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Relative Localization for Pairs of Robots Based on Unidentifiable Moving Features

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Abstract— **This paper presents a new method for relative localization of a pair of robots based on the trajectories described by unidentifiable moving objects. Our approach uses a Rao-Blackwellized particle filter to estimate both the relative location of the robots and the data associations between the moving objects around the robots. We describe our implementation on real robots and present experiments illustrating the robustness of our algorithm.**

I. INTRODUCTION

In the past cooperative robotics has received remarkable attention from the robotics community since the use of a team of robots instead of a single robot has several advantages. Teams of robots can solve problems which cannot be solved by a single robot. Additionally, robot teams can be more efficient due to the parallelism they introduce. Furthermore, teams of robots improve the robustness since the failure of a single system can often be compensated by its team members. However, the use of a team of robots also introduces new challenges for example in the areas of decision making or state estimation. In this paper we concentrate on the second aspect and consider the question of how to relatively localize a pair of robots in the absence of global pose information. In particular we are interested in estimating the relative pose of two robots based on information about moving objects in the field of views of the robots. This approach has certain advantages even for larger teams of robots. It allows robots to determine their relative pose in the absence of static information such as a map. It also provides a way to share a single global positioning sensor such as a GPS among a team of robots. Simultaneously, it does not require that the robots can identify each other or distinguish themselves from other moving objects.

In the last years several authors investigated the problem of tracking moving objects with a mobile robot [11], [7], [14]. The authors mainly focused on the problem of how to robustly keep track of the moving objects and investigated sample-based belief representations and proposed solutions for the data association problem between consecutive observations. If, however, one wants to relatively localize a pair of robots based on observed moving objects, one additionally has to deal with the potential data associations between the observations of the individual objects. The key idea of this paper is that, if two robots are tracking the same moving objects, the trajectories described by these moving objects, the robots included, must be compatible. The goal of the approach presented here is to accumulate evidence about the relative location of two robots based on the information about the trajectories of observed objects. This paper extends our previous work [8] in which we presented an approach to utilize the kinematic information about moving objects to establish correspondences between the moving objects tracked by different robots. We particularly investigate how to use the information provided by a tracking system to relatively localize a pair of robots.

The approach presented in this paper is purely probabilistic. We use a Rao-Blackwellized particle filter [3] to estimate the posterior over both the relative locations of the robots and the data associations between the moving features around them. Our approach includes an efficient way of managing the uncertainty about the data associations of the individual trackers. Our method has been implemented and evaluated on real robots. The results suggest that our approach can robustly localize the robots and deal with certain kinds of sensor limitations such as restricted fields of view.

The rest of the paper is organized as follows. After discussing related work Section III provides a full description of the problem. In sections IV and V we then present our algorithm for relative pose estimation based on moving features. Finally, we describe experimental results in Section VI.

II. RELATED WORK

The problem of multi-robot localization has been studied intensively in the past. In the algorithm proposed by Fox et al. [4] each robot maintains its own belief about its pose relative to a given map. When two robots meet each other they exchange their beliefs. A further approach to the multi-robot localization problem has been proposed by Rekleitis et al. [9]. In this work the authors use the robots as landmarks in order to reduce the odometry errors during exploration tasks. Roumeliotis [10] presents a distributed cooperative localization algorithm based on a single Kalman filter that jointly estimates the position of the robots of the team. The method provides a framework to estimate the global positions of the robots and to maintain the correlations between the poses of the different robots. Howard et al. [6] describe a multi-robot localization method where only the robots are used as landmarks. Thus, the robots do not localize themselves in a common static reference system. Rather each robot estimates the relatives poses of the other members of the team with respect to itself.

All these approaches assume that the identity of the robots is always known. The detection and identification of robots are achieved by special markers placed on the robots (such as color patches and fiducial codes) or by using an external system (such as a camera). This assumption avoids data association problems according to the lack of identities and greatly reduces the overall complexity of the state estimation problem. However, in certain situations the identity of the robots is not always available. For example, when a laser range sensor is used, when it is not possible to use cameras and/or artificial markers, when the landmarks are temporarily not visible, or in the presence of ambiguities.

Our framework is similar to that of [6] in the sense that the robots are not localized with respect to a global reference system. However, our approach allows to use all moving objects observed by the pair of robots to compute their relative location. Furthermore, it does not require that the identity of the observed objects is known to estimate the relative locations.

