Chapter 19
Continuous Petri Nets: Observability and Diagnosis

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Abstract Reconstructing the state of a system from available measurements is a fundamental issue in system theory. It may be considered as a self-standing problem, or it can be seen as a pre-requisite for solving a problem of different nature, such as stabilization, state-feedback control, diagnosis, etc. In the case of Continuous Petri Nets (CPNs), this problem has been studied for both untimed and timed models. In this chapter it is considered first the problem of observability of timed CPNs under infinite server semantics (or variable speed). It is assumed that the marking of some places are measured due to some sensors and the problem is to estimate the initial/actual state/marking. Three different concepts of observability are defined based on the knowledge of the firing rate vector and, algebraic and graph-based criteria are presented. In the last part, untimed CPNs are considered. Measuring/observing the firing amount in which some transitions are fired, it is shown that the set of possible markings in which the system may be is convex. Based on this characterization, some linear programming problems are presented permitting the computation of diagnosis states when some unobservable transitions model possible faults.

19.1 Introduction and Motivation

The observability problem for CPNs has been studied for both untimed and timed models. In the case of untimed systems, the state estimation is close to the one of discrete event systems since the firing of transitions can be assumed/seen as sequential and the corresponding events are not appearing simultaneously. In the case of timed systems, since the evolution can be characterized by a set of switching differential equations, the state estimation problem is more related to the linear and hybrid systems theory.

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In this chapter, we first study different aspects of observability of CPN under infinite server semantics. The notion of *redundant configurations* is presented together with a necessary and sufficient condition for a configuration to be redundant. In some cases, this permits to reduce the system dimension. Three different concepts of observability can be defined for timed CPN based on the firing rate vector $\boldsymbol{\lambda}$. The classical observability problem is the one where the question is to estimate the initial state/marking measuring only a subset of states, while assuming a constant value for the firing vector. In this case, the set of differential equations is time invariant and the concept is called *punctual* or *classical observation*. Observability criteria of *piecewise affine systems* [4] can be applied to CPN since CPN is a subclass of those systems.

As already known, observability of a hybrid system requires not only the estimation of the continuous states but also of the discrete ones. To characterize this, the notion of *distinguishable regions* is introduced and a quadratic programming problem (QPP) is given to check if two regions are or are not distinguishable. Then, an observability criterion is given for general CPN systems. Since the complexity of a potential algorithm based on this criterion may be high, some rules permitting the "deletion" of join transitions are given.

In many real systems it is impossible to have the exact values of the machine speeds. In the extreme case, nothing is known about the firing rate vector and the observability criteria of piecewise affine systems cannot be applied anymore. In this case, *structural observability* is defined and approaches based on the graph theory are used to study it. Only the knowledge of the system structure and the firing count vector (even if not constant) is assumed to be known. The idea is to determine which state variables can be estimated independently of the time values associated to transitions.

Finally, if one wants to estimate the system for "almost all" possible values of firing rate, *generic observability* is defined. In many cases, some punctual values of firing count vector can produce the loss of observability but it is not very important since it is observable outside a proper algebraic variety of the parameter space. Also here, graph based approaches are used. This concept is similar to the works on *linear structured systems* [7].

In the last part of the chapter, we present the effect of fluidization of Petri nets with respect to fault diagnosis. Untimed CPNs are considered and it is assumed that the amount in which some transitions are fired can be observed. It is pointed out that the set of potential marking after a sequence of observed transitions is convex. Based on this convexity, two linear programming problems (LPP) are given that permit us to assign three diagnosis states. The fluidization allows us to relax the assumption, common to all discrete event system diagnosis approaches, that there exists no cycle of unobservable transitions.

This chapter is mainly developed based on the theoretical results presented in [9, 14] and in the survey [19] for the observability of timed CPN under infinite server semantics. Theoretical results for state estimation and fault diagnosis for untimed CPN have been presented in [15].

19.2 A previous Technicality: Redundant Configurations

We assume that the reader is familiar with the notions and definition given in Chapter 18 where CPNs have been introduced.

The number of configurations of a CPN is exponential and upper bounded by $\prod_{t_j \in T} |{}^\bullet t_j|$. A necessary condition for the observability of a CPN system is the observability of all linear systems. Therefore, if some configurations are "removed", the complexity analysis of the observability may decrease. Notice that the notion of *implicit places* [20] and *time implicit arcs* [18] cannot be used in the context of observability since the implicitness in these cases is proved for a given initial marking and for a given time intepretation. In our case, the initial marking is assumed to be partially known. In this section we study a stronger concept, only depending on the net structure and valid for all possible initial markings $\boldsymbol{m}_0 \in \mathbb{R}^{|P|}_{\geq 0}$, concept called *redundant configuration*.

Definition 1. Let $\mathscr{R}_i$ be a region associated to a CPN system. If for all $\boldsymbol{m}_0$, $\mathscr{R}_i \subseteq \bigcup_{j \neq i} \mathscr{R}_j$ then $\mathscr{R}_i$ is a *redundant region* and the corresponding configuration a *redundant configuration*.

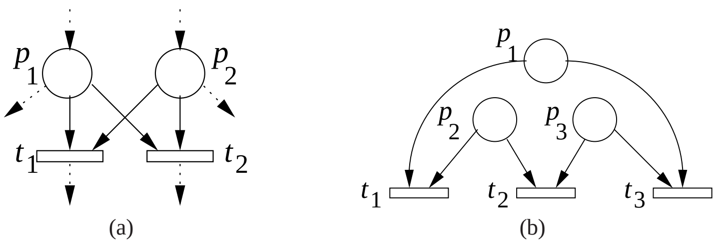


Fig. 19.1 Continuous PN with redundant regions.

Example 1. Let us consider the subnet in Fig. 1(a) and assume all $\boldsymbol{m} \in \mathbb{R}^{|P|}_{\geq 0}$ for which the enabling degree of t_1 is given by m_1. Therefore, the following inequality is satisfied: $m_1 \leq m_2$. Assume also that the enabling degree of t_2 is given by m_2. Hence, $m_2 \leq m_1$. Finally, let us assume that the enabling degree of all other transitions are given by the same places. Obviously, these markings belong to a region $\mathscr{R}_1$ such that for each marking $\boldsymbol{m} \in \mathscr{R}_1$ the following is true $m_1 = m_2$.

Let us consider now all markings $\boldsymbol{m} \in \mathbb{R}^{|P|}_{\geq 0}$ for which the enabling degrees of t_1 and t_2 are given by m_1 and the enabling degree of all other transitions is given by the same set of places as for markings belonging to $\mathscr{R}_1$. It is obvious that these markings belong to a region $\mathscr{R}_2$ for which $m_1 \leq m_2$.

