Representing Partial and Uncertain Sensorial Information Using the Theory of Symmetries*

Juan D. Tardós

Dpto. de Ingeniería Eléctrica e Informática
Centro Politécnico Superior, Universidad de Zaragoza
María de Luna 3, 50015 Zaragoza, SPAIN

Abstract

In this paper, we propose a general model for representing sensorial information and its uncertainty, the Symmetries and Perturbation model (SPmodel). In it, the intrinsic partiality of geometric information is represented in terms of the symmetries of the involved geometric elements. Location uncertainty due to sensor imprecision is represented by means of a local perturbation, expressed in a reference frame attached to the geometric element, with an associated probabilistic model.

Using the SPmodel, we develop a method for integrating geometric information that allows to estimate the location of a feature or an object from a set of partial and uncertain observations. The integration mechanism is based on extended Kalman filter theory.

1 Introduction

A simple sensorial observation informs about the location in space of local elements from the surface of objects, such as planes, edges, vertices, etc. We will call such elements *geometric features*. Each observation considered alone may *partially* locate the corresponding feature, and is always affected by *uncertainty* due to sensor imprecision. To complete the information about feature location, we need to integrate observations taken from different viewpoints or with other sensors. The process of recognizing and locating an object is based on the matching of geometric features observed in the scene with features of object models. The location of a feature also informs partially about the location of the object in space. For the recognition and location process to be robust, we must integrate information about several features.

To better visualize the notion of the partiality of geometric information, let us consider the example of figure 1. In it, several points of an edge are located using a mobile proximity sensor (for example, a laser proximity sensor mounted on the robot hand). The location of an edge, with unknown extremities, leaves two undetermined degrees of freedom (d.o.f.) on object location: position along the edge and orientation around it. Moreover, there are two alternative solutions, corresponding to the two opposite edge orientations. An intuitive explanation is that an edge has *translational symmetry* along it, *rotational symmetry* around it, and a *cyclic symmetry* of 180 degrees around any axis perpendicular to it. In a similar way, the constraints imposed by each point on the edge location are related to the symmetries of both geometric elements.

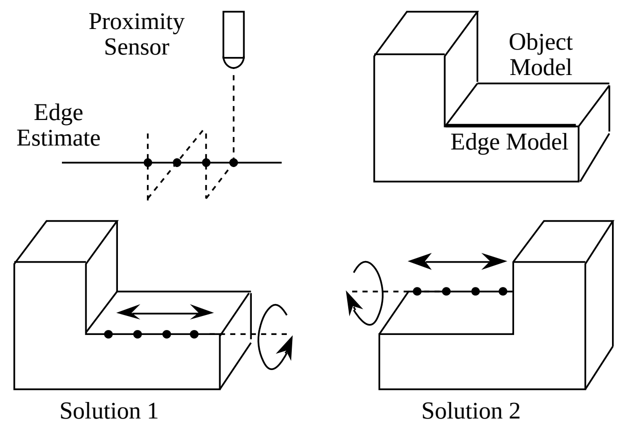

Figure 1: Observation of an edge, and partiality of information about object location.

This paper is related to [4] and [8], in which location constraints imposed by the matching of two equal or complementary features are represented using subgroups of transformations or symmetries. In [8], a fairly general method to find the solution of a set of

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constraints is proposed. It determines the existence and uniqueness of solution, and finds it in the majority of cases of practical interest. However, it is based on the *exact matching* of features and thus cannot be applied to calculate the location of an object from a set of observed features, due to the *uncertainty* of sensor observations.

The problem of representing and integrating uncertain sensorial information has been studied by several authors. The main approach proposed is to use optimal estimation techniques based on a probabilistic model of uncertainty (usually Gaussian). In sensor data fusion, the relationship between the location to estimate and some measure of it is in general nonlinear. In this case, suboptimal estimations can be obtained using an extended Kalman filter (EKF). Some relevant works following this approach are [1, 3, 5, 6].