From a different point of view, the general problem considered in this paper can be regarded as a particular network of mobile sensors. This network uses the same sensor to estimate the relative locations of the nodes of the network and to track moving objects around them. In this case one wants to jointly track the objects (data associations) and also discover and maintain the network topology (relative locations of the sensors). The problem is therefore a multi-hypothesis tracking problem from mobile platforms. Whereas several solutions have been proposed to the problem of multi-tracking [1], [13] there is, at least to the best of our knowledge, no work coupling the tracking problem with the estimation of the relative poses of the sensors. For example, Schulz et al. [12] recently presented a multi-hypothesis approach to people tracking with a network of location and ID-sensors. They assume, however, that the locations of the sensors are known and static.

Compared to previous work our approach is novel in the sense that it estimates the relative pose of two robots based on the kinematic about not identified moving objects. It combines techniques for tracking moving objects with an approach to jointly estimate the relative pose of the pair of robots and the correspondences between the tracked objects. Although in this work we have focused only on the use of kinematic information provided by the trackers, the method can be extended to include other types of information, i.e. shape, color, texture. This provides a framework to fuse information from different kind of sensors.

III. Problem Description, Notation and Assumptions

Throughout this paper we assume that each robot is equipped with sensors that provide odometry measurements and observations of the positions of moving objects in the vicinity of the robots. Suppose $\mathbf{Z}_i = \{\mathbf{z}_{i1}, \ldots, \mathbf{z}_{in}\}$ are the observations corresponding to the moving objects in the field of view of robot r_i and u_i the motion executed by r_i. In this paper we will use the superscript index k to refer to the set of variables up to time k and the subscript k to refer the variable at time k. To ease and shorten the notation all the variables with the suffix ij denote the set of variables of r_i and r_j. For instance, $u_{ij} = \{u_i, u_j\}$ is the set of actions of both robots r_i and r_j.

Our goal is to estimate the relative locations $\mathbf{x}^k = (x, y, \theta)$ of the two robots r_i and r_j given all information $\mathbf{Z}_{ij}^k$ and u_{ij}^{k-1}. Let η_{i_k} be the intra-robot data associations between the observations $\mathbf{Z}_{i_k}$ and $\mathbf{Z}_{i_{k-1}}$ and η_{j_k} those between $\mathbf{Z}_{j_k}$ and $\mathbf{Z}_{j_{k-1}}$. Let also η_{z_k} be the inter-robot data associations, i.e., the data associations between the observations $\mathbf{Z}_{i_k}$ and $\mathbf{Z}_{j_k}$. Mathematically, we want to estimate the joint distribution over the relative pose of the robots and the data associations

$$p(\mathbf{x}^k, \eta_z^k, \eta_i^k, \eta_j^k | \mathbf{Z}_{ij}^k, u_{ij}^{k-1}) \tag{1}$$

Finally, we assume that the robots are able to communicate independently of their relative positions. In a real scenario, however, the fact of being able to communicate already provides some prior information about the relative positions of the robots (wireless connection available, strength of the radio link, etc). Even though we do not explicitly address this issue in this paper, our algorithm can be easily extended to incorporate limited communication ranges in order to improve the performance of the method.

IV. From Observations to Tracks

To keep as much as possible of the computational burden on the individual robots, we track the moving objects around each platform independently on each robot. Thus, the robots only need to share the information about the trajectories of the moving objects and can compute their relative location based on the associations between these trajectories. In this section we show how we factorize Eq. (1) to use trackers and which information the trackers must provide. Accordingly, we compute the joint distribution as follows,

$$\begin{aligned} & p(\mathbf{x}^k, \eta_z^k, \eta_i^k, \eta_j^k | \mathbf{Z}_{ij}^k, u_{ij}^{k-1}) \\ &= p(\mathbf{x}^k, \eta_z^k | \eta_i^k, \eta_j^k, \mathbf{Z}_{ij}^k, u_{ij}^{k-1}) p(\eta_i^k, \eta_j^k | \mathbf{Z}_{ij}^k, u_j^{k-1}) \quad (2) \\ &= p(\mathbf{x}^k, \eta_z^k | \eta_i^k, \eta_j^k, \mathbf{Z}_{ij}^k, u_{ij}^{k-1}) \\ &\quad p(\eta_i^k | \mathbf{Z}_i^k, u_i^{k-1}) p(\eta_j^k | \mathbf{Z}_j^k, u_j^{k-1}). \quad (3) \end{aligned}$$

