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# On State Estimation of Timed Choice-Free Petri Nets ⋆

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**Abstract:** In this paper, we present an online algorithm for state estimation of timed choice-free Petri nets. We assume that the net structure and initial marking are known, and that the set of transitions is divided in *observable* and *unobservable* one. Given an observed word and assuming that the time durations associated to the unobservable transitions are unknown, our problem is to estimate the possible states in which the timed net system can be. This work extends the notion of *basis markings* defined for untimed Petri nets considering now the time information. The proposed algorithm deals with three main steps: (1) wait for a new observation and compute the set of basis markings without considering the time; (2) update the set of time equations that contain the time restriction for the unobservable transitions; (3) update the set of basis markings removing the time-inconsistent markings. The extension of the algorithm to general nets is discussed, as well.

## 1. INTRODUCTION

Reconstructing the state of a system from available measurements is a fundamental issue in several applications. State observation can be seen as a self-standing problem, but also as a pre-requisite for solving problems of different nature. This problem has been extensively investigated in time driven systems. On the contrary, despite the attention payed by several authors in the last years, there are relatively few works addressing this topic in discrete and hybrid systems, thus several related problems are still open.

In the case of discrete event systems modeled by Petri nets, different approaches for observability have been recently proposed. In [6] the problem was that of reconstructing the initial marking (assumed only partially known) from the observation of transition firings. In [8] this approach was extended to the observation and control of timed nets. In other works it was assumed that some of the transitions of the net are not observable [3] or undistinguishable [5], thus complicating the observation problem. In [1] the author has studied the possibility of defining the set of markings reached firing a "partially specified" step of transitions using logical formulas, without having to enumerate this set. In [9] the authors have discussed the problem of estimating the marking of a Petri net using a mix of transition firings and place observations.

In this paper, we study the problem of state estimation of discrete event systems modeled by timed Petri nets. We assume that the set of transitions is split into two subsets: *observable* and *unobservable*. The firing of the observable transitions can be detected, while the firing of the unobservable transitions cannot and the time durations associated to unobservable transitions are unknown. The main idea is to extend the notion of *basis markings* to timed nets. The set of *basis markings* is proposed in [7] to characterize the set of *consistent markings*, i.e., the set of possible markings of a PN after an observed word. Knowing the set of basic markings, the set of consistent markings is obtained from the first one by firing the unobservable transitions.

Using some reduction rules, we show how to reduce both the structure and the state space of the unobservable net. The reduction rules merge indistinguishable transitions, in order to simplify the estimation procedure. To reconstruct the marking of the original net it is necessary to determine the markings of the input/output places of merged transitions. These markings can be expressed as the solution of a linear system that expresses their dependence from the marking of the new places.

Assuming that the time durations of the unobservable transitions are not known, we compute together with the set of basis markings a set of *time equations*. This set represents the relation between the observation and time durations of unobservable firing sequences. The set of time equations is used after to reduce the set of basic markings since according to the time information.

The online algorithm that we propose estimates the state of a timed PN and is based on the following three main steps: (1) compute the set of basis markings; (2) compute the set of time equations; (3) reduce the set of basis markings according to the set of time equations.

This paper is organized as follows: a background on Petri nets are given in Section 2; in Section 3 we characterize the time duration of a firing sequences; reduction rules are in presented in Section 4; and, an online algorithm for state estimation of timed PN is introduced in Section 5.

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## 2. TIMED PETRI NET SYSTEMS WITH UNOBSERVABLE TRANSITIONS

In this section, we recall the basic definition of timed Petri net system (for a general introduction, see [10]).

*Definition 1.* A PN system is a pair $\langle \mathcal{N}, \boldsymbol{m}_0 \rangle$, where $\mathcal{N} = \langle P, T, \boldsymbol{Pre}, \boldsymbol{Post} \rangle$ is a net structure with a set of places $P$; a set of transitions $T$; the pre and post incidence matrices $\boldsymbol{Pre}, \boldsymbol{Post} \in \mathbb{N}_{\geq 0}^{|P| \times |T|}$; and $\boldsymbol{m}_0 \in \mathbb{N}_{\geq 0}^{|P|}$ is the initial marking, where $|P|$ is the number of places and $|T|$ is the number of transitions.

The incidence matrix is $\boldsymbol{C} = \boldsymbol{Post} - \boldsymbol{Pre}$. For every node $v \in P \cup T$, the set of its input and output nodes are denoted as $^\bullet v$ and $v^\bullet$, respectively. A *directed circuit* of PN is a sequence $p_{i1}t_{i1}p_{i2}t_{i2} \cdots p_{in}t_{in}$, where $p_{ij} \in P, t_{ij} \in T, p_{ij} \in {}^\bullet t_{ij}, t_{ij} \in {}^\bullet p_{i,j+1}$, and $\forall j \neq k, p_{ij} \neq p_{ik}$. A net having no directed circuits is called *acyclic*.

A transition $t \in T$ is enabled at a marking $\boldsymbol{m}$ if and only if $\boldsymbol{m} \geq \boldsymbol{Pre}[\cdot, t]$. If a marking $\boldsymbol{m'}$ is reachable from $\boldsymbol{m}$ by firing a sequence $\sigma = t_{i1}t_{i2} \cdots t_{in}$, where $t_{ij} \in T, j = 1, 2, \ldots, n$: the fundamental state equation can be written as $\boldsymbol{m'} = \boldsymbol{m} + \boldsymbol{C} \cdot \boldsymbol{\sigma}$, where $\boldsymbol{\sigma} \in \mathbb{N}_{\geq 0}^{|T|}$ is the firing count vector of $\sigma$; $\boldsymbol{m}[\sigma\rangle$ denotes that $\sigma$ is firable from $\boldsymbol{m}$, while $\boldsymbol{m}[\sigma\rangle \boldsymbol{m'}$ means the firing of $\sigma$ drives $\boldsymbol{m}$ to $\boldsymbol{m'}$.

