

Lesson 7: Regions

1. Introduction
2. Region growing
3. Split and Merge

Bibliography:

RM Haralick, LG Shapiro: **Computer and Robot Vision**, Addison-Wesley, 1992



1. Region Segmentation

- Goal: divide an image I into n regions R_i *connected* and *homogeneous* according to a criterion $H(R_i)$, such that:

$$1. I = \bigcup_{i=1}^n R_i$$

$$2. R_i \cap R_j = \emptyset \quad \forall i \neq j$$

$$3. H(R_i) = \text{true} \quad \forall i$$

$$4. H(R_i \cup R_j) = \text{false} \quad \forall R_i, R_j \text{ neighbouring}, i \neq j$$

- Criterion $H(R_i)$ measures the homogeneity of a certain property $P(\text{pixel})$, that depends on the application:
 - Gray level, colour, texture,...
- The way of measuring $H(R_i)$ also depends on the application



Homogeneity Criteria

- Distance to a reference value:
 - $|P(q) - \text{ref_value}| < \text{threshold}$
- Distance to the mean value:
 - $|P(q) - \text{mean}_p(R_i)| < \text{threshold}$
- Small standard deviation:
 - $\sigma_p(R_i) < \text{threshold}$
- Hypothesis test:
 - Can the values of $P(q)$ found in the region come from a Gaussian distribution $N(m_p, \sigma_p)$?



2. Region Growing

$$1. I = \bigcup_{i=1}^n R_i$$

$$2. R_i \cap R_j = \emptyset \quad \forall i \neq j$$

$$3. H(R_i) = \text{true} \quad \forall i$$

$$4. H(R_i \cup R_j) = \text{false} \quad \forall R_i, R_j \text{ neighbouring}, i \neq j$$

- Group similar pixels or regions into bigger regions
- Region growing by adding pixels
 - Start with a set of seed pixels (they comply with 3)
 - Add neighbouring pixels to each region while complying with 3
 - When no more additions are possible, 4 is fulfilled
 - #regions = #seeds + 1 (background)
- Example: contour following with hysteresis thresholding
 - Seeds: magnitude > th_sup
 - Criterion: magnitude > th_inf



Region Growing

- Example: $P(i,j) = f(i,j)$ (intensity)
 - Criterion $H(R)$: $|f(i,j) - f(\text{seed})| < 3$

0	0	5	6	7
1	1	5	8	7
0	1	6	7	7
2	0	7	6	6
0	1	5	6	5

seeds

0	0	5	6	7
1	1	5	8	7
0	1	6	7	7
2	0	7	6	6
0	1	5	6	5

result

- Criterion $H(R)$: $|f(i,j) - f(\text{seed})| < 8$

0	0	5	6	7
1	1	5	8	7
0	1	6	7	7
2	0	7	6	6
0	1	5	6	5

seeds

0	0	5	6	7
1	1	5	8	7
0	1	6	7	7
2	0	7	6	6
0	1	5	6	5

result

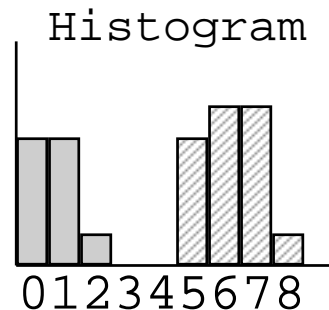


Choosing the Seeds

- Using a priori information of the type of regions we are interested in:
 - Contour: high magnitude
 - Find oranges in a tree: pixels with orange colour
 - Find the road: grey pixels in the lower part of the image
- Computing properties of all pixels and searching for clusters:

seeds = cluster centres

0	0	5	6	7
1	1	5	8	7
0	1	6	7	7
2	0	7	6	6
0	1	5	6	5



Seeds: 1 y 6



Hypothesis Test for Region Fusion

- Suppose that property $P(\cdot)$ follows inside each region a Gaussian distribution: $N(\mu_i, \sigma_i^2)$
- Given two neighbouring regions R_1 and R_2 , with n_1 and n_2 pixels, there are two alternative hypothesis:

H_0 : The values of $P(\cdot)$ possibly come from the same Gaussian, with: $\mu_1 = \mu_2$ and $\sigma_1 = \sigma_2$.