Eq. (3) follows from Eq. (2) under the assumption of independence between the observations and movements of the two robots. From Eq. (3) we see that the distributions of the intra-robot data associations η_i^k and η_j^k only depend on the observations and movements of the corresponding robot and therefore can be computed independently. This is what a tracking system actually does on each individual robot. It solves the intra-robot data associations computing the set of possible tracks which implicitly represent a history of data associations.

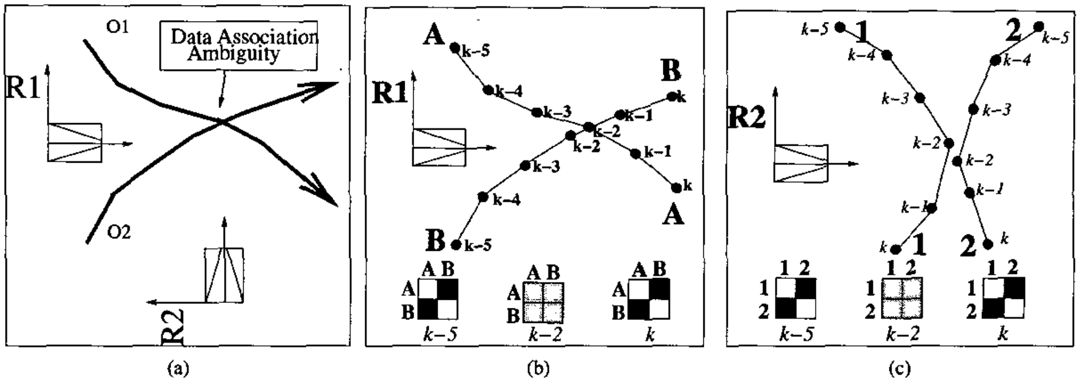


Fig. 1. A possible scenario: (a) shows the global situation. For the sake of clarity in this representation the robots R1 and R2 are static and we only use the two moving objects O1,O2. The objects cross their trajectories producing an ambiguity in the intra-robot data associations. (b) and (c) show, respectively, the observations obtained by the robots at each time step and the intra-robot data associations made by the individual trackers. The matrices represent the probability of flipping tracks at different time steps (black 0, white 1). Note that the robots make different data associations at time step $k-2$. Consequently the estimated trajectories are different. However, the flipping matrices reflect the ambiguity and allow us to correctly match the trajectories by changing the inter-robot data association at that time step from $\{1B, 2A\}$ to $\{1A, 2B\}$.

In order to match the trajectories the robots must share the information generated by their tracking systems. Unfortunately, from the multi-tracking literature it is known that keeping the set of all possible tracks has an exponential complexity due to the combinatorial explosion of possible data associations. In the presence of ambiguities this requires to keep an increasing number of possible tracks. To illustrate this consider the situation depicted in Figure 1(a) where two robots observe two moving objects. A data association ambiguity arises at time step $k-2$ for the individual trackers of each robot when the two moving objects intersect. Each robot should keep both possible sets of tracks (figures 1(b) and (c)) requiring to communicate an unbounded number of tracks and their probabilities.

We, therefore, need a way to represent the set of all possible trajectories with a constant cost on the number of tracked objects. To do this we use what we call a flipping matrix. Each element of a flipping matrix represents the probability of switching the identity of a pair of tracks at a certain point in time (see figures 1(b) and (c)). Thus, the tracking system of each robot computes the maximum a posteriori (MAP) set of tracks together with the corresponding flipping matrices. The set of all matrices up to time k allow us to approximate from the MAP estimated tracks the set of all possible tracks corresponding to all the possible data associations.