From the above definition of $\mathscr{R}_1$ and $\mathscr{R}_2$, it is obvious that $\mathscr{R}_1 \subseteq \mathscr{R}_2$ for all $\boldsymbol{m} \in \mathbb{R}^{|P|}_{\geq 0}$. Therefore, $\mathscr{R}_1$ (and the corresponding configuration) can be ignored in the analysis of the CPN system. ■

According to Def. 1, a region $\mathscr{R}_i$ is non-redundant if it is a full-dimensional convex polytope in $\mathbb{R}^{|P|}_{\geq 0}$. Therefore, for a given region we need to check if the inequalities composing its definition are strictly satisfied. If for a join t_j with $p_i, p_k \in {}^{\bullet}t_j$ does not exists $\boldsymbol{m} \in \mathbb{R}^{|P|}_{\geq 0}$ such that $\frac{\boldsymbol{m}[p_i]}{\boldsymbol{Pre}[p_i,t_j]} < \frac{\boldsymbol{m}[p_k]}{\boldsymbol{Pre}[p_k,t_j]}$ then the linear systems of the regions containing in its definition $\frac{\boldsymbol{m}[p_i]}{\boldsymbol{Pre}[p_i,t_j]} \leq \frac{\boldsymbol{m}[p_k]}{\boldsymbol{Pre}[p_k,t_j]}$ are redundant.

Proposition 1. *Let $\mathscr{N}$ be a timed CPN system. The region $\mathscr{R}_i$ with the corresponding configuration $\mathscr{C}_i$ is redundant iff $\nexists \boldsymbol{m} \in \mathbb{R}^{|P|}_{\geq 0}$ solution of the following system of strict inequalities of the form $\frac{\boldsymbol{m}[p_k]}{\boldsymbol{Pre}[p_k,t_j]} < \frac{\boldsymbol{m}[p_u]}{\boldsymbol{Pre}[p_u,t_j]}$, one for each $\frac{\boldsymbol{m}[p_k]}{\boldsymbol{Pre}[p_k,t_j]} \leq \frac{\boldsymbol{m}[p_u]}{\boldsymbol{Pre}[p_u,t_j]}$ defining $\mathscr{R}_i$.*

The existence of a solution for the system of strict inequalities in Prop. 1 can be checked solving a linear programming problem using a variable ε. For each $\frac{\boldsymbol{m}[p_k]}{\boldsymbol{Pre}[p_k,t_j]} < \frac{\boldsymbol{m}[p_u]}{\boldsymbol{Pre}[p_u,t_j]}$, a constraint of the following form is added: $\frac{\boldsymbol{m}[p_k]}{\boldsymbol{Pre}[p_k,t_j]} + \varepsilon \leq \frac{\boldsymbol{m}[p_u]}{\boldsymbol{Pre}[p_u,t_j]}$. The objective function will be to maximize ε. If the resulting LPP is infeasible or has as solution $\varepsilon = 0$ then $\mathscr{R}_i$ is a redundant region.

A pre-arc (an arc connecting a place p_i with a transition t_j) is called *implicit* in an untimed system, if for any reachable marking, the marking of p_i is never constraining/restricting the firing of t_j. If the system is under a timed interpretation, it is called *timed implicit*. It may seem that if a mode is redundant, a set of arcs has to be implicit or timed implicit, since they cannot define the enabling. However, it is not true, since it is not that an arc never defines the enabling, but that a combination of arcs may never define the enabling. For example, in the net in Fig. 1(a), none of the arcs is implicit, although a region (the one corresponding to $m_1 = m_2$) is reduced to its borders. In this example, the redundant mode could also have been avoided by fusing transitions t_1 and t_2 into a single one [18]. However, this kind of transformation cannot always be applied, as shown in the following example.

Example 2. Let us consider the CPN in Fig. 1(b) and let us consider the region $\mathscr{R}_1 = \{m_2 \leq m_1, m_3 \leq m_2, m_1 \leq m_3\}$ that it is equivalent to assume that the enabling degree of t_1 is given by m_2, the one of t_2 by m_3 and of t_3 by m_1. Applying Proposition 1 we want to check if $\mathscr{R}_1$ is redundant. We have to consider the following system:

$$\begin{cases} m_2 < m_1 & (1) \\ m_3 < m_2 & (2) \\ m_1 < m_3 & (3). \end{cases} \tag{19.1}$$

Combining 19.1(2) and 19.1(3) we obtain $m_1 < m_2$ that is in contradiction with 19.1(1). Therefore, region $\mathscr{R}_1$ and configuration $\mathscr{C}_1$ are redundant. ∎

The same problem of reducing dimension of a CPN under infinite server semantics has been studied in [16] using the concept of symmetry. It is shown that such a symmetry leads to a permutation of the regions and to equivariant dynamics (dynamical systems that have symmetries). This can be used for reductions to systems of smaller dimension.

19.3 Observability Criteria

Let us assume that the marking of some places $P_o \subseteq P$ can be measured, i.e., the token load at every time instant is known, due to some sensors. The observability problem is to estimate the other marking variables using these measurements. Defining $\boldsymbol{A}_i = \boldsymbol{C} \cdot \boldsymbol{\Lambda} \cdot \boldsymbol{\Pi}_i$ (see Chapter 18 for the definitions of $\boldsymbol{\Lambda}$ and $\boldsymbol{\Pi}_i$), the system dynamic is given by:

$$\begin{cases} \dot{\boldsymbol{m}}(\tau) = \boldsymbol{A}_i \cdot \boldsymbol{m}(\tau), & \boldsymbol{m} \in \mathscr{R}_i \\ \boldsymbol{y}(\tau) = \boldsymbol{S} \cdot \boldsymbol{m}(\tau) \end{cases} \tag{19.2}$$

where $\boldsymbol{S}$ is a $|P_o| \times |P|$ matrix, each row of $\boldsymbol{S}$ has all components zero except the one corresponding to the i^{th} measurable place that is 1. Observe that the matrix $\boldsymbol{S}$ is the same for all linear systems since the measured places are characteristic to the CPN system. Here it is considered that all linear systems are deterministic, i.e., noise-free.

Definition 2. Let $\langle \mathscr{N}, \boldsymbol{\lambda}, \boldsymbol{m}_0 \rangle$ be a timed CPN system with infinite server semantics and $P_o \subseteq P$ the set of measurable places. $\langle \mathscr{N}, \boldsymbol{\lambda}, \boldsymbol{m}_0 \rangle$ is *observable in infinitesimal time* if it is always possible to compute its initial state $\boldsymbol{m}_0$ in any time interval $[0, \varepsilon)$.

Let us first assume that the system is a Join Free (JF) CPN (a CPN is JF if there is no synchronization, i.e., $\forall t_j \in T, |{}^\bullet t_j| = 1$). Therefore, it is a linear system and let us assume that its dynamical matrix is denoted by $\boldsymbol{A}$. In Systems Theory a very well-known observability criterion exists which allows us to decide whatever a continuous time invariant linear system is observable or not. Besides, several approaches exist to compute the initial state of a continuous time linear system that is observable. The output of the system and the *observability matrix* are:

$$\boldsymbol{y}(\tau) = \boldsymbol{S} \cdot e^{\boldsymbol{A} \cdot \tau} \cdot \boldsymbol{m}(\tau_0) \tag{19.3}$$

$$\vartheta = \left[\boldsymbol{S}^T, (\boldsymbol{S}\boldsymbol{A})^T, \ldots, (\boldsymbol{S}\boldsymbol{A}^{n-1})^T \right]^T. \tag{19.4}$$

Proposition 2. *[11, 17] Eq.* (19.3) *is solvable for all* $\boldsymbol{m}(\tau_0)$ *and for all* $\tau > 0$ *iff the observability matrix* ϑ *has full rank (in our case, rank*$(\vartheta) = |P|$*).*