Then, merge both regions

H_1 : The values of $P(\cdot)$ come from two different Gaussians
Then, the two regions cannot be merged



Test for Region Fusion

$$m_k = \frac{1}{n_k} \sum_{q \in R_k} P(q)$$

$$S_k^2 = \frac{1}{n_k - 1} \sum_{q \in R_k} [P(q) - m_k]^2$$

$$t = \frac{m_1 - m_2}{S_c \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

Follows a Student's t-distribution with $n_1 + n_2 - 2$ degrees of freedom

$$S_c^2 = \frac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{(n_1 + n_2 - 2)}$$

Estimated joint variance

- Reject H_0 (do not merge) if: $|t| > t_{1-\alpha/2, (n_1+n_2-2)}$
- Test significance level: α

Probability of rejecting H_0 , being true

Probability of not merging, being possible

Typical values: $\alpha = 0.01 - 0.05$



Test for adding one pixel

Special case of the test for adding one pixel q ($n_2=1$) to a region R_k with n_k pixels:

$$m_k = \frac{1}{n_k} \sum_{r \in R_k} P(r)$$

$$S_k^2 = \frac{1}{n_k - 1} \sum_{r \in R_k} [P(r) - m_k]^2$$

$$t = \frac{P(q) - m_k}{S_k \sqrt{\frac{n_k + 1}{n_k}}}$$

Follows a Student's t distribution with $n_k - 1$ degrees of freedom

- Add pixel if: $|t| \leq t_{1-\alpha/2, n_k-1}$
- If pixel accepted, update the mean and variance



Test for adding one pixel

- Test significance: α
Probability of rejecting a good pixel
Typical value $\alpha = 0.01 - 0.05$
- Example, for $\alpha = 0.05$
 - $n_k = 10$, $t_{0.975, 9} = 2.262$
 - $n_k = 50$, $t_{0.975, 49} = 2.009$
- When starting a region ($n_k = 1$), use an a priori threshold for $|P(q) - m_k|$