In our current system we use a multi-target tracker based on a set of independent Extended Kalman Filters (EKF) and the nearest neighbor principle to compute the MAP set of tracks $0_i = \{o_{i1}, ..., o_{in}\}$. Each element of O_i is an independent Kalman filter that estimates the location and velocity of a moving object. To efficiently compute the flipping probabilities between the tracks of a single robot r_i we use Markov Chain Monte Carlo (MCMC) techniques. We construct a Markov chain to approximate the distribution of possible flips among the tracks. A flip represents an association between a pair of tracks of O_i that exchange their identities and its likelihood is a function of the distance between the tracks. The chains are generated using smart chain flipping, a specific version of the Metropolis-Hasting algorithm proposed by Dellaert et al. [2] for the data association problem. The smart chain flipping technique proposes new mappings among the tracks based on the individual likelihoods of all the possible flips. We use the samples of the MCMC to compute the probability of the flips between each pair of tracks. The probability of a flip is its frequency in the chain, i.e. the number of occurrences of the flip divided by the number of samples of the chain.

In the following we will denote t_{i_k} to all the information provided by the tracker of robot r_i at time k. Thus, t_{i_k} contains the estimated EKFs O_i at time k, the flipping probabilities among the tracks at this point in time and the odometry reading u_{i_k}. As before t_i^k represents the set of t_{i_k} up to time k. The t_i^k can be regarded as a sufficient statistics for the set of η_i^k, $\mathbf{Z}_i^k$ and u_i^{k-1}. Substituting them in Eq. (3) we obtain,

$$p(\mathbf{x}^k, \eta_z^k, \eta_i^k, \eta_j^k \mid \mathbf{Z}_{ij}^k, u_{ij}^{k-1}) = p(\mathbf{x}^k, \eta_t^k \mid t_{ij}^k) \qquad (4)$$

where $t_{ij}^k = \{t_i^k, t_j^k\}$ and η_t^k are the data associations between the tracks of the individual robots. A inter-robot data association is a set of pairs $\{< o_{ih}, o_{jl} >\}$ representing that tracks o_{ih} and o_{jl} correspond to the same object. The terms $p(\eta_i^k \mid \mathbf{Z}_i^k, u_i^{k-1})$ and $p(\eta_j^k \mid \mathbf{Z}_j^k, u_j^{k-1})$ of Eq. (3) do not appear on Eq. 4 as the flipping probabilities already contain the information associated to these terms. We are now able to define how to use the information provided by the tracking systems to estimate the inter-robot data associations and the relative pose of the vehicles.

V. Estimation of Relative Pose and Correspondences

Figure 2(a) shows the graphical representation of our problem using individual trackers. According to this graphical model we realize that given the data associations

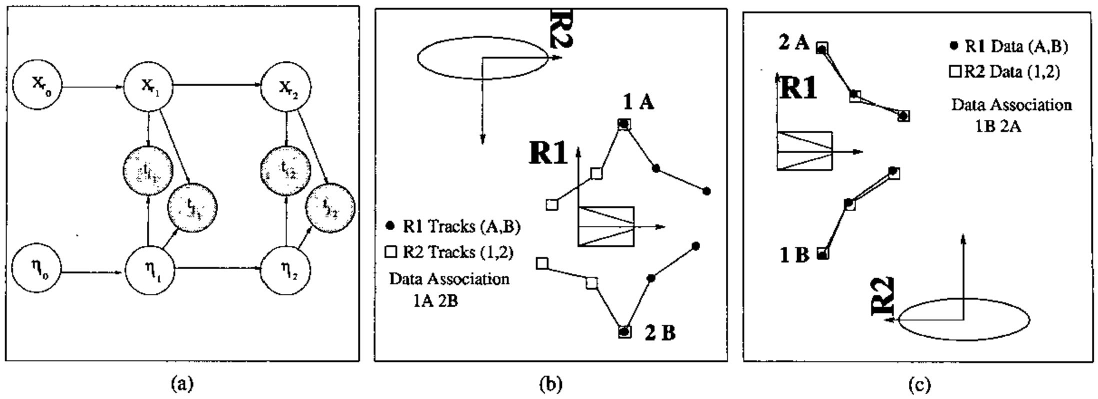


Fig. 2. (a) Graphical model for relative pose estimation based on trackers. Each robot tracks the objects around them t_i and t_j. The hidden states are the relative location of the robots $\mathbf{x}$ and the correspondences between the tracked objects η_t. (b) and (c) show two different data associations for the example of Fig.1. The wrong data association (b) computes an erroneous relative location and consequently the trajectories of the objects are not compatible. On the other hand the correct inter-robot data association (c) computes the correct relative location and the trajectories of the moving objects are compatible.