A basic drawback of these approaches is that they use a different set of parameters to represent the location of each type of geometric element, according to the number of d.o.f. that determine its location. In this paper, we briefly introduce the theory of symmetries (§2), and we use it to propose a probabilistic model of location and its uncertainty that is completely general: the SPmodel (§3). Based on this model and on Kalman filter theory, we develop an integration method (§4) that allows to obtain a suboptimal estimation of the location of a feature or an object from a set of partial and uncertain observations.

2 Theory of symmetries

2.1 Transformations: notation

A transformation is a bijective mapping from Euclidean space $I\!R^3$ in itself, that preserves distance, and handedness of reference systems. The set of transformations T, with the composition operation, forms a noncommutative group: composition is associative, with identity element $1\!\!1$, and every element t has an inverse t^{-1}. A transformation t_{AB} that defines the location of a reference B relative to a reference A may be represented by a vector composed of three cartesian coordinates and three Roll-Pitch-Yaw angles, that we call *location vector:*

$$\mathbf{x}_{AB} = (x,\ y,\ z,\ \psi,\ \theta,\ \phi)^T$$

$$t_{AB} = Transl(x, y, z)\ Rot(z, \phi)\ Rot(y, \theta)\ Rot(x, \psi)$$

The composition and inversion of location vectors are represented with operators $\oplus$ and $\ominus$ respectively.

2.2 Symmetries and subgroups of transformations

In this section we formally introduce the concept of symmetry, and we present some results relating symmetries, geometric relations, and subgroups of transformations.

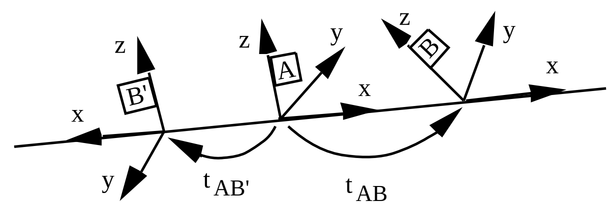


Figure 2: Examples of symmetries of an infinite edge.

Definition 1 Symmetries of a geometric element. *Given a geometric element defined by a set of points $E \subseteq I\!R^3$, its symmetries are the set of transformations that preserve E.*

We express such symmetries in a reference attached to the geometric element, and we denote them by $\mathcal{S}_\alpha$, where α represents the type of geometric element. For example, the symmetries of an infinite edge (in practice, an edge with unknown extremities), using an associated reference A with its x axis aligned with the edge (figure 2), are:

$$\mathcal{S}_{edge} = \{Transl(a, 0, 0)\ Rot(y, k\pi)\ Rot(x, \psi) \mid a \in I\!R;\ \psi \in (-\pi, \pi];\ k \in \{0, 1\}\}$$

Intuitively, the symmetries of an element are the relative transformations between all possible associated references that represent the same location of the element in space. In this sense, symmetries represent the intrinsic *partiality* of information related to the location of a geometric element. This idea can be formalized as follows:

Proposition 1 *Two geometric elements E and F of type α are coincident iff $t_{EF} \in \mathcal{S}_\alpha$, where t_{EF} is the relative transformation between their associated references.*

The next proposition states the relationship between equivalence geometric relations and subgroups of transformations. It can be easily proven using the definitions of subgroup and equivalence relation [7].

Proposition 2 *Let E_α be the set of geometric elements of type α, S a nonempty subset of T, and $\Re_s$ a binary geometric relation defined in E_α by:*

$$\forall E, F \in E_\alpha,\ E\ \Re_s\ F \Leftrightarrow t_{EF} \in S$$

Then, $\Re_s$ is an equivalence relation iff S is a subgroup of T.

This result states that all equivalence relations between elements of the same type, which depend on their relative location, define a subgroup of transformations. A particular case of such relation is *coincidence*, that can be expressed by the condition $t_{EF} \in \mathcal{S}_\alpha$. This fact leads to the following result:

Corollary 1 *The symmetries $\mathcal{S}_\alpha$ of a type of geometric element form a subgroup of T.*

There are other interesting equivalence geometric relations, as for example *parallelism*: according to proposition 2, the set of relative transformations between parallel planes or edges are subgroups of T.