T-Distribution

$$\alpha = 0.05$$

df	75%	80%	85%	90%	95%	97.5%	99%	99.5%	99.75%	99.9%	99.95%
1	1.000	1.376	1.963	3.078	6.314	12.71	31.82	63.66	127.3	318.3	636.6
2	0.816	1.061	1.386	1.886	2.920	4.303	6.965	9.925	14.09	22.33	31.60
3	0.765	0.978	1.250	1.638	2.353	3.182	4.541	5.841	7.453	10.21	12.92
4	0.741	0.941	1.190	1.533	2.132	2.776	3.747	4.604	5.598	7.173	8.610
5	0.727	0.920	1.156	1.476	2.015	2.571	3.365	4.032	4.773	5.893	6.869
6	0.718	0.906	1.134	1.440	1.943	2.447	3.143	3.707	4.317	5.208	5.959
7	0.711	0.896	1.119	1.415	1.895	2.365	2.998	3.499	4.029	4.785	5.408
8	0.706	0.889	1.108	1.397	1.860	2.306	2.896	3.355	3.833	4.501	5.041
9	0.703	0.883	1.100	1.383	1.833	2.262	2.821	3.250	3.690	4.297	4.781
10	0.700	0.879	1.093	1.372	1.812	2.228	2.764	3.169	3.581	4.144	4.587
11	0.697	0.876	1.088	1.363	1.796	2.201	2.718	3.106	3.497	4.025	4.437
12	0.695	0.873	1.083	1.356	1.782	2.179	2.681	3.055	3.428	3.930	4.318
13	0.694	0.870	1.079	1.350	1.771	2.160	2.650	3.012	3.372	3.852	4.221
14	0.692	0.868	1.076	1.345	1.761	2.145	2.624	2.977	3.326	3.787	4.140
15	0.691	0.866	1.074	1.341	1.753	2.131	2.602	2.947	3.286	3.733	4.073
16	0.690	0.865	1.071	1.337	1.746	2.120	2.583	2.921	3.252	3.686	4.015
17	0.689	0.863	1.069	1.333	1.740	2.110	2.567	2.898	3.222	3.646	3.965
18	0.688	0.862	1.067	1.330	1.734	2.101	2.552	2.878	3.197	3.610	3.922
19	0.688	0.861	1.066	1.328	1.729	2.093	2.539	2.861	3.174	3.579	3.883
20	0.687	0.860	1.064	1.325	1.725	2.086	2.528	2.845	3.153	3.552	3.850
21	0.686	0.859	1.063	1.323	1.721	2.080	2.518	2.831	3.135	3.527	3.819
22	0.686	0.858	1.061	1.321	1.717	2.074	2.508	2.819	3.119	3.505	3.792
23	0.685	0.858	1.060	1.319	1.714	2.069	2.500	2.807	3.104	3.485	3.767
24	0.685	0.857	1.059	1.318	1.711	2.064	2.492	2.797	3.091	3.467	3.745
25	0.684	0.856	1.058	1.316	1.708	2.060	2.485	2.787	3.078	3.450	3.725
26	0.684	0.856	1.058	1.315	1.706	2.056	2.479	2.779	3.067	3.435	3.707
27	0.684	0.855	1.057	1.314	1.703	2.052	2.473	2.771	3.057	3.421	3.690
28	0.683	0.855	1.056	1.313	1.701	2.048	2.467	2.763	3.047	3.408	3.674
29	0.683	0.854	1.055	1.311	1.699	2.045	2.462	2.756	3.038	3.396	3.659
30	0.683	0.854	1.055	1.310	1.697	2.042	2.457	2.750	3.030	3.385	3.646
40	0.681	0.851	1.050	1.303	1.684	2.021	2.423	2.704	2.971	3.307	3.551
50	0.679	0.849	1.047	1.299	1.676	2.009	2.403	2.678	2.937	3.261	3.496
60	0.679	0.848	1.045	1.296	1.671	2.000	2.390	2.660	2.915	3.232	3.460
80	0.678	0.846	1.043	1.292	1.664	1.990	2.374	2.639	2.887	3.195	3.416
100	0.677	0.845	1.042	1.290	1.660	1.984	2.364	2.626	2.871	3.174	3.390
120	0.677	0.845	1.041	1.289	1.658	1.980	2.358	2.617	2.860	3.160	3.373
∞	0.674	0.842	1.036	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291



Recursive Region Growing

```
procedure recursive_growing
  choose_seeds
  for p in seeds
    create_region(R)
    add_pixel(p,R)
  endfor
end recursive_growing

procedure add_pixel(p,R)
  label(p,R)
  for q in neighbours(p)
    if not_labelled(q) and test_criterion(q,R) then
      add_pixel(q,R)
    endif
  endfor
end add_pixel
```

- Choosing appropriate seeds, the algorithm works in the interesting regions.
- Some pixels will remain unlabeled (background)



Iterative Region Growing

```
procedure one_pass_growing
  for i:= 1 to rows
    for j:= 1 to cols
      Rleft:= Region(i, j-1)
      Cleft:= test_criterion(i, j, Rleft)
      Rsup:= Region(i-1, j)
      Csup:= test_criterion(i, j, Rsup)
      (C, R):=min(Cleft, Rleft, Csup, Rsup)
      if Cleft<=th and Csup<=th
        and test_fusion(Rleft, Rsup) then
          R=fusion(Rleft, Rsup)
        endif
      if C<=th then
        add_pixel(i, j, R)
      else
        create_region(i, j)
      endif
    endfor
  endfor
end one_pass_growing
```