between the tracks η_t^k we can estimate the relative location of the robots. Furthermore the full posterior $p(\mathbf{x}^k, \eta_t^k \mid t_{ij}^k)$ in Eq. (4) can be factorized as follows:

$$p(\mathbf{x}^k, \eta_t^k | t_{ij}^k) = p(\mathbf{x}^k | \eta_t^k, t_{ij}^k) p(\eta_t^k | t_{ij}^k). \tag{5}$$

The data association problem is known to be a hard problem with combinatorial complexity in the number of objects. Besides, the mutual exclusion restrictions between the objects make it analytically intractable. In this paper we implement Eq. (5) using a Rao-Blackwellized particle filter [3]. The key idea is to sample over possible data associations $p(\eta_t^k \mid t_{ij}^k)$ and to compute the relative position of the robots based on the sampled data associations. Given a data association sample $\eta_t^{k,s}$, the relative pose can be analytically estimated using a single Kalman filter [1]. Thus, each data association sample $\eta_t^{k,s}$ has associated a Kalman filter representing the relative position of the robots based on the specific data associations history of this sample.

Figures 2 (b) and (c) show two inter-robot data associations and the corresponding relative location for the example of Figure 1. The first data association, and accordingly its associated relative position, is wrong. Therefore, the trajectories of the moving objects diverge. On the other hand, Figure 2 (c) represent the correct correspondence and relative position. In this case the trajectories described for the objects are compatible.

A. Estimation of correspondences

The estimation of the distribution $p(\eta_t^k \mid t_{ij}^k)$ is done sequentially based on the previous one $p(\eta_t^{k-1} \mid t_{ij}^{k-1})$,

$$p(\eta_t^k \mid t_{ij}^k) \propto p(t_{ij_k} \mid \eta_t^k, t_{ij}^{k-1}) p(\eta_t^k \mid t_{ij}^{k-1}) \tag{6}$$

$$= p(t_{ij_k} \mid \eta_t^k, t_{ij}^{k-1}) p(\eta_{t_k} \mid \eta_t^{k-1}, t_{ij}^{k-1}) p(\eta_t^{k-1} \mid t_{ij}^{k-1}) \tag{7}$$

$$= p(t_{ij_k} | \eta_{t_k}, \mathbf{x}_{k-1}) p(\eta_{t_k} | \eta_{t_{k-1}}, t_{ij_{k-1}}) p(\eta_t^{k-1} | t_{ij}^{k-1}) \tag{8}$$

The previous derivation is obtained applying Bayes and substituting the set of tracks t_{ij}^{k-1} and data associations η_t^{k-1} by the relative pose of the robots $\mathbf{x}_{k-1}$. The latter comes out from the fact that knowing both, the relative pose of the robots is a sufficient statistics for the data associations. Note that we assume a Markov process for the evolution of the data associations $p(\eta_{t_k} \mid \eta_{t_{k-1}}, t_{ij_{k-1}})$. This assumption does not hold for pure observations and is a consequence of using individual trackers for each robot. It allows us to use only the information corresponding to the last step, $t_{ij_{k-1}}$ and $\eta_{t_{k-1}}$, discarding previous states.

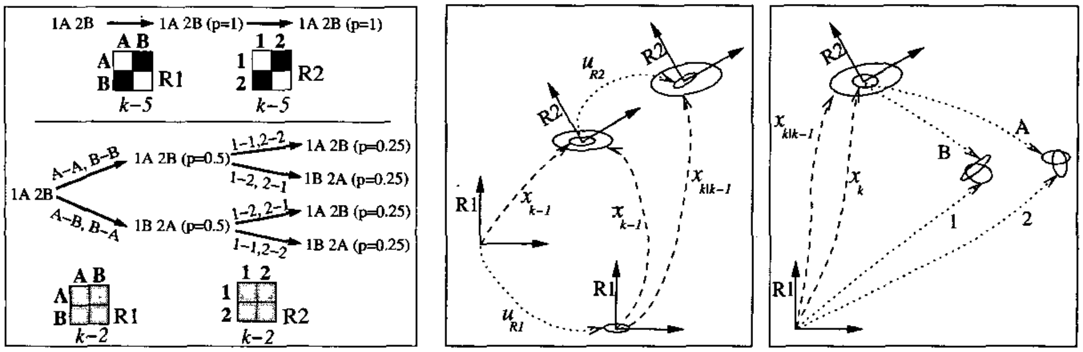
The usual way to compute Eq. (8) for particle filters is to propagate the samples of the previous step S_{k-1} according to the prediction model $p(\eta_{t_k} \mid \eta_{t_{k-1}}, t_{ij_{k-1}})$ and weigh them according to the likelihood $p(t_{ij_k} \mid \eta_{t_k}, \mathbf{x}_{k-1})$. The new set S_k is obtained by sampling the predicted sample set according to the weights.