The set of transitions $T$ is partitioned into two sets: $T_o$ and $T_u$, where $T_o$ is the set of *observable* transitions, whose firing can be detected by an external observer, and $T_u$ is the set of *unobservable* transitions. The firing sequence $\sigma^o$ is an observable firing sequence, if $t \in \sigma^o$, then $t \in T^o$; $\sigma^u$ is an unobservable firing sequence, if $t \in \sigma^u$, then $t \in T^u$. An observation function $\lambda : \sigma \rightarrow T_o^*$, where $T_o^*$ is the Kleene closure of $T_o$, extracts a sequence of observable transitions $\lambda(\sigma)$ from $\sigma$. Let $\sigma = \sigma_1^u \sigma_1^o \sigma_2^u \sigma_2^o \cdots \sigma_n^u$, then $\lambda(\sigma) = \sigma_1^o \sigma_2^o \cdots \sigma_{n-1}^o$. Observable transitions are represented as white rectangles, while unobservable ones as black rectangles.

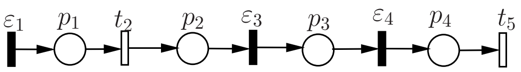


Fig. 1. Example of $w = \lambda(\sigma)$

*Example 2.* For the PN in Fig. 1, observable transitions are $t_2, t_5$, and unobservable transitions are $\varepsilon_1, \varepsilon_3, \varepsilon_4$. Let $\sigma = \varepsilon_1 t_2 \varepsilon_3 \varepsilon_4 t_5$, then the observed word of $\sigma$ is $w = \lambda(\sigma) = t_2 t_5$.

*Definition 3.* A timed PN system is a triple $\langle \mathcal{N}, \boldsymbol{\theta}, \boldsymbol{m}_0 \rangle$, where $\langle \mathcal{N}, \boldsymbol{m}_0 \rangle$ is a PN system and $\boldsymbol{\theta} \in \mathbb{R}_{\geq 0}^{|T|}$ is the time vector that associates to each transition $t_j$ a constant time delay, $\theta_j = \boldsymbol{\theta}[t_j]$.

The time duration of a transition is deterministic, i.e., if a transition $t$ is enabled at time $\tau$, $t$ is fired at $\tau + \theta[t]$. The single server semantic is used, which means a transition cannot be enabled simultaneously more than once.

We make the following assumptions: (A1) the initial marking and net structure are known; (A2) the unobservable induced subnet is acyclic; (A3) The time durations of observable transitions are known, while the time durations of unobservable transitions are unknown.

The second assumption implies that there are not spurious solutions in the unobservable subnet, i.e., all markings, solution of the state equation are reachable. Therefore, the set of basis markings can be characterized using the state equation.

Even if the initial marking is known, because of the partial observation, the state of timed PN's cannot be determined by observation. To characterize the possible set of markings we use a subset of it, which is called the set of *basis markings*. Knowing this set of basis markings, the consistent markings, which are the possible markings in the net system, can be obtained by simply firing the unobservable transitions from the basis markings.

*Definition 4.* [7] Given a marking $\boldsymbol{m}$ and an observable transition $t \in T_o$, we define the set of *explanations* of $t$ at $\boldsymbol{m}$ as $\Sigma(\boldsymbol{m}, t) = \{\sigma \in T_u^* | \boldsymbol{m}[\sigma\rangle \boldsymbol{m'}, \boldsymbol{m'} \geq \boldsymbol{Pre}[\cdot, t]\}$.

The set of *minimal explanations* of $t$ at $\boldsymbol{m}$ as $\Sigma_{min}(\boldsymbol{m}, t) = \{\sigma \in \Sigma(\boldsymbol{m}, t) | \nexists \sigma' \in \Sigma(\boldsymbol{m}, t) : \boldsymbol{\sigma'} \lneqq \boldsymbol{\sigma}\}$, where $\boldsymbol{\sigma'} \lneqq \boldsymbol{\sigma}$ means that for every $t$, $\boldsymbol{\sigma'}[t] \lneqq \boldsymbol{\sigma}[t]$ and there exists $t$ such that $\boldsymbol{\sigma'}[t] < \boldsymbol{\sigma}[t]$.

In the following, the set of basis markings without time is introduced. The set of basis markings of observation $w$ is $\mathcal{M}_b(w)$ and denotes the possible markings according to $w$.

*Definition 5.* The set of basis markings of observation $w = vt$ is defined as $\mathcal{M}_b(w) = \{\boldsymbol{m} \in \mathbb{N}_{\geq 0}^{|P|} | \forall \boldsymbol{m'} \in \mathcal{M}_b(v) : \forall \sigma \in \Sigma_{min}(\boldsymbol{m'}, t), \boldsymbol{m'}[\sigma t\rangle \boldsymbol{m}\}$. For empty word $\epsilon$, $\mathcal{M}_b(\epsilon) = \{\boldsymbol{m}_0\}$.

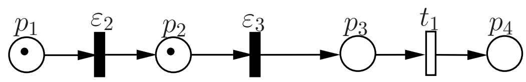


Fig. 2. Example of the set of basis markings

*Example 6.* Let us consider the PN's in Fig. 2 with $\boldsymbol{m}_0 = [1, 1, 0, 0]^T$. The unobservable transitions are $\varepsilon_2$ and $\varepsilon_3$, while the observable transition is $t_1$. Assume $t_1$ has been observed.