The initial state can be obtained solving the following system of equations that has a unique solution under the rank condition:

$$\begin{bmatrix} \boldsymbol{y}(0) \\ \frac{d}{dt}\boldsymbol{y}(0) \\ \frac{d^2}{dt^2}\boldsymbol{y}(0) \\ \vdots \\ \frac{d^{n-1}}{dt^{n-1}}\boldsymbol{y}(0) \end{bmatrix} = \vartheta \cdot \boldsymbol{m}(0). \tag{19.5}$$

The observability of a JF CPN systems has been considered in [9], where an interesting interpretation of the observability at the graph level. Let us assume that a place p_i is measured, therefore $\boldsymbol{m}_i(\tau)$ and its variation $\dot{\boldsymbol{m}}_i(\tau)$ are known at every time moment τ. Because the net has no join, the flow of all its output transitions t_j of p_i is the product of λ_j and $\boldsymbol{m}_i(\tau)$ according to the server semantics definition. Assume that p_i is not an attribution (a place $p \in P$ is an attribution if $|{}^\bullet p| \geq 1$). Hence has at most one input transition t_k. Knowing the derivative and the output flows, the input flow through the input transition t_k is estimated. Applying again the server semantics definition, $f[t_k] = \lambda_k \cdot \boldsymbol{m}[{}^\bullet t_k]$ ($|{}^\bullet t_k| = 1$ since the net is join free). Obviously, the marking of ${}^\bullet t_k$ can be computed immediately. Observe that this is a *backward* procedure: measuring p_i, ${}^\bullet({}^\bullet p_i)$ is estimated in the absence of joins and attributions.

The problem of state estimation of general CPN systems and not only JF net systems is not so easy. In this case, a very important problem for the observability is the determination of the configuration, also called *discrete state*, i.e, the linear system that governs the system evolution. It may happen that the continuous state estimation fits with different discrete states, i.e., observing some places, it may happen that more than one linear system satisfies the observation. If the continuous states are on the border of some regions, it is not important which linear system is assigned, but it may happen that the solution corresponds to interior points of some regions and it is necessary to distinguish between them.

Example 3. Let us consider the timed CPN in Fig. 2(a). Assume the firing rate of all transitions equal to 1 and $P_o = \{p_3\}$ implying $\boldsymbol{S} = [0\ 0\ 1]^T$. This system has two configurations corresponding to two linear systems:

$$\Sigma_i = \begin{cases} \dot{\boldsymbol{m}}(\tau) = \boldsymbol{A}_i \cdot \boldsymbol{m}(\tau) \\ \boldsymbol{y}(\tau) = [0\ 0\ 1] \cdot \boldsymbol{m}(\tau) \end{cases}, i = 1, 2 \qquad (19.6)$$

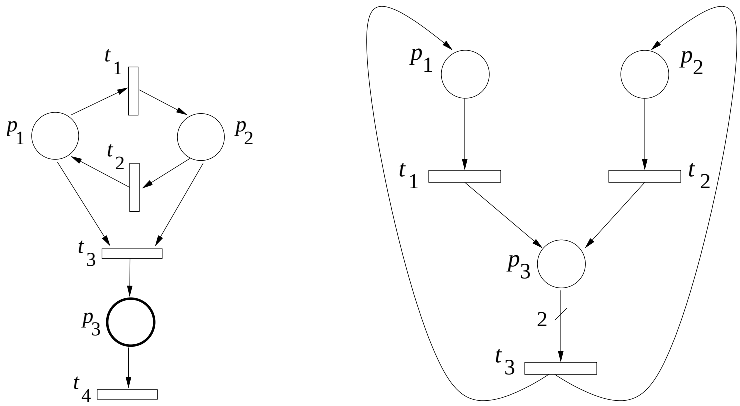
where A_1 is the dynamic matrix corresponding to the configuration in which the marking of p_1 is defining the flow of t_3 while for A_2, the marking of p_2 is giving the flow of t_3.

The observability matrices of these two linear systems are:

$$\vartheta_1 = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & -1 \\ -3 & 1 & 1 \end{bmatrix}; \quad \vartheta_2 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & -1 \\ 1 & -3 & 1 \end{bmatrix}.$$

Both have full rank, meaning that both linear systems are observable. Let us take $\boldsymbol{m}_1 = [1\ 2\ 0]^T \in \mathscr{R}_1 \setminus \mathscr{R}_2$ and $\boldsymbol{m}_2 = [2\ 1\ 0]^T \in \mathscr{R}_2 \setminus \mathscr{R}_1$. As it is well-known, the corresponding observations are $\vartheta_i \cdot \boldsymbol{m}_i(\tau) = [\boldsymbol{y}(\tau)\ \dot{\boldsymbol{y}}(\tau)\ \ldots]^T$. Nevertheless, for the selected markings we have that $\vartheta_1 \cdot \boldsymbol{m}_1 = \vartheta_2 \cdot \boldsymbol{m}_2 = [0\ 1\ -1]^T$. Therefore, it is impossible to distinguish between $\boldsymbol{m}_1$ and $\boldsymbol{m}_2$. ■

Definition 3. Two configurations i and j of a CPN system are *distinguishable* if for any $\boldsymbol{m}_1 \in \mathscr{R}_1 \setminus \mathscr{R}_2$ and any $\boldsymbol{m}_2 \in \mathscr{R}_2 \setminus \mathscr{R}_1$ the observation $\boldsymbol{y}_1(\tau)$ for the trajectory through $\boldsymbol{m}_1$ and the observation $\boldsymbol{y}_2(\tau)$ for the trajectory through $\boldsymbol{m}_2$ are *different* on an interval $[0, \varepsilon)$.



(a) A CPN that has undistinguishable configurations for some particular values of $\boldsymbol{\lambda}$

(b) A JF net that is not observable if p_3 is measured and $\lambda_1 = \lambda_2$

Fig. 19.2 Two CPN.

Remark that we remove the solutions at the border $\mathscr{R}_1 \cap \mathscr{R}_2$ since for those points both linear systems lead to identical behavior, therefore it is not important which one is chosen. If all pairs of modes are distinguishable, it is always possible to uniquely assign a *configuration* (or region) to an observed continuous state. Assuming that the linear systems corresponding to all configurations are observable, a QPP per pair of regions can be proposed to check their distinguishability.

$$\begin{array}{ll} z = \max & \boldsymbol{\beta}^T \cdot \boldsymbol{\beta} \\ \text{s.t.} & \vartheta_1 \cdot \boldsymbol{m}_1 - \vartheta_2 \cdot \boldsymbol{m}_2 = 0 \\ & \boldsymbol{\beta} = \boldsymbol{m}_1 - \boldsymbol{m}_2 \\ & \boldsymbol{m}_1 \in \mathscr{R}_1 \\ & \boldsymbol{m}_2 \in \mathscr{R}_2 . \end{array} \tag{19.7}$$

First, let us observe that if the feasible set of (19.7) is empty (i.e., the problem is infeasible), linear systems are distinguishable. If in QPP (19.7) $z = 0$, using the fact that both systems are observable, i.e., ϑ_1 and ϑ_2 have both full rank, $\boldsymbol{m}_1 = \boldsymbol{m}_2$ is obtained. Therefore, there exist no interior markings $\boldsymbol{m}_1 \in \mathscr{R}_1$ and $\boldsymbol{m}_2 \in \mathscr{R}_2$ with the same observation, i.e., $\vartheta_1 \cdot \boldsymbol{m}_1 = \vartheta_2 \cdot \boldsymbol{m}_2$, and the modes are distinguishable. Finally, if the solution is $z > 0$ the linear systems are undistinguishable being the same evolution in a small interval starting from two markings belonging to different regions. Finally, if the solution is $z > 0$ we cannot say nothing about distinguishability of the linear systems. Moreover, the exact solution of (19.7) is not necessary to be computed and if a feasible solution with $z > \delta$, with δ a small positive number, is found the search can be stopped.