When generating the proposal distribution, we must take into account the flipping probabilities introduced in the previous section. Neglecting these terms would allow wrong data associations and their associated relative pose to continuously evolve to fit the observations based only on the current state of the tracks at time k. Intuitively, in the absence of ambiguity in the tracking systems, the inter-robot assignments cannot change. If there exist ambiguities, the inter-robot data associations η_{t_k} can only evolve to accommodate possible flips on the trackers of each robot.

Therefore, we sample potential track flips based on the flipping matrices computed by each robot. Each flip exchanges the identity of the tracks of two pairings of the previous data association sample $\eta_{t_{k-1}}^s$. For instance, figure 3(a) show the possible data associations generated from association {1A,2B} and their probability according to the flipping matrices.

Once the data association η_{t_k} is known the likelihood $p(t_{ij_k} \mid \eta_{t_k}, \mathbf{x}_{k-1})$ of a given sample is easily computed based on the distances between the positions of the associated tracks given the relative pose $\mathbf{x}_{k-1}$ (see figure 3(c)).

In this work we only use the kinematic information generated by the tracking systems. The framework, how-



(a) (b) (c)

Fig. 3. One step of the algorithm. (a) Sampling the data associations. The proposed data associations evolve according to the flip probabilities of each robot. The figure shows the possible results of the sampling procedure and their probabilities for two different cases of figure 1. (b) Prediction of the relative pose of the robots using odometry. The uncertainty of the relative location increases due to the motion of both vehicles. In the figure robot 1 acts as reference system and x represents the relative location of robot 2. (c) Likelihood of the data associations and update step of the relative location. The likelihood $p(t_{ij_k} \mid \eta_{t_k}, \mathbf{x}_{k-1})$ measures how well the inter-robot data association $\eta_{t,k}$ explains the location of the objects observed by the robots at time k. Those samples with wrong data associations ($\{1A\ 2B\}$ in this case) will have a lower likelihood and will be removed during the resampling step. On the other hand the update step reduces the uncertainty of the relative location $\mathbf{x}_k$.

ever, allows to extend the information included in the tracks t_{ij}^k to include other types of information. This new information will modify the inter-robot and intra-robot data association models and the likelihood function. For instance, color or shape information could be used to ease the data associations modifying the probabilities of the possible correspondences.

B. Relative Pose Estimation

As mentioned before, the relative position associated to each data association sample is estimated using an Extended Kalman Filter. This Kalman filter represents the location of one of the robots with respect to the other based on a given data association history. The relative pose estimation must take into account the fact that the reference system is moving. Consequently, the relative position must be transformed to the new robot position at each time step,

$$\bar{\mathbf{x}}_{k-1} = \mathbf{x}_{k-1} \oplus u_{k-1} \tag{9}$$

where $\bar{\mathbf{x}}_{k-1}$ is the estimated relative position at time $k-1$ referenced to the robot position at k. $\oplus$ represents the composition of two uncertain locations: the previous relative position estimation $\mathbf{x}_{k-1}$ and the displacement of the robot acting as reference system between the last two steps u_{k-1}.

We apply the classical EKF equations to estimate the position of the tracked robot. In our case the motion model, $\mathbf{x}_{k|k-1} = f(\bar{\mathbf{x}}_{k-1}, u_{k-1})$, computes the predicted relative position based on the odometry readings of the tracked robot (see figure 3(b)). The observations are integrated through the joint observation model $h(t_{ij_k}, \mathbf{x}_{k|k-1}) = 0$ in the update step to compute $\mathbf{x}_k$. This model is a function of the positions of the objects tracked by each robot, the correspondences between these objects at this time step and the relative pose of the robots (figure 3(c)). In other words, the observation model represents the fact that, given the correct correspondences and its relative pose, the trajectories described by the objects do not diverge. If they are not correct, the resulting pose will be wrong and the trajectories will not match. This sample will be deleted in the resampling step due to its low importance factor.