The set of basis markings before any observation is $\mathcal{M}_b(\epsilon) = \{\boldsymbol{m}_0\}$, where $\epsilon$ is the empty word. When $w = t_1$ is observed, the set of explanations is $\Sigma(\boldsymbol{m}_0, w) = \{\sigma_1 = \varepsilon_3, \sigma_2 = \varepsilon_2\varepsilon_3\}$. Therefore, the set of minimal explanations is $\Sigma_{min}(\boldsymbol{m}_0, w) = \{\sigma_1\}$. By firing $\sigma_1 t_1$, the marking $\boldsymbol{m}_1 = [1, 0, 0, 1]^T$ is obtained and the new set of basis marking is $\mathcal{M}_b(t_1) = \{\boldsymbol{m}_1\}$.

For a marking $\boldsymbol{m}$ in the set of basis markings, there exists $\sigma$ such that $\boldsymbol{m}_0[\sigma\rangle \boldsymbol{m}$. The sequence $\sigma$ is composed by the observable transitions and unobservable firing sequences, which are minimal explanations. In order to represent the firing sequences that drive the marking from $\boldsymbol{m}_0$ to $\boldsymbol{m}$, based on the set of minimal explanation, we present the set of minimal firing sequences.

*Definition 7.* Given a marking $\boldsymbol{m}$ and an observed word $w = t_{i1}t_{i2} \cdots t_{i,n-1}t_{in}$, we define the set of *firing sequences consistent with* $w$ as $\Gamma(\boldsymbol{m}, w) = \{\sigma \in T^* | \sigma = \sigma_1^u t_{i1} \sigma_2^u t_{i2} \cdots t_{i,n-1} \sigma_n^u t_{in}, \boldsymbol{m}_0[\sigma\rangle \boldsymbol{m}\}$.

Based on $\Gamma(\boldsymbol{m}, w)$, we define the set of *minimal firing sequences* as $\Gamma_{min}(\boldsymbol{m}, w) \subseteq \Gamma(\boldsymbol{m}, w)$, that $\forall \sigma^u \in \sigma$, $\sigma^u$ is a minimal explanation of corresponding marking and observation.

*Definition 8.* The set of basis markings at time $\tau$ of a timed Petri net is defined as $\mathcal{M}_b(w,\tau) = \{\boldsymbol{m} \in \mathcal{M}_b(w) | \exists \sigma \in \Gamma_{min}(\boldsymbol{m}, w), \sigma = \sigma' t, \lambda(\sigma t) = w, t$ is observed at $\tau\}$.

The firing sequences consistent with $w$ defines firing sequences whose observation word is $w$ and lead the system to marking $\boldsymbol{m}$. The set of basis markings at time $\tau$ describes markings obtained from sequences in $\Gamma_{min}(\boldsymbol{m}, w)$.

## 3. TIME DURATION OF FIRING SEQUENCE

In order to estimate the state of a timed PN, it is important to know the time duration of a firing sequence. In this section, we define and analyze such time duration.

Let us consider a firing sequence $\sigma = t_1 t_2 \cdots t_n$. The time duration of $\sigma$ is denoted by $\iota(\sigma)$ and it is defined as the time duration from the enabling of $t_1$ to the firing of $t_n$:

$$\iota(\sigma) = \tau_n - (\tau_1 - \theta_1). \tag{1}$$

*Proposition 9.* Let $\sigma = t_1 t_2 \cdots t_n$, the following equation is satisfied:

$$max\{\theta_1, \ldots, \theta_n\} \leq \iota(\sigma) \leq \sum_{i=1}^{n} \theta_i. \tag{2}$$

If one and only one transition from $\sigma$ is enabled at each time instant, then

$$\iota(\sigma) = \sum_{i=1}^{n} \theta_i \tag{3}$$

**Proof.** If there exists overlapping of time durations, the time duration of the firing sequence is less than the sum of the time durations of all transitions (2). If there is no overlapping, then (3) holds. □

The previous proposition can be generalized to sequences that can be partitioned into subsequences. For example, if $\sigma = \sigma_1 \sigma_2 \cdots \sigma_n$ and at each time moment, the enabled transitions belong to one and only one subsequence $\sigma_i$, then:

$$\iota(\sigma) = \iota(\sigma_1) + \iota(\sigma_2) + \cdots + \iota(\sigma_n). \tag{4}$$

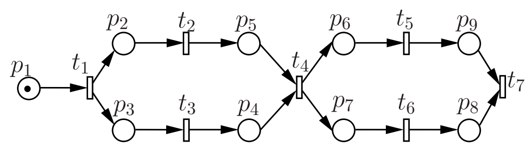


Fig. 3. Example of $\iota(\sigma) = \iota(\sigma_1) + \iota(\sigma_2) + \cdots + \iota(\sigma_n)$

*Example 10.* Let us consider the PN in Fig. 3 with $\boldsymbol{m}_0 = [1,0,0,0,0,0,0,0,0]^T$ and $\boldsymbol{\theta} = [1,2,3,1,2,3,1]^T$. Since it is a deterministic PN, the following observed word is obtained $w = t_1 t_2 t_3 t_4 t_5 t_6 t_7$ at the following time instants $1,3,4,5,7,8,9$.