Example 4. In Example 3, for the timed CPN in Fig. 2(a) it is shown that $\vartheta_1 \cdot \boldsymbol{m}_1 = \vartheta_2 \cdot \boldsymbol{m}_2 = [0, 1, -1]^T$. Solving QPP (19.7), the problem is found to be unbounded, thus the linear systems Σ_1 and Σ_2 are undistinguishable. For the interpretation of this result, let us consider the equations that govern the evolution of the system:

$$f_3 = \lambda_3 \cdot \min\{m_1, m_2\} \tag{19.8}$$
$$\dot{m}_1 = \lambda_2 \cdot m_2 - \lambda_1 \cdot m_1 - f_3 \tag{19.9}$$
$$\dot{m}_2 = \lambda_1 \cdot m_1 - \lambda_2 \cdot m_2 - f_3. \tag{19.10}$$

Summing (19.9) and (19.10) and integrating, we obtain

$$(m_1 + m_2)(\tau) = (m_1 + m_2)(0) - 2\int_0^\tau f_3(\theta) \cdot d\theta \tag{19.11}$$

Obviously, if p_3 is measured, f_3 can be estimated since $f_3(\tau) = \dot{m}_3(\tau) + \lambda_4 \cdot m_3(\tau)$. Therefore, according to (19.8), the minimum between m_1 and m_2 is estimated. Moreover, due to (19.11) their sum is also known. Nevertheless, these two equations are not enough to compute the markings, i.e., we have the values but it is impossible to distinguish which one corresponds to which place.

We use the same CPN system to illustrate that if the solution of LPP (19.7) is $z > 0$ or unbounded we cannot decide. Let us take now $\boldsymbol{\lambda} = [2\ 1\ 1\ 1]^T$. In this case, the dynamical matrices are:

$$\boldsymbol{A}_1 = \begin{bmatrix} -3 & 1 & 0 \\ 1 & -1 & 0 \\ 1 & 0 & -1 \end{bmatrix}, \quad \boldsymbol{A}_2 = \begin{bmatrix} -2 & 0 & 0 \\ 2 & -2 & 0 \\ 0 & 1 & -1 \end{bmatrix},$$

and the observability matrices (assuming also $P_o = \{p_3\}$):

$$\vartheta_1 = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & -1 \\ -4 & 1 & 1 \end{bmatrix}; \quad \vartheta_2 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & -1 \\ 2 & -3 & 1 \end{bmatrix}.$$

Let $\boldsymbol{m}_1 = [1\ 5\ 1]^T \in \mathscr{R}_1 \setminus \mathscr{R}_2$ and $\boldsymbol{m}_2 = [2\ 1\ 1]^T \in \mathscr{R}_2 \setminus \mathscr{R}_1$. Making the computations, we have: $\vartheta_1 \boldsymbol{m}_1 = \vartheta_2 \boldsymbol{m}_2 = [1\ 0\ 2]^T$. So, we have the same observations for these two markings at a time τ but the modes are distinguishable. To see this let us assume the marking at $\tau + \delta$, where δ is a very small value. Being a small time variation, we can consider that the flow of the transitions are constant during the time interval $(\tau, \tau + \delta)$ and we can write:

$$\boldsymbol{m}_1'(\tau + \delta) = \boldsymbol{m}_1(\tau) + \boldsymbol{A}_1 \boldsymbol{m}_1(\tau)\delta = [1 + 2\delta\ \ 5 - 4\delta\ \ 1]^T,$$

and

$$\boldsymbol{m}_2'(\tau + \delta) = \boldsymbol{m}_2(\tau) + \boldsymbol{A}_2 \boldsymbol{m}_2(\tau)\delta = [2 - 4\delta\ \ 1 + 2\delta\ \ 1]^T.$$

The corresponding observations for these markings are: $\vartheta_1 \boldsymbol{m}_1' = [1\ 2\delta\ 2 - 12\delta]^T \neq \vartheta_2 \boldsymbol{m}_2' = [1\ 2\delta\ 2 - 14\delta]^T$. Since in all linear systems the set of measured places is the same and the firing rates are also the same can be observed immediately that any $\boldsymbol{m}_1'' \in \mathscr{R}_1$, $\boldsymbol{m}_2'' \in \mathscr{R}_1$ with $\vartheta_1 \boldsymbol{m}_1''(\tau) = \vartheta_2 \boldsymbol{m}_2''(\tau)$ it holds that $\vartheta_1 \boldsymbol{m}_1''(\tau+\delta) \neq \vartheta_2 \boldsymbol{m}_2''(\tau+\delta)$. Therefore, according to Def. 3, the modes are distinguishable. ■

Using the notion of distinguishable modes, an immediate criterion for observability in infinitesimal time is:

Theorem 1. *A timed CPN system $\langle \mathscr{N}, \boldsymbol{\lambda}, \boldsymbol{m}_0 \rangle$ under infinite server semantics is observable in infinitesimal time iff:*

1. *All pairs of configurations are distinguishable,*
2. *For each region, the associated linear system is observable.*

A complementary observability problem is presented in [10]. For the discrete-time model and measuring some places, the problem is to estimate the firing flow (speed) of the transitions and not the marking of the other places. Since the flow of a transition is the product between its firing rate (constant value) and the enabling degree, in some cases, measuring places or transitions is equivalent. Anyhow, in order to compute the flow through joins it is necessary to measure all of its input places. Moreover, we may also have different markings that have the same firing flow.

19.4 Reducing Complexity

Theorem 1 provides a criterion of observability of a CPN system. Observe that the complexity of an algorithm to check this property is not small. The algorithm based on this criterion should be linear in the number of subsystems (for each subsystem the observability matrix and its rank should be computed) but this number is exponential in the number of joins. Moreover, for each pair of subsystems, their distinguishability is necessary to be checked. For this reason, some results have been proposed in order to “delete” the joins without affecting the observable space. After that, observability can be checked using only the observability matrix. This reduction can be done under some general conditions if the net system is attribution free (AF - a net is attribution free if there exists no place $p \in P$ such that $|{}^\bullet p| \geq 2$) or equal conflict (EQ - a net is equal conflict if for any $t_1, t_2 \in T$ such that ${}^\bullet t_1 \cap {}^\bullet t_2 \neq \emptyset$ then $\boldsymbol{Pre}[\cdot, t_1] = \boldsymbol{Pre}[\cdot, t_2]$) [14].

