\mathcal{A}+\mathcal{B}} \end{aligned}$$

The innovation matrix

$$\mathbf{S}_{\mathcal{H}} = \mathbf{H}_{\mathcal{H}} \; \mathbf{P}^{-}_{\mathcal{A}+\mathcal{B}} \; \mathbf{H}^{T}_{\mathcal{H}}$$

rxr   *rxc*   *nxn*   *cxr*



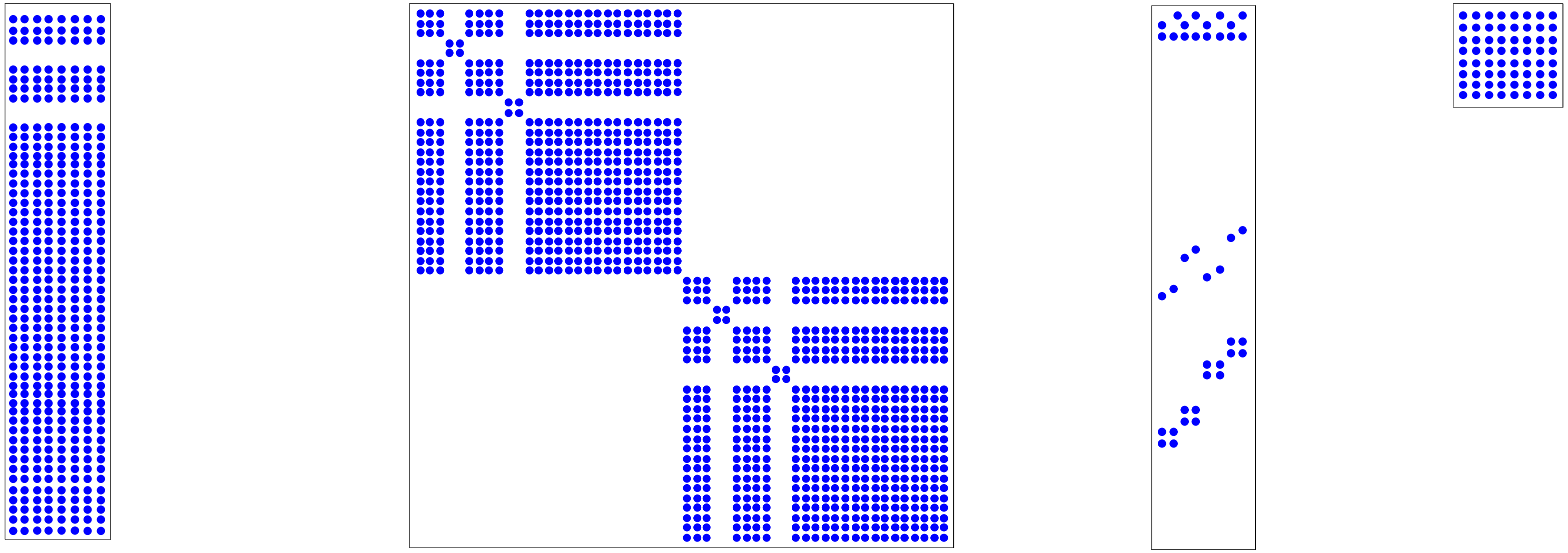
O(rn)

ROBOTICS UNIVERSIDAD DE ZARAGOZA

The gain matrix

$$\mathbf{K}_{\mathcal{H}} = \mathbf{P}^{-}_{\mathcal{A}+\mathcal{B}} \; \mathbf{H}^{T}_{\mathcal{H}} \; (\mathbf{S}_{\mathcal{H}})^{-1}$$

$\mathbf{K}_{\mathcal{H}}$	$\mathbf{P}^{-}_{\mathcal{A}+\mathcal{B}}$	$\mathbf{H}^{T}_{\mathcal{H}}$	$(\mathbf{S}_{\mathcal{H}})^{-1}$
nxr	*nxn*	*cxr*	*rxr*



O(r²n)

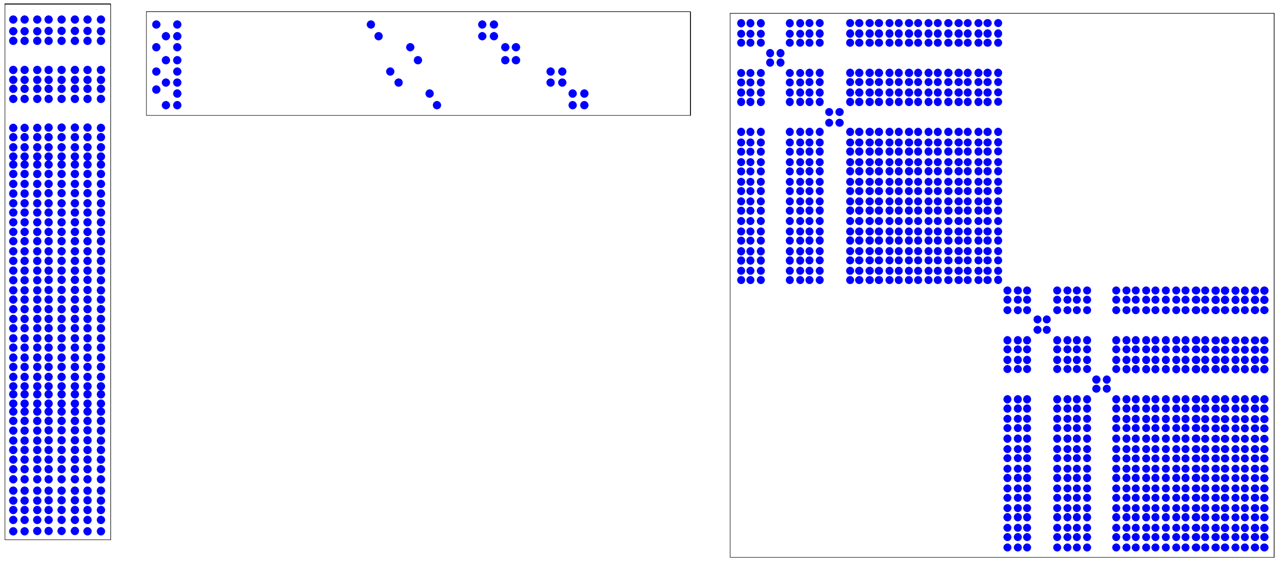
The update

$$\mathbf{P}_{\mathcal{A}+\mathcal{B}} = (\mathbf{I} - \mathbf{K}_{\mathcal{H}}\mathbf{H}_{\mathcal{H}})\,\mathbf{P}^{-}_{\mathcal{A}+\mathcal{B}}$$

$$\ldots\ldots = \ldots\ldots\ \mathbf{K}_{\mathcal{H}} \quad \mathbf{H}_{\mathcal{H}} \quad \mathbf{P}^{-}_{\mathcal{A}+\mathcal{B}}$$

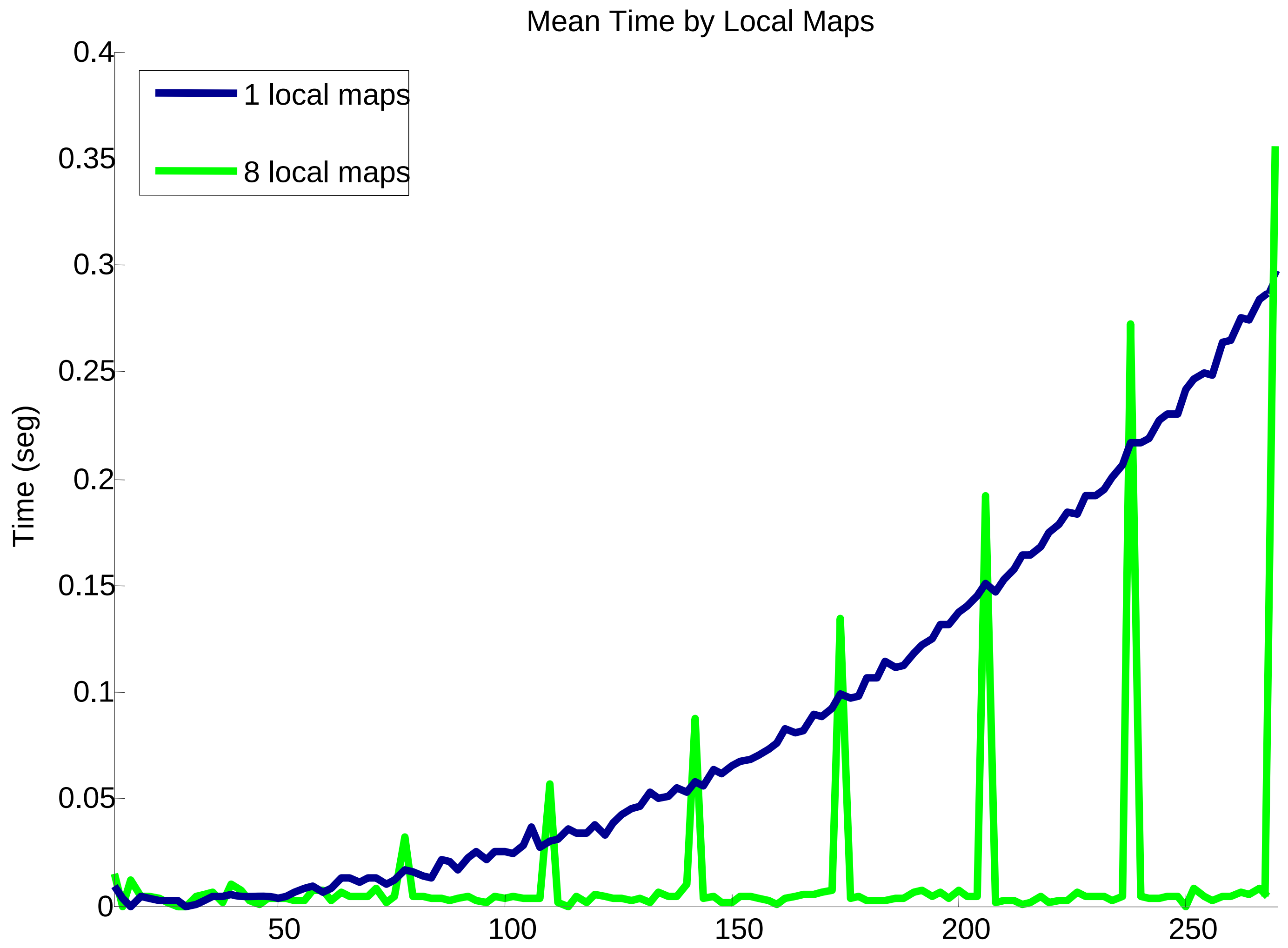
nxr *rxc* *nxn*

....