Let us write $w$ as $w = \sigma = \sigma_1 \sigma_2 \sigma_3 \sigma_4 \sigma_5$, with $\sigma_1 = t_1, \sigma_2 = t_2 t_3, \sigma_3 = t_4, \sigma_4 = t_5 t_6, \sigma_5 = t_7$. According to (1), the time durations are $\iota(\sigma) = 9, \iota(\sigma_1) = 1, \iota(\sigma_2) = 3, \iota(\sigma_3) = 1, \iota(\sigma_4) = 3, \iota(\sigma_5) = 1$. Since the condition in(4) is satisfied,

$$\begin{aligned}\iota(\sigma) &= \iota(\sigma_1) + \iota(\sigma_2) + \iota(\sigma_3) + \iota(\sigma_4) + \iota(\sigma_5) \\ &= 1 + 3 + 1 + 3 + 1 = 9.\end{aligned}$$

## 4. REDUCTION RULES

The firing of unobservable transitions cannot be distinguished by observation. In order to reduce the state space of the unobservable subnet, reductions can be used. In this section, based on [2], reduction rules are proposed to unobservable subnet of an ordinary timed Petri net system. The rules should be applied before state estimation to reduce complexity.

### *4.1 First Reduction Rule*

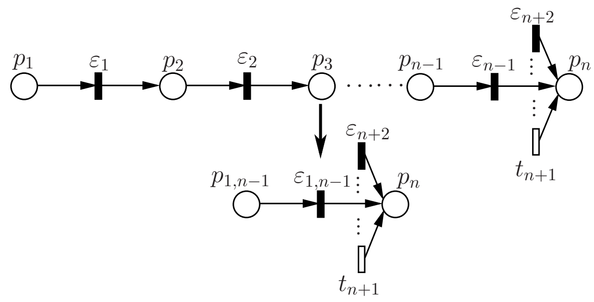

Fig. 4. Illustration of the reduction rule # 1

In Fig. 4, $\varepsilon_1, \cdots, \varepsilon_{n-1}$ are unobservable and, $|{p_1}^\bullet| = 1$; $|^\bullet p_i| = |{p_i}^\bullet| = |^\bullet \varepsilon_j| = |{\varepsilon_j}^\bullet| = 1, i = 2, \ldots, n-1, j = 1, \ldots, n-1$. The unobservable firing sequence $\varepsilon_1 \varepsilon_2 \cdots \varepsilon_{n-1}$ moves a token from $p_1$ to $p_n$ and can be merged into one transition $\varepsilon_{1,n-1}$, such that, in the reduced net, $\boldsymbol{m}[p_{1,n-1}] = \sum_{i=1}^{n-1} \boldsymbol{m}[p_i], \theta_{1,n-1} = \sum_{i=1}^{n-1} \theta_i$.

### *4.2 Second Reduction Rule*

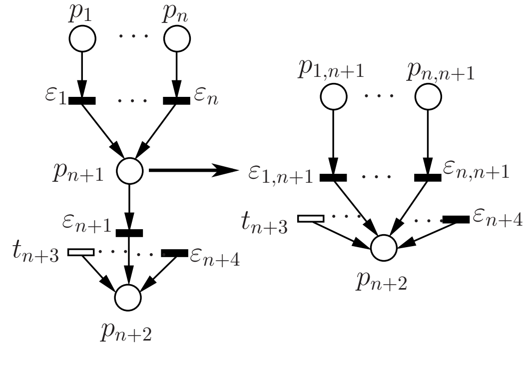

Fig. 5. Illustration of the reduction rule # 2

In Fig. 5, $\varepsilon_1, \cdots, \varepsilon_{n+1}$ are unobservable transitions and $|{p_i}^\bullet| = 1, i = 1, \ldots, n$; $|^\bullet p_{n+1}| = n$ and $|{p_{n+1}}^\bullet| = 1$. The unobservable firing sequence $\varepsilon_1 \varepsilon_{n+1}$ $(\ldots,\ \varepsilon_n \varepsilon_{n+1})$ moves a token from $p_1$ $(\ldots,\ p_n)$ to $p_{n+2}$. Therefore, $\varepsilon_1$ and $\varepsilon_{n+1}$ $(\ldots,\ \varepsilon_n$ and $\varepsilon_{n+1})$ can be merged into one transition $\varepsilon_{1,n+1}$ $(\ldots,\ \varepsilon_{n,n+1})$, such that, $\boldsymbol{m}[p_{i,n+1}] = \boldsymbol{m}[p_i] + \boldsymbol{m}[p_{n+1}], \theta_{i,n+1} = \theta_i + \theta_{n+1}$.

### 4.3 Third Reduction Rule

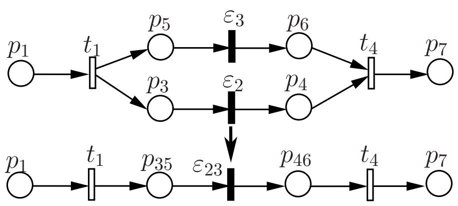


Fig. 6. Illustration of the rule # 3

In Fig. 6, unobservable transitions $\varepsilon_2$ and $\varepsilon_3$ cannot be distinguished in the firing sequence $t_1\varepsilon_2\varepsilon_3t_4$ or $t_1\varepsilon_3\varepsilon_2t_4$. Therefore, $\varepsilon_2$ and $\varepsilon_3$ can be merged into one transition $\varepsilon_{23}$, such that, the time duration is $\theta_{23} = max\{\theta_2, \theta_3\}$. The marking of the reduced net satisfies:

- $\boldsymbol{m}[p_{35}] = \boldsymbol{m}[p_3] + \boldsymbol{m}[p_5], \boldsymbol{m}[p_{46}] = 0$.
- $\boldsymbol{m}[p_{35}] = 0, \boldsymbol{m}[p_{46}] = \boldsymbol{m}[p_4] + \boldsymbol{m}[p_6]$.

## 5. ESTIMATE THE STATE OF CHOICE-FREE NETS

The state estimation mainly includes three steps: (1) the set of basis markings is computed without considering time; (2) the set of time equations is obtained; (3) the set of basis markings is reduced based on the time information.