Definition 4. Let $\mathscr{N} = \langle P, T, \boldsymbol{Pre}, \boldsymbol{Post} \rangle$ be a net and $\mathscr{N}' = \langle F, T', \boldsymbol{Pre'}, \boldsymbol{Post'} \rangle$ a subnet of $\mathscr{N}$, i.e., $F \subseteq P$, $T' \subseteq T$ and $\boldsymbol{Pre'}, \boldsymbol{Post'}$ are the restrictions of $\boldsymbol{Pre}, \boldsymbol{Post}$ to F and T'. $\mathscr{N}'$ is a strongly connected p-component of $\mathscr{N}$ if for all $p_1, p_2 \in F$ there exists a path from p_1 to p_2 of the form $\langle p_1, t_1, p_i, t_i, \ldots, t_j, p_j, t_2, p_2 \rangle$ with $t_1 \in p_1{}^\bullet$, $p_i \in t_1{}^\bullet$, ..., $p_j \in t_j{}^\bullet$, $t_2 \in p_j{}^\bullet$, $p_2 \in t_2{}^\bullet$.

Further, a strongly connected p-component $\mathscr{N}' = \langle F, T', \boldsymbol{Pre'}, \boldsymbol{Post'} \rangle$ is called *terminal* if for all $p \in F$ it holds that: there exists a path from p to other place p' implies $p' \in F$.

Proposition 3. *[14] Let $\langle \mathcal{N}, \boldsymbol{\lambda} \rangle$ be a timed AF CPN and assume that for any join t_i there exists no strongly connected p-component containing all $^\bullet t_i$. Let $\mathcal{N}'$ be the net obtained from $\mathcal{N}$ by just removing all join transitions together with its input and output arcs. $\mathcal{N}$ is observable iff $\mathcal{N}'$ is observable.*

Observe that the net in Fig. 2(a) is not satisfying the conditions of the previous theorem since $^\bullet t_3 = \{p_1, p_2\}$ belongs to a strongly connected p-component. However, if the net has attributions, joins cannot be removed in general.

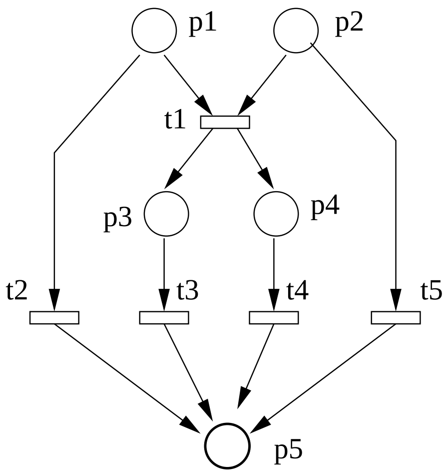


Fig. 19.3 CPN used in Ex. 5.

Example 5. Let us consider the CPN system in Fig. 19.3 with $\boldsymbol{\lambda} = [a, 1, 2, 3, 4]^T$, $a \in \mathbb{R}_{>0}$ and p_5 the measured place. This net has an attribution in place p_5 and has a join in t_1. The linear system obtained by removing the join t_1 is observable and p_1 and p_2 do not belong to a strongly connected p-component. However, the join transition t_1 cannot be removed without affecting the observability space. The dynamical matrices of the two linear systems are:

$$\boldsymbol{A}_1 = \begin{bmatrix} -1-a & 0 & 0 & 0 & 0 \\ -a & -4 & 0 & 0 & 0 \\ a & 0 & -2 & 0 & 0 \\ a & 0 & 0 & -3 & 0 \\ 1 & 2 & 3 & 4 & 0 \end{bmatrix}, \quad \boldsymbol{A}_2 = \begin{bmatrix} -1 & -a & 0 & 0 & 0 \\ 0 & -4-a & 0 & 0 & 0 \\ 0 & a & -2 & 0 & 0 \\ 0 & a & 0 & -3 & 0 \\ 1 & 2 & 3 & 4 & 0 \end{bmatrix}.$$

Computing the determinants of the corresponding observability matrices, we have:

$$det(\vartheta_1) = 192 \cdot a^3 - 912 \cdot a^2 + 720 \cdot a + 288,$$

which has two positive real roots ($a_1 = 3.5885$ and $a_2 = 1.4498$), and

$$\det(\vartheta_2) = -96 \cdot a^3 - 408 \cdot a^2 - 216 \cdot a + 288,$$

with one positive real root ($a_3 = 0.5885$). Obviously, if λ_1 is equal to one of these roots, the CPN system will not be observable since one of the corresponding linear system will not be observable.

Hence, for some particular values of $\boldsymbol{\lambda}$, the system obtained removing the join is observable but the original system (with join) is not observable. ■

Proposition 4. *[14] Let $\langle \mathcal{N}, \boldsymbol{\lambda}, \boldsymbol{m}_0 \rangle$ be a timed EQ continuous Petri net system and $\mathcal{N}'$ obtained from $\mathcal{N}$ by just removing all join transitions together with its input and output arcs. $\mathcal{N}$ is observable iff $\mathcal{N}'$ is observable.*

The previous two propositions provide necessary and sufficient conditions to "reduce non-linearity" and study the observability of a non-linear system on an equivalent, w.r.t. the observability space, linear system. Hence, it is enough to check the rank of only one observability matrix in order to decide the observability of these CPN net systems.

19.5 Structural and generic observability

In this section the main results of structural and generic observability of CPN are presented. First we will illustrate by an example that the presence of an attribution may lead to the loss of the observability. For this reason, the main result for structural observability has been given assuming that the net has no attribution while generic observability may be studied easily in the case of nets with attributions.

From the previous graph-based interpretation (the backward strategy) of the observability, it is obvious that the output connectedness is required for a place p to be estimated from an observation. For those places for which there is no path to an output, their marking cannot be estimated. Therefore, the *terminal strongly connected p-components* present a special interest because any place of the net should be connected to those components in order to be able to be estimated.

Definition 5. A strongly connected p-component $\mathcal{N}' = \langle F, T', \boldsymbol{Pre'}, \boldsymbol{Post'} \rangle$ of a net $\mathcal{N}$ is said to be *terminal* if there is no path from a place belonging to F to a place not in F.

Strongly connected p-components of a PN can be computed immediately, adapting the classical polynomial time algorithms (for example the one in [6]) to a bipartite graph.

Definition 6. Let $\langle \mathcal{N}, \boldsymbol{\lambda}, \boldsymbol{m}_0 \rangle$ be a CPN system and P_o the set of measured places. $\mathcal{N}$ is *structurally observable* if $\langle \mathcal{N}, \boldsymbol{\lambda}, \boldsymbol{m}_0 \rangle$ is observable for all values of $\boldsymbol{\lambda} \in \mathbb{R}^{|T|}_{>0}$.

Proposition 5. *[14] Let $\mathcal{N}$ be a join and attribution free CPN. $\mathcal{N}$ is structurally observable iff at least one place from each terminal strongly connected component is measured.*

Let us consider now attributions and see that this construction can lead to the loss of observability. Assume the CPN system in Fig. 2(b) where p_3 (an attribution place) is the measured place. Writing down the differential equation we have:

$$\dot{m}_3(\tau) = \lambda_1 \cdot m_1(\tau) + \lambda_2 \cdot m_2(\tau) - \lambda_3 \cdot m_3(\tau).$$

From the previous equation, $\lambda_1 \cdot m_1(\tau) + \lambda_2 \cdot m_2(\tau)$ can be computed since the other variables are known. However, if $\lambda_1 = \lambda_2$, will be impossible to distinguish between $m_1(\tau)$ and $m_2(\tau)$ and the system is not observable. In general, if there exist two input transitions to an attribution place with the same firing rate, the system is not *observable* [14]. Nevertheless, this is not a general rule since the observability is a global property.