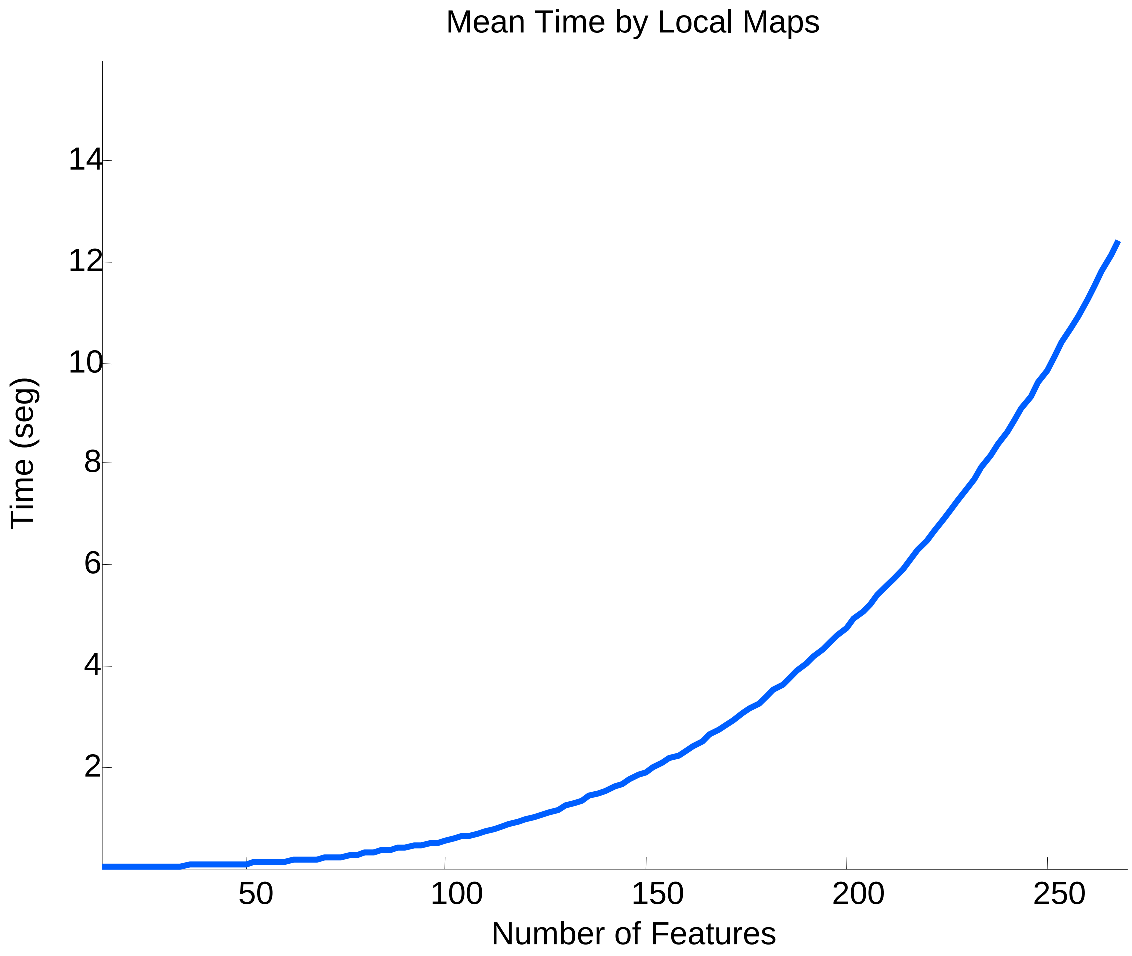


Matching and fusion is $O(n^2)$

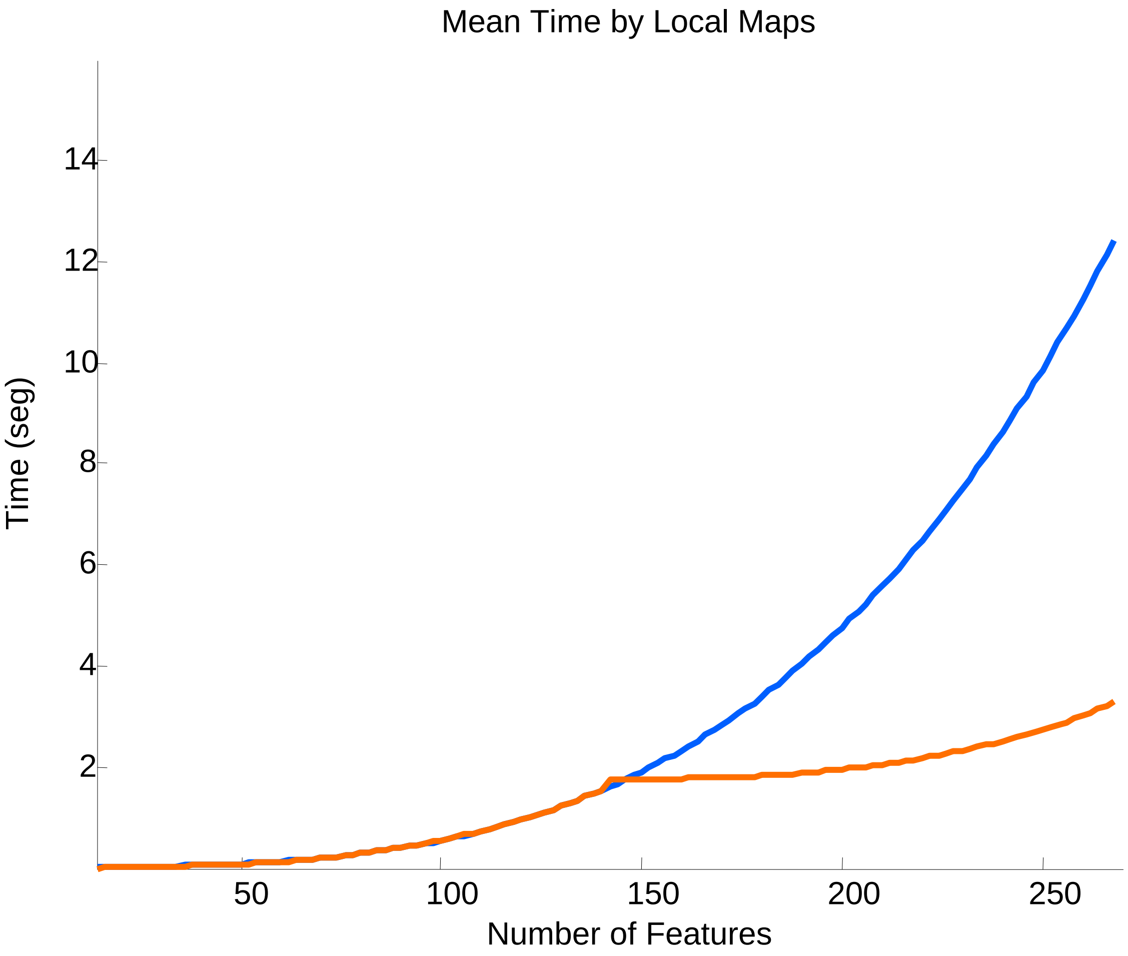
Cost of Map Joining per step



Total SLAM with Map Joining

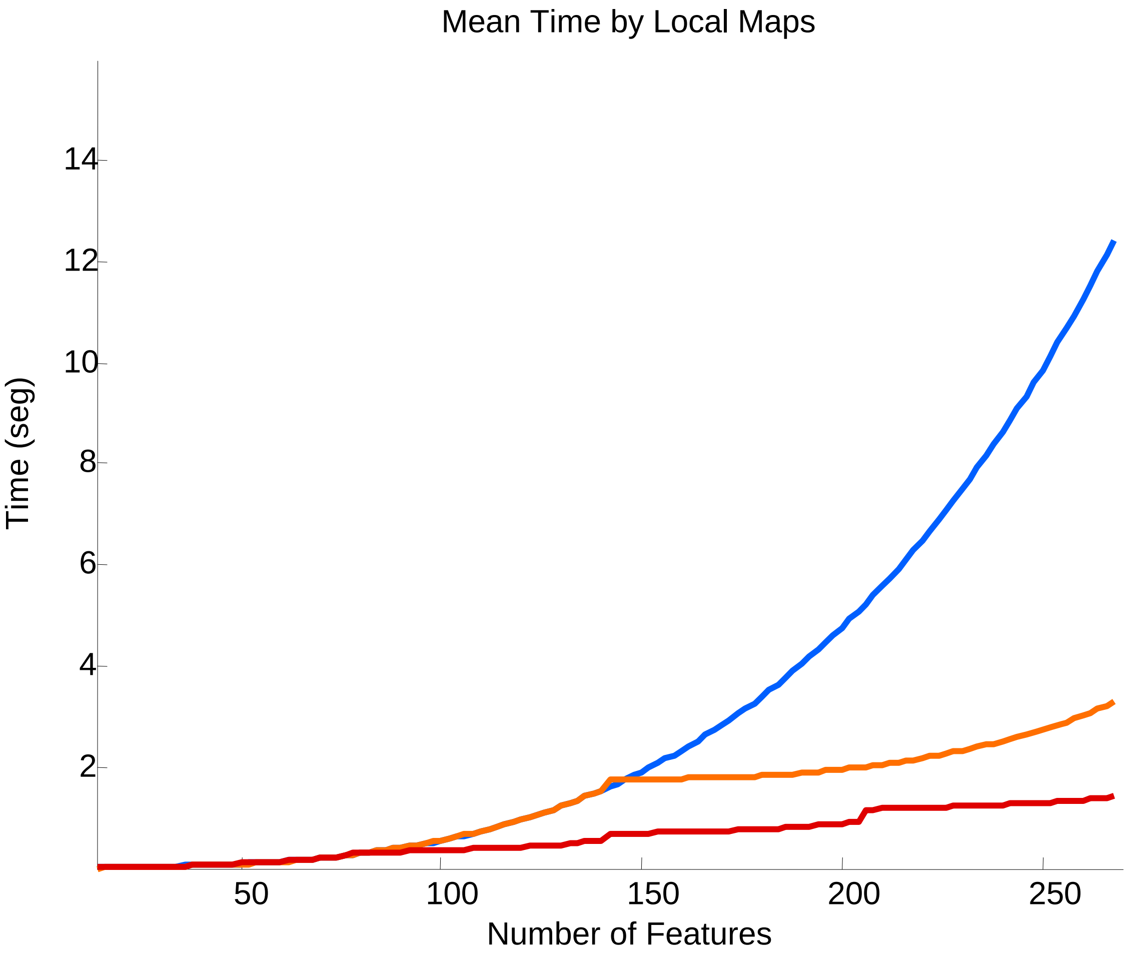


Total SLAM with Map Joining

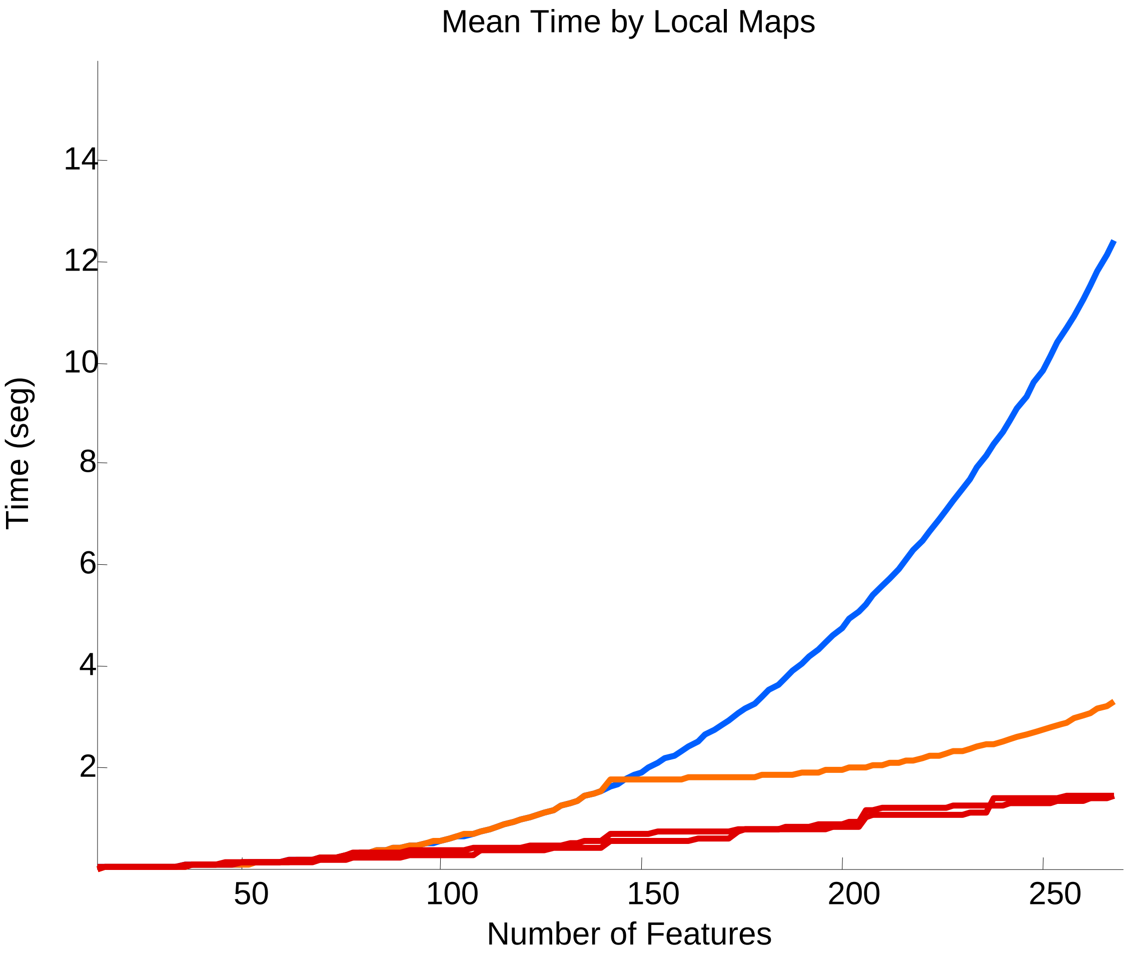


ROBOTICS
UNIVERSIDAD DE ZARAGOZA

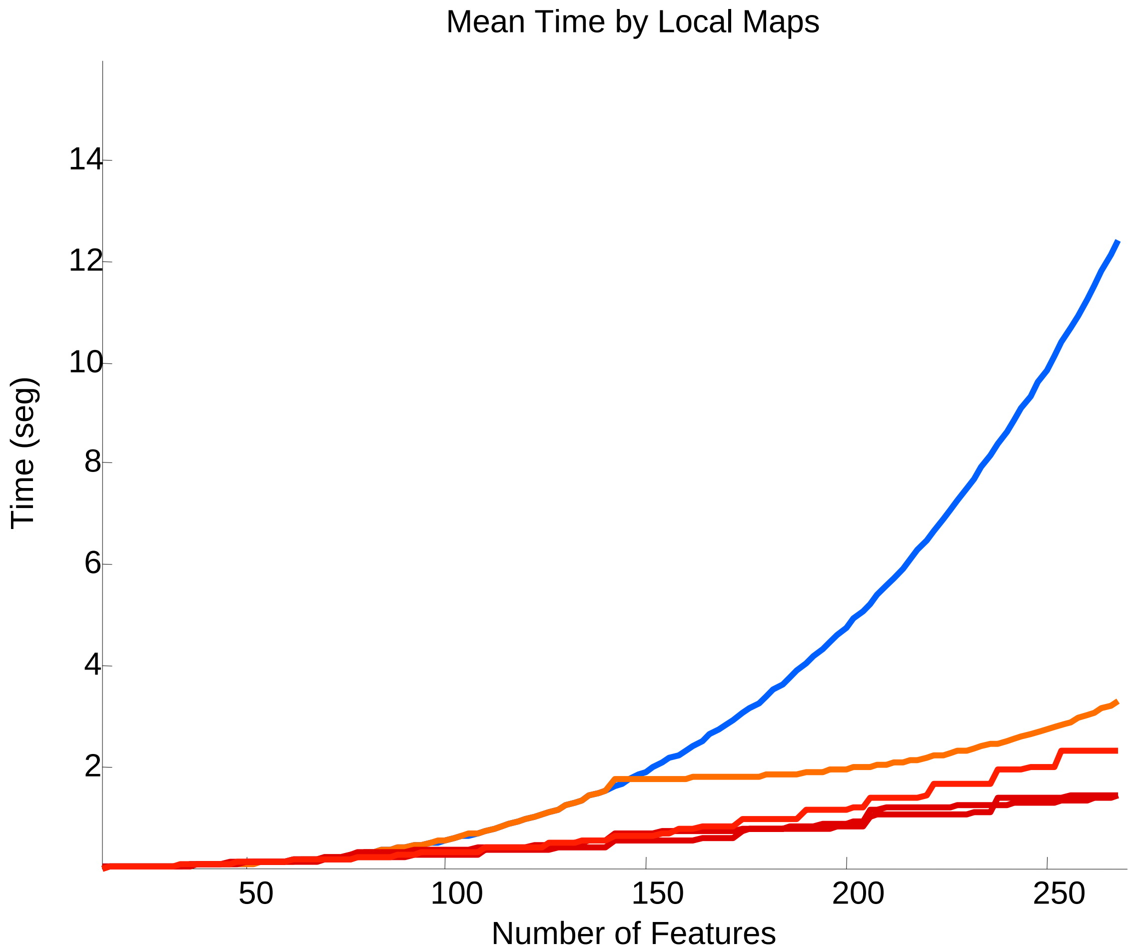
Total SLAM with Map Joining



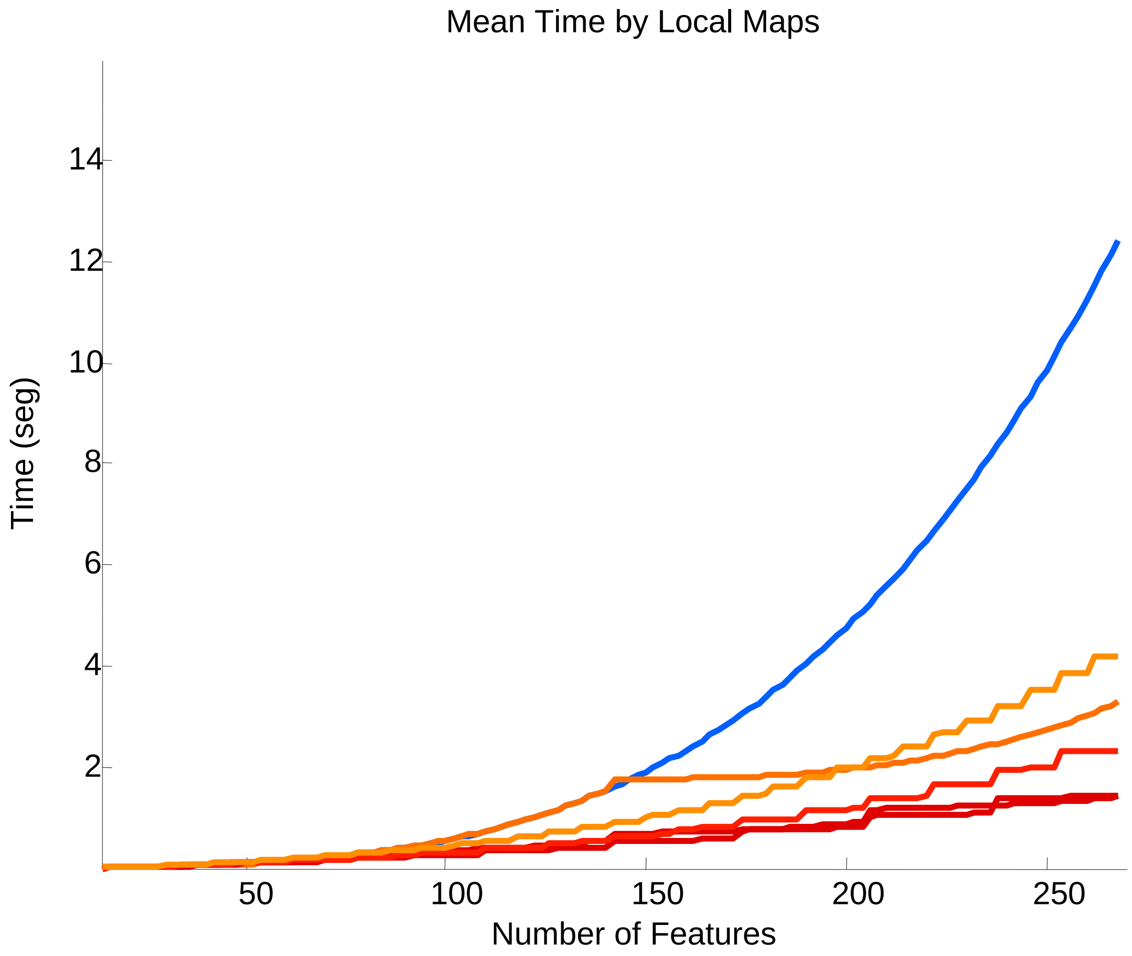
Total SLAM with Map Joining



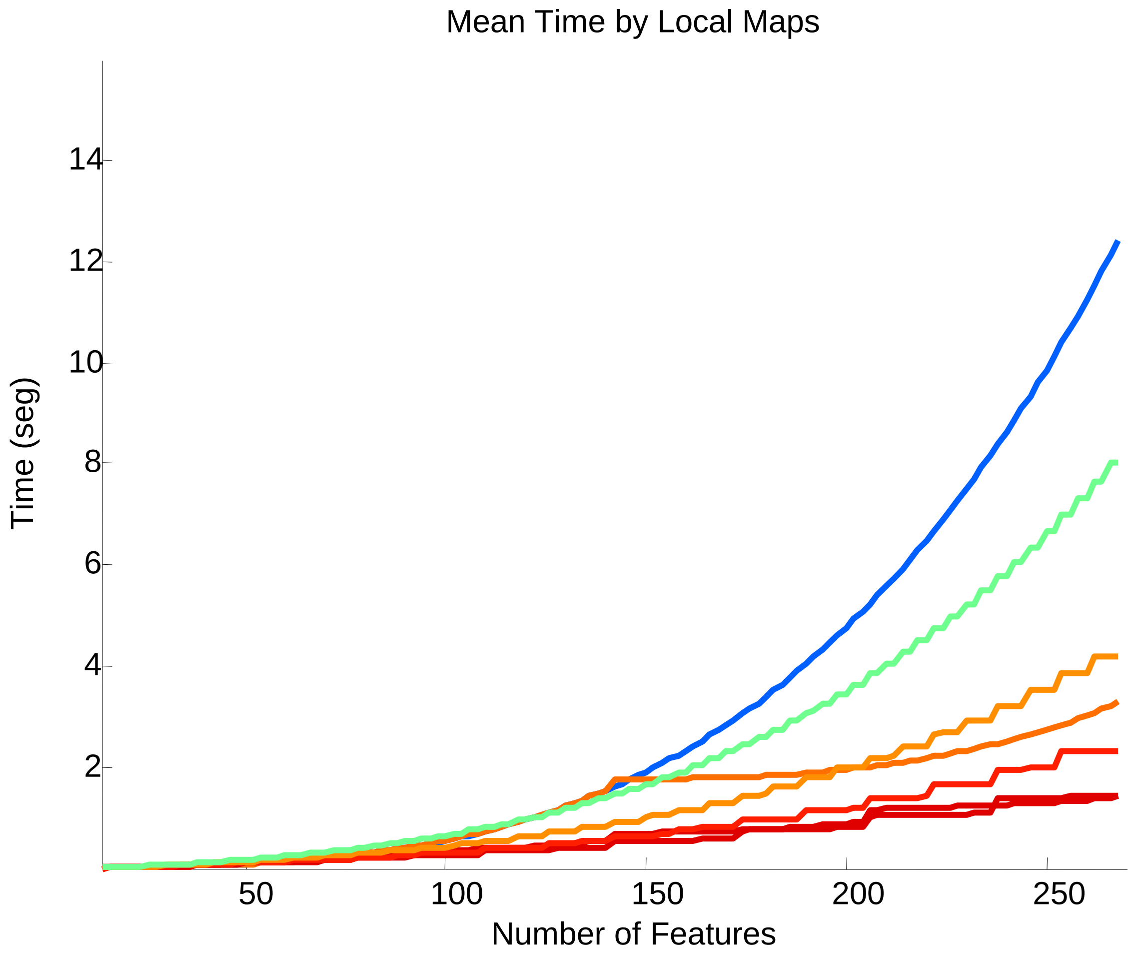
Total SLAM with Map Joining



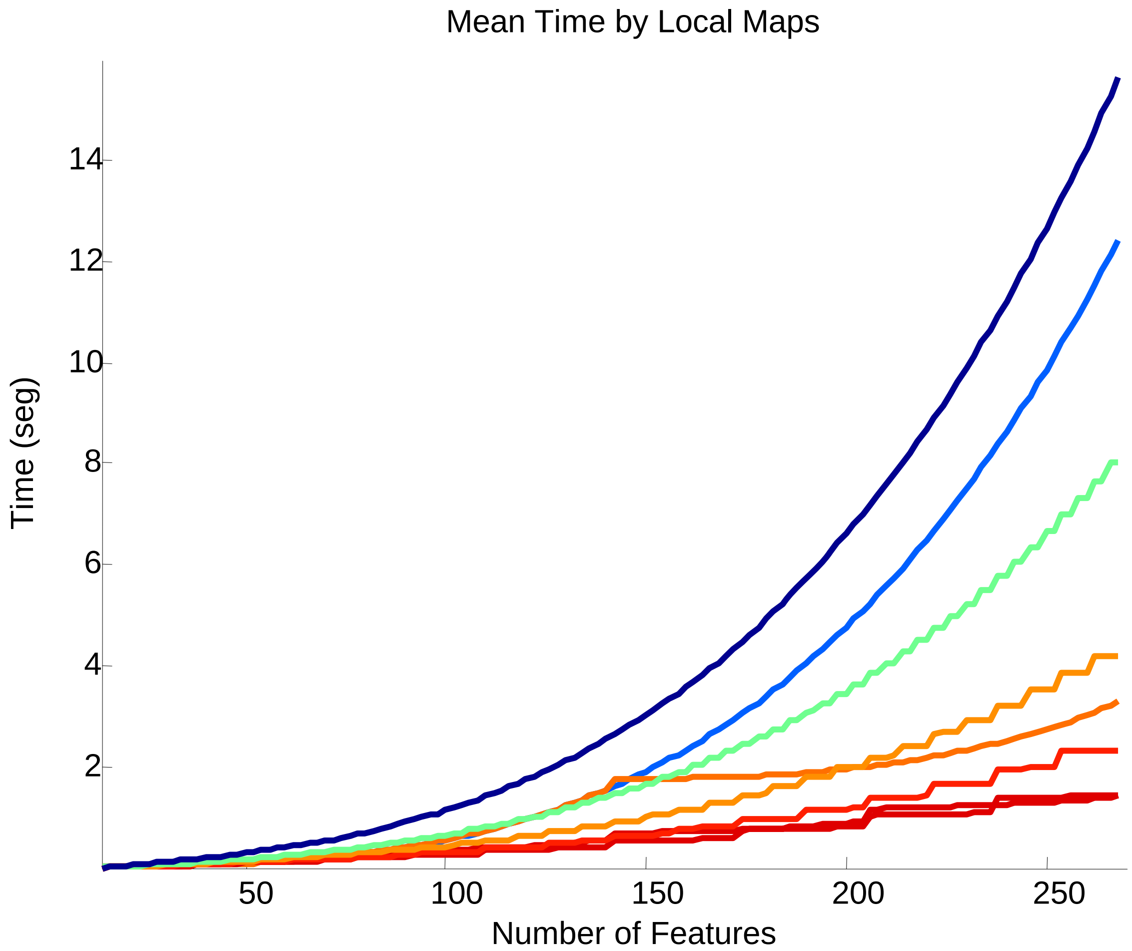
Total SLAM with Map Joining



Total SLAM with Map Joining

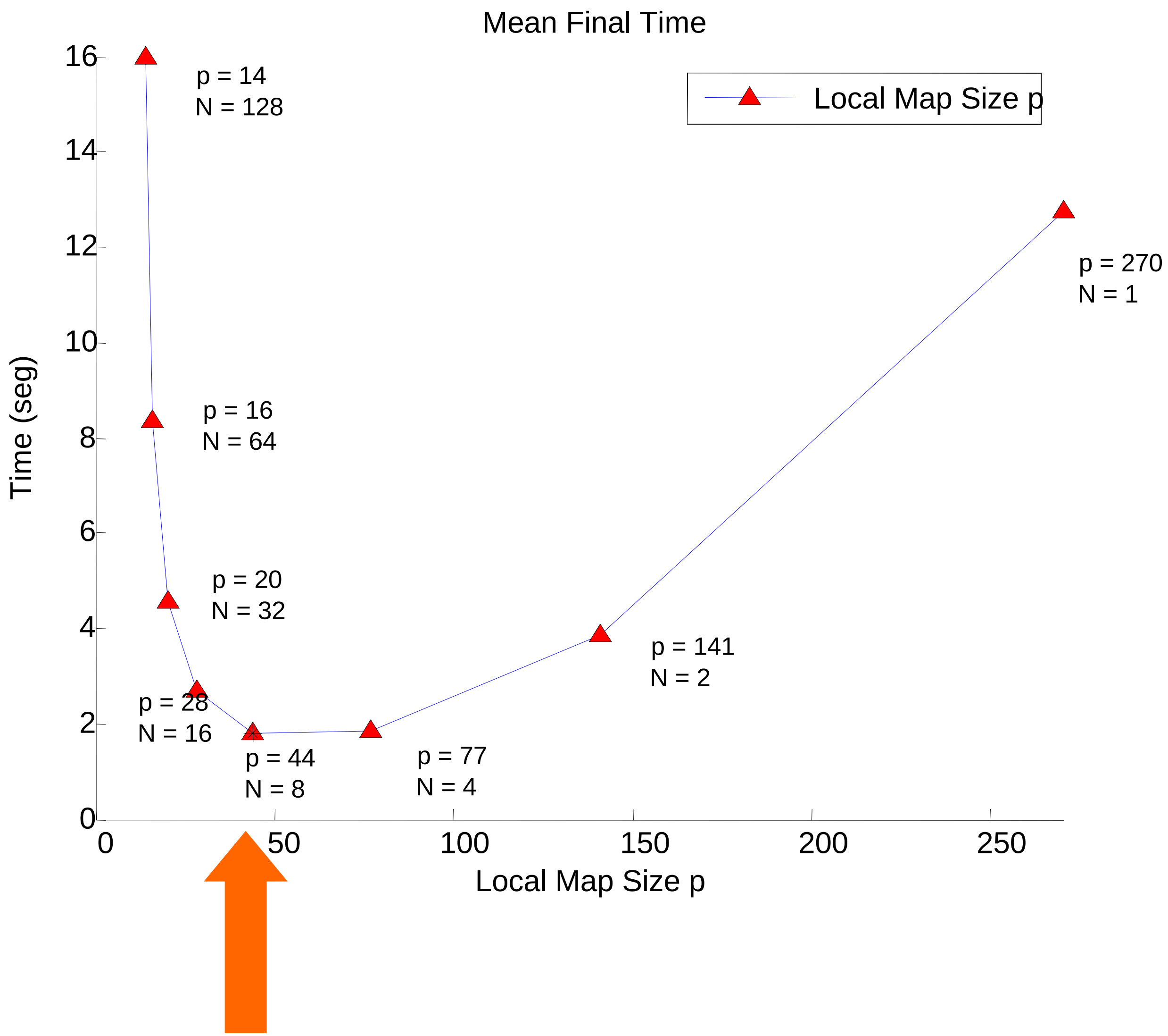


Total SLAM with Map Joining

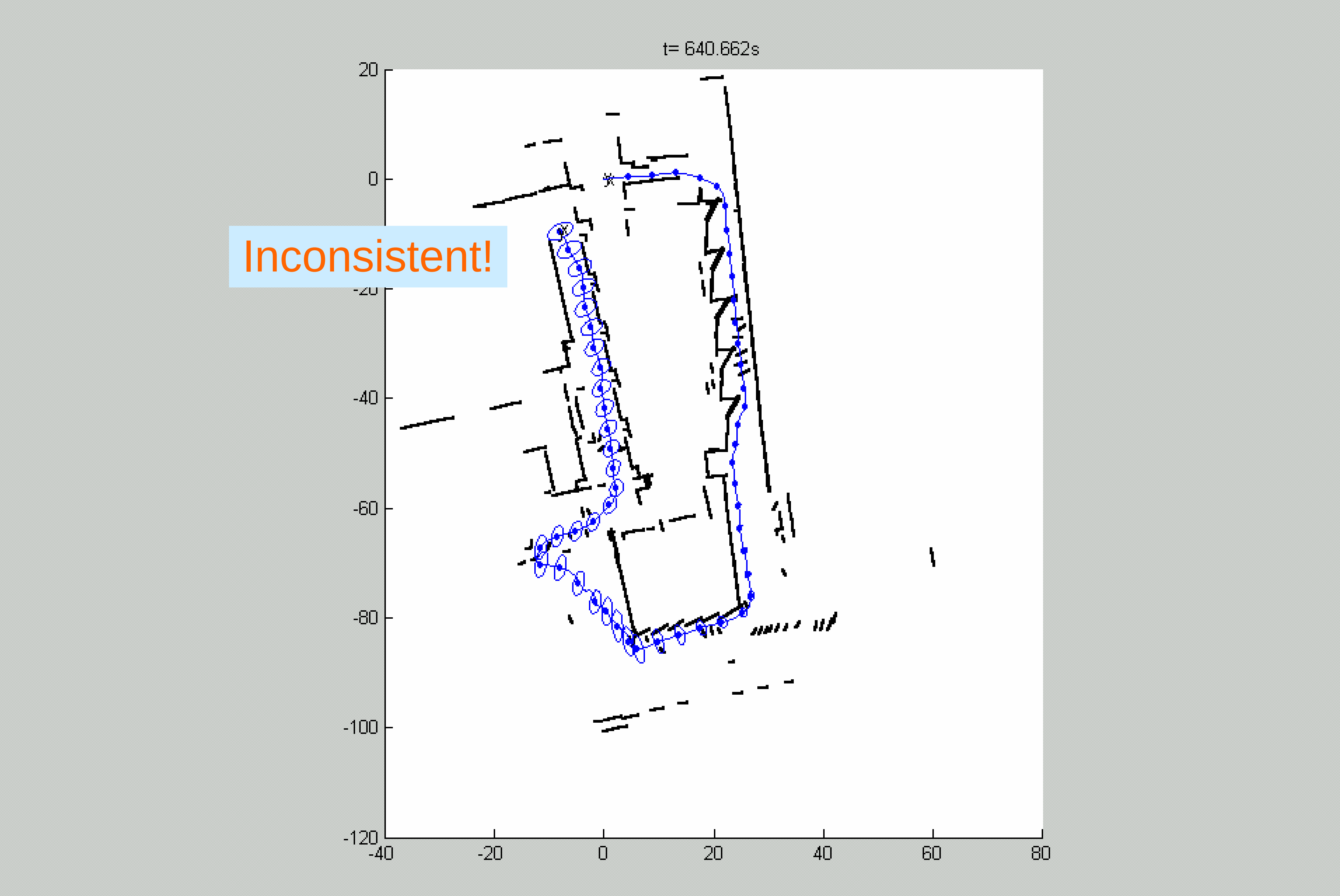


There is an optimal submap size

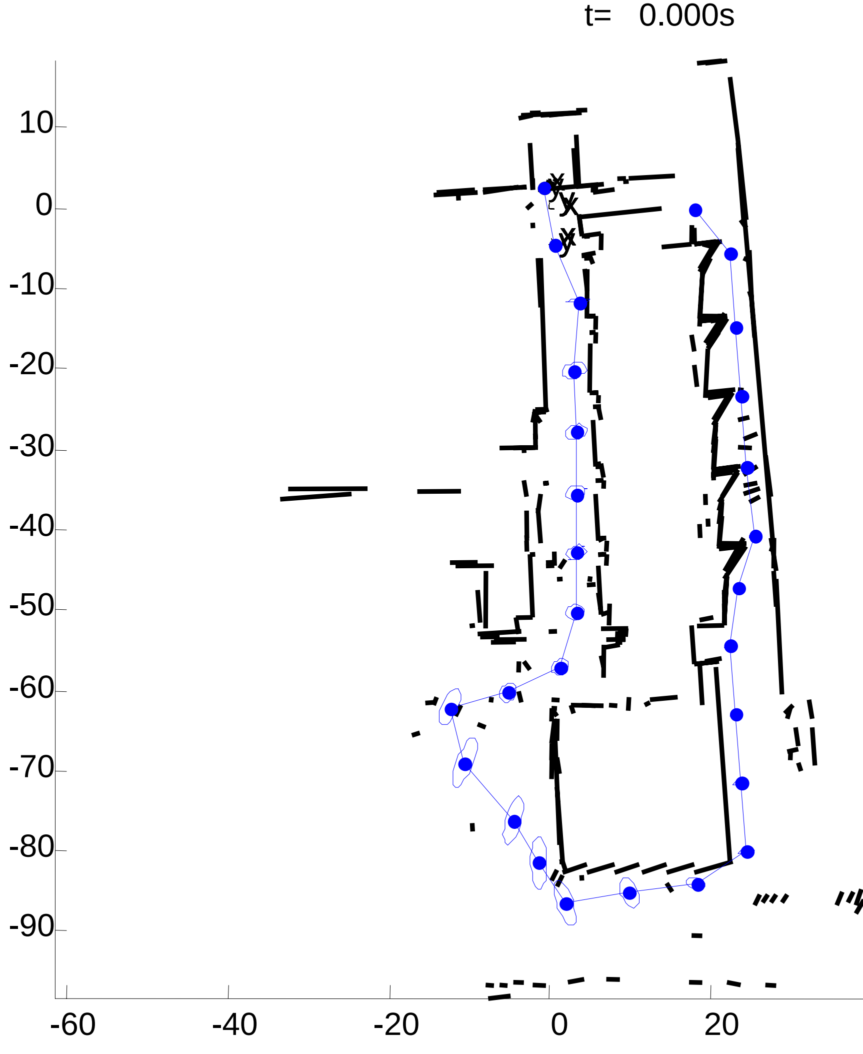
Optimal local map size



EKF-SLAM: remember?



Map Joining closes the loop!