### 5.1 Computation of $\mathcal{M}_b(wt_j, \tau_j)$

The set of basis markings at time $\tau = 0$ is $\mathcal{M}_b(\epsilon, 0) = \{\boldsymbol{m}_0\}$. Let us assume that the current set of basis markings at time $\tau$ is $\mathcal{M}_b(w, \tau)$, where $w$ is the actual observation. When the firing of a new transition $t_j$ is observed at time $\tau_j$, the following operations should be performed in order to compute $\mathcal{M}_b(wt_j, \tau_j)$.

(1) Let $\mathcal{M}_b(wt_j, \tau_j) = \emptyset$,
(2) For each $\boldsymbol{m} \in \mathcal{M}_b(w, \tau)$,
  (a) compute $\Sigma_{min}(\boldsymbol{m}, t_j)$,
  (b) let $\mathcal{M}' = \{\boldsymbol{m}' | \boldsymbol{m}[\sigma t_j\rangle \boldsymbol{m}', \sigma \in \Sigma_{min}(\boldsymbol{m}, t_j)\}$,
  (c) let $\mathcal{M}_b(wt_j, \tau_j) = \mathcal{M}_b(wt_j, \tau_j) \cup \mathcal{M}'$.

For each basis marking $\boldsymbol{m}$ of the previous set, the set of minimal explanations is computed in $\Sigma_{min}(\boldsymbol{m}, t_j)$. Therefore, when $t_j$ is observed after the firing of the minimal explanations of $\Sigma_{min}(\boldsymbol{m}, t_j)$ from $\boldsymbol{m}$, the new set of basic markings is obtained.

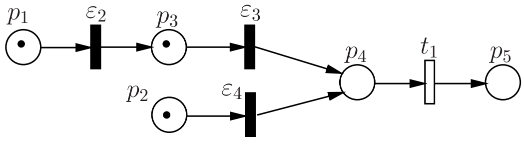


Fig. 7. PN system used in Example 11

*Example 11.* Let us consider the PN's in Fig. 7 with $\theta_1 = 1$ and $\boldsymbol{m}_0 = [1, 1, 1, 0, 0]^T$. The set of minimal firing sequences for the empty word is $\Gamma_{min}(\boldsymbol{m}_0, \epsilon) = \emptyset$, and the set of basis marking at time 0 is $\mathcal{M}_b(\epsilon, 0) = \{\boldsymbol{m}_0\}$.

If $w = t_1$ is observed at time 4, $\mathcal{M}_b(t_1, 4)$ is computed as follows: (1) $\mathcal{M}_b(t_1, 4) = \emptyset$; (2) $\Sigma_{min}(\boldsymbol{m}_0, t_1) = \{\varepsilon_3, \varepsilon_4\}$; (3) $\mathcal{M}' = \{\boldsymbol{m}_1 = [1, 0, 1, 0, 1]^T, \boldsymbol{m}_2 = [1, 1, 0, 0, 1]^T\}$, where $\boldsymbol{m}_0[\varepsilon_4 t_1\rangle \boldsymbol{m}_1$, $\boldsymbol{m}_0[\varepsilon_3 t_1\rangle \boldsymbol{m}_2$; (4) $\mathcal{M}_b(t_1, 4) = \{\boldsymbol{m}_1, \boldsymbol{m}_2\}$.The sets of minimal firing sequences are $\Gamma_{min}(\boldsymbol{m}_1, w) = \{\varepsilon_4 t_1\}, \Gamma_{min}(\boldsymbol{m}_2, w) = \{\varepsilon_3 t_1\}$.

### 5.2 Obtention of the set of time equations

The set of basis markings in the previous section is computed without considering any time consideration. Assuming that the time durations associated to the unobservable transitions are not known, in this section we provide a procedure to obtain a set of equations to characterize all possible time durations associated to these unobservable transitions. It will be shown also how this set of time equations can be used to remove those time-inconsistent markings from the set of basis markings.

Let us assume that the time instant at which $t_j$ was observed is $\tau_j$, while the current set of basis markings is $\mathcal{M}_b(wt_j, \tau_j)$. To each set of basis markings we associate a *set of time equations.* These equations are obtained as the union of different equations. Let $\Gamma = \bigcup_{\boldsymbol{m} \in \mathcal{M}_b(wt_j, \tau_j)} \Gamma_{min}(\boldsymbol{m}, wt_j)$ be the set of all minimal firing sequences of all basis markings. The following time equation is obtained: $min\{\iota(\Gamma)\} = \tau_j$, where $\iota(\Gamma)$ is the set of time durations of each sequence in $\Gamma$. The time equation obtained at time $\tau$ is marked as $o_\tau$.

*Example 12.* In Example 11, the set of basis markings at time 4 has been computed. The set of minimal firing sequences are $\Gamma_{min}(\boldsymbol{m}_1, t_1) = \{\varepsilon_4 t_1\}$ and $\Gamma_{min}(\boldsymbol{m}_2, t_1) = \{\varepsilon_3 t_1\}$. Therefore, $\Gamma = \{\varepsilon_3 t_1, \varepsilon_4 t_1\}$ and the time equation is $o_4 = \min\{\iota(\varepsilon_3 t_1), \iota(\varepsilon_4 t_1)\} = 4$.

This has the following intepretation: because $t_1$ has been fired at 4 and since for its firing, $\varepsilon_3$ or $\varepsilon_4$ should fire the firing delay of at least one of the following sequences $\varepsilon_3 t_1$ and $\varepsilon_4 t_1$ should be 4.