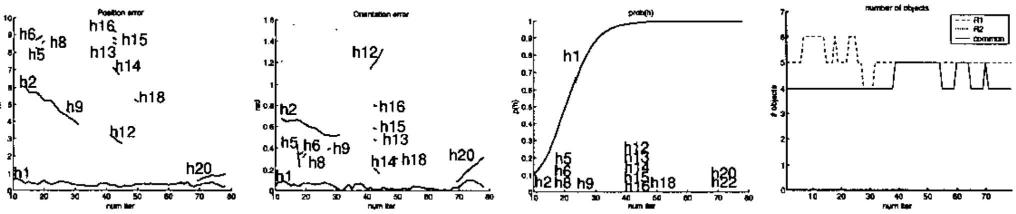
(a) (b)

Fig. 4. Pioneer 3 with two SICK lasers used in the experiments (a) and a snapshot of one of the experiments (b).

VI. Experiments

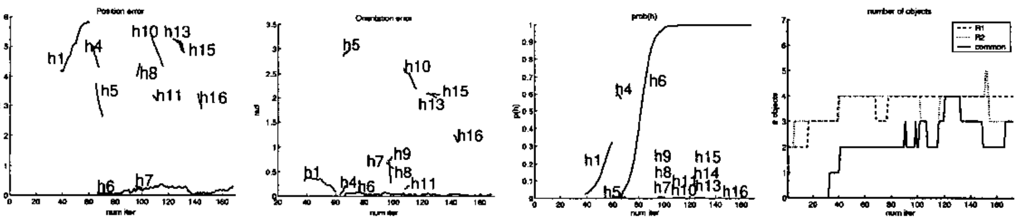
The technique described above has been implemented and tested on data collected with real robots. In this section we present results demonstrating the ability of our method to correctly determine initial relative locations and to reliably track them.

The real data experiments have been performed using two Pioneer-3 robots as the one shown in Figure 4. Both robots are equipped with two sick LMS291 laser range scanners which together provide a 360 degrees field of view. We carried out 5 experiments in the main lobby of the lecture hall at the computer science department of the University of Freiburg. The size of this hall is approximately 51 by 18 meters. In each experiment the robots were manually driven in this hall for about 3 to



(a) (b) (c) (d)

Fig. 5. Results of a real robot experiment: Position (a) and orientation (b) error for each hypothesis, probabilities of the individual hypotheses (c), and number of objects being tracked (d).



(a) (b) (c) (d)

Fig. 6. Results of a real robot experiment with no common evidence in the very beginning: Position (a) and orientation (b) error for each hypothesis, probabilities of the individual hypotheses (c), and number of objects being tracked (d).

4 minutes while up to 5 people walked around them. The right image of Figure 4 shows a typical situation encountered during this experiment. To evaluate the results of our algorithm we computed the ground truth using the localization system developed by Hähnel et al. [5]. The real data were recorded and processed off-line. We run each real data set starting at different points in time to increase the number of initial configurations.

Figures 5 and 6 show the behavior of our algorithm in two different situations. For the sake of clarity we clustered the data association samples using the relative location associated to them. This way we are able to plot only hypotheses corresponding to different locations independently of the current data associations. The probability of each hypothesis was computed sequentially based on the likelihood of the samples belonging to the corresponding cluster and using a fixed likelihood for the hypothesis of not being tracking the correct associations. Moreover, hypotheses are only plotted after surviving more than ten iterations. The figures show the relative error in terms of distance and orientation and the probability for all such hypotheses. In addition, we plot the number of objects tracked by each robot, and the number of common objects (without taking into account occlusions) according to the ground truth.

In the first situation shown in Figure 5, the robots started close to each other and had four moving objects in common. The correct relative pose was estimated based on the correct data associations for the tracks of these objects, which is represented by the hypothesis h1 in this figure. Some new hypotheses are introduced when new objects appear (for instance in iterations 5 and 17). In this experiment the robots are moving close to each other. Therefore, the probability of the correct hypothesis h1 quickly converges to 1 while the others are deleted during the resampling step. As depicted in Figure 5(a) and (b), the position and orientation error with respect to the ground truth for h1 is always under 0.6 meters and 4 degrees respectively. Note that the initial error for h1 is small since our algorithm generates it based on the right data association hypothesis.