Montecarlo Simulations

- Simulation conditions
  - Perfect data association
  - Ideal odometry and measurement noise
    - » white, Gaussian, known covariance

- Advantages of simulation:
  - Consistency can be tested against the true map
  - A simulation with noise=0 gives the theoretical map covariance (without linearization errors)
  - Run several algorithms using the same data
  - Carry out a large amount of runs

Consistency Testing

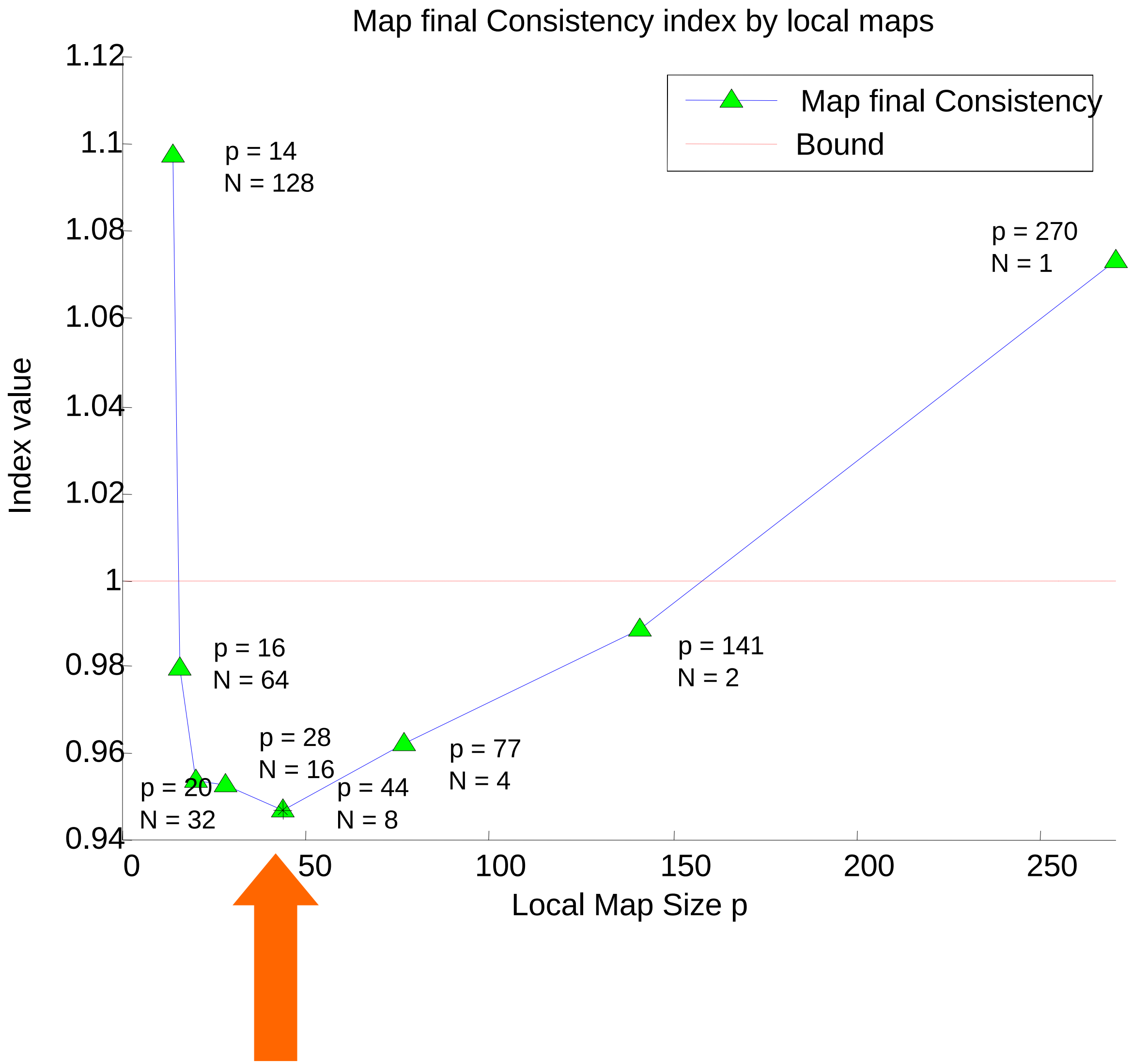
1. Define the consistency index CI as:

$$CI = \frac{D^2}{\chi^2_{r,1-\alpha}},$$

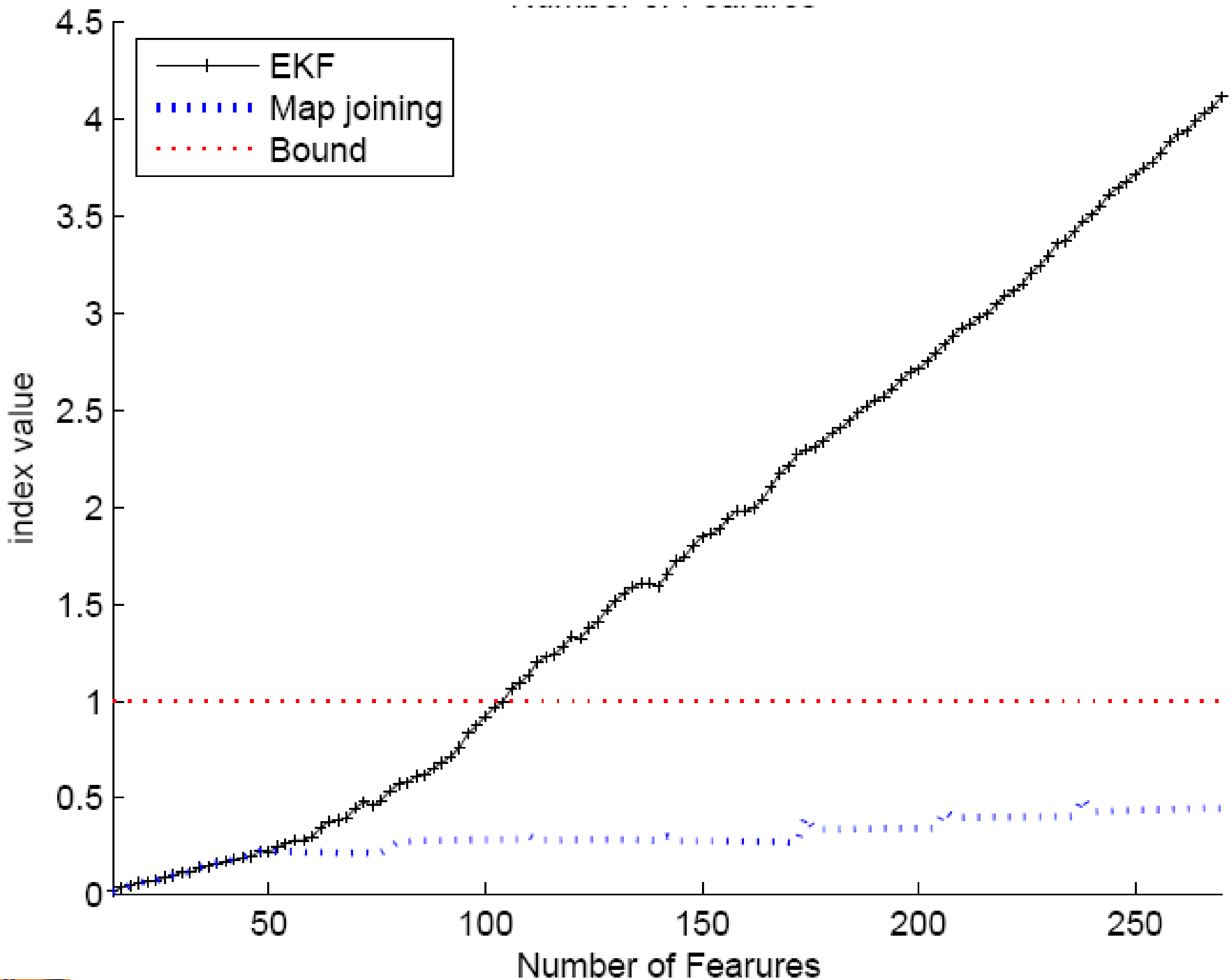
2. When CI < 1, the estimation is consistent with ground truth

3. When CI > 1, the estimation is inconsistent

Mean map consistency

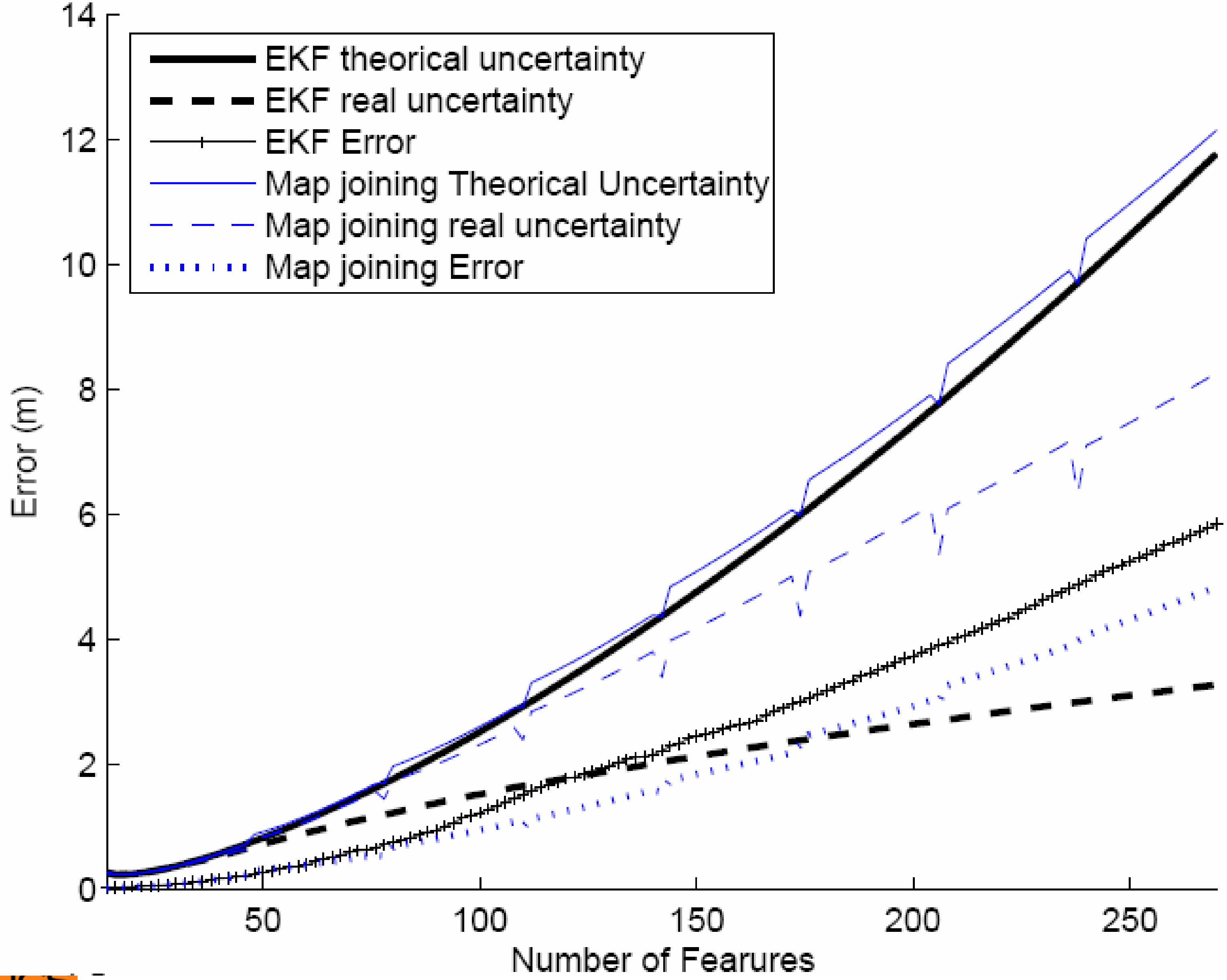


Mean feature location



ROBOTICS
UNIVERSIDAD DE ZARAGOZA

Mean feature location



ROBOTICS UNIVERSIDAD DE ZARAGOZA

Map Joining is a 'more for less' algorithm:

- It improves the consistency of the resulting map
- It reduces the computational cost

Limitations:

- Map joining is still $O(n^2)$
- Environment size must be known

Overcoming limitations:

- Can we have EKF-SLAM updates in $O(n)$?

ROBOTICS
UNIVERSIDAD DE ZARAGOZA

Outline

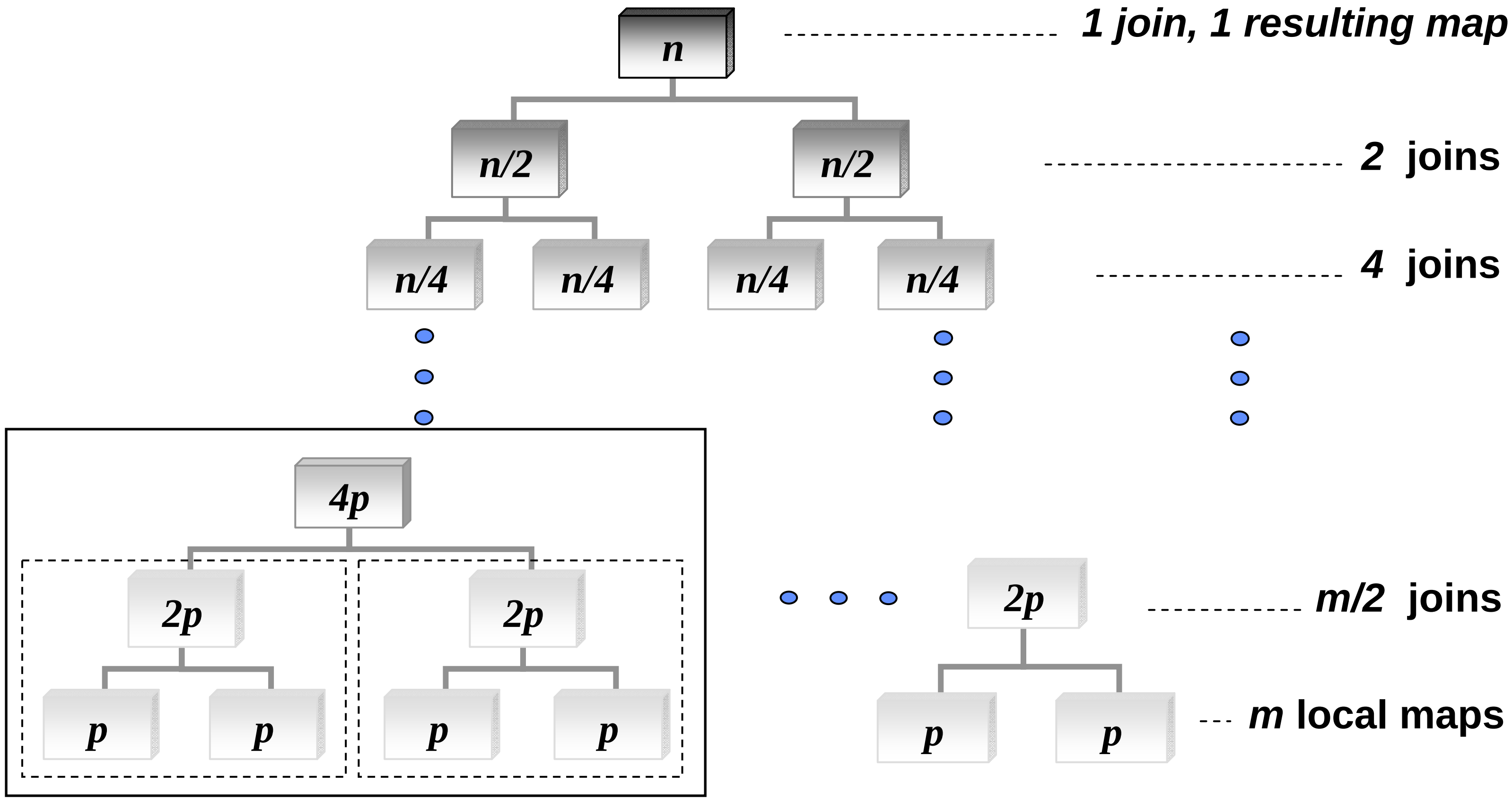
1. Basic EKF SLAM
   1. Introduction: the need for SLAM
   2. The basic EKF SLAM algorithm
   3. Feature extraction
   4. Data Association

2. The Data Association Problem
   1. Continuous Data Association
   2. The Loop Closing Problem
   3. The Global Localization Problem

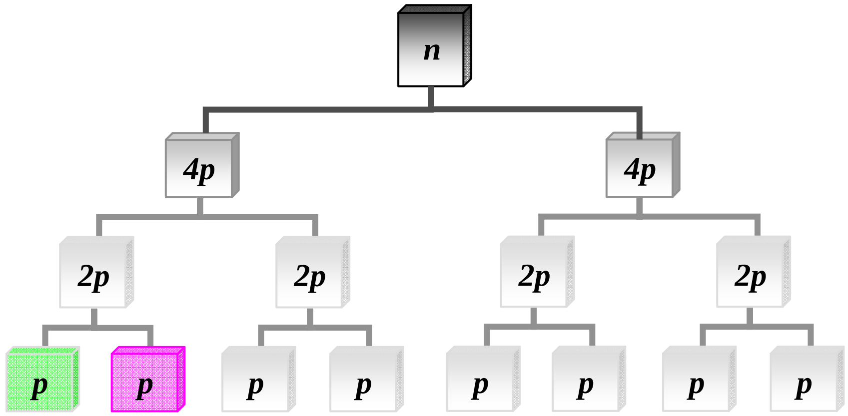
3. Advanced EKF SLAM
   1. Computational complexity of EKF SLAM
   2. Consistency of the EKF SLAM
   3. SLAM using local maps
      1. Sequential Map Joining
      2. **Divide and Conquer SLAM**
      3. Hierarchical SLAM

D&C: Map Hierarchy

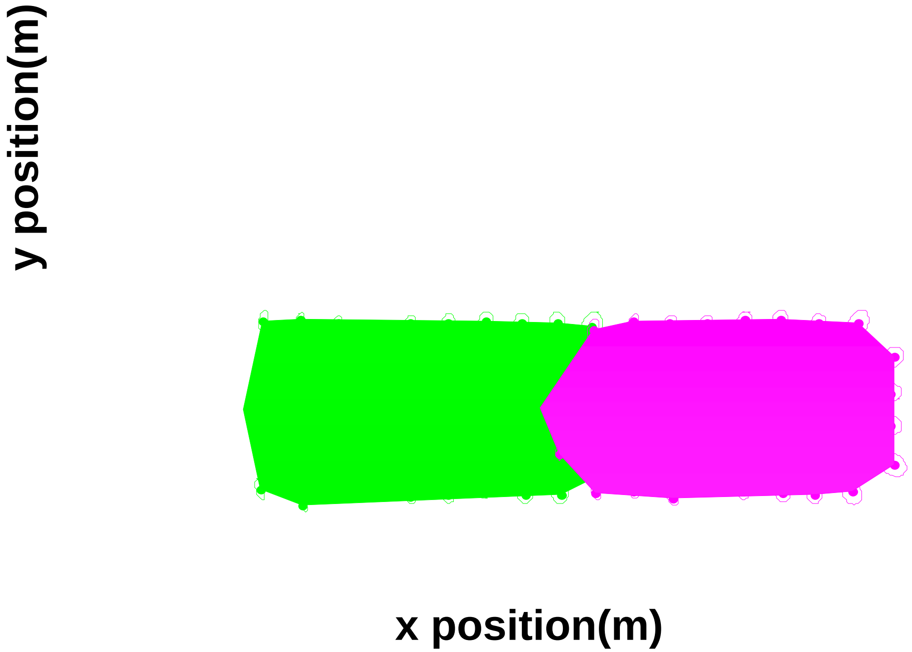
- Tree of local maps of half the size their ancestors
- Leaves: $m = n / P$ local maps of limited size P