If not added, used as new seed

- » Labels the whole image
- » Uninteresting regions may appear



3. Split and Merge

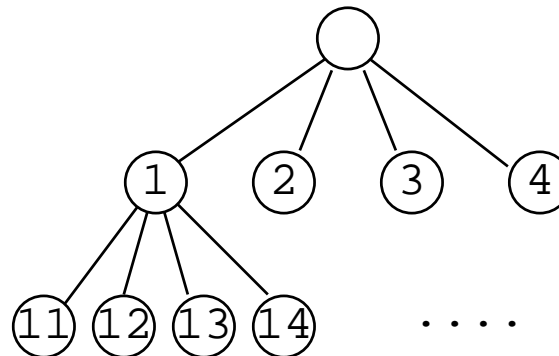
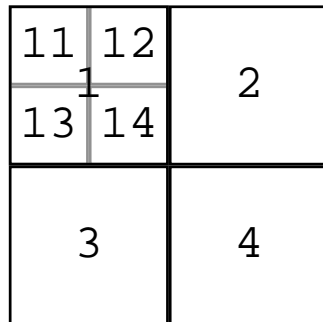
$$1. I = \bigcup_{i=1}^n R_i$$

$$2. R_i \cap R_j = \emptyset \quad \forall i \neq j$$

$$3. H(R_i) = \text{true} \quad \forall i$$

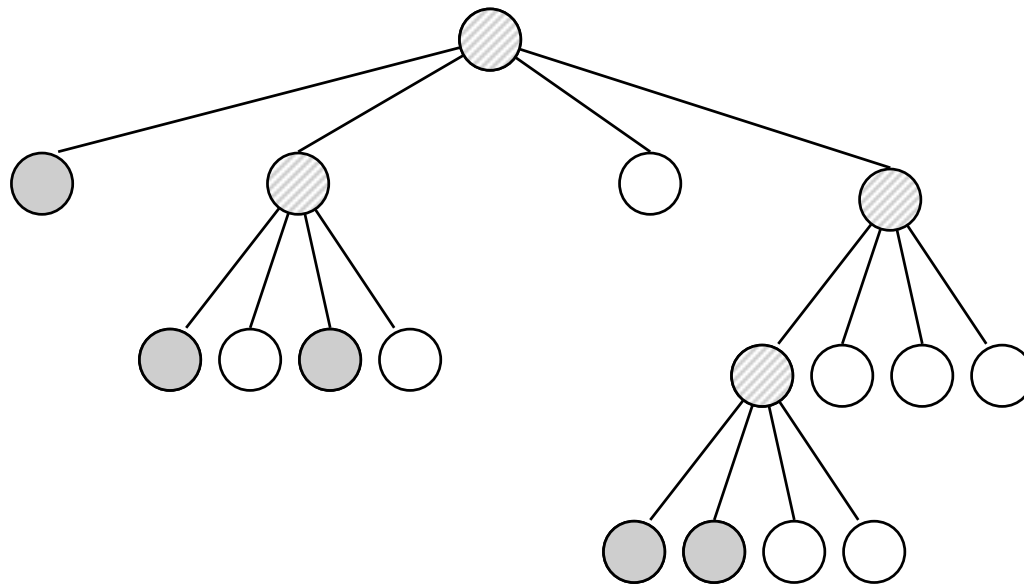
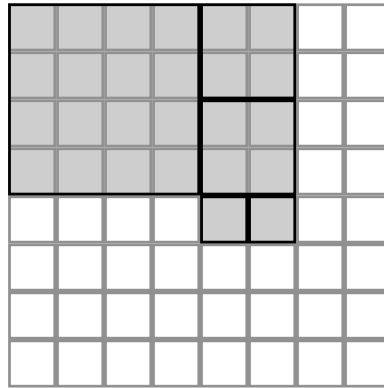
$$4. H(R_i \cup R_j) = \text{false} \quad \forall R_i, R_j \text{ neighbouring}, i \neq j$$

- Start with the whole image
- Split the regions until 3 is attained
- Merge neighbouring regions until 4 is attained
- Quad-tree:



Split and Merge

- Quad-tree example:



Test for splitting in four

Hypothesis test:

- Suppose that property $P(\cdot)$ follows inside each region a Gaussian distribution: $N(\mu_i, \sigma_i^2)$
- Given a region R with $4n$ pixels, choose between two hypothesis:

H_0 : The means and variances of $P(q)$ on the four regions can be the same

Do not split the region

H_1 : The means and variances of $P(q)$ on the four regions cannot be the same

Split the region in four



Test for splitting in four

$$m_k = \frac{1}{n} \sum_{p \in R_k} P(p) \quad k = 1..4$$

$$m = \frac{1}{4} \sum_{k=1}^4 m_k$$

$$f = \frac{n \sum_{k=1}^4 (m_k - m)^2 / 3}{\sum_{k=1}^4 \sum_{p \in R_k} [P(p) - m_k]^2 / 4(n-1)}$$

Follows a F-distribution with 3 and $4(n-1)$ degrees of freedom

- Reject H_0 (split in four) if: $|f| > f_{1-\alpha, 3, 4(n-1)}$
- Test significance level: α
Probability of splitting a region being unnecessary
Typical values: $\alpha = 0.01 - 0.05$
- Example: region 16×16 , $n = 64$, $\alpha = 0.05$
 $f_{0.95, 3, 252} = 2.6049$



F-distribution for $\alpha = 0.05$

df2/df1	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	INF
1	161.44	199.50	215.70	224.58	230.16	233.98	236.76	238.88	240.54	241.88	243.90	245.94	248.01	249.05	250.09	251.14	252.19	253.25	254.31
2	18.512	19.000	19.164	19.246	19.296	19.329	19.353	19.371	19.384	19.395	19.412	19.429	19.445	19.454	19.462	19.470	19.479	19.487	19.495
3	10.128	9.5521	9.2766	9.1172	9.0135	8.9406	8.8867	8.8452	8.8123	8.7855	8.7446	8.7029	8.6602	8.6385	8.6166	8.5944	8.5720	8.5494	8.5264
4	7.7086	6.9443	6.5914	6.3882	6.2561	6.1631	6.0942	6.0410	5.9988	5.9644	5.9117	5.8578	5.8025	5.7744	5.7459	5.7170	5.6877	5.6581	5.6281
5	6.6079	5.7861	5.4095	5.1922	5.0503	4.9503	4.8759	4.8183	4.7725	4.7351	4.6777	4.6188	4.5581	4.5272	4.4957	4.4638	4.4314	4.3985	4.3650
6	5.9874	5.1433	4.7571	4.5337	4.3874	4.2839	4.2067	4.1468	4.0990	4.0600	3.9999	3.9381	3.8742	3.8415	3.8082	3.7743	3.7398	3.7047	3.6689
7	5.5914	4.7374	4.3468	4.1203	3.9715	3.8660	3.7870	3.7257	3.6767	3.6365	3.5747	3.5107	3.4445	3.4105	3.3758	3.3404	3.3043	3.2674	3.2298
8	5.3177	4.4590	4.0662	3.8379	3.6875	3.5806	3.5005	3.4381	3.3881	3.3472	3.2839	3.2184	3.1503	3.1152	3.0794	3.0428	3.0053	2.9669	2.9276
9	5.1174	4.2565	3.8625	3.6331	3.4817	3.3738	3.2927	3.2296	3.1789	3.1373	3.0729	3.0061	2.9365	2.9005	2.8637	2.8259	2.7872	2.7475	2.7067
10	4.9646	4.1028	3.7083	3.4780	3.3258	3.2172	3.1355	3.0717	3.0204	2.9782	2.9130	2.8450	2.7740	2.7372	2.6996	2.6609	2.6211	2.5801	2.5379
11	4.8443	3.9823	3.5874	3.3567	3.2039	3.0946	3.0123	2.9480	2.8962	2.8536	2.7876	2.7186	2.6464	2.6090	2.5705	2.5309	2.4901	2.4480	2.4045
12	4.7472	3.8853	3.4903	3.2592	3.1059	2.9961	2.9134	2.8486	2.7964	2.7534	2.6866	2.6169	2.5436	2.5055	2.4663	2.4259	2.3842	2.3410	2.2962
13	4.6672	3.8056	3.4105	3.1791	3.0254	2.9153	2.8321	2.7669	2.7144	2.6710	2.6037	2.5331	2.4589	2.4202	2.3803	2.3392	2.2966	2.2524	2.2064
14	4.6001	3.7389	3.3439	3.1122	2.9582	2.8477	2.7642	2.6987	2.6458	2.6022	2.5342	2.4630	2.3879	2.3487	2.3082	2.2664	2.2229	2.1778	2.1307