2.3 Subgroups of transformations

In this section we present a classification of the subgroups of T in conjugation classes. Given a subgroup S_1 of T, and a constant transformation t, the set of transformations

$$S_2 = t\ S_1\ t^{-1} = \{t\ s_1\ t^{-1} \mid s_1 \in S_1\}$$

is a *conjugate subgroup* of S_1. The conjugate subgroups of S_1 form an equivalence class named *conjugation class*. As the mapping defined by $f_t(s_1) = t\ s_1\ t^{-1}$ is an isomorphism, all the subgroups in a conjugation class have the same structure and properties. Intuitively, S_1 and S_2 represent the same type of motions expressed in two different references, related by transformation t.

Table 1 shows a classification of subgroups of T in conjugation classes, that summarizes the classifications used in [4] and [8], with some changes in notation. We can distinguish three types of subgroups:

- *Translational and rotational subgroups* ($\mathcal{M}_\alpha$). These subgroups correspond to continuous motions, that can always be expressed as sets of translations and rotations in the axis of a cartesian reference.
- *Cyclic subgroups* ($\mathcal{C}_\alpha$). They comprise the revolution symmetries of regular polygons, the *reverse* subgroup corresponding to the change in orientation of an edge or a plane, and the prismatic symmetries that are a combination of both.
- *Other subgroups.* There are other subgroups corresponding to the symmetries of helicoids, lattices, and regular polyhedrals, that we do not consider, because such elements are not geometric features that can be extracted with simple perception operations.

The classification of table 1 is not exhaustive, in the sense that there are other subgroups formed by the composition of two subgroups of different types. For example, $T_x R_x C_{2y}$ is a subgroup corresponding to the symmetries of an infinite edge.

Notation	Type of Transformations	D.o.f.	
$\mathcal{M}_\alpha$	*Translations and Rotations*	T	R
$\mathbb{1}$	Identity transformation	0	0
R_x	Rotation around an edge	0	1
R_{xyz}	Rotation around a point	0	3
T_x	Translation on an edge	1	0
$T_x R_x$	Movement on an edge	1	1
T_{xy}	Translation on a plane	2	0
$T_{xy} R_z$	Movement on a plane	2	1
T_{xyz}	Generic translation	3	0
$T_{xyz} R_z$	Movement of a parallel plane	3	1
$T_{xyz} R_{xyz}$	Generic transformation	3	3
$\mathcal{C}_\alpha$	*Cyclic Symmetries*	Elem.	
C_{nx}	Polygonal symmetry	n	
C_{2y}	Reverse	2	
$C_{nx} C_{2y}$	Prismatic symmetry	2n	
...	*Helicoidal, Crystallographic,*		
...	*and Polyhedric*		

$T_x = \{Transl(a,0,0) \mid a \in \mathbb{R}\}$
$T_{xy} = T_x T_y = \{Transl(a,b,0) \mid a,b \in \mathbb{R}\}$
$T_{xyz} = T_x T_y T_z = \{Transl(a,b,c) \mid a,b,c \in \mathbb{R}\}$
$R_x = \{Rot(x,\psi) \mid \psi \in (-\pi,\ \pi]\}$
$R_{xyz} = R_x R_y R_z = \{Rot(\mathbf{u},\theta) \mid \mathbf{u} \in \mathbb{R}^3, \theta \in (-\pi,\pi]\}$
$C_{nx} = \{Rot(x, 2\pi k/n) \mid k \in \{0,\ldots,n-1\}\ \}$
$C_{2y} = \{Rot(y, k\pi) \mid k \in \{0,1\}\ \}$

Table 1: Classification of transformation subgroups.

2.4 Symmetries of features and sensorial observations

According to the preceding section, the symmetries of a geometric feature of type α can be expressed as:

$$\mathcal{S}_\alpha = \mathcal{M}_\alpha \mathcal{C}_\alpha$$

where $\mathcal{M}_\alpha$ is a symmetry or subgroup of continuous motion (translation and rotation) and $\mathcal{C}_\alpha$ is a cyclic symmetry. The first represents the d.o.f. not determined by the feature location. The second, a finite set of different ways of matching an observed feature with a feature in the model of an object. This gives different hypotheses of location that must be verified with more sensorial information. Figure 3 shows several types of geometric features with their symmetries, expressed in their associated references.