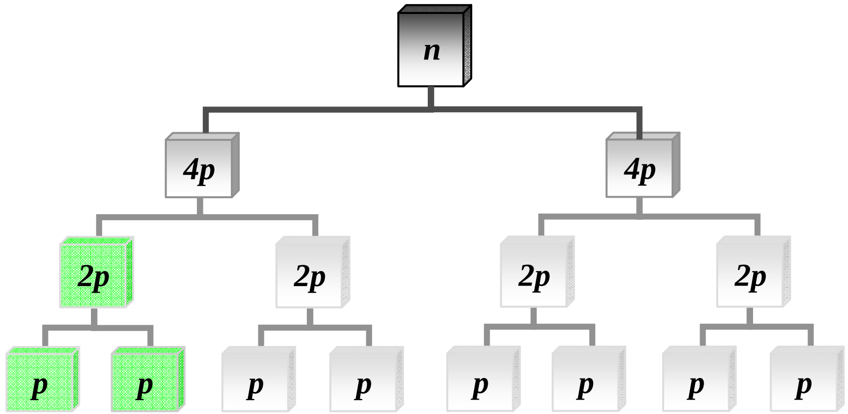
Map Hierarchy



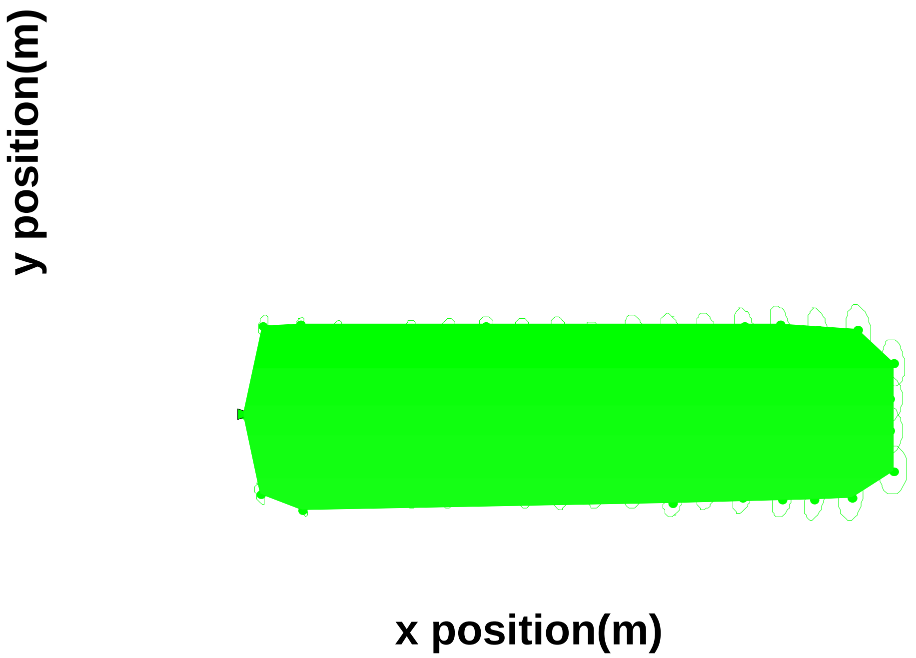
Number of Maps : 2



Map Hierarchy

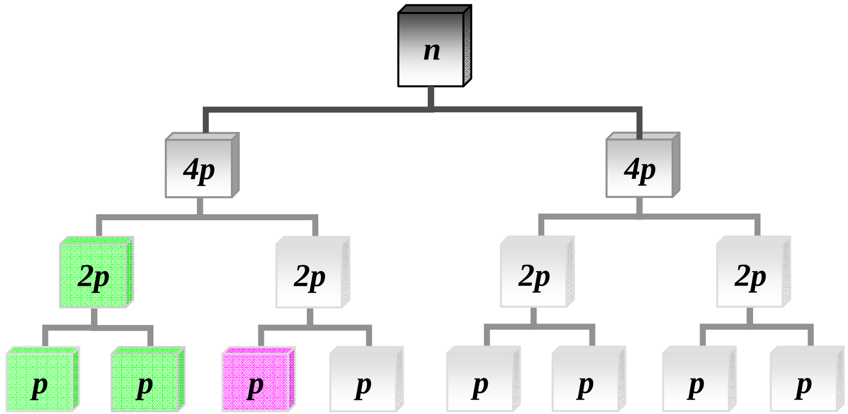


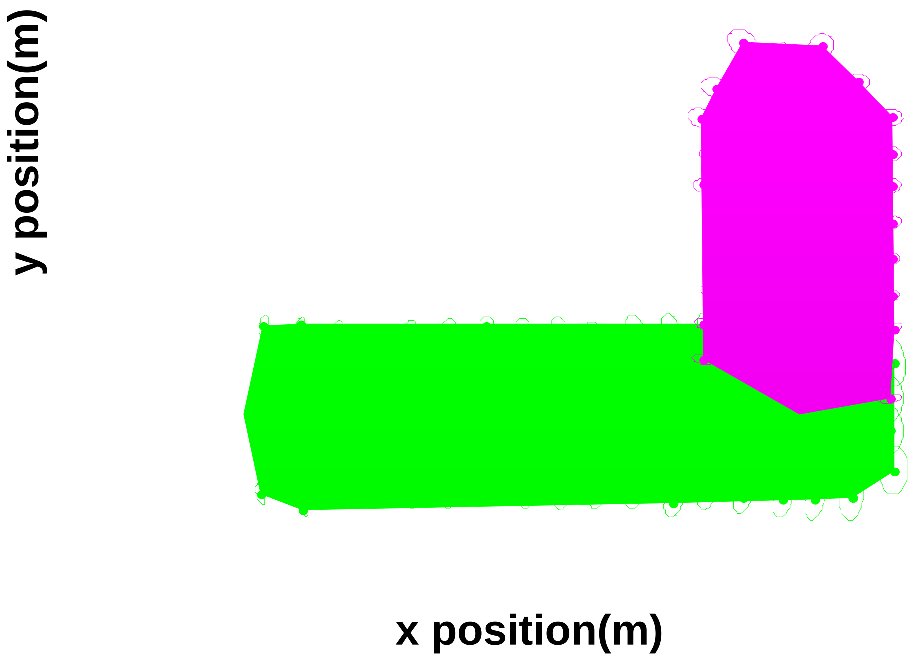
Number of Maps : 1



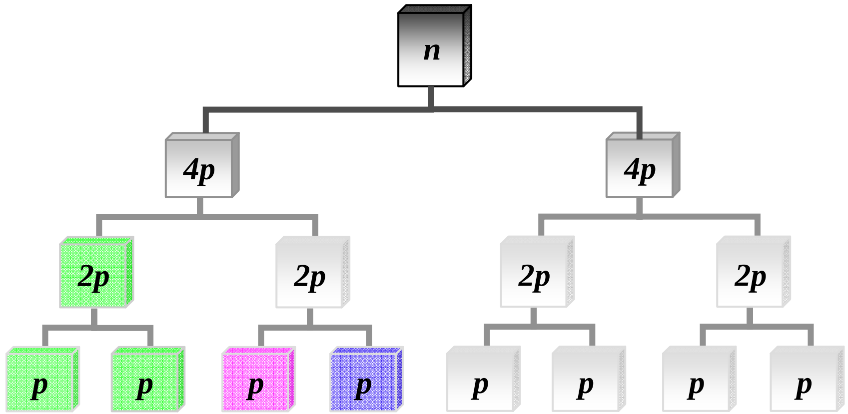
Map Hierarchy

Number of Maps : 2

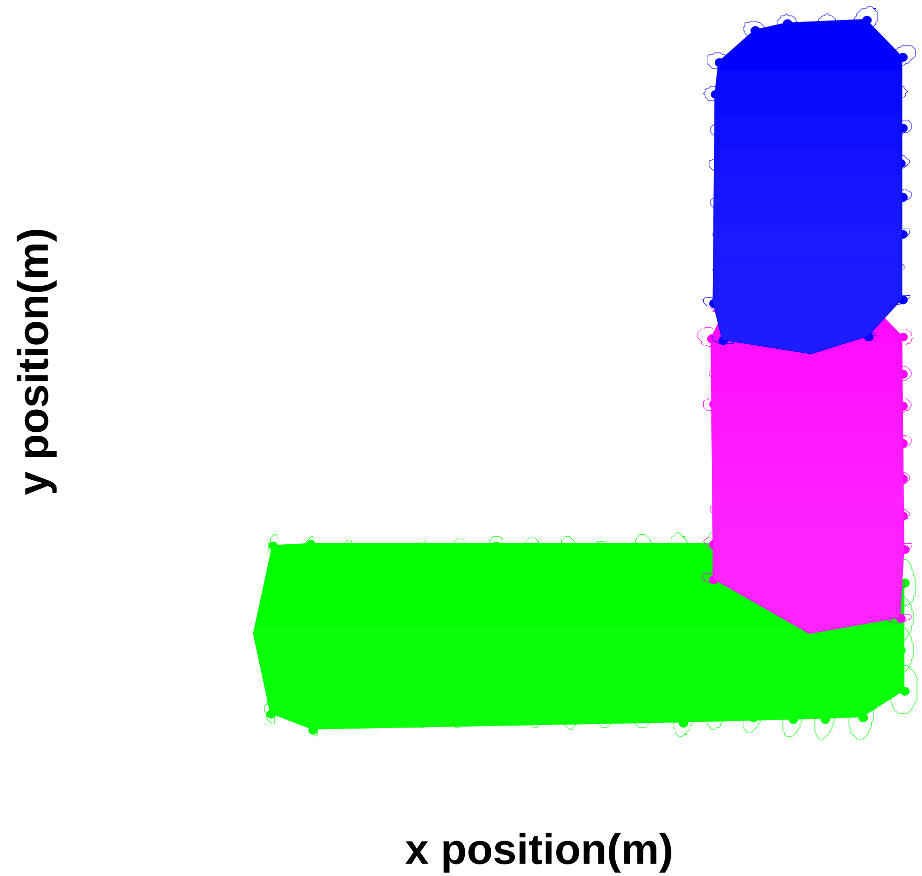




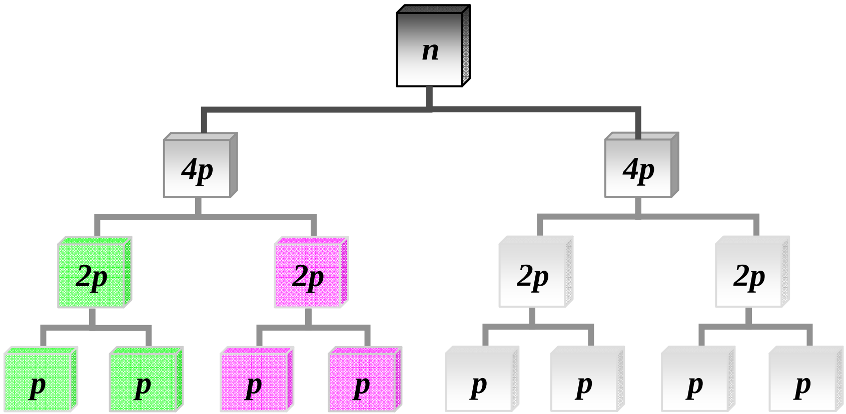
Map Hierarchy

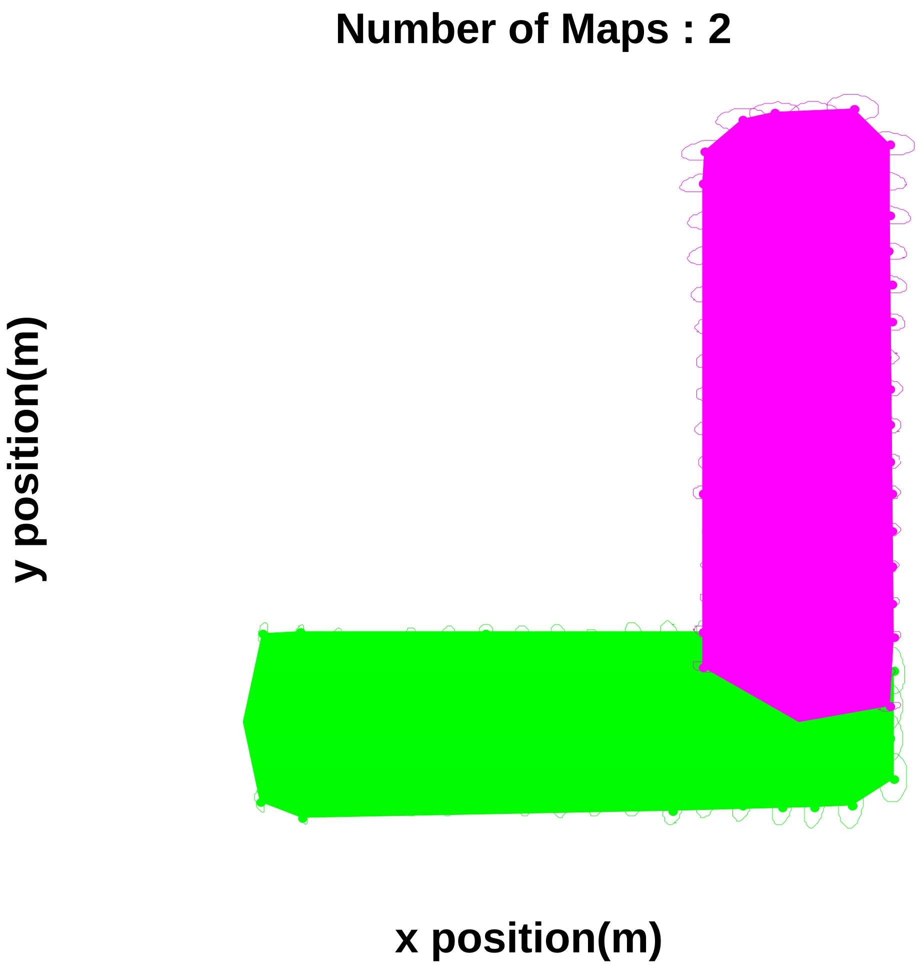


Number of Maps : 3

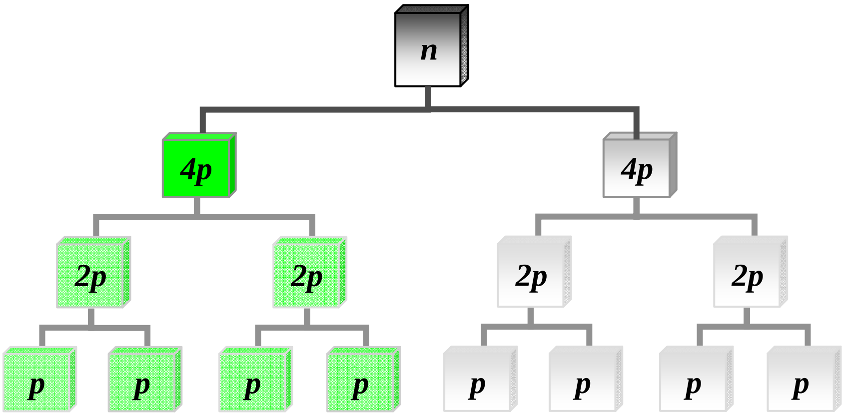


Map Hierarchy

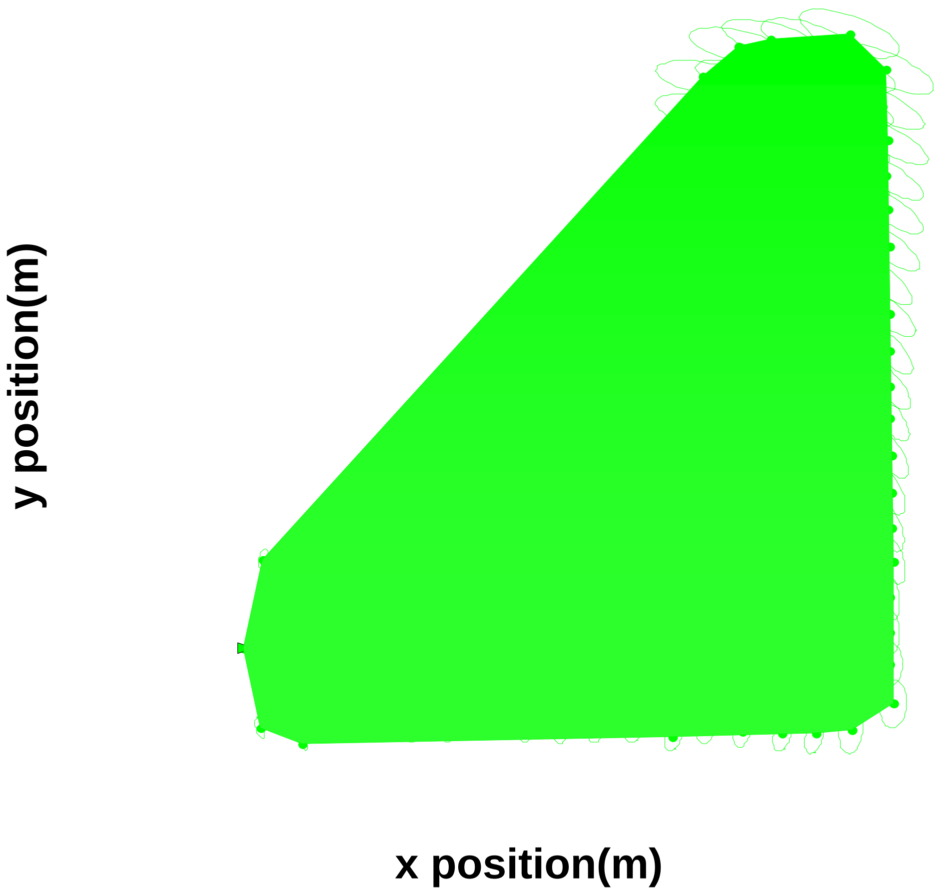




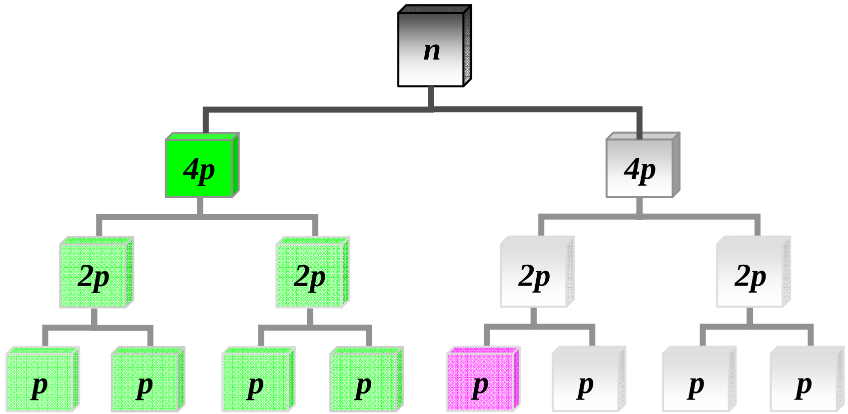
Map Hierarchy

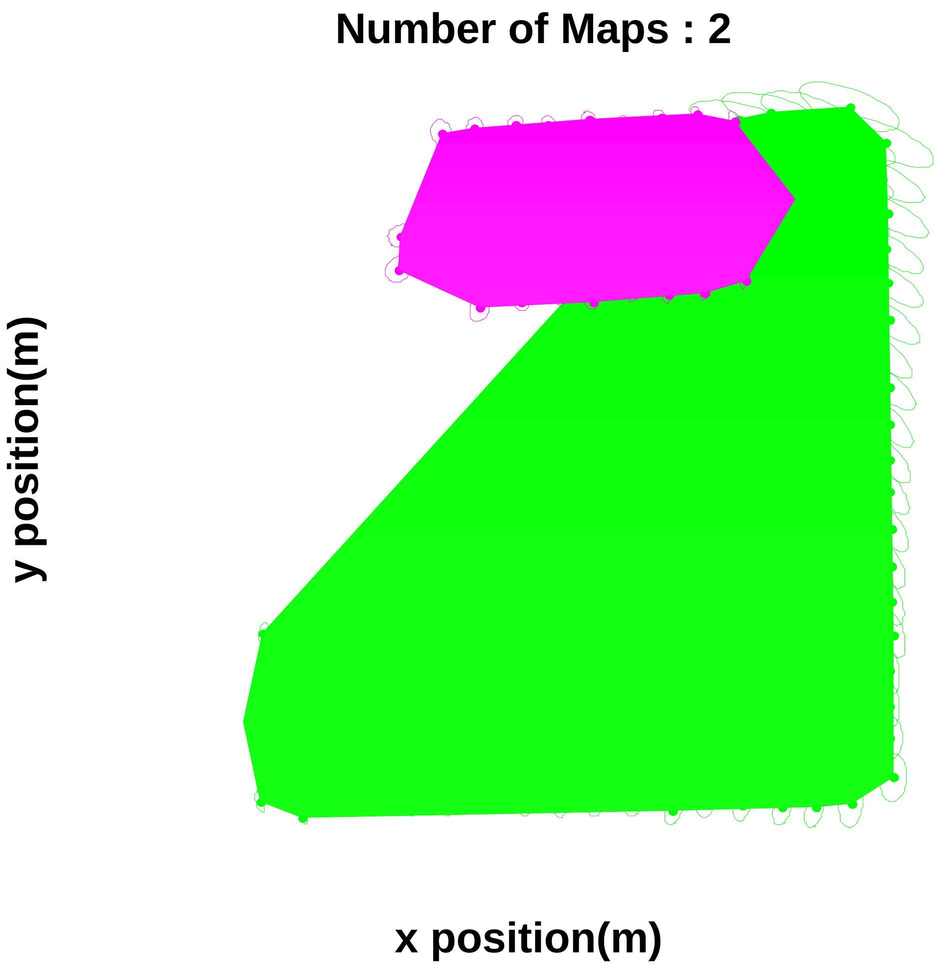


Number of Maps : 1

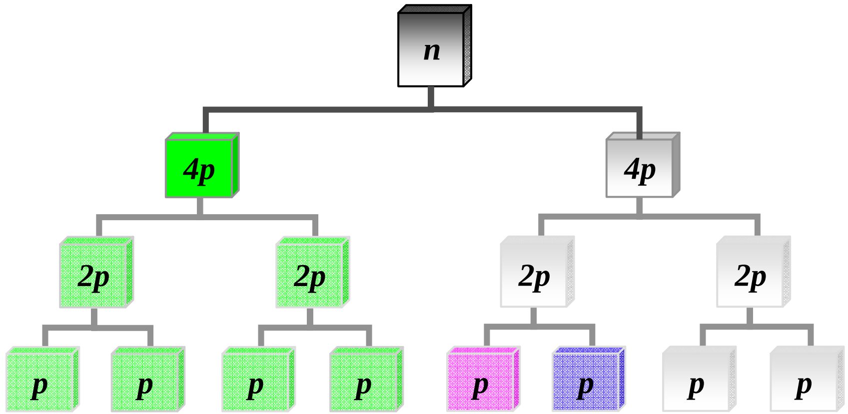


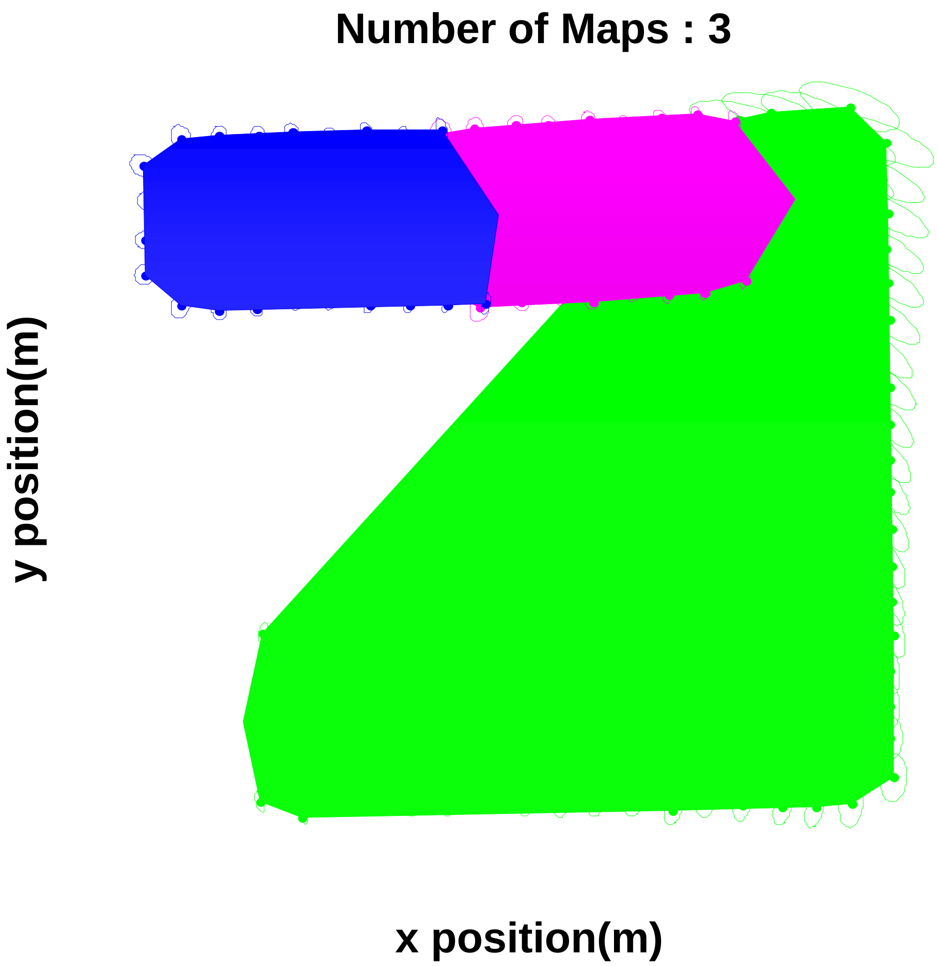
Map Hierarchy



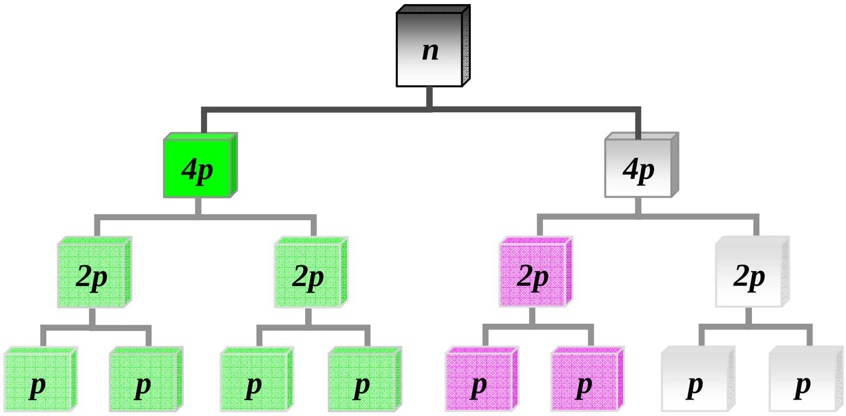


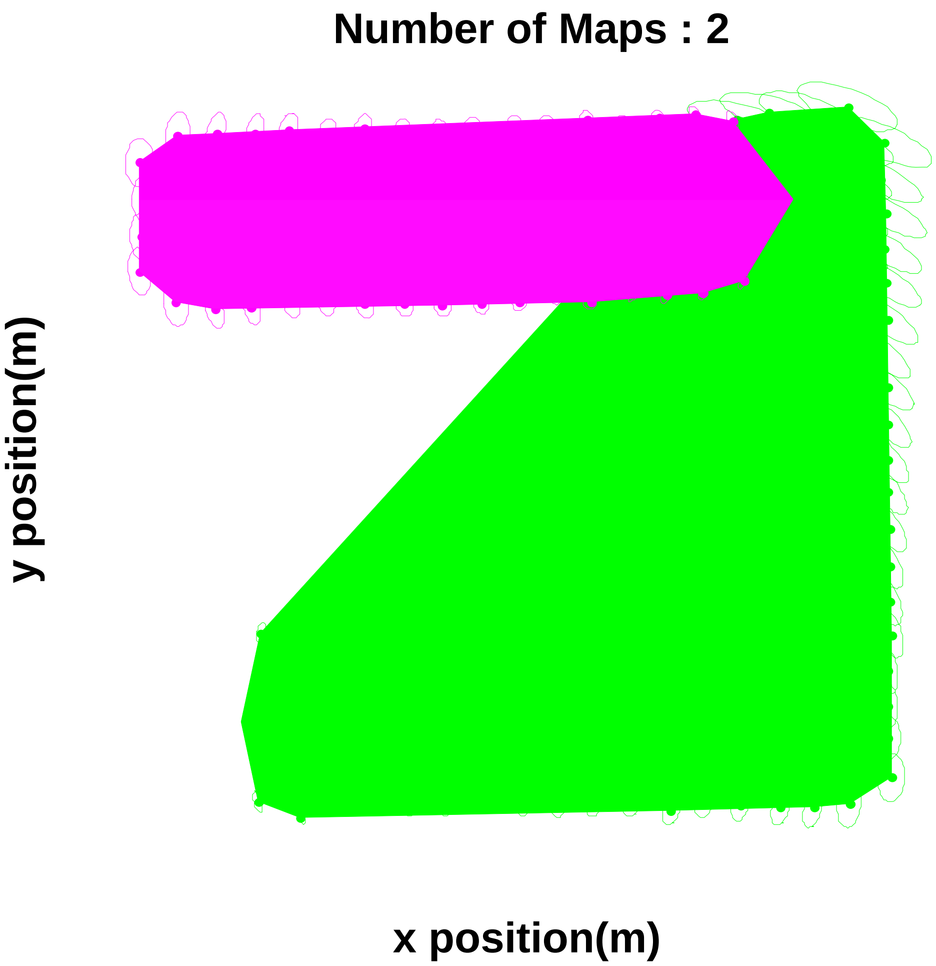
Map Hierarchy



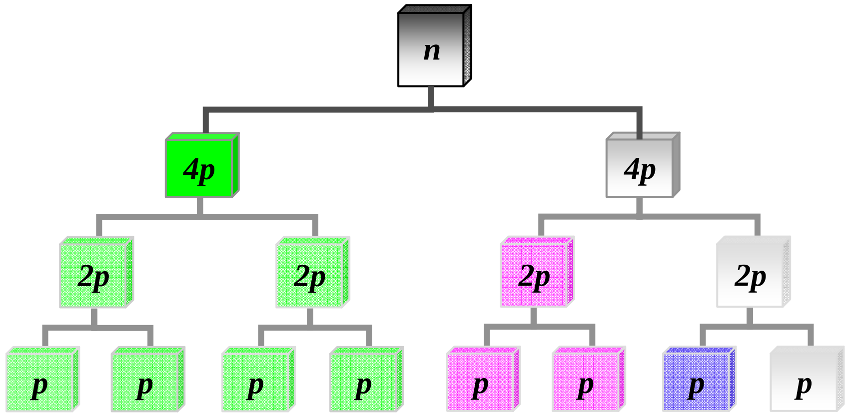


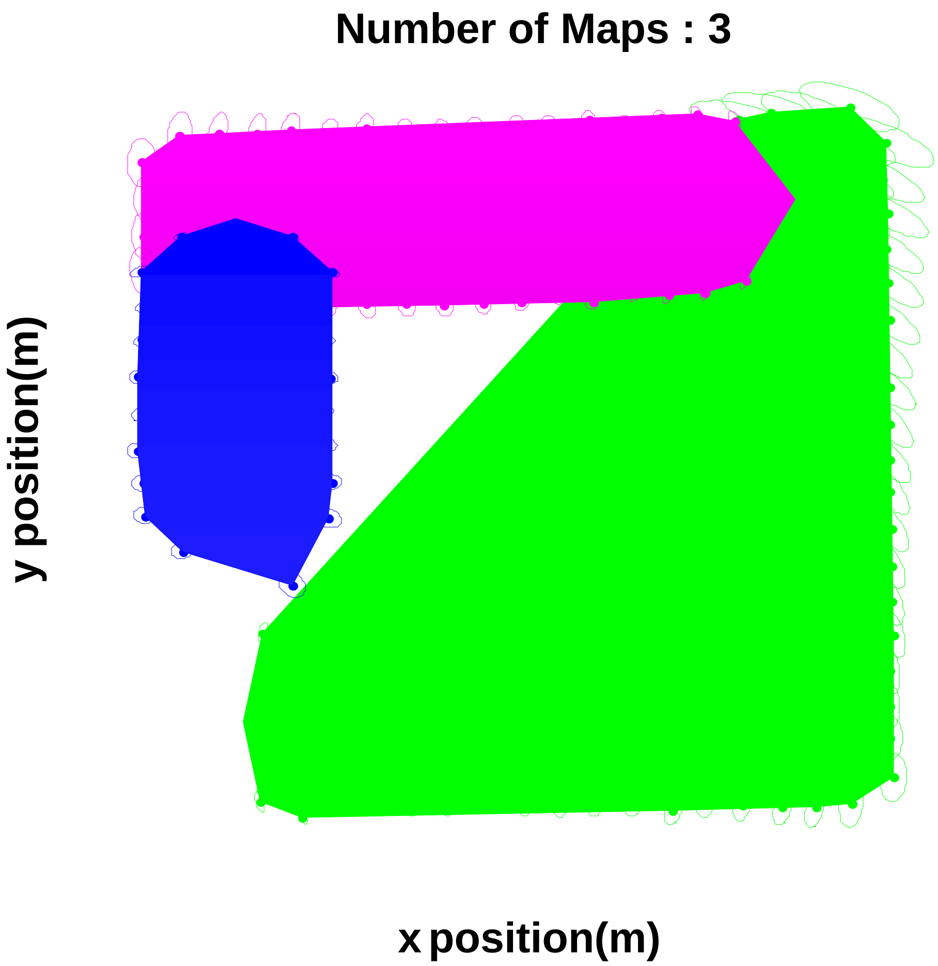
Map Hierarchy



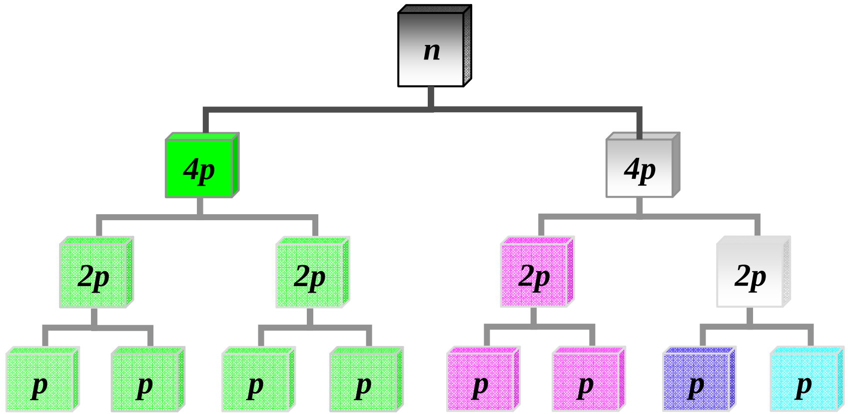


Map Hierarchy

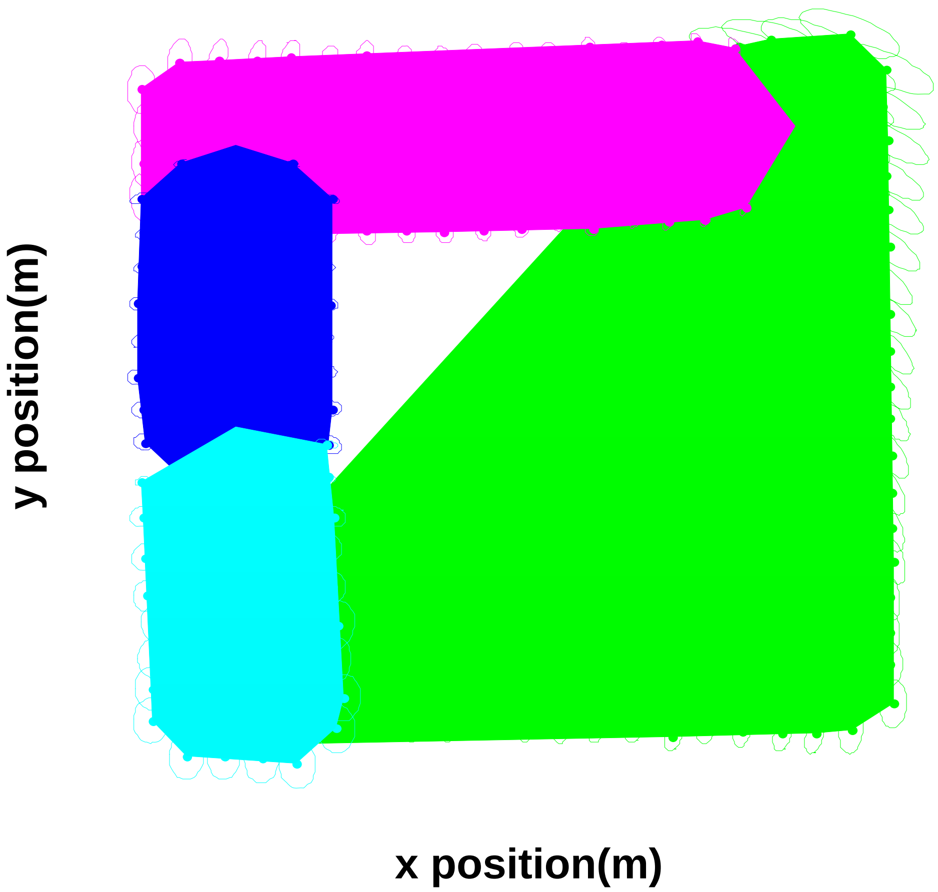




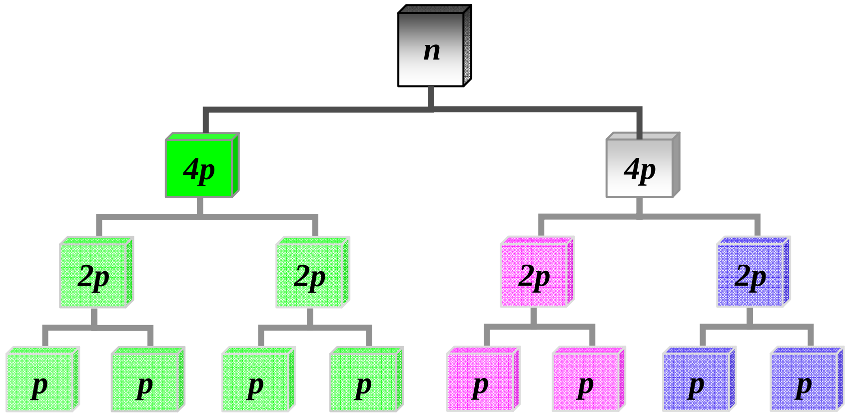
Map Hierarchy

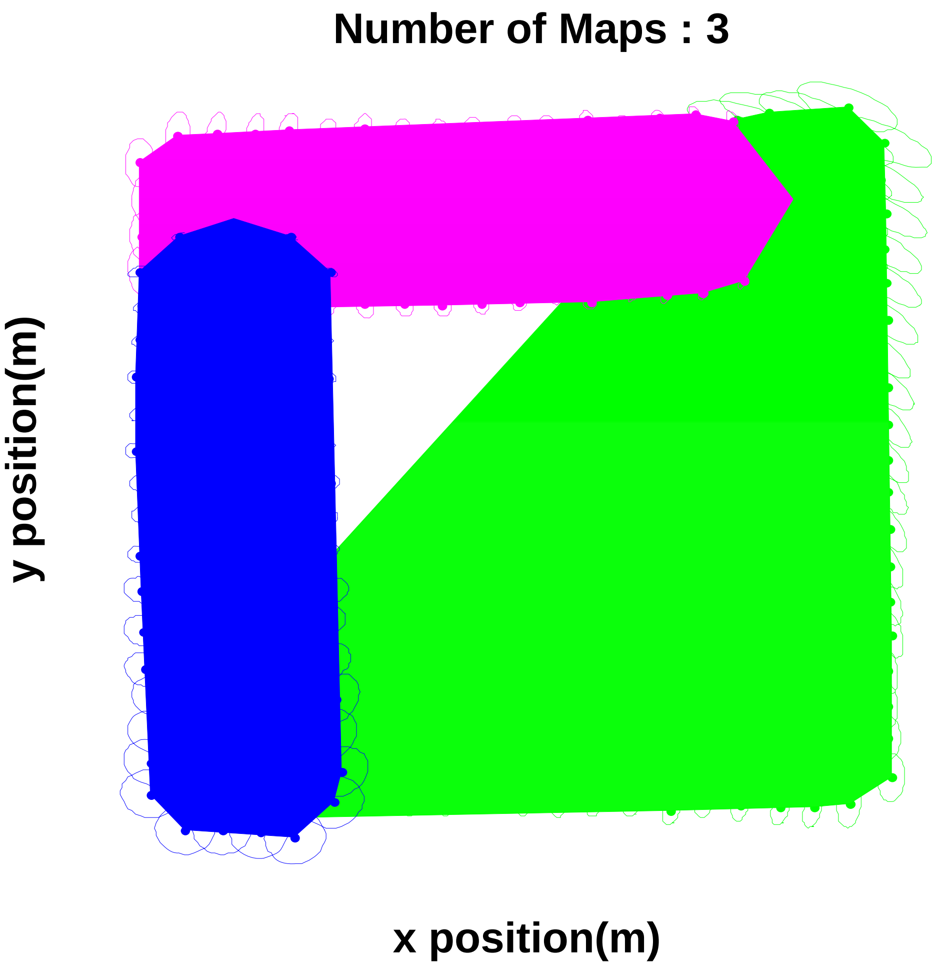


Number of Maps : 4