15	4.5431	3.6823	3.2874	3.0556	2.9013	2.7905	2.7066	2.6408	2.5876	2.5437	2.4753	2.4034	2.3275	2.2878	2.2468	2.2043	2.1601	2.1141	2.0658
16	4.4940	3.6337	3.2389	3.0069	2.8524	2.7413	2.6572	2.5911	2.5377	2.4935	2.4247	2.3522	2.2756	2.2354	2.1938	2.1507	2.1058	2.0589	2.0096
17	4.4513	3.5915	3.1968	2.9647	2.8100	2.6987	2.6143	2.5480	2.4943	2.4499	2.3807	2.3077	2.2304	2.1898	2.1477	2.1040	2.0584	2.0107	1.9604
18	4.4139	3.5546	3.1599	2.9277	2.7729	2.6613	2.5767	2.5102	2.4563	2.4117	2.3421	2.2686	2.1906	2.1497	2.1071	2.0629	2.0166	1.9681	1.9168
19	4.3807	3.5219	3.1274	2.8951	2.7401	2.6283	2.5435	2.4768	2.4227	2.3779	2.3080	2.2341	2.1555	2.1141	2.0712	2.0264	1.9795	1.9302	1.8780
20	4.3512	3.4928	3.0984	2.8661	2.7109	2.5990	2.5140	2.4471	2.3928	2.3479	2.2776	2.2033	2.1242	2.0825	2.0391	1.9938	1.9464	1.8963	1.8432
21	4.3248	3.4668	3.0725	2.8401	2.6848	2.5727	2.4876	2.4205	2.3660	2.3210	2.2504	2.1757	2.0960	2.0540	2.0102	1.9645	1.9165	1.8657	1.8117
22	4.3009	3.4434	3.0491	2.8167	2.6613	2.5491	2.4638	2.3965	2.3419	2.2967	2.2258	2.1508	2.0707	2.0283	1.9842	1.9380	1.8894	1.8380	1.7831
23	4.2793	3.4221	3.0280	2.7955	2.6400	2.5277	2.4422	2.3748	2.3201	2.2747	2.2036	2.1282	2.0476	2.0050	1.9605	1.9139	1.8648	1.8128	1.7570
24	4.2597	3.4028	3.0088	2.7763	2.6207	2.5082	2.4226	2.3551	2.3002	2.2547	2.1834	2.1077	2.0267	1.9838	1.9390	1.8920	1.8424	1.7896	1.7330
25	4.2417	3.3852	2.9912	2.7587	2.6030	2.4904	2.4047	2.3371	2.2821	2.2365	2.1649	2.0889	2.0075	1.9643	1.9192	1.8718	1.8217	1.7684	1.7110
26	4.2252	3.3690	2.9752	2.7426	2.5868	2.4741	2.3883	2.3205	2.2655	2.2197	2.1479	2.0716	1.9898	1.9464	1.9010	1.8533	1.8027	1.7488	1.6906
27	4.2100	3.3541	2.9604	2.7278	2.5719	2.4591	2.3732	2.3053	2.2501	2.2043	2.1323	2.0558	1.9736	1.9299	1.8842	1.8361	1.7851	1.7306	1.6717
28	4.1960	3.3404	2.9467	2.7141	2.5581	2.4453	2.3593	2.2913	2.2360	2.1900	2.1179	2.0411	1.9586	1.9147	1.8687	1.8203	1.7689	1.7138	1.6541
29	4.1830	3.3277	2.9340	2.7014	2.5454	2.4324	2.3463	2.2783	2.2229	2.1768	2.1045	2.0275	1.9446	1.9005	1.8543	1.8055	1.7537	1.6981	1.6376
30	4.1709	3.3158	2.9223	2.6896	2.5336	2.4205	2.3343	2.2662	2.2107	2.1646	2.0921	2.0148	1.9317	1.8874	1.8409	1.7918	1.7396	1.6835	1.6223
40	4.0847	3.2317	2.8387	2.6060	2.4495	2.3359	2.2490	2.1802	2.1240	2.0772	2.0035	1.9245	1.8389	1.7929	1.7444	1.6928	1.6373	1.5766	1.5089
60	4.0012	3.1504	2.7581	2.5252	2.3683	2.2541	2.1665	2.0970	2.0401	1.9926	1.9174	1.8364	1.7480	1.7001	1.6491	1.5943	1.5343	1.4673	1.3893
120	3.9201	3.0718	2.6802	2.4472	2.2899	2.1750	2.0868	2.0164	1.9588	1.9105	1.8337	1.7505	1.6587	1.6084	1.5543	1.4952	1.4290	1.3519	1.2539
INF	3.8415	2.9957	2.6049	2.3719	2.2141	2.0986	2.0096	1.9384	1.8799	1.8307	1.7522	1.6664	1.5705	1.5173	1.4591	1.3940	1.3180	1.2214	1.0000