In the second situation the robots started far away from each other and therefore were not tracking any common moving object. Anyway the robots try to estimate their relative position using the moving objects they perceive and generate wrong hypotheses (h1, h4, and h5). This situation lasts until iteration 40. Then the right data association (hypothesis h6) was generated and started being tracked. As more common objects appeared, some other hypotheses were created but they were removed after some steps. This is depicted in Figure 6.

Finally, Figure 7 shows the results for a complete experiment where there were always common features. Fig 7(a) plots the ground truth trajectory of one robot as a solid line. Additionally it contains the trajectory of the same robot computed based on the ground truth trajectory of the second robot and the relative pose between the two robots (dotted line). The dashed line corresponds to the trajectory of the first robot computed based on its initial pose and the odometry data. Figure 7(b) plots the positioning error of the first robot with respect to the ground truth. As the figure shows, the accuracy is quite high and the robots stay well localized over the whole experiment despite the fact of using moving features only. Finally, Figure 7(c) show the trajectories of all the moving objects seen by each robot based on our pose estimation. According to this figure,

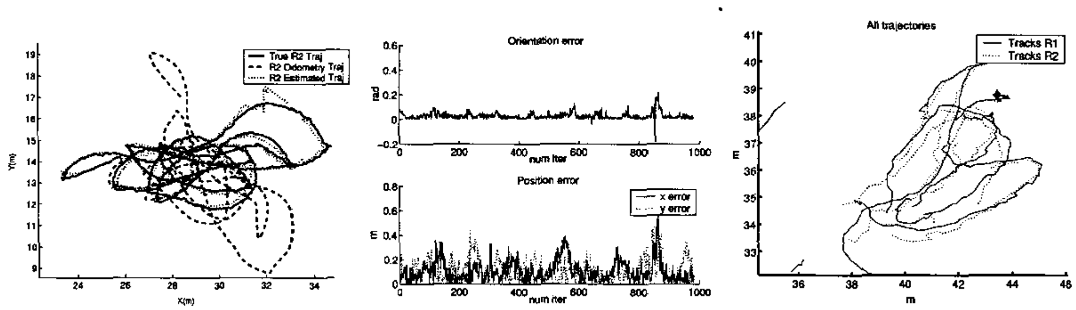


Fig. 7. (a) Trajectories of one robot assuming the trajectory of the other robot is known. The error between the estimated trajectory (dotted line) and the ground truth (solid). Just using odometry (dashed line) accumulates error over time. (b) Error of position for the estimated relative trajectories. (c) All moving objects trajectories matched during 100 steps.

the moving objects are also tracked quite accurately. Thus our algorithm has in fact determined the correct matching between the trajectories of the moving objects.

Despite the fact of using not identifiable moving landmarks, in the presence of common moving objects our algorithm is able to track the relative pose of the robots with a similar precision to the method presented in [6] at the same time that it estimates the data associations between the objects. However, the results depend a lot on the precision of the observations. This makes it difficult to compare our algorithm to other systems using different methods to obtain the robot positions as the quality of the data can differ largely. The number of samples used in the experiments was 50. This number is large enough to manage all the initial location hypotheses generated from the observations and the data association ambiguities of our experiments. The number of moving objects present in the field of view of the sensor is usually limited. In our case the maximum number of objects was six. The computation times of our non optimized Matlab implementation on a PentiumIV are under the sensor rate of 220 milliseconds. Therefore, the algorithm can be executed in real time.

VII. Conclusions

In this paper we presented a method to estimate the relative position of a pair of robots based only on the trajectories described by unidentifiable moving objects. The method first computes the correspondences between the moving objects observed by each individual robot. Then it estimates the relative position based on these correspondences. It uses a Rao-Blackwellized particle filter to sample over potential data associations and then efficiently computes the relative poses corresponding to these data associations. Special techniques are included to overcome the combinatorial number of possible tracks due to data association ambiguities.

Our algorithm has been implemented and evaluated in practical experiments. The results suggest that our approach can robustly estimate the relative pose of pairs of moving robots based on observations of moving objects.

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