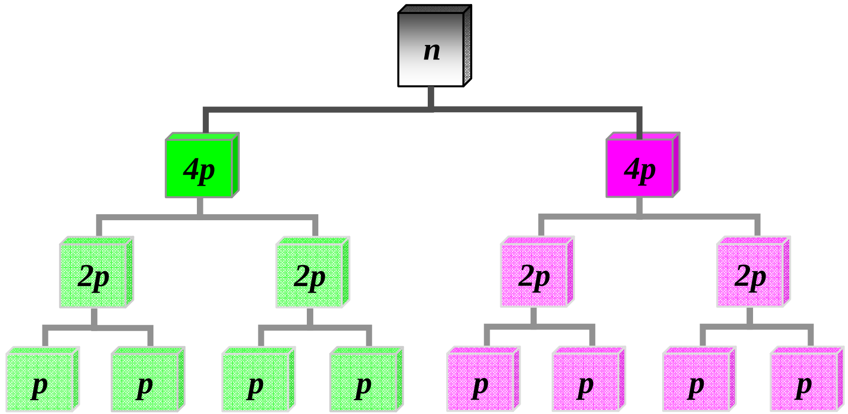


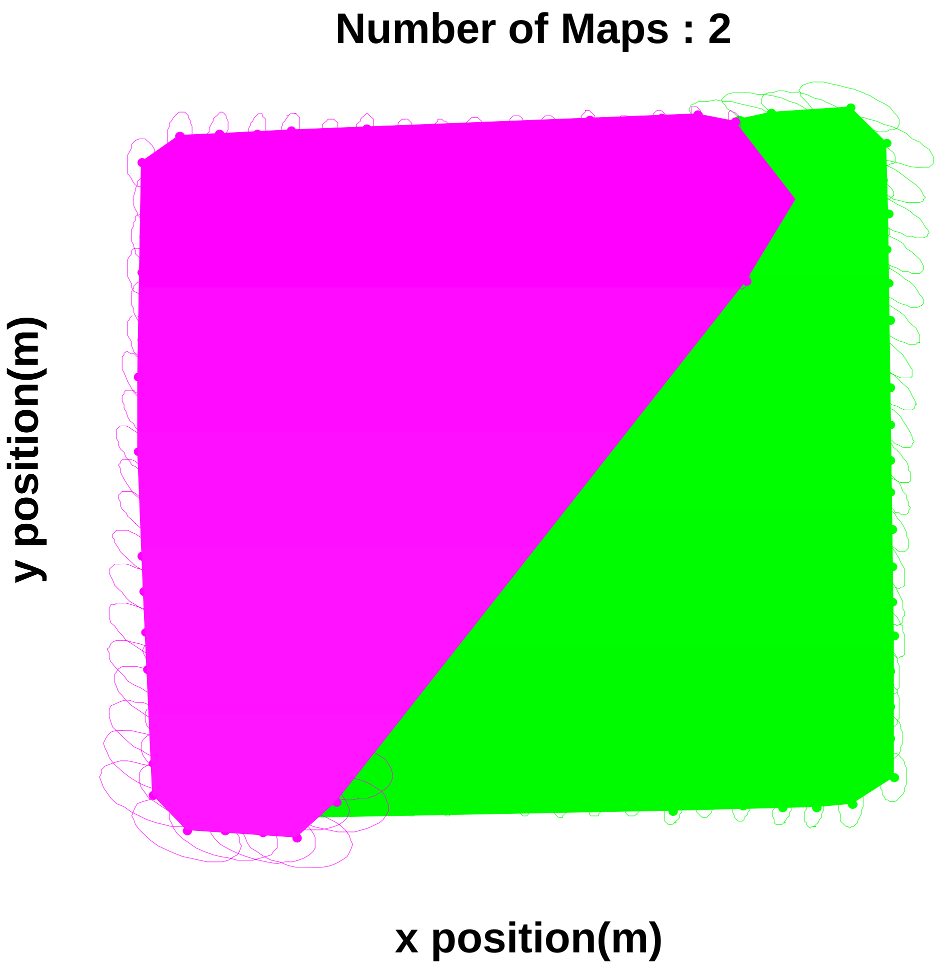
Map Hierarchy



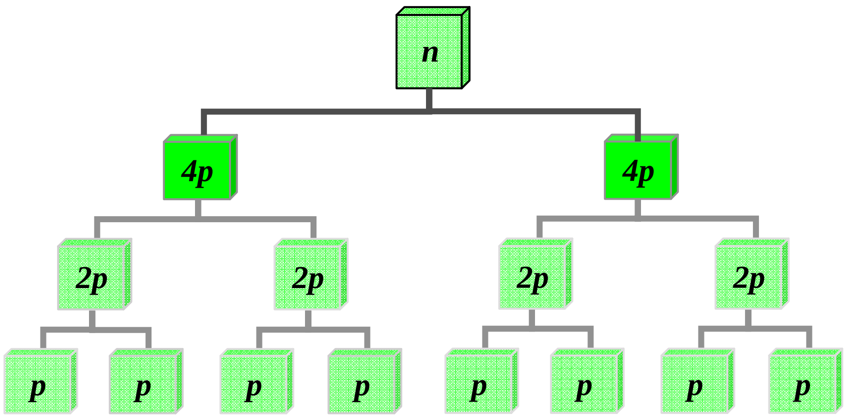


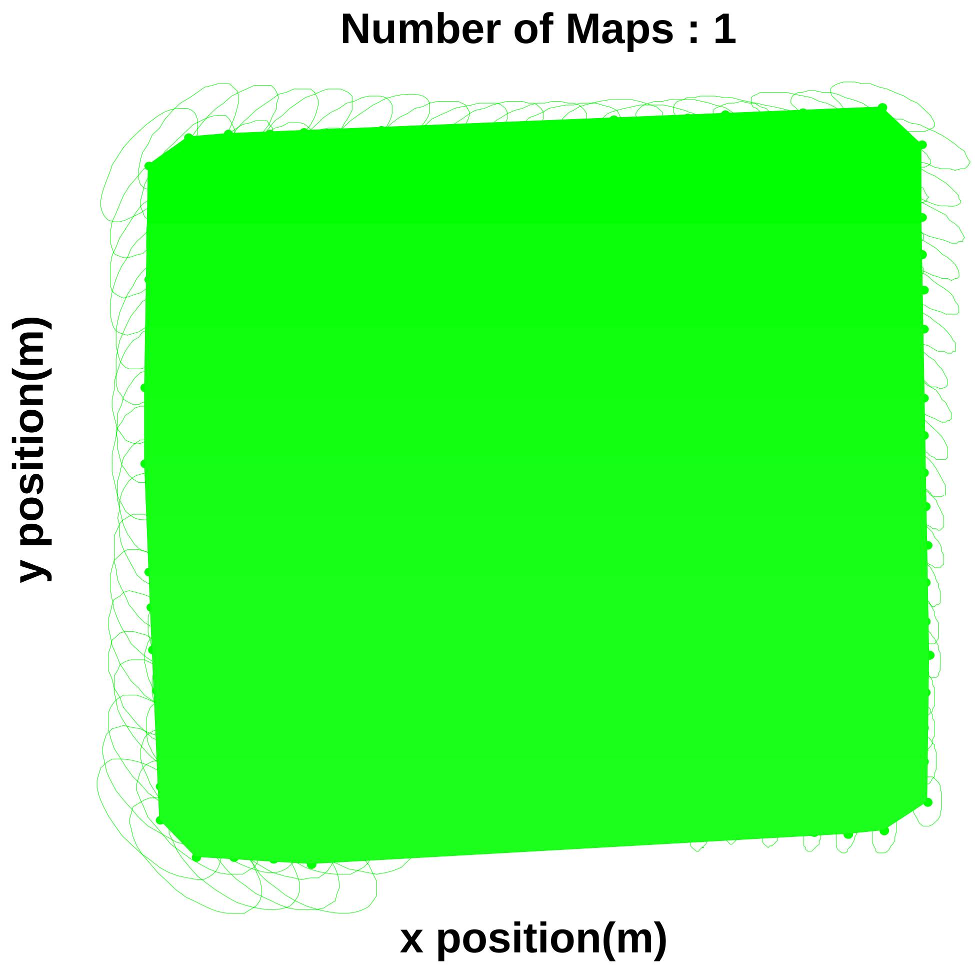
Map Hierarchy





Map Hierarchy





Divide and Conquer SLAM

- ***m*** maps of size ***P***, cost ***O(P³)*** each
- joined into ***m/2*** maps of size ***2P,*** cost ***O((2P)²)***
- joined into ***m/4*** maps of size ***4P,*** cost ***O((4P)²)***
- ... joined into ***1*** local map of size ***n,*** cost ***O(n²)***

$$
\begin{aligned}
C &= O\left(P^3 m + \sum_{i=1}^{\log_2 m} \frac{m}{2^i}(2^i P)^2\right) \\
&= O\left(P^3 n/P + \sum_{i=1}^{\log_2 n/P} \frac{n/P}{2^i}(2^i P)^2\right) \\
&= O\left(P^2 n + \sum_{i=1}^{\log_2 n/P} P \frac{n}{2^i}(2^i)^2\right) \\
&= O\left(P^2 n + P\, n \sum_{i=1}^{\log_2 n/P} 2^i\right)
\end{aligned}
$$

$$m = n/P:$$

Divide and Conquer SLAM

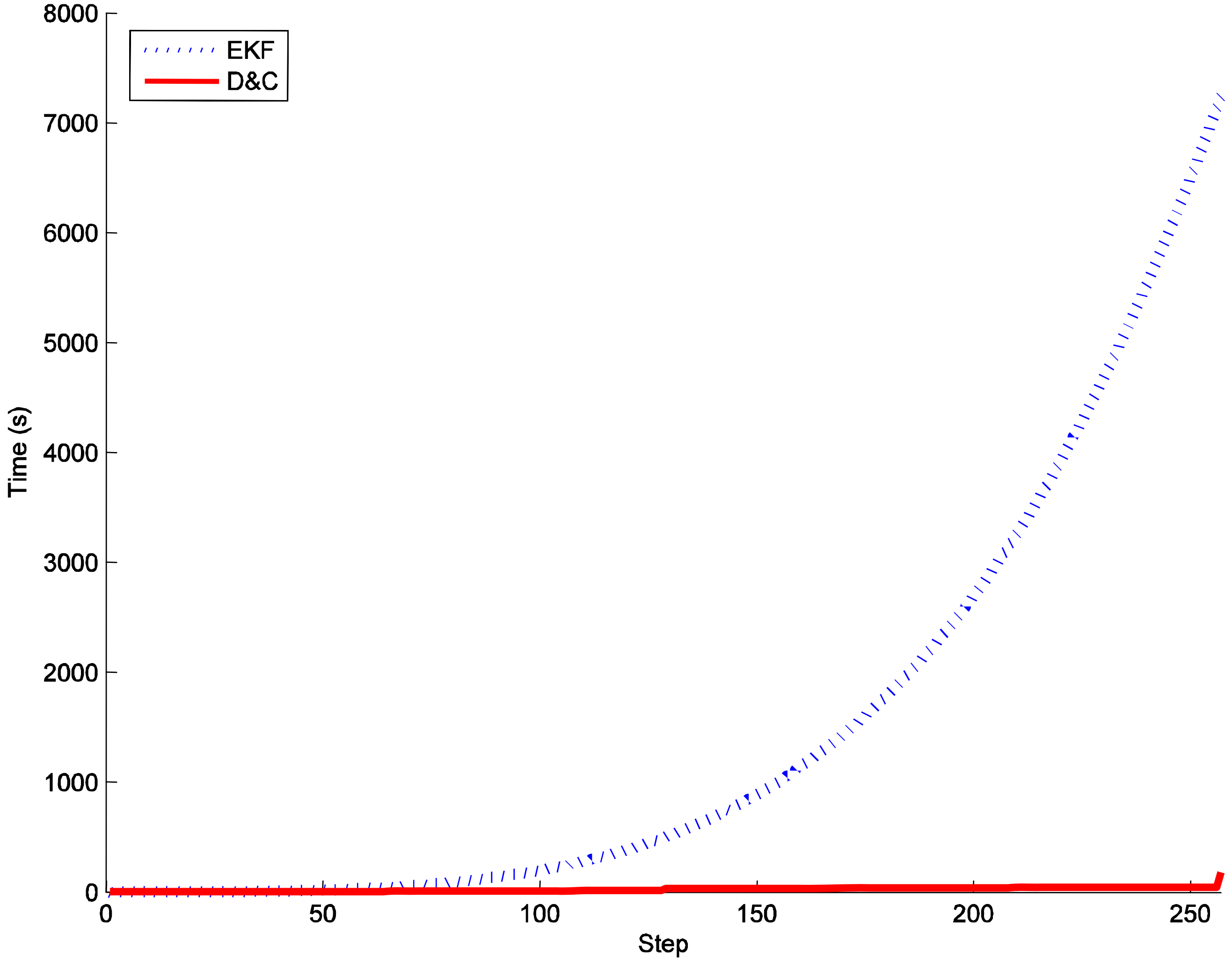
- This is a geometric progression of the type:

$$\sum_{i=1}^{k} r^i = \frac{r^{k+1} - r}{r - 1}$$

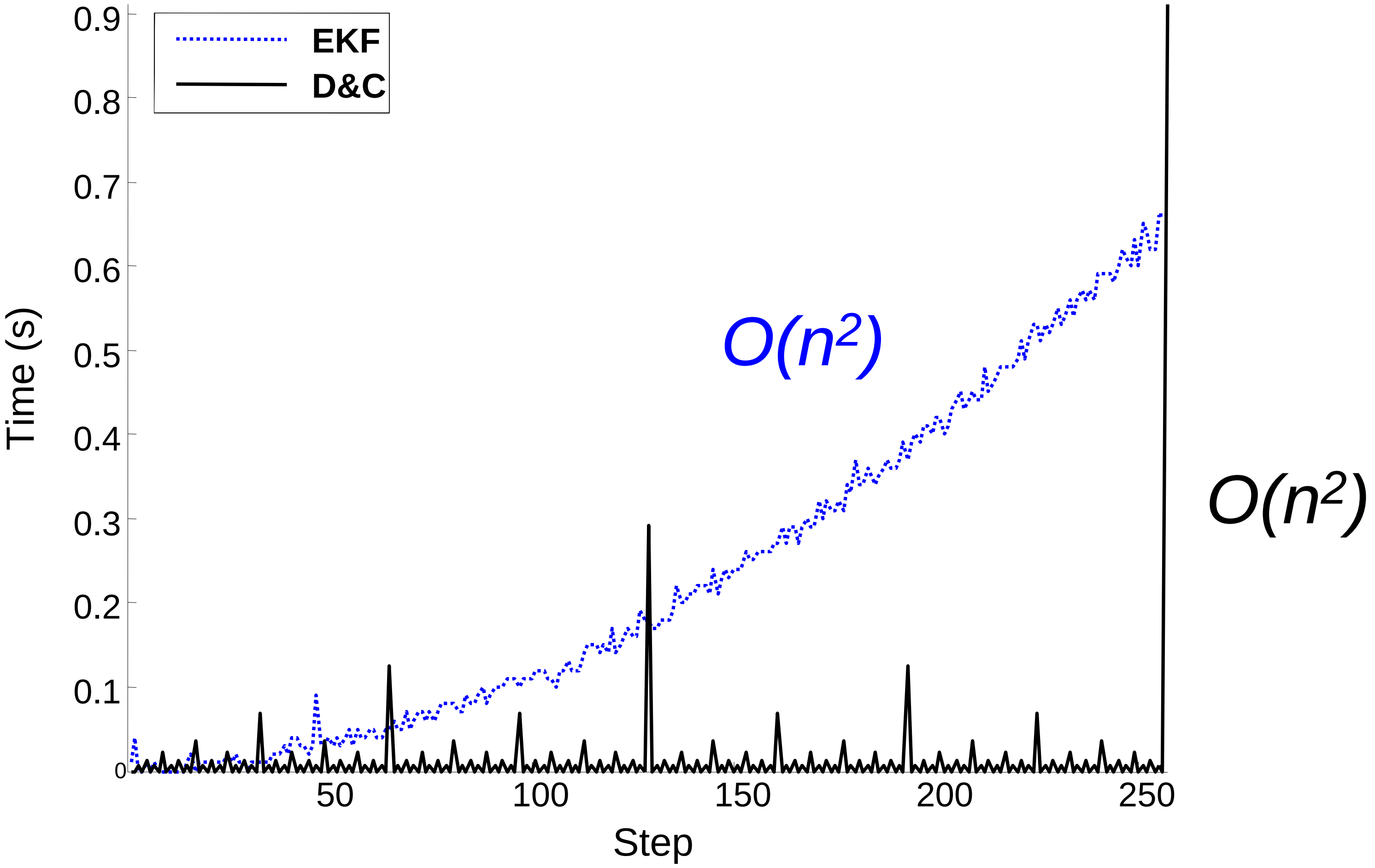
- Thus in this case:

$$\begin{aligned} C &= O\left(P^2 n + P\,n \frac{2^{\log_2 n/P+1} - 2}{2-1}\right) \\ &= O\left(P^2 n + P\,n(2\,n/P - 2)\right) \\ &= O\left(P^2 n + 2n^2 - 2Pn\right) \\ &= O(n^2) \end{aligned}$$