F-distribution for $\alpha = 0.01$

df2/df1	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	INF
1	4052.1	4999.5	5403.3	5624.5	5763.6	5858.9	5928.3	5981.0	6022.4	6055.8	6106.3	6157.2	6208.7	6234.6	6260.6	6286.7	6313.0	6339.3	6365.8
2	98.503	99.000	99.166	99.249	99.299	99.333	99.356	99.374	99.388	99.399	99.416	99.433	99.449	99.458	99.466	99.474	99.482	99.491	99.499
3	34.116	30.817	29.457	28.710	28.237	27.911	27.672	27.489	27.345	27.229	27.052	26.872	26.690	26.598	26.505	26.411	26.316	26.221	26.125
4	21.198	18.000	16.694	15.977	15.522	15.207	14.976	14.799	14.659	14.546	14.374	14.198	14.020	13.929	13.838	13.745	13.652	13.558	13.463
5	16.258	13.274	12.060	11.392	10.967	10.672	10.456	10.289	10.158	10.051	9.888	9.722	9.553	9.466	9.379	9.291	9.202	9.112	9.020
6	13.745	10.925	9.780	9.148	8.746	8.466	8.260	8.102	7.976	7.874	7.718	7.559	7.396	7.313	7.229	7.143	7.057	6.969	6.880
7	12.246	9.547	8.451	7.847	7.460	7.191	6.993	6.840	6.719	6.620	6.469	6.314	6.155	6.074	5.992	5.908	5.824	5.737	5.650
8	11.259	8.649	7.591	7.006	6.632	6.371	6.178	6.029	5.911	5.814	5.667	5.515	5.359	5.279	5.198	5.116	5.032	4.946	4.859
9	10.561	8.022	6.992	6.422	6.057	5.802	5.613	5.467	5.351	5.257	5.111	4.962	4.808	4.729	4.649	4.567	4.483	4.398	4.311
10	10.044	7.559	6.552	5.994	5.636	5.386	5.200	5.057	4.942	4.849	4.706	4.558	4.405	4.327	4.247	4.165	4.082	3.996	3.909
11	9.646	7.206	6.217	5.668	5.316	5.069	4.886	4.744	4.632	4.539	4.397	4.251	4.099	4.021	3.941	3.860	3.776	3.690	3.602
12	9.330	6.927	5.953	5.412	5.064	4.821	4.640	4.499	4.388	4.296	4.155	4.010	3.858	3.780	3.701	3.619	3.535	3.449	3.361
13	9.074	6.701	5.739	5.205	4.862	4.620	4.441	4.302	4.191	4.100	3.960	3.815	3.665	3.587	3.507	3.425	3.341	3.255	3.165
14	8.862	6.515	5.564	5.035	4.695	4.456	4.278	4.140	4.030	3.939	3.800	3.656	3.505	3.427	3.348	3.266	3.181	3.094	3.004
15	8.683	6.359	5.417	4.893	4.556	4.318	4.142	4.004	3.895	3.805	3.666	3.522	3.372	3.294	3.214	3.132	3.047	2.959	2.868
16	8.531	6.226	5.292	4.773	4.437	4.202	4.026	3.890	3.780	3.691	3.553	3.409	3.259	3.181	3.101	3.018	2.933	2.845	2.753