If $t_1$ is observed again at time 6, the sets of minimal explanations are $\Sigma_{min}(\boldsymbol{m}_1, t_1) = \{\varepsilon_3\}$, $\Sigma_{min}(\boldsymbol{m}_2, t_1) = \{\varepsilon_4, \varepsilon_2\varepsilon_3\}$, implying the set of basis markings is $\mathcal{M}_b(t_1t_1, 6) = \{\boldsymbol{m}_3 = [1, 0, 0, 0, 2]^T, \boldsymbol{m}_4 = [0, 1, 0, 0, 2]^T\}$. and the sets of firing sequences consistent with $w = t_1t_1$ are $\Gamma_{min}(\boldsymbol{m}_3, t_1) = \{\varepsilon_4 t_1 \varepsilon_3 t_1\}$ and $\Gamma_{min}(\boldsymbol{m}_4, t_1) = \{\varepsilon_3 t_1 \varepsilon_4 t_1, \varepsilon_3 t_1 \varepsilon_2 \varepsilon_3 t_1\}$, while the corresponding time equation is $o_6 = \min\{\iota(\varepsilon_4 t_1 \varepsilon_3 t_1), \iota(\varepsilon_3 t_1 \varepsilon_4 t_1), \iota(\varepsilon_3 t_1 \varepsilon_2 \varepsilon_3 t_1)\} = 6$.

Let us analyze the time durations of the sequences in $o_6$. First of all, according to the definition of the time duration of a sequence, $\iota(\varepsilon_4 t_1 \varepsilon_3 t_1)$ and $\iota(\varepsilon_3 t_1 \varepsilon_4 t_1)$ provides the same information. The time durations of the two firing sequences are the same. Hence one of this sequence can be removed from $o_6$. Removing for example the second one, we obtain $o_6 = \min\{\iota(\varepsilon_4 t_1 \varepsilon_3 t_1), \iota(\varepsilon_3 t_1 \varepsilon_2 \varepsilon_3 t_1)\} = 6$.

According to $o_4$, $\theta_3 \geq 4 - \theta_1 = 3$. We will show that in $o_6$, $\iota(\varepsilon_3 t_1 \varepsilon_2 \varepsilon_3 t_1) > 6$ hence it is never the one that gives the minimum and can be removed.

$$\iota(\varepsilon_3 t_1 \varepsilon_2 \varepsilon_3 t_1) \geq \theta_3 + \theta_3 + \theta_1 = 2\theta_3 + \theta_1 \geq 7$$

Therefore, $\varepsilon_3 t_1 \varepsilon_2 \varepsilon_3 t_1$ is inconsistent with the time information. It can be deleted from $o_6$, so $o_6 = \iota(\varepsilon_4 t_1 \varepsilon_3 t_1) = 6$, and the corresponding basis marking should be removed, i.e., $\mathcal{M}_b(t_1t_1, 6) = \{\boldsymbol{m}_3 = [1, 0, 0, 0, 2]^T\}$.

As it was illustrated by the previous example, some basis markings are time-inconsistent with the observation. On the other hand, some time equations that are obtained can be redundant.

In order to remove an element $\iota(\sigma_j)$ from a minimum function $o_j$ the following procedure can be used: (i) let $\sigma_j = \sigma_j^1\sigma_j^2\ldots\sigma_j^r$ such that (4) is satisfied, i.e., the time duration of $\sigma_j$ is the sum of time durations of the sub-sequences: $\iota(\sigma_j) = \iota(\sigma_j^1) + \iota(\sigma_j^2) + \ldots, +\iota(\sigma_j^r)$; (ii) find $\sigma_{k,l}^i, i = 1,\ldots$ in $O$ such that they are subsequences of $\sigma_j^l, l = 1,\ldots,r$; according to (2), $\iota(\sigma_j^l) \geq \iota(\sigma_{k,l}^i), \forall i$; (iii) if $\sum_i \iota(\sigma_{k,l}^i) > \tau_j$, where $\tau_j$ is the time instant when $o_j$ is computed, $\iota(\sigma_j)$ should be removed from $o_j$.

*Proposition 13.* Let $O$ be the current set of time equations, where

$$O = \left\{ \begin{array}{l} min\{\iota(\sigma_{1,1}), \iota(\sigma_{1,2}), \ldots, \iota(\sigma_{1,k_1})\} = \tau_1, \\ min\{\iota(\sigma_{2,1}), \iota(\sigma_{2,2}), \ldots, \iota(\sigma_{2,k_2})\} = \tau_2, \\ \vdots \\ min\{\iota(\sigma_{q,1}), \iota(\sigma_{q,2}), \ldots, \iota(\sigma_{q,k_q})\} = \tau_q, \end{array} \right\}$$

and let $o_j$ be the time equation obtained at time $\tau_j > \tau_q$, where $o_j : min\{\iota(\sigma_{j,1}), \iota(\sigma_{j,2}), \ldots, \iota(\sigma_{j,k_j})\} = \tau_j$, with $q, k_q, j \in \mathbb{N}_{>0}$.

Let $\iota(\sigma_j) \in \{\iota(\sigma_{j,1}), \iota(\sigma_{j,2}), \ldots, \iota(\sigma_{j,k_j})\}$ and decompose $\sigma_j$ as $\sigma_j = \sigma_j^1\sigma_j^2\ldots\sigma_j^r$. Find all $\sigma_{k,l}^i$ in $O$ such that $\sigma_{k,l}^i$ is a subsequence of a $\sigma_j^l$ and $\mathrm{f}\iota(\sigma_{k,j}^i) \geq \sigma_j^l, \forall l$. If $\sum_i \iota(\sigma_{k,j}^i) > \tau_j$ then remove $\iota(\sigma_j)$ from $o_j$.