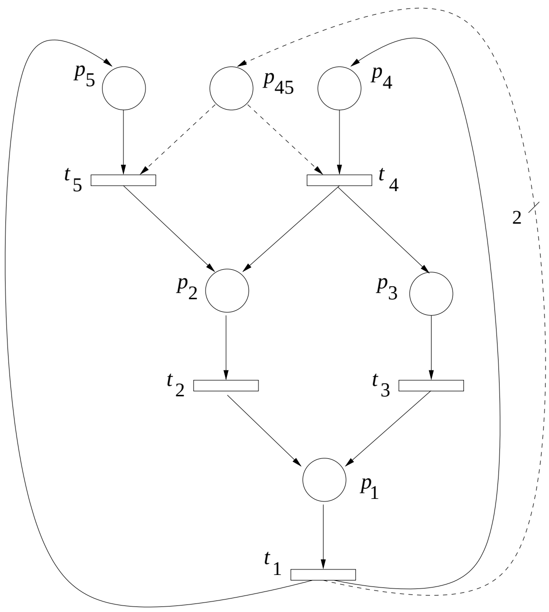


Fig. 19.4 A JF net that is observable measuring the attribution place p_2 even if $\lambda_4 = \lambda_5 = \lambda_2 = \lambda_3$

Let us consider the timed CPN is Fig. 19.4 with $\boldsymbol{\lambda} = \mathbf{1}$, assume that p_2 is measured and let us see if the system is observable using the backward strategy presented before. Then m_4 and m_5 cannot be estimated directly, but their sum (a linear combination of them) is computable (place p_{45} in the figure). Going backwards, m_1 is estimated and, even although m_1 is an attribution, since m_2 is measured, then m_3 can also be estimated. Using m_3, now m_4 is estimated and, through the linear combination of p_{45}, m_5 as well. Therefore, by measuring p_2 the system is observable for all $\boldsymbol{\lambda}$, i.e., structurally observable.

Observe that this loss of the observability is due to the presence of attributions happens for very specific values of $\boldsymbol{\lambda}$. If the firing rates of the transitions are chosen randomly in $\mathbb{R}_{>0}$, the probability to have such a loss of observability is almost null. Hence, a concept weaker than structural observability can be studied. It is similar with the concept of "structural observability" defined in [5, 7] for linear systems.

Definition 7. Let $\langle \mathcal{N}, \boldsymbol{\lambda}, \boldsymbol{m}_0 \rangle$ be a CPN system and P_o the set of measured places. $\mathcal{N}$ is *generically observable* if $\langle \mathcal{N}, \boldsymbol{\lambda}, \boldsymbol{m}_0 \rangle$ is observable for all values of $\boldsymbol{\lambda}$ outside a proper algebraic variety of the parameter space.

The relation between structural and generic observability is immediate. If $\mathcal{N}$ is structurally observable then it is generically observable. In general, the reverse is not true.

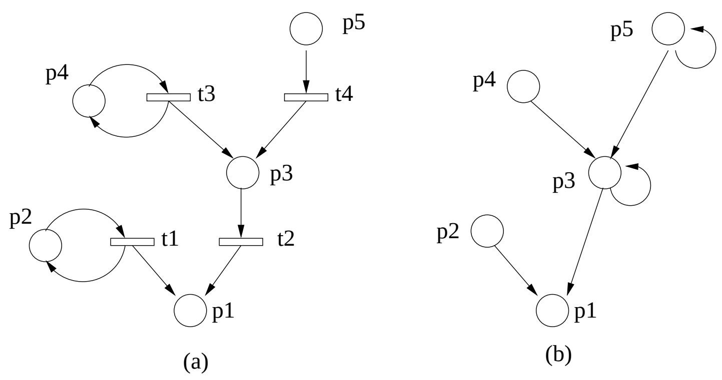


Fig. 19.5 (a) A JF ContPN; (b) Its associated graph.

In [5], generic observability is studied for structured linear systems using an *associated graph*; observability is guaranteed when there exists a state-output connection for every state variable (the system is said to be *output connected*) and no *contraction* exists. The transformation of a JF net into its corresponding *associated directed graph* is straightforward (see Fig. 19.5 for an example).

Using the associated graph and Proposition 1 in [5], the following result has been obtained to characterize the generic observability.

Corollary 1. *[14] Let $\mathcal{N}$ be a pure JF CPN. $\mathcal{N}$ is generically observable iff at least one place from each terminal strongly connected p-component is measured.*

The previous result can be extended immediately to general CPNs, i.e., it is not true only for JF nets. In Example 3 is given a CPN system containing two undistinguishable configurations. Then, changing the firing rates of the transitions in Example 4, these modes become distinguishable. Obviously, two configurations are undistinguishable when the path from states (markings) to the outputs are identical in both linear systems. This happens for some particular values of firing rates, e.g., $\lambda_1 = \lambda_2$ in the CPN of Fig. 2(a). If the firing rates are chosen randomly, the backward paths cannot be identical. Therefore, any pair of subsystems are distinguishable.

Corollary 2. *[14]Let $\mathcal{N}$ be a pure CPN. $\mathcal{N}$ is generically observable iff at least one place from each terminal strongly connected p-component is measured.*

For example, the net in Fig. 2(a) is not observable (hence neither structurally observable) but it is generically observable.

19.6 Observers Design

JF nets lead to linear systems, for which, Luenberger's observers [11, 17] are frequently used for the estimation of the states. Such an observer for a JF PN, i.e., with a single linear system, can be expressed as:

$$\dot{\tilde{\boldsymbol{m}}}(\tau) = \boldsymbol{A} \cdot \tilde{\boldsymbol{m}}(\tau) + \boldsymbol{K} \cdot (\boldsymbol{y}(\tau) - \boldsymbol{S} \cdot \tilde{\boldsymbol{m}}(\tau)),$$

where $\tilde{\boldsymbol{m}}(\tau)$ is the marking estimation, $\boldsymbol{A}$ and $\boldsymbol{S}$ are the matrices defining the evolution of the marking of the system and its output in continuous time, $\boldsymbol{y}(\tau)$ is the output of the system, and $\boldsymbol{K}$ is a design matrix of parameters.

At a particular time instant, a CPN evolves according to a given linear system. Thus, an online estimation can be performed by designing one (Luenberger) linear observer per each potential linear system of the PN (in a similar way to [8] for a class of piecewise linear systems) and selecting the one that accomplishes certain properties. The "goodness" of an estimate can be measured by means of a *residual* [3]. Let us use the 1-norm $||\cdot||_1$, which is defined as $||\boldsymbol{x}||_1 = |\boldsymbol{x}_1| + \ldots + |\boldsymbol{x}_n|$. The residual at a given instant, $r(\tau)$, is the distance between the output of the system and the output that the observer's estimate, $\tilde{\boldsymbol{m}}(\tau)$, yields

$$r(\tau) = ||\boldsymbol{S} \cdot \tilde{\boldsymbol{m}}(\tau) - \boldsymbol{z}(\tau)||_1.$$

In order to be *suitable*, the estimations of the observers must verify the following conditions:

- The residual must tend to zero.
- The estimations of the places in a synchronization have to be *coherent* with the operation mode for which they are computed.