17	8.400	6.112	5.185	4.669	4.336	4.102	3.927	3.791	3.682	3.593	3.455	3.312	3.162	3.084	3.003	2.920	2.835	2.746	2.653
18	8.285	6.013	5.092	4.579	4.248	4.015	3.841	3.705	3.597	3.508	3.371	3.227	3.077	2.999	2.919	2.835	2.749	2.660	2.566
19	8.185	5.926	5.010	4.500	4.171	3.939	3.765	3.631	3.523	3.434	3.297	3.153	3.003	2.925	2.844	2.761	2.674	2.584	2.489
20	8.096	5.849	4.938	4.431	4.103	3.871	3.699	3.564	3.457	3.368	3.231	3.088	2.938	2.859	2.778	2.695	2.608	2.517	2.421
21	8.017	5.780	4.874	4.369	4.042	3.812	3.640	3.506	3.398	3.310	3.173	3.030	2.880	2.801	2.720	2.636	2.548	2.457	2.360
22	7.945	5.719	4.817	4.313	3.988	3.758	3.587	3.453	3.346	3.258	3.121	2.978	2.827	2.749	2.667	2.583	2.495	2.403	2.305
23	7.881	5.664	4.765	4.264	3.939	3.710	3.539	3.406	3.299	3.211	3.074	2.931	2.781	2.702	2.620	2.535	2.447	2.354	2.256
24	7.823	5.614	4.718	4.218	3.895	3.667	3.496	3.363	3.256	3.168	3.032	2.889	2.738	2.659	2.577	2.492	2.403	2.310	2.211
25	7.770	5.568	4.675	4.177	3.855	3.627	3.457	3.324	3.217	3.129	2.993	2.850	2.699	2.620	2.538	2.453	2.364	2.270	2.169
26	7.721	5.526	4.637	4.140	3.818	3.591	3.421	3.288	3.182	3.094	2.958	2.815	2.664	2.585	2.503	2.417	2.327	2.233	2.131
27	7.677	5.488	4.601	4.106	3.785	3.558	3.388	3.256	3.149	3.062	2.926	2.783	2.632	2.552	2.470	2.384	2.294	2.198	2.097
28	7.636	5.453	4.568	4.074	3.754	3.528	3.358	3.226	3.120	3.032	2.896	2.753	2.602	2.522	2.440	2.354	2.263	2.167	2.064
29	7.598	5.420	4.538	4.045	3.725	3.499	3.330	3.198	3.092	3.005	2.868	2.726	2.574	2.495	2.412	2.325	2.234	2.138	2.034
30	7.562	5.390	4.510	4.018	3.699	3.473	3.304	3.173	3.067	2.979	2.843	2.700	2.549	2.469	2.386	2.299	2.208	2.111	2.006
40	7.314	5.179	4.313	3.828	3.514	3.291	3.124	2.993	2.888	2.801	2.665	2.522	2.369	2.288	2.203	2.114	2.019	1.917	1.805
60	7.077	4.977	4.126	3.649	3.339	3.119	2.953	2.823	2.718	2.632	2.496	2.352	2.198	2.115	2.028	1.936	1.836	1.726	1.601
120	6.851	4.787	3.949	3.480	3.174	2.956	2.792	2.663	2.559	2.472	2.336	2.192	2.035	1.950	1.860	1.763	1.656	1.533	1.381
INF	6.635	4.605	3.782	3.319	3.017	2.802	2.639	2.511	2.407	2.321	2.185	2.039	1.878	1.791	1.696	1.592	1.473	1.325	1.000