**Proof.** Obviously, If the previous conditions are satisfied, $\iota(\sigma_j) > \tau_j$. Hence it is not timed-consistent with the observation. □

### 5.3 Algorithm for state estimation

In this section, we present an algorithm for state estimation of systems modeled by timed PN's. When a new observation is available, the four steps in Algorithm 1 are performed.

**Algorithm 1** Estimate the state of timed PN's

1: Compute the set of basis markings $\mathcal{M}_b(wt_j, \tau_j)$ based on the current observation $t_j$ at $\tau_j$.
2: Compute the time equation $o_j$.
3: Reduce $o_j$ based on Prop. 13.
4: Reduce the set of basis markings $\mathcal{M}_b(wt_j, \tau_j)$ accordingly.

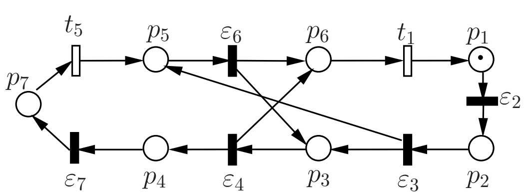


Fig. 8. Example of the algorithm

*Example 14.* Let us consider the PN in Fig. 8. with observable transitions $t_1$ and $t_5$, $\theta_1 = \theta_5 = 1$, and the initial marking $\boldsymbol{m}_0 = [p_1, p_2, p_3, p_4, p_5, p_6, p_7]^T = [1, 0, 0, 0, 0, 0, 0]^T$. Apply reduction rule # 1, transitions $\varepsilon_2$ and $\varepsilon_3$ are merged into $\varepsilon_{23}$, and places $p_1$ and $p_2$ are merged into $p_{12}$. Fig. 9 shows the reduced model. The initial marking $\boldsymbol{m}_0 = [p_{12}, p_3, p_4, p_5, p_6, p_7]^T = [1, 0, 0, 0, 0, 0]^T$

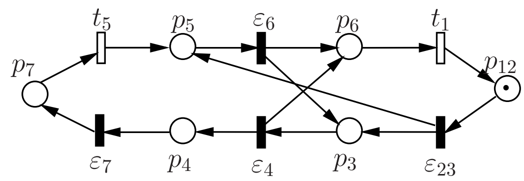


Fig. 9. A PN system to illustrate the steps of the state estimation algorithms.

The state estimation algorithm is applied on the reduced PN in Fig. 9. Let us assume the following observations: $t_1$ at 5, 9 and $t_5$ at 10.

- At time 0, the set of basis markings is $\mathcal{M}_b(\epsilon, 0) = \{\boldsymbol{m}_0\}$ and the set of time equations is $O = \emptyset$.

- At time 6, $t_1$ is observed ($w = t_1$). The set of minimal explanations is $\Sigma_{min} = (\boldsymbol{m}_0, t_1) = \{\sigma_1 = \varepsilon_{23}\varepsilon_6, \sigma_2 = \varepsilon_{23}\varepsilon_4\}$, meaning that $\sigma_1$ or $\sigma_2$ has been fired in order to enable $t_1$. By firing $\sigma_1 t_1$ and $\sigma_2 t_1$, the set of basis markings is obtained as $\mathcal{M}_b(w, 6) = \{\boldsymbol{m}_1 = [1, 2, 0, 0, 0, 0]^T, \boldsymbol{m}_2 = [1, 0, 1, 1, 0, 0]^T$, and the sets of minimal firing sequences are $\Gamma_{min}(\boldsymbol{m}_1, w) = \{\sigma_1 t_1\}$ and $\Gamma_{min}(\boldsymbol{m}_2, w) = \{\sigma_2 t_1\}$. The time equation at time 6 is $min\{\iota(\sigma_1 t_1), \iota(\sigma_2 t_1)\} = 6$, the only equation that will compose $O$.

- At time 9, $w = t_1 t_1$ and the sets of minimal explanations are $\Sigma_{min}(\boldsymbol{m}_1, t_1) = \{\sigma_1, \varepsilon_4\}, \Sigma_{min}(\boldsymbol{m}_2, t_1) = \{\sigma_2, \varepsilon_6\}$.

By firing $\sigma_1 t_1$ and $\varepsilon_4 t_1$ from $\boldsymbol{m}_1$, we obtain $\boldsymbol{m}_3 = [1, 4, 0, 0, 0, 0]^T$ and $\boldsymbol{m}_4 = [2, 1, 1, 0, 0, 0]^T$, respectively; by firing $\sigma_2 t_1$ and $\varepsilon_6 t_1$ from $\boldsymbol{m}_2$, $\boldsymbol{m}_4$ and $\boldsymbol{m}_5 = [1, 0, 2, 2, 0, 0]^T$ are obtained. Therefore, $\mathcal{M}_b(w, 9) = \{\boldsymbol{m}_3, \boldsymbol{m}_4, \boldsymbol{m}_5\}$ and

$$\begin{array}{l} \Gamma_{min}(\boldsymbol{m}_3, w) = \{\sigma_3 = \sigma_1 t_1 \sigma_1 t_1\}, \\ \Gamma_{min}(\boldsymbol{m}_4, w) = \{\sigma_4 = \sigma_1 t_1 \varepsilon_4 t_1, \sigma_6 = \sigma_2 t_1 \varepsilon_6 t_1\}, \\ \Gamma_{min}(\boldsymbol{m}_5, w) = \{\sigma_5 = \sigma_2 t_1 \sigma_2 t_1\}. \end{array}$$

From previous sets the time equation at time 9 is obtained as $min\{\iota(\sigma_3), \iota(\sigma_4), \iota(\sigma_5), \iota(\sigma_6)\} = 9$.