To represent the location of a sensorial observation we also use an associated reference. We call *symmetries of an observation* the set of relative transformations between references corresponding to *equivalent* locations. By proposition 2, such symmetries also form a subgroup of transformations.

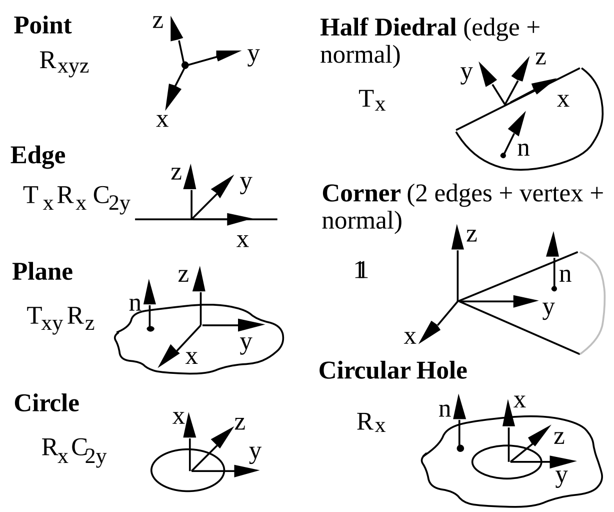


Figure 3: Examples of features and their symmetries.

In the case of observations that correspond directly to a geometric element, as for example a *point* on an edge obtained with a proximity sensor, the symmetries of the observation are those of the element. Otherwise, the observation has its own symmetries. For example, if we obtain only the orientation of a plane using photometric stereo, and we associate to it a reference with its z axis along the normal of the plane, the symmetries of the observation are $T_{xyz}R_z$.

2.5 Pairing geometric elements

The basic integration scheme for geometric sensorial information follows two steps:

1. Integration of several observations to estimate the location of a feature.

2. Integration of several estimated features to recognize and locate an object.

Both steps are based on the pairing of two geometric elements. In the first case, each observation is paired with a feature. In the second, each estimated feature is paired with a feature in the object model. Pairing two such elements imposes constraints on their relative location.

We call *pairing relation* $\mathcal{P}_{\alpha\beta}$ the set of possible values of the relative transformation between two paired geometric elements of type α and β. In this section we propose a method to represent pairing constraints using *binding matrices*.

Definition 2 Binding matrix. *We call binding matrix a row-selection matrix B of dimensions $n \times 6$ and maximal range, formed by n rows of the identity matrix I_6, taken in order, with $n \leq 6$.*

Definition 3 Self-binding matrix. *Given a geometric feature or a sensorial observation of type α, its self-binding matrix is a binding matrix B_α such that its motion symmetries are:*

$$\mathcal{M}_\alpha = \{t_{AB} \in T \mid B_\alpha \, \mathbf{x}_{AB} = 0\}$$

It is important to note that such matrix exists for *all subgroups* in table 1, and thus for *all possible types of features and observations.* B_α can be obtained by selecting the rows of I_6 corresponding to the d.o.f. not included in the subgroup $\mathcal{M}_\alpha$. For example, in the case of a plane, $M_\alpha = T_{xy}R_z$, and

$$B_\alpha = \begin{pmatrix} 0\,0\,1\,0\,0\,0 \\ 0\,0\,0\,1\,0\,0 \\ 0\,0\,0\,0\,1\,0 \end{pmatrix}$$

Definition 4 Binding matrix of a pairing. *Given a pairing relation $\mathcal{P}_{\alpha\beta}$ between two geometric elements of type α and β, we call binding matrix of this pairing (provided it exists) a binding matrix $B_{\alpha\beta}$ such that*

$$\mathcal{P}_{\alpha\beta} = \{t_{AB} \in T \mid B_{\alpha\beta} \, \mathbf{x}_{AB} = 0\}$$