Thus, at a given time instant, only coherent estimations are suitable. Moreover, a criterion must be established to decide which coherent estimation is, at a given time instant, the most appropriate. An adequate *heuristics* is to choose the coherent estimation with minimum residual.

Consider the CPN system in Fig. 19.6. Let its output be the marking of place p_1, i.e., $\boldsymbol{S} = [1\ 0\ 0]$. The net has two configurations: $\mathscr{C}_1 = \{(p_1,t_1),(p_1,t_2),(p_3,t_3)\}$ and

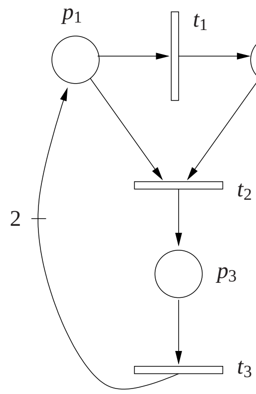


Fig. 19.6 A CPN with two linear systems.

$\mathscr{C}_2 = \{(p_1,t_1),(p_2,t_2),(p_3,t_3)\}$. For the linear system corresponding to $\mathscr{C}_1$, m_2 is not observable. However, for the linear system corresponding to $\mathscr{C}_2$, the marking of all the places can be estimated. Let $\boldsymbol{\lambda} = [0.9\ 1\ 1]^T$ and $\boldsymbol{m}_0 = [3\ 0\ 0]^T$. The marking evolution of this system is depicted in Fig. 7(a).

One observer per linear system is designed. Let the initial state of observer 1 be $\boldsymbol{e}_{01} = [1\ 2]^T$ and its eigenvalues be $[-12+2\cdot\sqrt{3}\cdot i,\ -12-2\cdot\sqrt{3}\cdot i]$. Since observer 1 can only estimate m_1 and m_3, the first component of its state vector corresponds to the estimation of m_1, and its second component to the estimation of m_3. For observer 2, let the initial state be $\boldsymbol{e}_{02} = [1\ 0\ 2]^T$ and its eigenvalues be $[-15,\ -12+2\cdot\sqrt{3}\cdot i,\ -12-2\cdot\sqrt{3}\cdot i]$. The evolution of the coherent estimation with minimum residual is shown in Fig. 7(a).

The resulting estimation can be improved by taking into account some considerations. When the first system switch happens, the estimation becomes discontinuous and, what is more undesirable, the estimation for the marking of p_3 becomes worse. A similar effect happens when the second system switch occurs. Another undesirable phenomenon is that, after the first switch, the estimation of m_2 just disappears (since it is unobservable in configuration $\mathscr{C}_1$).

One way to avoid discontinuities in the resulting estimation, is to use the estimation of the observer that is going to be filtered out in order to update the estimation of the observer that is not going to be filtered out. This estimation update must be done when a system switch is detected. In order not to loose the estimation of the marking of a place when it was "almost perfectly" estimated (recall the case of m_2 when the first switch happened) a simulation of the system can be launched. The initial marking of this simulation is the estimation of the system just before the observability of the marking is lost. Such a simulation can be seen as an estimation for those markings that are not observable by the observer being considered. The simulation should only be carried out when an estimation for all the places exists and the residual is not significant. Fig. 7(b) shows the evolution of the estimation obtained by this strategy.

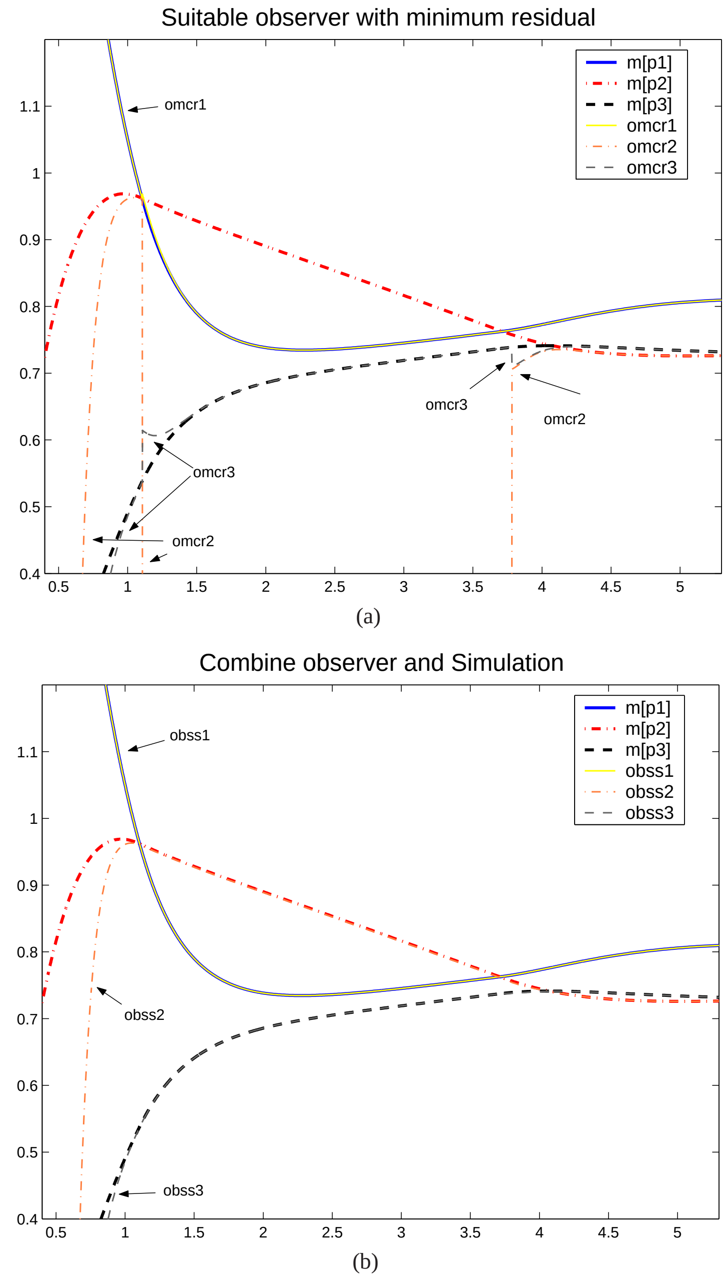


Fig. 19.7 The marking evolution is given by $(\boldsymbol{m}[p_1], \boldsymbol{m}[p_2], \boldsymbol{m}[p_3])$. (a) The estimate of the minimum residual and coherent observer is $(omcr1, omcr2, omcr3)$. (b) The estimate of the observer that makes use of a simulation is $(obss1, obss2, obss3)$.

One of the main advantages is that the residual does not increase sharply when the mode of the system changes. Another interesting feature is that the use of a simulation allows one to estimate the marking of places that in some modes are in principle not observable: in Fig. 7(b) it can be seen that the marking of p_2 can be estimated, even when it is unobservable due to configuration $\mathscr{C}_1$ being active.