Observe that $\sigma_3 = \sigma_1(t_1\sigma_1)t_1$ satisfying Prop. 13, and $\iota(\sigma_3) = \iota(\sigma_1) + \iota(t_1\sigma_1) + \iota(t_1)$. Form the equations of $O$ can be observed immediately that $\iota(t_1\sigma_1) \geq 6$ and $\iota(\sigma_1) = \iota(t_1\sigma_1) - \theta_1 \geq 5$. Therefore, $\iota(\sigma_3) \geq 5 + 6 + 1 = 12 > 11$. Hence, $\iota(\sigma_3)$ should be removed. For the same reason, $\iota(\sigma_5)$ is also redundant and can be removed. The set of time equations becomes:

$$O = \left\{ \begin{array}{l} min\{\iota(\sigma_1 t_1), \iota(\sigma_2 t_1)\} = 6, \\ min\{\iota(\sigma_4), \iota(\sigma_6)\} = 9. \end{array} \right\}$$

The set of basis markings is reduced to $\mathcal{M}_b(w, 9) = \{\boldsymbol{m}_4\}$.

- At time 10, $t_5$ is observed ($w = t_1 t_1 t_5$). The set of minimal explanations is $\Sigma_{min} = (\boldsymbol{m}_4, t_5) = \{\varepsilon_7\}$. Firing $\varepsilon_7 t_5$, the set of basis markings is obtained as $\mathcal{M}_b(w, 10) =$

$\{\boldsymbol{m}_6 = [2,1,0,1,0,0]^T\}$, and the set of minimal firing sequences as $\Gamma_{min}(\boldsymbol{m}_6, w) = \{\sigma_7 = \sigma_4\varepsilon_7 t_5, \sigma_8 = \sigma_6\varepsilon_7 t_5\}$. The time equation obtained at this time moment is $min\{\iota(\sigma_7), \iota(\sigma_8)\} = 10$. Hence, the set of time equations is

$$O = \left\{ \begin{array}{l} min\{\iota(\sigma_1 t_1), \iota(\sigma_2 t_1)\} = 6, \\ min\{\iota(\sigma_4), \iota(\sigma_6)\} = 9, \\ min\{\iota(\sigma_7), \iota(\sigma_8)\} = 10. \end{array} \right\}$$

Being an online procedure, seems that the set of time equations is growing indefinitely. However, dealing only with time deterministic Petri nets, this is not true and there exists a moment from which any other time equation does not provide new information and the set of time equations is not updated anymore.

In the following, we discuss the time in a structurally live (SL) and structurally bounded (SB) choice-free net with a minimal T-semiflow $\boldsymbol{x}$. We assume the upper bound of time duration of every transition is $u$, and then the upper bound of a firing vector $\sigma$ is $u(\boldsymbol{\sigma}) = u \cdot \sum_{i=1}^{|T|} \boldsymbol{\sigma}[i]$. Let $\boldsymbol{m}_h$ be *home state*, i.e., it can be reached from every reachable marking[4]. Based on [11], $\boldsymbol{m}_h$ will be reached by a firing sequence $\sigma_h$, with $\boldsymbol{\sigma}_h \leq \boldsymbol{x}$.

*Proposition 15.* In a SL&SB choice-free net with minimal T-semiflow $\boldsymbol{x}$, if the initial marking is live, it is not necessary to update the set of time equations after the time instant $2 \cdot u(\boldsymbol{x})$.

**Proof.** Because the net is SL&SB and the initial marking is live, then there exists a circle in the reachability graph and a home state $\boldsymbol{m}_h$. From $\boldsymbol{m}_0$, after firing $\sigma_h$, the home state is reached and the system behavior starts to repeat. Therefore, from this moment, it is not necessary to update the set of time equations. □

### 5.4 Extension to nets with choices

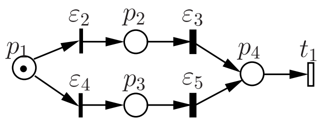


Fig. 10. Example of PN's with choice

Let us consider the PN in Fig. 10 with $\varepsilon_2$ and $\varepsilon_4$ immediate transitions, i.e., $\theta_2 = \theta_4 = 0, \theta_1 = 1$, and $\boldsymbol{m}_0 = [1,0,0,0]^T$. Assume $t_1$ is observed at time 4. Obviously, $\varepsilon_2\varepsilon_3$ or $\varepsilon_4\varepsilon_5$ has been fired to enable $t_1$, but we don't know exactly which one. Since $t_1$ has been observed at 4, we can say that $\iota(\varepsilon_2\varepsilon_3 t_1)$ or $\iota(\varepsilon_4\varepsilon_5 t_1)$ is 4, but we cannot say nothing about the time duration of the other. Hence, we cannot say that the minimum of $\iota(\varepsilon_2\varepsilon_3 t_1)$ and $\iota(\varepsilon_4\varepsilon_5 t_1)$ is 4.

Therefore, to apply the algorithm to general nets, there exist two possibilities: (1) reduce the net using the reduction rules, to obtain a choice-free one (2) treat each choice separated, i.e., enumerate all possible combinations of firing sequences. This approach is similar with the one of state estimation of untimed PN's.

## 6. CONCLUSIONS AND FUTURE WORK

In this paper, we provide an algorithm for state estimation of timed choice-free PNs. First, an algorithm to compute the set of consistent markings is given and then, the time information are grouped into a set of time equations that is used to reduce the set of consistent markings. Some reduction rules are presented that can be used also to reduce the state space of the timed systems merging the indistinguishable transitions. Finally, we discuss the general case, i.e., nets with choices, and we show that the procedure is similar with the standard one of untimed Petri nets. As a future work we plan to extend these rules and also to implement the algorithm in MATLAB.

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