Total cost of D&C SLAM



Computational cost per step



Computational cost per step

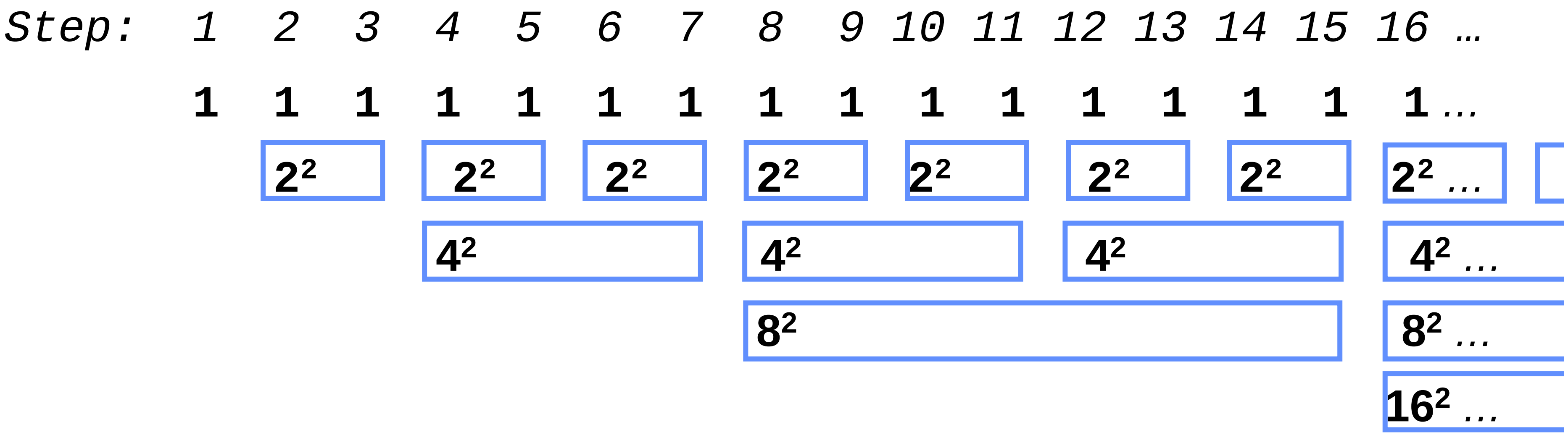
Step:	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	…
	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	…
		2^2		2^2		2^2		2^2		2^2		2^2		2^2		2^2	…
				4^2				4^2				4^2				4^2	…
								8^2								8^2	…
																16^2	…

Computational cost per step

Step:	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	…
	1	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	…
		2^2		2^2		2^2		2^2		2^2		2^2		2^2		2^2	…
				4^2				4^2				4^2				4^2	…
								8^2								8^2	…
																16^2	…

Not needed until…

Computational cost per step



Amortized cost per step

- First idea:

Step:	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16*	*…*
	1	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**…**
		2	**2**	**2**	**2**	**2**	**2**	**2**	**2**	**2**	**2**	**2**	**2**	**2**	**2**	**2**	**…**
				4	**4**	**4**	**4**	**4**	**4**	**4**	**4**	**4**	**4**	**4**	**4**	**4**	**…**
								8	**8**	**8**	**8**	**8**	**8**	**8**	**8**	**8**	**…**
																16	

Amortized cost per step

- First idea:

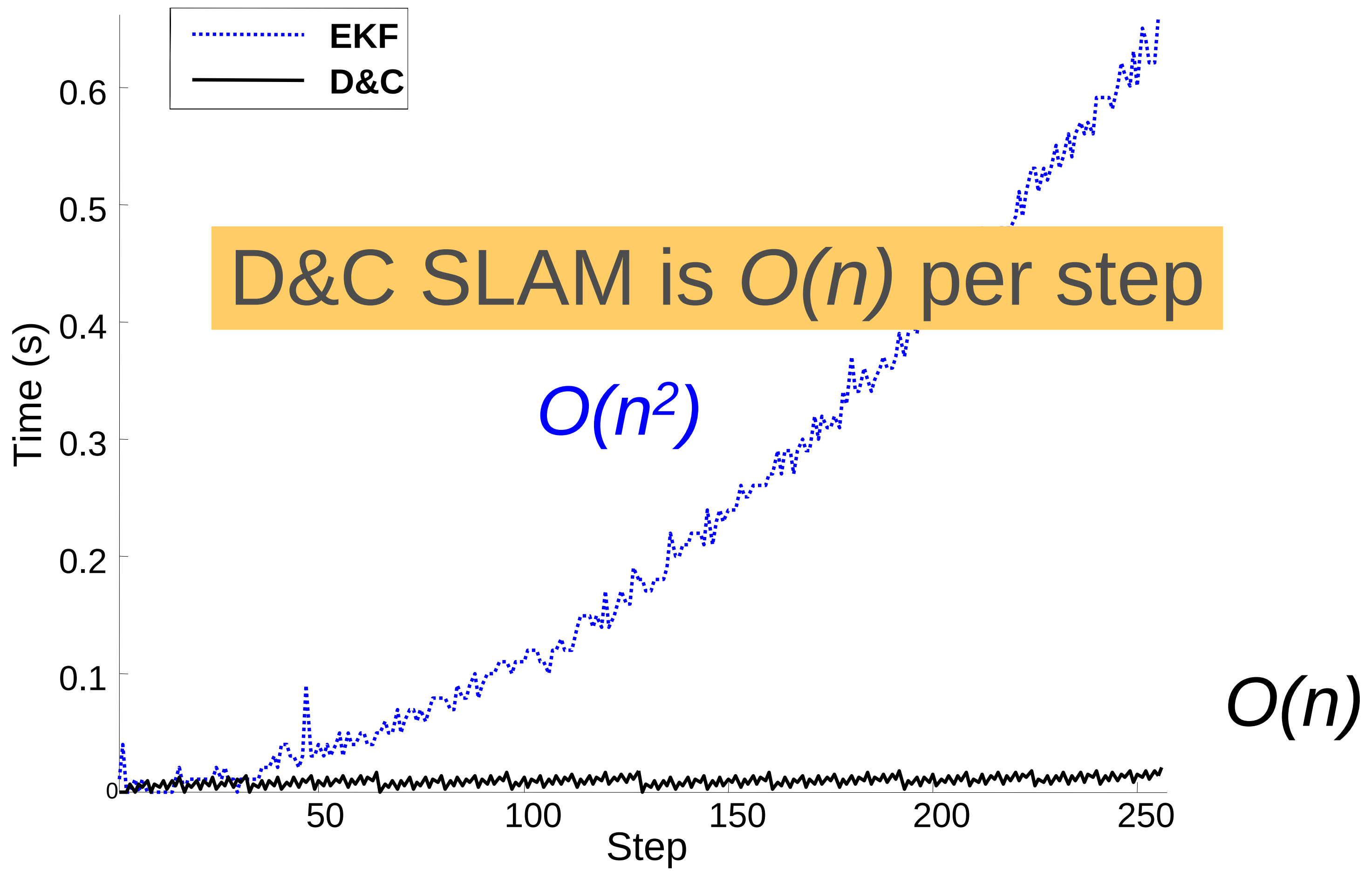
Step:	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	…
	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	…
		2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	…
				4	4	4	4	4	4	4	4	4	4	4	4	4	…
								8	8	8	8	8	8	8	8	8	…
																16	

Amortized cost per step

- Better idea:

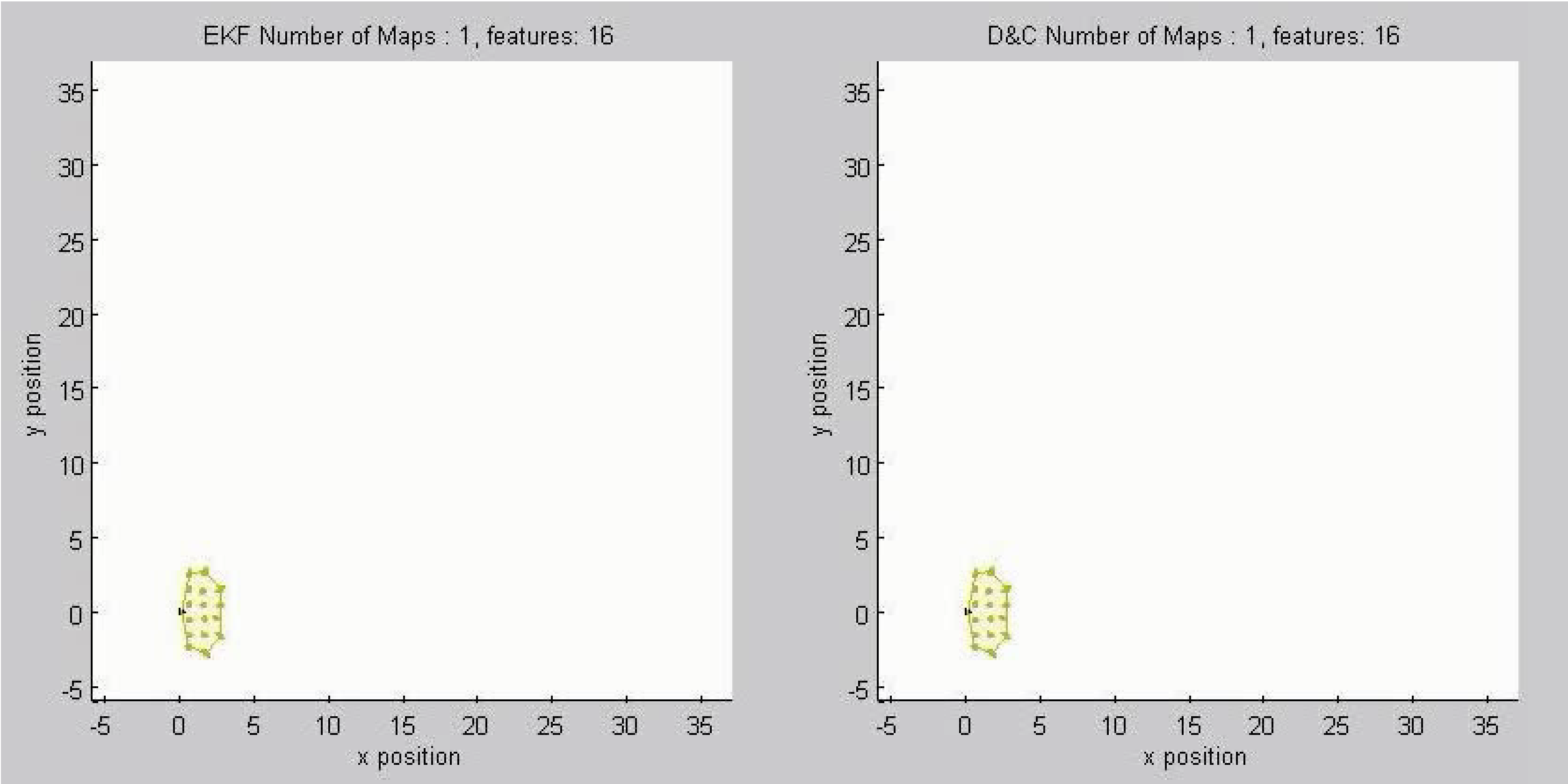
Step:	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	…
	1	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**…**
			4		**4**		**4**		**4**		**4**		**4**		**4**		**…**
						8	**8**			**8**	**8**			**8**	**8**		**…**
												16	**16**	**16**	**16**		**…**

Amortized cost per step

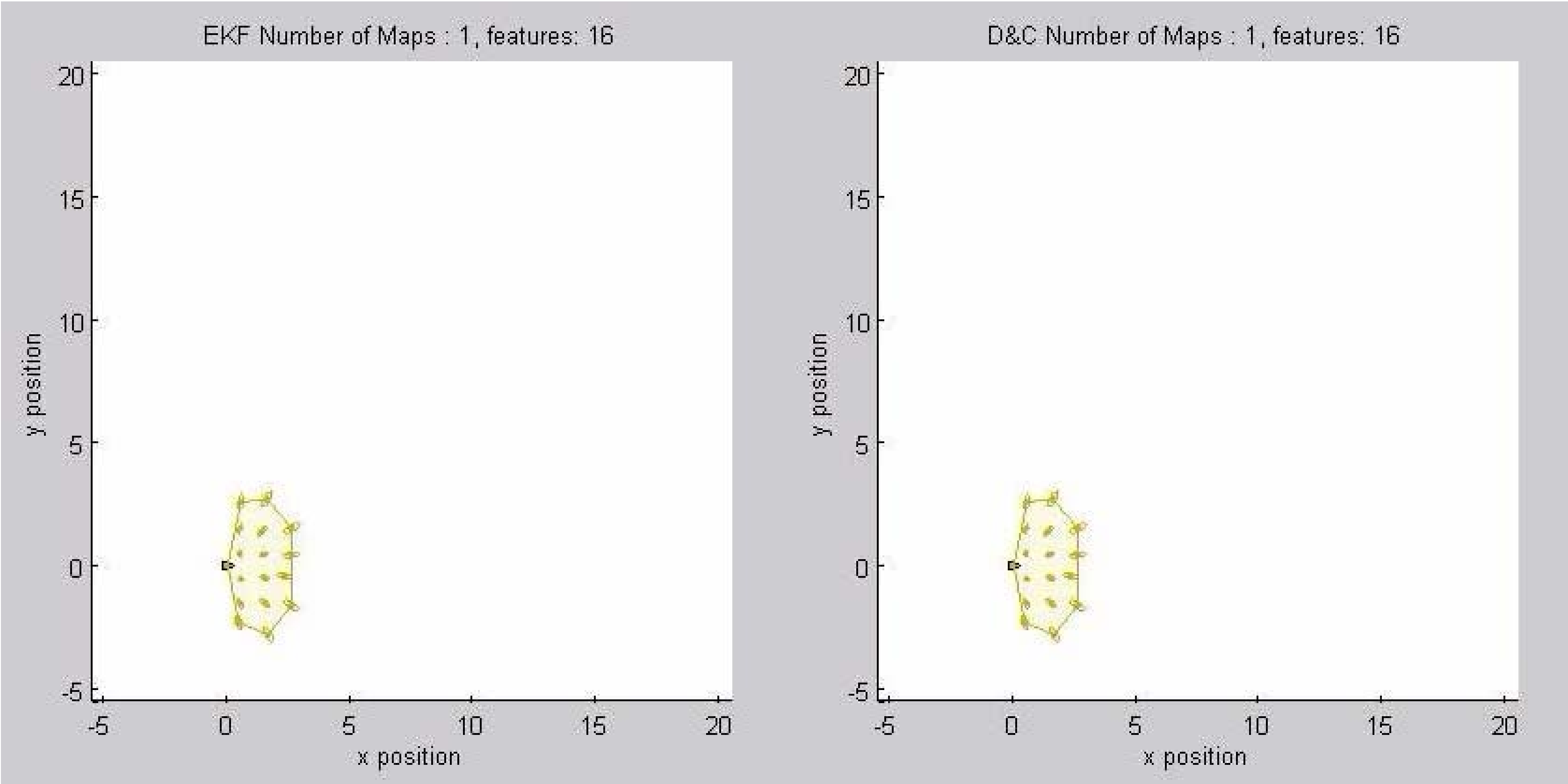


- The full map can be recovered at any time in a single $O(n^2)$ step
- But no part of the algorithm or process requires it

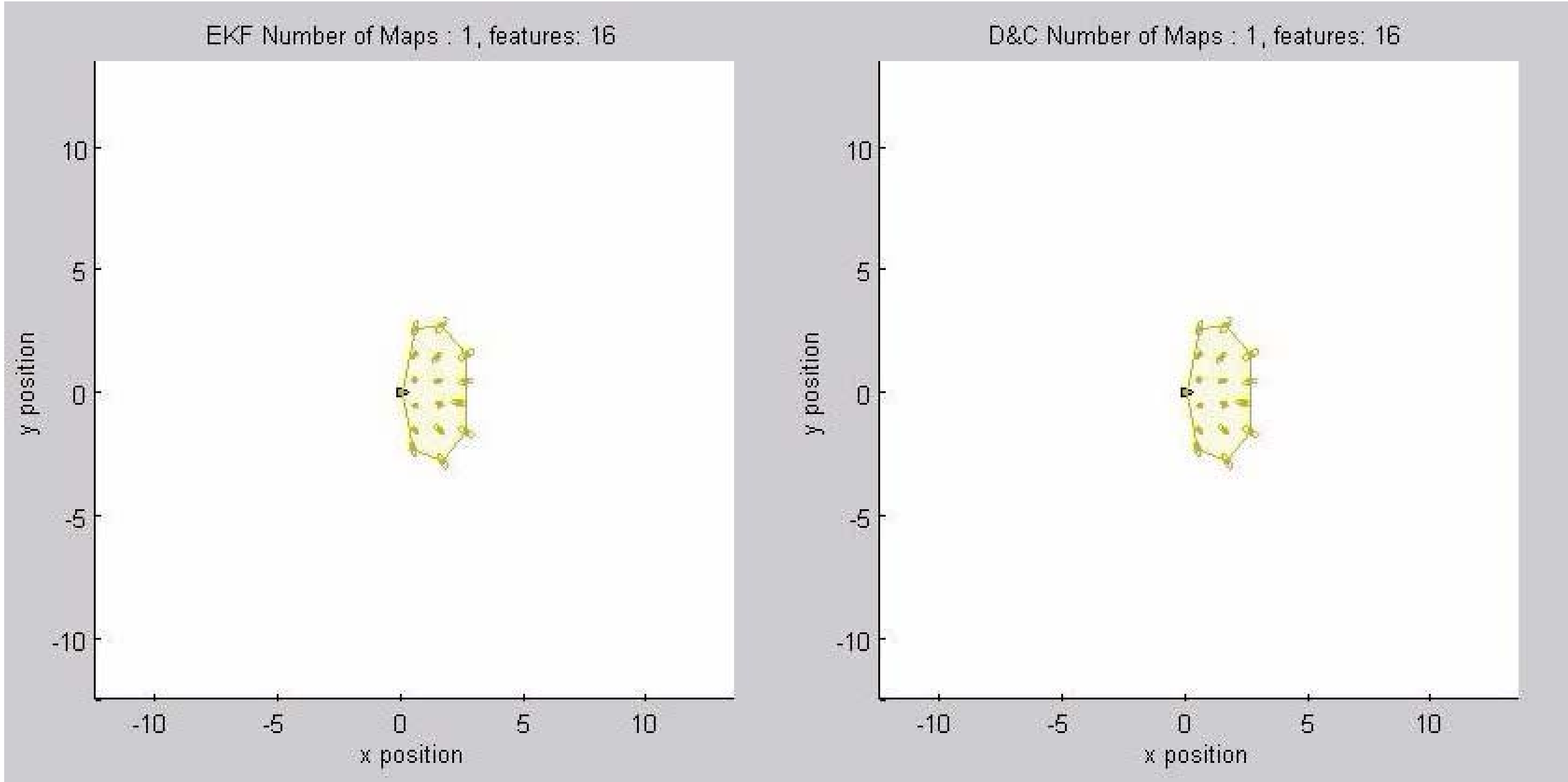
Consistency: LOOP



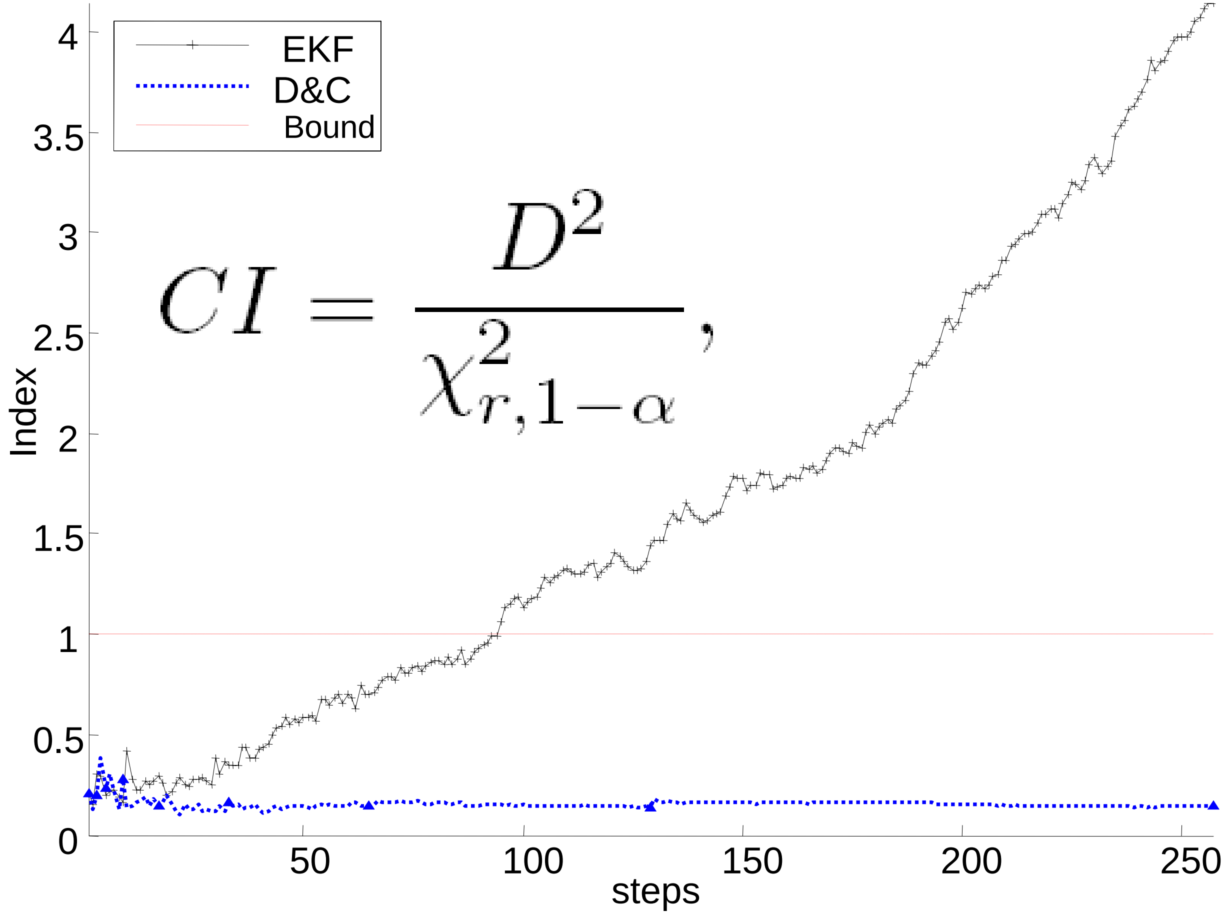
Consistency: LAWN



Consistency : SNAIL



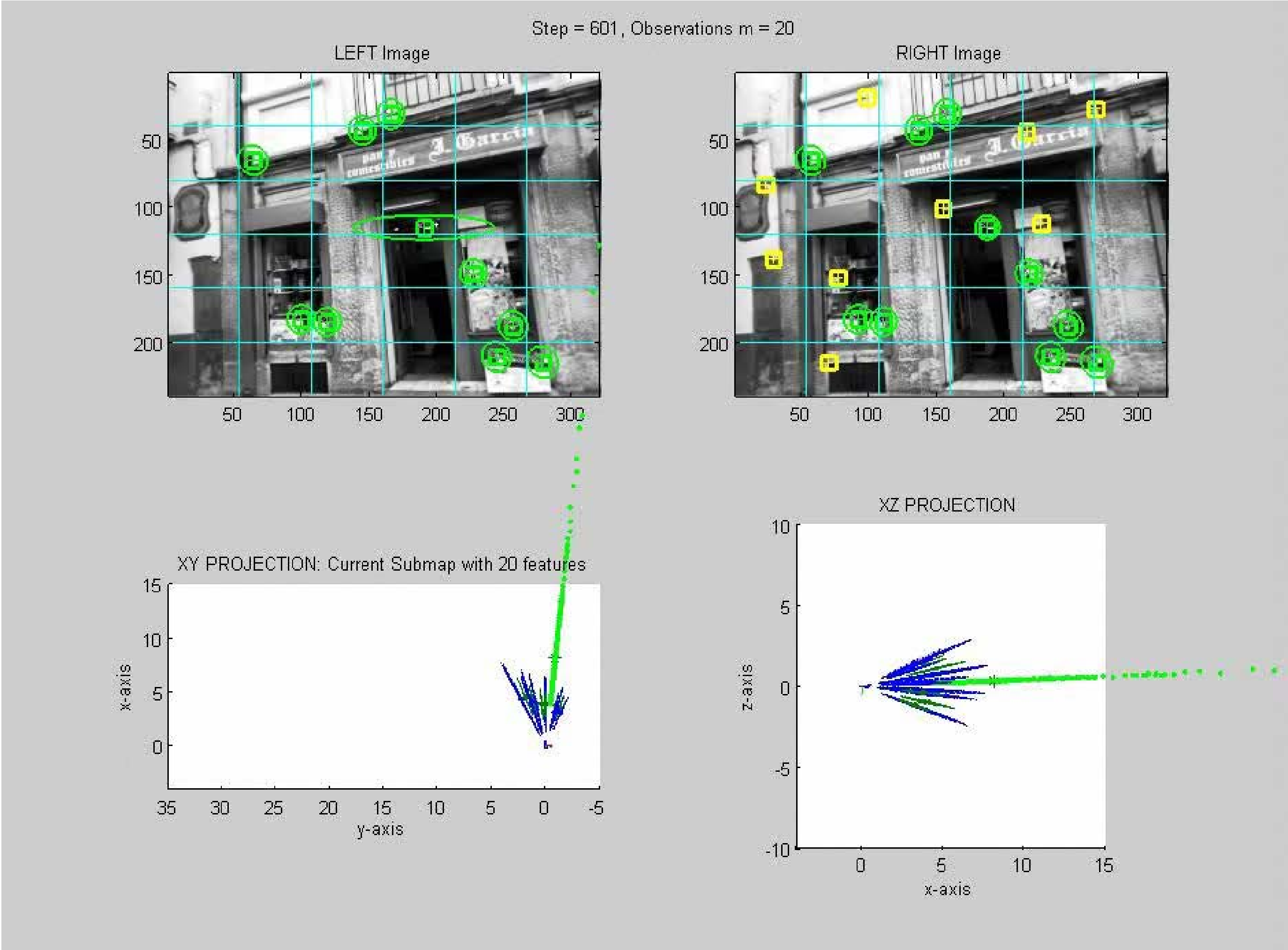
Consistency: Montecarlo runs mean consistency