19.7 Diagnosis using Untimed CPNs

Let us now consider untimed CPN (see Chapter 18 for a short introduction and the differences among the timed and untimed models). Observability and state estimation problems in systems modeled by an *untimed* CPN have also been studied [15]. Nevertheless, in this case it is assumed that the initial marking is known (and not unknown as in previous sections) and the set of transitions is partitioned in two subsets: *observable* ($T_o \subseteq T$) and *unobservable* transitions ($T_u \subseteq T$, $T_o \cap T_u = \emptyset$) (hence transitions are observed and not places). When an observable transition fires, its firing quantity is measured/observed. From the initial marking and given a sequence of observed transitions each one with a given firing amount, it is impossible to uniquely determine the actual marking because the unobservable transitions can fire intercalated with the observable transitions. All markings in which the net may be given the actual observation is called the *set of consistent markings*.

Proposition 6. *[15] Let $\langle \mathscr{N}, \boldsymbol{m}_0 \rangle$ be a CPN system where $\mathscr{N} = \langle P, T, \boldsymbol{Pre}, \boldsymbol{Post} \rangle$ and $T = T_o \cup T_u$. Assume that the net system obtained from $\mathscr{N}$ removing all transitions T_o has no spurious solution (solution of the state equation but corresponding to unreachable markings). Given an observed word $t_1(\alpha_1)t_2(\alpha_2)\ldots t_k(\alpha_k)$ with $t_i \in T_o$ $\forall i = 1, \ldots, k$, the set of consistent markings is convex.*

Based on this proposition, an iterative algorithm has be derived [15] in order to characterize the set of consistent markings after an observation word w. The main idea of the algorithm is to start from each vertex of the previous set and compute the vertices of some polytopes. Taking the convex hull of all new vertices, the new set of consistent markings is obtained. The computational complexity of the algorithm is exponential because requires the computation of vertices, but the compact representation as a convex polytope is a real advantage. The fluidization allows us to relax the assumption, common to all the discrete event system diagnosis approaches, that there exist no cycle of unobservable transitions.

Fault diagnosis problem has been considered in Chapter 13 in the case of discrete Petri nets. Similarly, let us assume that a certain number of *anomalous* (or *fault*) behaviors may occur in the system. The occurrence of a fault behavior corresponds to the firing of an unobservable transition, but there may also be other transitions that are unobservable as well, but whose firing corresponds to regular behaviors. Then, assume that fault behaviors may be divided into r main classes (*fault classes*), and we are not interested in distinguishing among fault events in the same class. Usually, fault transitions that belong to the same fault class are transitions that represent similar physical faulty behavior.

This is modeled in PN terms assuming that the set of unobservable transitions is partitioned into two subsets

$$T_u = T_f \cup T_{reg},$$

where T_f includes all *fault* transitions and T_{reg} includes all transitions relative to unobservable but *regular events*. The set T_f is further partitioned into r subsets, namely,

$$T_f = T_f^1 \cup T_f^2 \cup \ldots \cup T_f^r$$

where all transitions in the same subset correspond to the same fault class. We will say that the i-th fault has occurred when a transition in T_f^i has fired.

Definition 8. Let $\langle \mathscr{N}, \boldsymbol{m}_0 \rangle$ be a CPN system, $T = T_o \cup T_u$ and w an observed word. A *diagnoser* is a function

$$\Delta : T_o^* \times \{T_f^1, T_f^2, \ldots, T_f^r\} \to \{N, U, F\}$$

(where T_o^* denotes the possible sequences obtainable combining elements in T_o, where each sequence is characterized by the firing amounts of all the transitions in it) that associates to each observation w and to each fault class T_f^i, $i = 1, \ldots, r$, a *diagnosis state*.

- $\Delta(w, T_f^i) = N$ if the i^{th} fault *cannot have occurred*. This is true if *none* of the firing sequences consistent with the observation contains fault transitions of class i.
- $\Delta(w, T_f^i) = U$ if a fault transition of class i *may have occurred or not*, i.e., it is uncertain, and we have no criteria to draw a conclusion in this respect.
- $\Delta(w, T_f^i) = F$ if the i^{th} *fault has occurred* since all fireable sequences consistent with the observation contain at least one fault transition of class i. ■

Thus, states N and F correspond to "certain" states: the fault has not occurred or it has occurred for sure; on the contrary state U is an "uncertain" state: the fault may either have occurred or not. Given an observation, the diagnosis state is computed solving two LPPs. Since the set of consistent marking is convex, it can be characterized by a set of vertices. Each vertex of the set of consistent markings is reached from the initial marking by firing the observed word w plus, eventually, some unobservable transitions. Moreover, after the observation w, other unobservable transitions may fire. For a given observed word w, the vectors of unobservable transitions that are fired in order to enable transition in w or after w are called *fireable firing sequences consistent with the observation* w and are denoted by $Y(\boldsymbol{m}_0, w)$.

Proposition 7. *[15] Consider an observed word* $w \in T_o^*$ *and* $Y(\boldsymbol{m}_0, w)$ *be the polytope containing all fireable sequences consistent with the observation* w. *Let*

$$\begin{cases} l_i = \min \sum_{t_j \in T_f^i} \rho(t_j) \\ s.t. \\ \boldsymbol{\rho} \in Y(\boldsymbol{m}_0, w) \end{cases} \quad \begin{cases} u_i = \max \sum_{t_j \in T_f^i} \rho(t_j) \\ s.t. \\ \boldsymbol{\rho} \in Y(\boldsymbol{m}_0, w). \end{cases} \tag{19.12}$$

It holds:

$$\begin{array}{l} \Delta(w, T_f^i) = N \Leftrightarrow u_i = 0 \\ \Delta(w, T_f^i) = U \Leftrightarrow l_i = 0 \wedge u_i > 0 \\ \Delta(w, T_f^i) = F \Leftrightarrow l_i > 0. \end{array}$$

19.8 Further readings

For timed continuos Petri nets under infinite server semantics the problem of sensor placement has been considered in [12]. It is assumed that each place can be measured using a sensor, each sensor having associated a cost. The problem is to decide the set of places with minimum cost ensuring the observability of the system. Since the observability is a global property, the brute force algorithm has an exponential complexity because has to consider all combinations of places. Some properties permitting to reduce this complexity have been proved in [12]. A similar problem but using a geometrical approach has been considered in [1] where some results in [12] received a different perspective. An observability problem for this firing semantics has been considered also in [10] using a discrete-time model. In this case the problem was to estimate the firing flow of transitions and not the marking of the places.

In the case of timed CPN under *finite server semantics* the problem has not been considered in literature. However, for a similar semantics, the continuous part of so called *First Order Hybrid Petri Nets* [2], a timed reachability problem has been considered in [13]. The observation problem reduces to determining the set of markings, in which the net may be at a given time. It is shown under which conditions the reachability set of the timed net under finite server semantics coincides with that of the untimed one and a procedure to compute the minimum time ensuring that the set of *consistent markings* is equal to the reachability set of untimed system is given for some net classes.

Different problems regarding observability of CPNs deserve a more deep study. For example, to check the distinguishability of two configurations, there exists no necessary and sufficient criterion. Moreover, the concept can be extended to more than two configurations. In the case of redundant regions, the structural symmetry can be considered and, in many cases, such symmetry will conduct to redundant linear systems. In the case of state estimation of untimed CPN, new approaches can be studied in order to decrease the complexity of the actual algorithms.

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