$$CI = \frac{D^2}{\chi^2_{r,1-\alpha}},$$

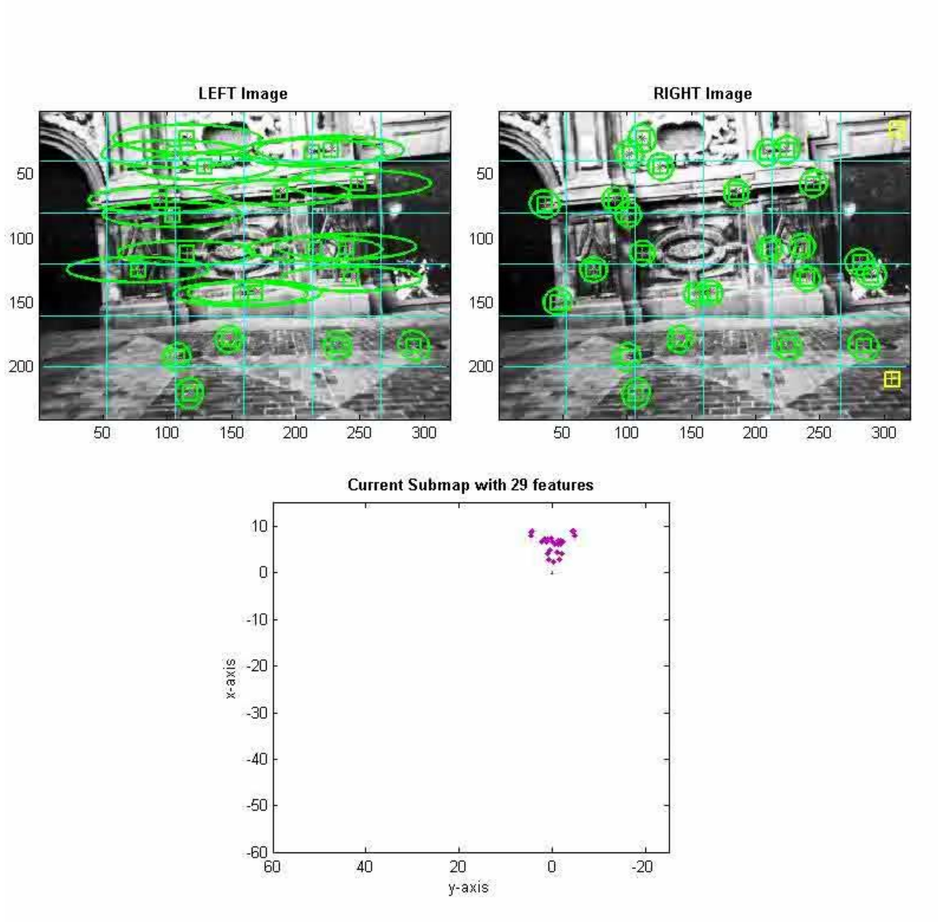
D&C SLAM is always more consistent

6DOF SLAM with stereo



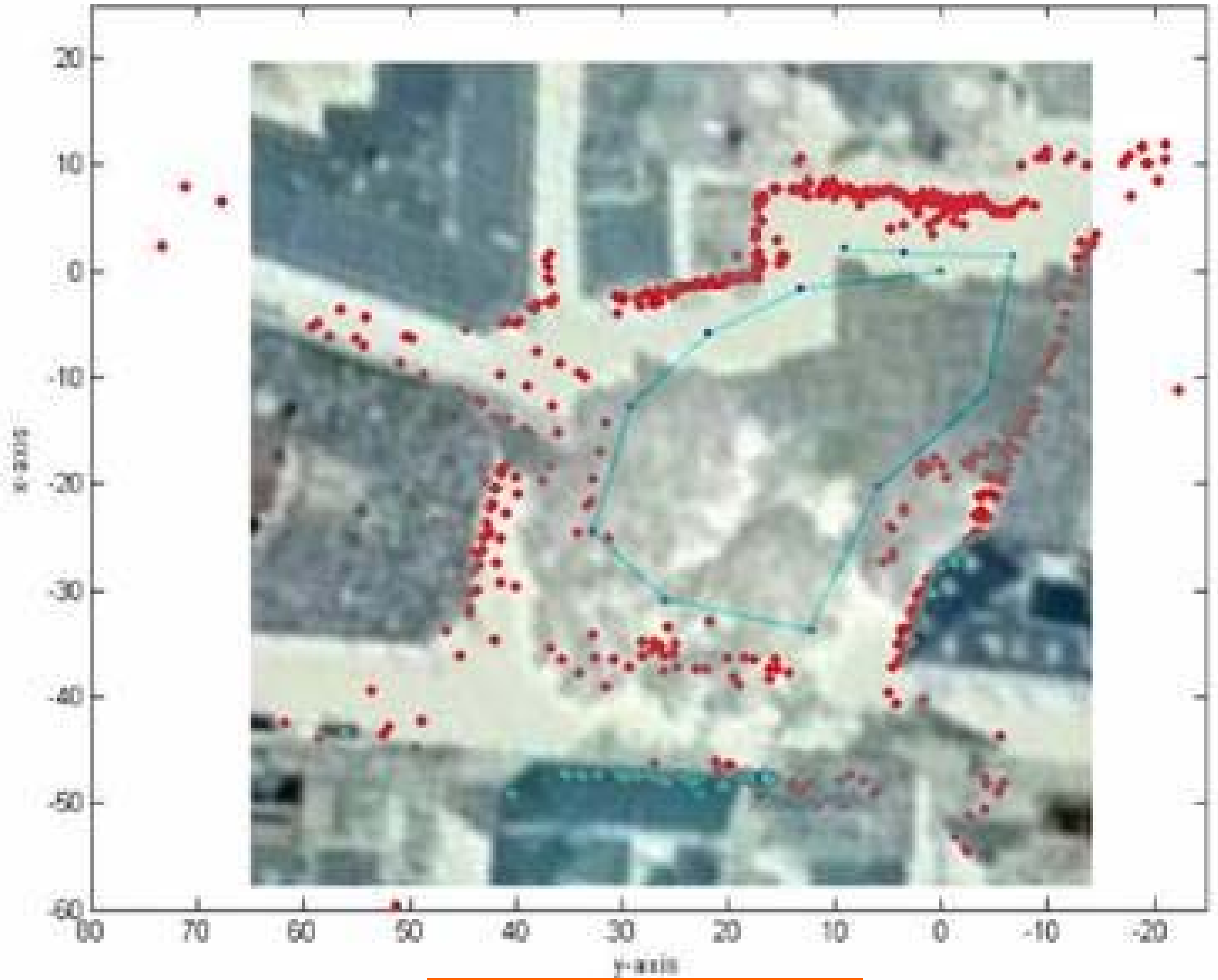
L. Paz, P. Piníes, J. Neira and J.D. Tardós **Large Scale 6DOF SLAM with Stereo-in-Hand.** Conditionally accepted, IEE Transactions on Robotics, 2008.

6DOF SLAM with stereo



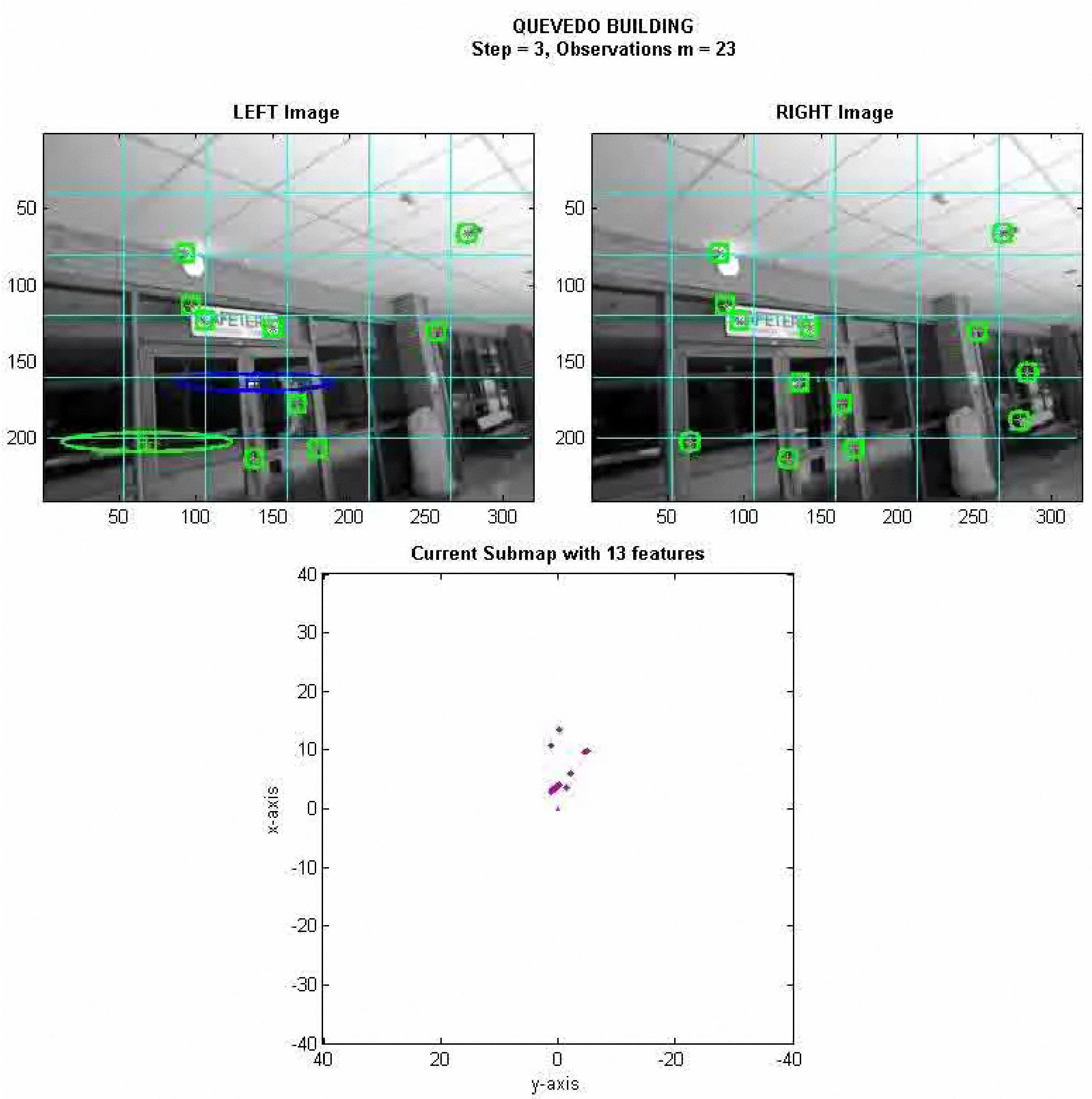
L. Paz, P. Piníes, J. Neira and J.D. Tardós **Large Scale 6DOF SLAM with Stereo-in-Hand.** Conditionally accepted, IEE Transactions on Robotics, 2008.

6Dof Stereo SLAM, outdoors

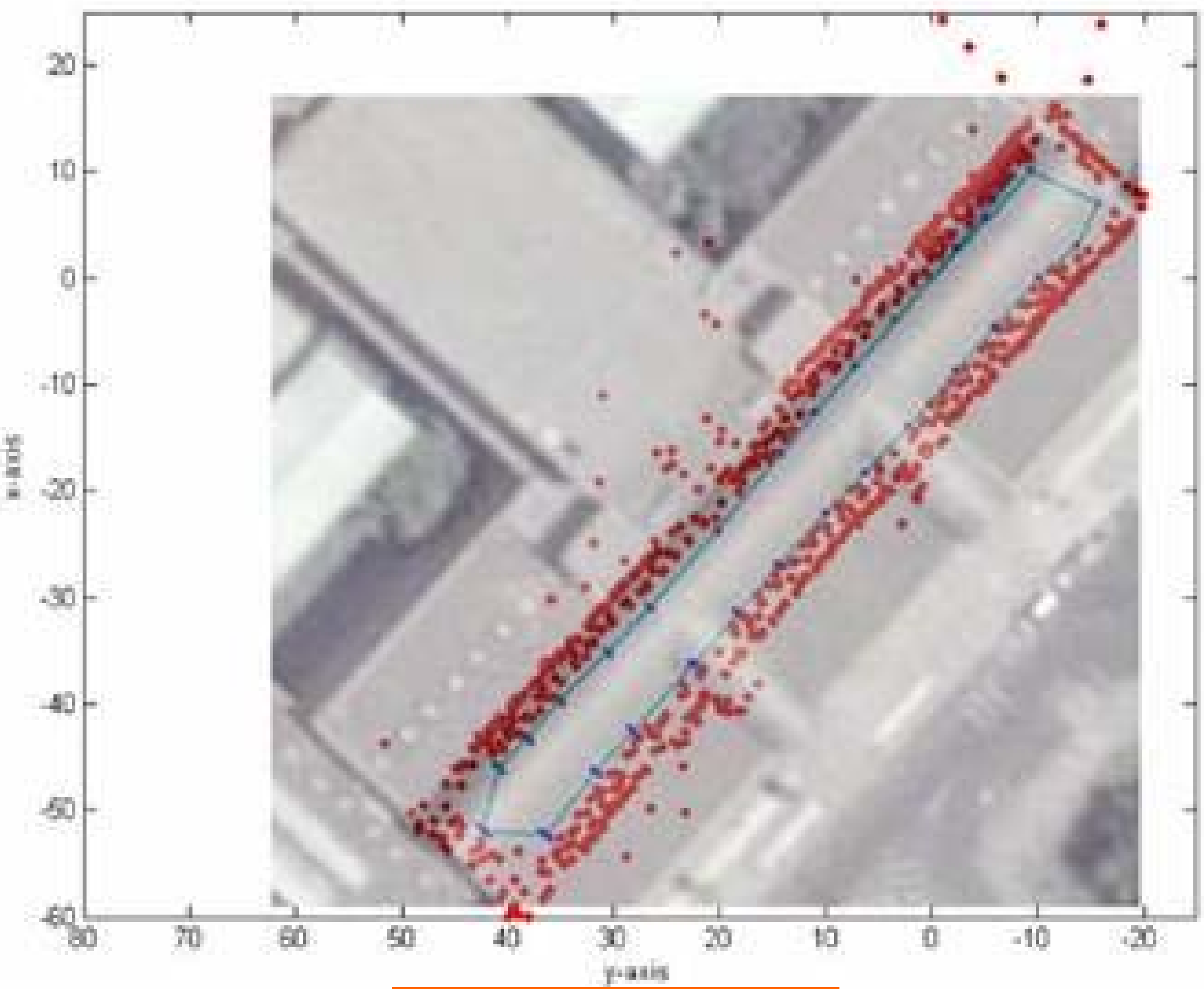


150 m loop

6DOF SLAM with stereo



6Dof Stereo SLAM, indoors



180 m loop

Outline

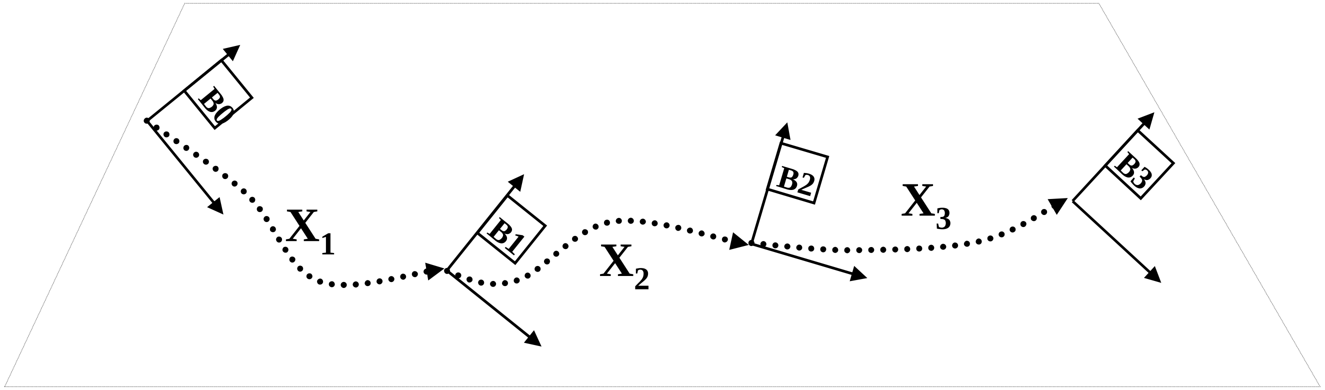
1. Basic EKF SLAM
    1. Introduction: the need for SLAM
    2. The basic EKF SLAM algorithm
    3. Feature extraction
    4. Data Association

2. The Data Association Problem
    1. Introduction
    2. Continuous Data Association
    3. The Loop Closing Problem
    4. The Global Localization Problem

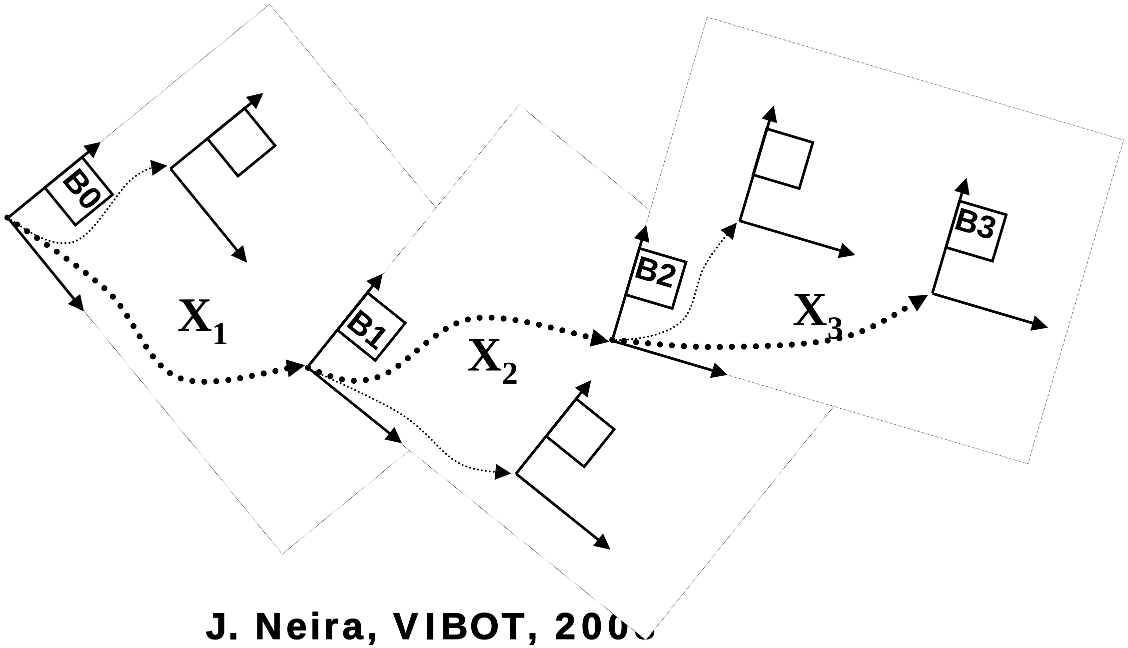
3. Advanced EKF SLAM
    1. Computational complexity of EKF SLAM
    2. Consistency of the EKF SLAM
    3. SLAM using local maps
        1. Sequential Map Joining
        2. Divide and Conquer SLAM
        3. **Hierarchical SLAM**

Hierarchical SLAM

- Global level: adjacency graph and relative stochastic map



- Local level: statistically independent local maps



ROBOTICS UNIVERSIDAD DE ZARAGOZA

Hierarchical SLAM

- Local maps:

$$\hat{\mathbf{x}}_{\mathcal{F}}^{B} = \begin{bmatrix} \hat{\mathbf{x}}_{R}^{B} \\ \vdots \\ \hat{\mathbf{x}}_{F_n}^{B} \end{bmatrix} ; \; \mathbf{P}_{\mathcal{F}} = \begin{bmatrix} \mathbf{P}_{R}^{B} & \cdots & \mathbf{P}_{RF_n}^{B} \\ \vdots & \ddots & \vdots \\ \mathbf{P}_{F_nR}^{B} & \cdots & \mathbf{P}_{F_nF_n}^{B} \end{bmatrix}$$

- Global relative map:

$$\hat{\mathbf{x}} = \begin{bmatrix} \vdots \\ \hat{\mathbf{x}}_i \\ \vdots \end{bmatrix} ; \; \mathbf{P} = \begin{bmatrix} . & 0 & 0 \\ 0 & \mathbf{P}_i & 0 \\ 0 & 0 & . \end{bmatrix}$$

Block diagonal

Hierarchical SLAM

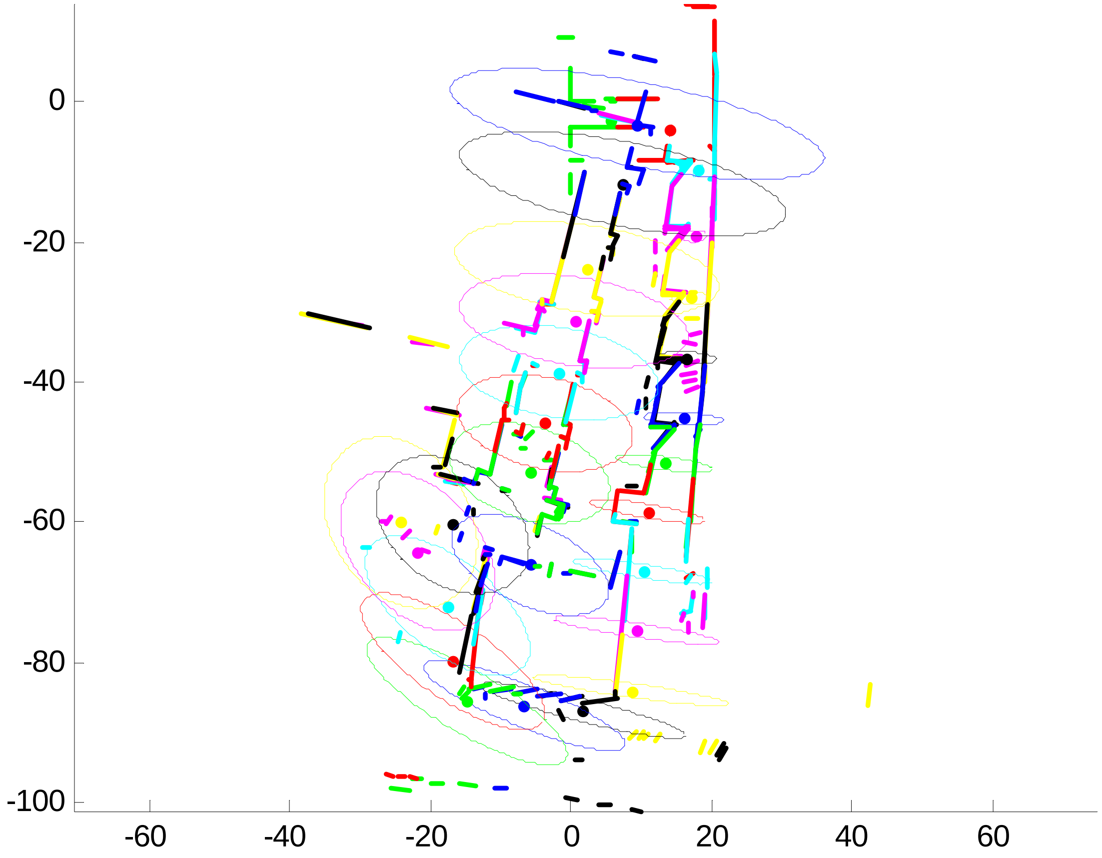
- Global relative map before loop closing:

$$\hat{\mathbf{x}} = \begin{bmatrix} \vdots \\ \hat{\mathbf{x}}_{i+1} \\ \vdots \\ \hat{\mathbf{x}}_j \end{bmatrix} ; \; \mathbf{P} = \begin{bmatrix} . & 0 & 0 & 0 \\ 0 & \mathbf{P}_{i+1} & 0 & 0 \\ 0 & 0 & . & 0 \\ 0 & 0 & 0 & \mathbf{P}_j \end{bmatrix}$$

- After loop closing:

$$\hat{\mathbf{x}} = \begin{bmatrix} \vdots \\ \hat{\mathbf{x}}_{i+1} \\ \hat{\mathbf{x}}_{ij} \\ \vdots \\ \hat{\mathbf{x}}_j \end{bmatrix} ; \; \mathbf{P} = \begin{bmatrix} . & 0 & 0 & 0 & 0 \\ 0 & \mathbf{P}_{i+1} & \mathbf{P}_{i+1,ij} & 0 & 0 \\ 0 & \mathbf{P}_{i+1,ij}^T & \mathbf{P}_{ij} & 0 & 0 \\ 0 & 0 & 0 & . & 0 \\ 0 & 0 & 0 & 0 & \mathbf{P}_j \end{bmatrix}$$

Imposing loop constraints



$$\mathbf{h}(\mathbf{x}) \equiv \mathbf{x}_1 \oplus \mathbf{x}_2 \oplus \cdots \oplus \mathbf{x}_{n-1} \oplus \mathbf{x}_n$$

Nonlinear constrained optimization

- Minimize corrections to the global map, subject to the loop constraint:

$$\min_{\mathbf{x}} \frac{1}{2}(\mathbf{x}-\hat{\mathbf{x}})^T \mathbf{P}^{-1}(\mathbf{x}-\hat{\mathbf{x}})$$
$$\mathbf{h}(\mathbf{x}) = \mathbf{0}$$

- Sequential Quadratic Programming (SQP) :

$$\mathbf{H}_i = \left[\left.\frac{\partial \mathbf{h}}{\partial \mathbf{x}_1}\right|_{\hat{\mathbf{x}}_i} \left.\frac{\partial \mathbf{h}}{\partial \mathbf{x}_2}\right|_{\hat{\mathbf{x}}_i} \cdots \left.\frac{\partial \mathbf{h}}{\partial \mathbf{x}_{n-1}}\right|_{\hat{\mathbf{x}}_i} \left.\frac{\partial \mathbf{h}}{\partial \mathbf{x}_n}\right|_{\hat{\mathbf{x}}_i} \right]$$

$$\begin{aligned} \mathbf{P}_i &= \mathbf{P}_0 - \mathbf{P}_0\mathbf{H}_i^T \left(\mathbf{H}_i\mathbf{P}_0\mathbf{H}_i^T\right)^{-1} \mathbf{H}_i\mathbf{P}_0 \\ \hat{\mathbf{x}}_{i+1} &= \hat{\mathbf{x}}_i - \mathbf{P}_i\mathbf{P}_0^{-1}(\hat{\mathbf{x}}_i - \hat{\mathbf{x}}_0) - \mathbf{P}_0\mathbf{H}_i^T \left(\mathbf{H}_i\mathbf{P}_0\mathbf{H}_i^T\right)^{-1} \hat{\mathbf{h}}_i \end{aligned}$$

» Iterate until convergence

ROBOTICS UNIVERSIDAD DE ZARAGOZA

Nonlinear constrained optimization

- A more efficient version:

$$\hat{\mathbf{x}}_{i+1} = \hat{\mathbf{x}}_0 + \mathbf{P}_0 \mathbf{H}_i^T \left(\mathbf{H}_i \mathbf{P}_0 \mathbf{H}_i^T \right)^{-1} \left(\mathbf{H}_i \left(\hat{\mathbf{x}}_i - \hat{\mathbf{x}}_0 \right) - \hat{\mathbf{h}}_i \right)$$

» Iterate until convergence

- Complexity:
  - P_0 is block diagonal
  - H_i is sparse with nonzeros only for the maps in the loop
  - The iteration is linear with the number of maps in the loop
- Convergence:
  - Converges in 2 or 3 iterations (for loops around 300m)
  - For bigger errors, may it converge to a local minimum ??

Nonlinear constrained optimization

- Generalization to closing several loops simultaneously:

$$\mathbf{h}_j(\mathbf{x}) = \mathbf{x}_{j_1} \oplus \mathbf{x}_{j_2} \oplus \cdots \oplus \mathbf{x}_{j_{n_j - 1}} \oplus \mathbf{x}_{j_{n_j}} = 0$$

$$\mathbf{h} = \begin{bmatrix} \mathbf{h}_1 \\ \mathbf{h}_2 \\ \vdots \\ \mathbf{h}_l \end{bmatrix}$$

Iterated Extended Kalman Filter

- Jacobian of the measurement function:

$$\mathbf{H}_i = \left[\left.\frac{\partial \mathbf{h}}{\partial \mathbf{x}_1}\right|_{\hat{\mathbf{x}}_i} \left.\frac{\partial \mathbf{h}}{\partial \mathbf{x}_2}\right|_{\hat{\mathbf{x}}_i} \cdots \left.\frac{\partial \mathbf{h}}{\partial \mathbf{x}_{n-1}}\right|_{\hat{\mathbf{x}}_i} \left.\frac{\partial \mathbf{h}}{\partial \mathbf{x}_n}\right|_{\hat{\mathbf{x}}_i} \right]$$

- Iterated EKF equations:

$$\begin{aligned} \mathbf{P}_i &= \mathbf{P}_0 - \mathbf{P}_0\mathbf{H}_i^T \left[\mathbf{H}_i\mathbf{P}_0\mathbf{H}_i^T + \mathbf{P}_z\right]^{-1} \mathbf{H}_i\mathbf{P}_0 \\ \hat{\mathbf{x}}_{i+1} &= \hat{\mathbf{x}}_i - \mathbf{P}_i\mathbf{P}_0^{-1}(\hat{\mathbf{x}}_i - \hat{\mathbf{x}}_0) + \mathbf{P}_0\mathbf{H}_i^T \left(\mathbf{H}_i\mathbf{P}_0\mathbf{H}_i^T + \mathbf{P}_z\right)^{-1} \left(\mathbf{z} - \hat{\mathbf{h}}_i\right) \end{aligned}$$

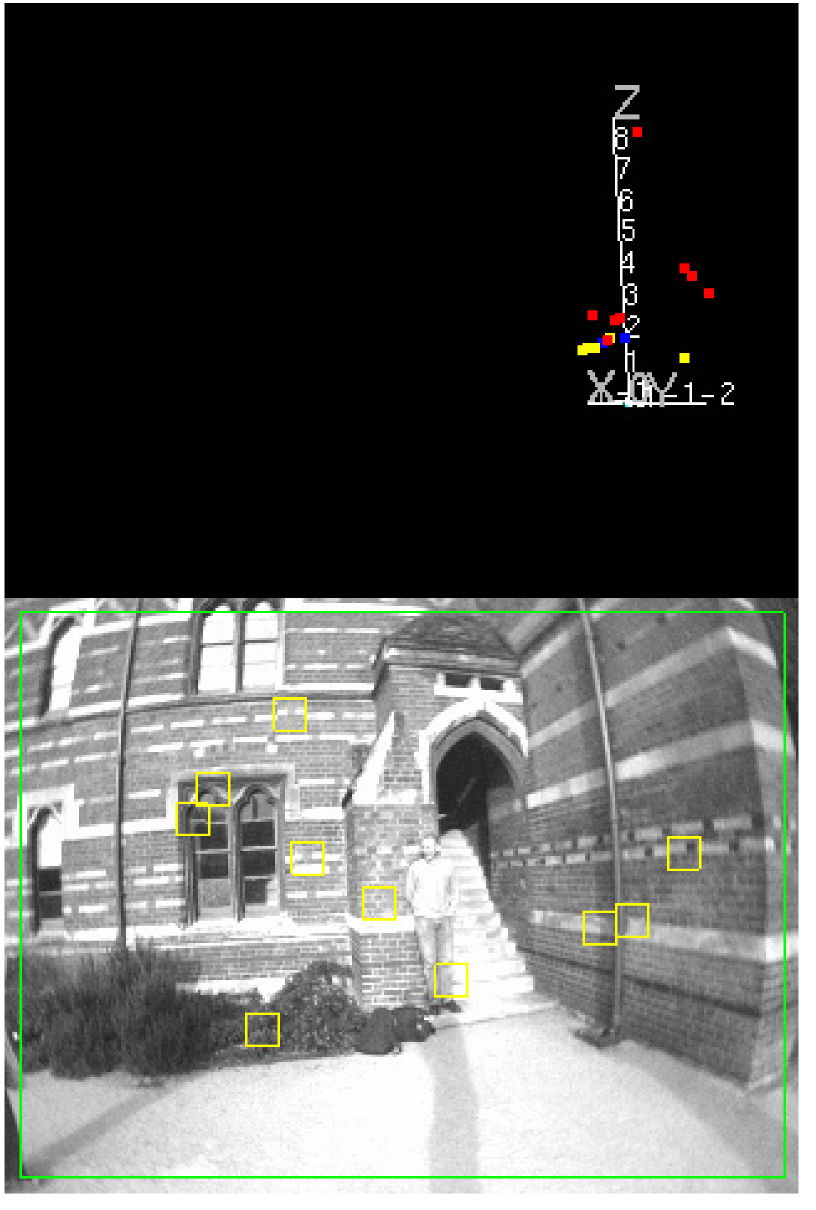
- With exact loop constraint, z = 0 and P_z = 0, IEKF is equivalent to nonlinear optimization with SQP

C. Estrada, J. Neira, J.D. Tardós, Hierarchical SLAM: real-time accurate map-ping of large environments. To appear in the IEEE Transactions on Robotics

Experiment

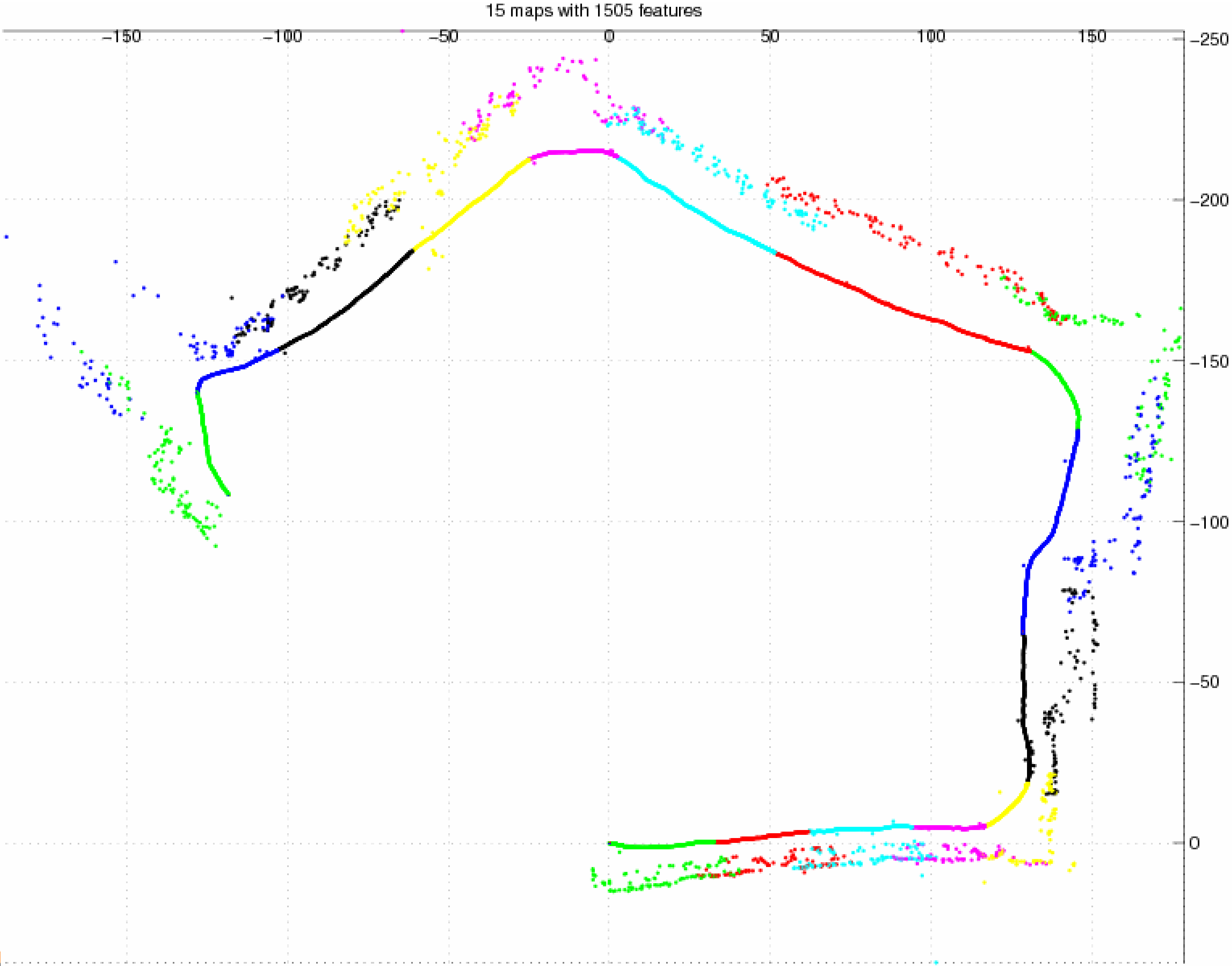
L. Clemente, A. Davison, I. Reid, J. Neira and J.D. Tardós
Mapping Large Loops with a Single Hand-Held Camera
2007 Robotics: Science and Systems, June 27-30, Atlanta, USA

Keble College, Oxford

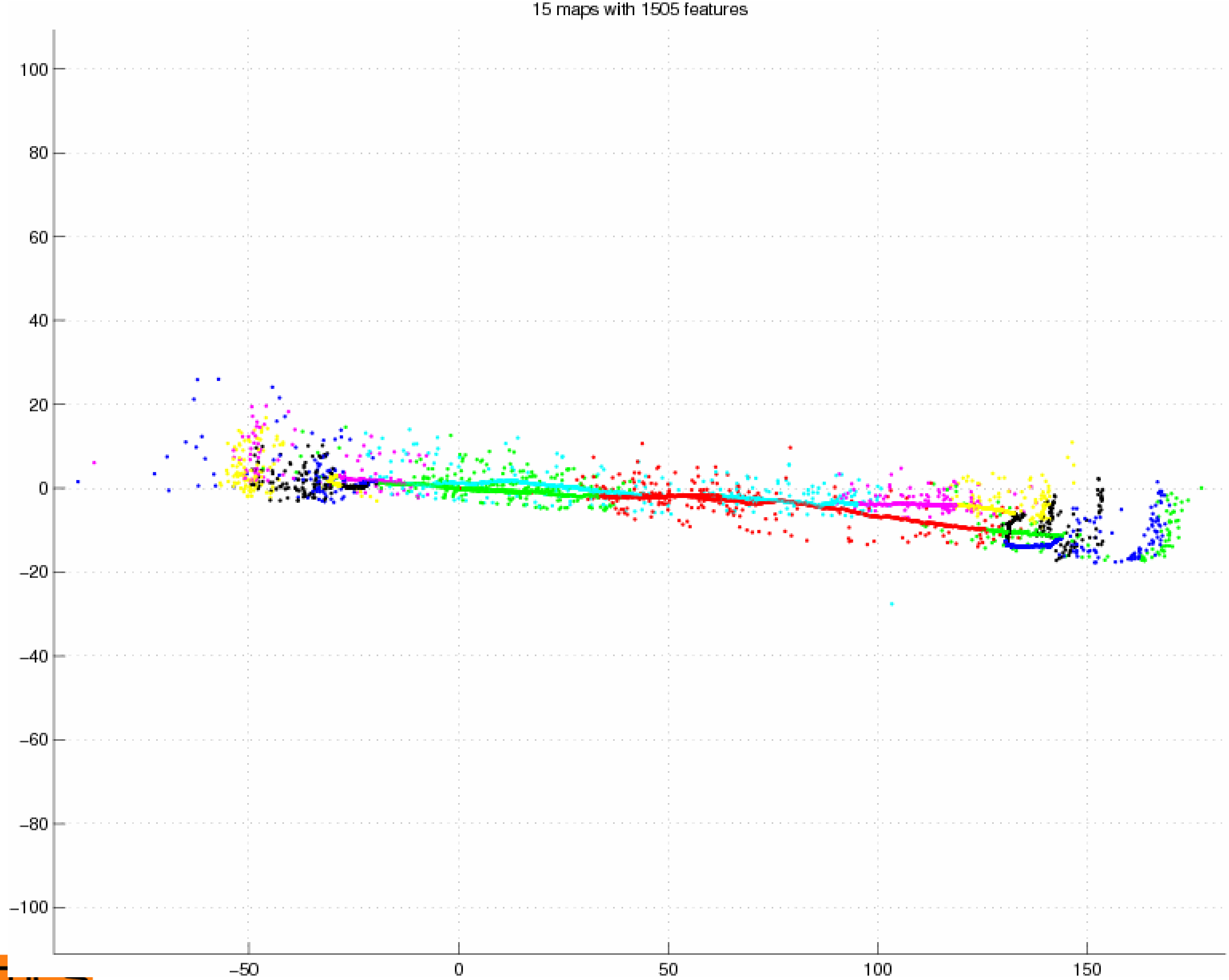



© 2007 Infoterra Ltd & Bluesky
©2007 Google™
Puntero 51°45'32.58" N 1°15'28.30" O
Secuencia |||||||||| 100%
Alt. ojo 116 m

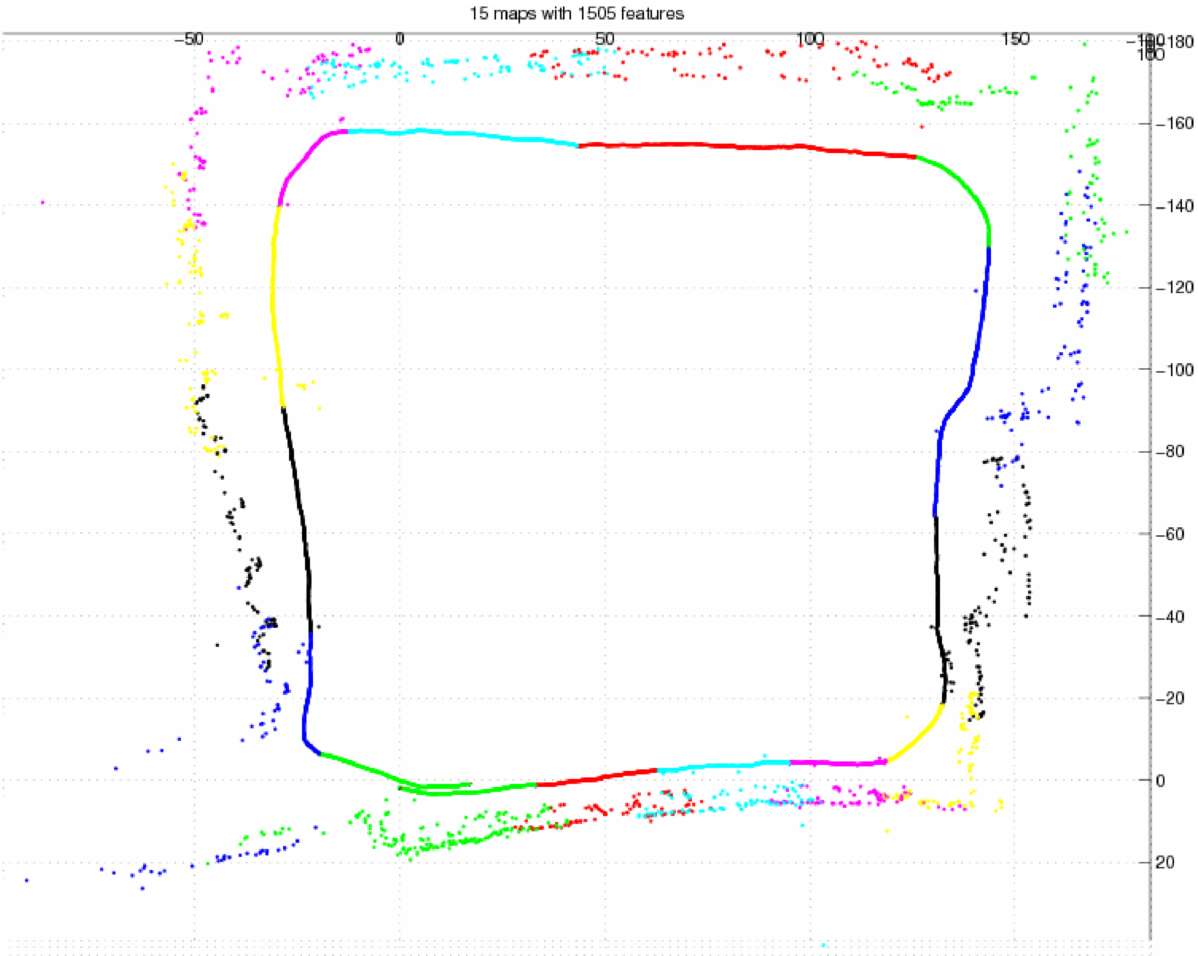
Sequence of local maps



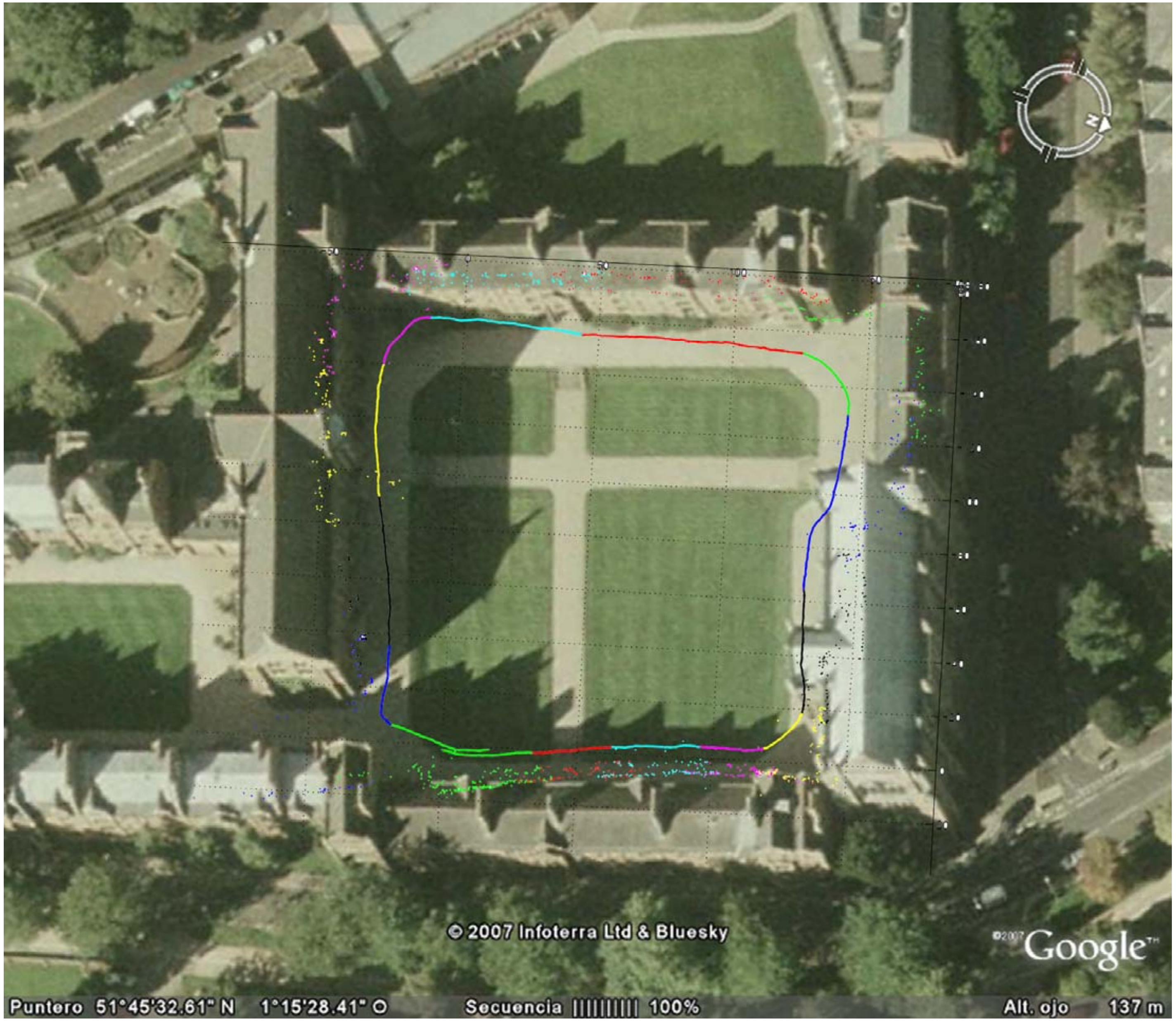
Sequence of local maps



Loop closing



Keble College, Oxford (290m)



Results

- Local Map building in real-time @30Hz
  - 60 features per map using inverse depth
  - Bigger maps if converted to (x,y,z)
- Joint Compatibility search adds only 2ms in the worst case
- Map-to-map matching in 1s (in Matlab)
  - With a new algorithm based on graph theory
- Loop optimization takes 800ms (6 iterations)
- The scale drifts along the map

Conclusions

	Loop 30m	Loop 300m	Longer loop
EKF-SLAM Nearest Neighbor	weak	--- inconsistent	---
EKF-SLAM Joint Compatib.	very good	--- inconsistent	---
Map Joining Joint Compatib.	**excellent**	weak	---
Hierarchical SLAM Relocation	overkill	**excellent**	future work

Recommended Readings

- J.D. Tardós, J. Neira, P. Newman, and J. Leonard. **Robust Mapping and Localization in Indoor Environments using Sonar Data**, Int. J. Robotics Research, Vol. 21, No. 4, April 2002, pp 311 –330
- J.A. Castellanos, J. Neira, J.D. Tardós, **Limits to the Consistency of EKF-based SLAM**, 5th IFAC Symposium on Intelligent Autonomous Vehicles, Lisbon, July 2004.
- C. Estrada, J. Neira, J.D. Tardós, **Hierarchical SLAM: real-time accurate mapping of large environments**. To appear in the IEEE Transactions on Robotics
- Grimson, W. E. L., **Object Recognition by Computer: The Role of Geometric Constraints,** The MIT Press, Cambridge, Mass., 1990
- José A. Castellanos and Juan D. Tardós, **Mobile Robot Localization and Map Building: A Multisensor Fusion Approach**, Kluwer Ac. Pub., Boston, 1999
- J. Neira and J.D. Tardós, **Data Association in Stochastic Mapping Using the Joint Compatibility Test**, IEEE Trans. Robotics and Automation, vol. 17, no. 6, pp. 890-897, Dec 2001.
- P.M. Newman, J.J. Leonard, J. Neira and J.D. Tardós: **Explore and Return: Experimental Validation of Real Time Concurrent Mapping and Localization**. IEEE Int. Conf. Robotics and Automation, May, 2002.
- J. Neira, J.D. Tardós, J.A. Castellanos, **Linear time vehicle relocation in SLAM**. 2003 IEEE Int. Conf. Robotics and Automation, Taipei, Taiwan, May, 2003.
- D. Ortín, J. Neira, J.M.M. Montiel, **Relocation using Laser and Vision.** 2004 IEEE Int. Conf. Robotics and Automation, New Orleans, USA, April, 2004.

INFORMATION

1st Summer School 2002, Stockholm

http://www.cas.kth.se/SLAM/

2nd Summer School 2004, Toulouse

http://www2.laas.fr/SLAM/

3rd Summer Schhol 2006, Oxford

http://www.robots.ox.ac.uk/~SSS06/

BIBLIOGRAPHY