When pairing an estimated feature E with a model feature M, both of type α, the pairing relation corresponds to the continuous motion symmetries of this feature $\mathcal{P}_{\alpha\alpha} = \mathcal{M}_\alpha$. In this case, the pairing binding matrix *always* exists, and coincides with the self-binding matrix of the feature. The pairing constraint can be written in either of these ways:

$$B_\alpha \, \mathbf{x}_{EM} = 0 \quad ; \quad B_\alpha \, \mathbf{x}_{ME} = 0$$

When pairing an observation P with an estimated feature E, in the *majority of cases of practical interest* it is possible to find at least one of the two possible binding matrices B_{EP} or B_{PE} (a more formal analysis of this issue can be found in [7]). Therefore, the pairing constraint can be written in one of these ways:

$$B_{EP} \, \mathbf{x}_{EP} = 0 \quad \text{(direct constraint)}$$

$$B_{PE} \, \mathbf{x}_{PE} = 0 \quad \text{(inverse constraint)}$$

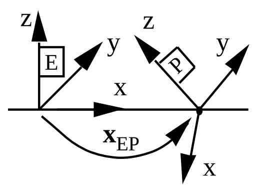
Two examples are shown in figure 4. The first one represents the pairing of an edge with a point belonging to it. The second one represents the pairing between an edge and its 2D observation made with a vision system. In this case, the observation is represented by the plane that contains the edge and the camera's optical center, with an associated reference with origin on the plane, and z axis along its normal.

Point on edge

$\mathcal{P} = T_x R_{xyz}$

$\mathbf{x}_{EP} = (x\ 0\ 0\ \psi_x\ \theta_y\ \phi_z)^T$

$B_{EP} = \begin{pmatrix} 0\,1\,0\,0\,0\,0 \\ 0\,0\,1\,0\,0\,0 \end{pmatrix}$



Edge observed by 2D vision

$\mathcal{P} = T_{xy} R_z R_x$

$\mathbf{x}_{PE} = (x\ y\ 0\ \psi_x\ 0\ \phi_z)^T$

$B_{PE} = \begin{pmatrix} 0\,0\,1\,0\,0\,0 \\ 0\,0\,0\,0\,1\,0 \end{pmatrix}$

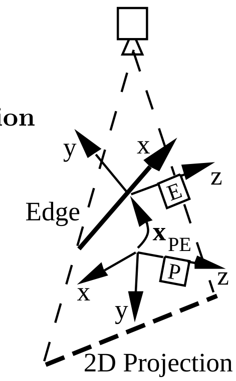


Figure 4: Examples of pairing binding matrices.

3 Representing uncertain geometric information

3.1 Classic probabilistic model

In previous works about integration of geometric information, the location of a geometric element E relative to a reference A is represented by a parameter vector $\boldsymbol{\theta}_{AE}$, and the knowledge about this location, by the mean and covariance of a probability distribution function associated to it [1, 2, 3, 6]:

$$\hat{\boldsymbol{\theta}}_{AE} = E[\boldsymbol{\theta}_{AE}] \quad ; \quad C_{AE} = Cov(\boldsymbol{\theta}_{AE})$$

The mean represents the estimated location, and the covariance its uncertainty. This type of representation has several drawbacks:

- A different parameter vector is used for each type of geometric element. For example, the location of an edge may be given by a point and its director vector [1, 2], or by the four parameters appearing in its equation [3]. The location of a plane may be represented by its normal vector and a point [2], the normal and the distance to the origin [1], or by the three parameters defining its equation [3]. Furthermore, the laws of parameter transformation between references are different in each case and usually quite complex.

- The value of uncertainty depends on the base reference used. Thus, the magnitude of values in the covariance matrix does not reflect the importance of errors. For example, a small uncertainty in the orientation of an object gives a very big or a very small variance in the distance of its planes to the origin, depending on the proximity of the object to the origin.

- The majority of representations used have *singularities* for some parameter values. Near these values, the precision of computations made with covariance matrices decreases drastically [6].

- Some representations are overparameterized, and may pose two problems when integrating information: either the value of some parameters is undetermined, giving infinite values in covariance matrices, or the set of parameters must verify several constraints (usually nonlinear), giving singular covariance matrices.

All these drawbacks have lead us to define a new probabilistic model that allows to represent the location of any type of geometric feature or sensorial observation, and its uncertainty: the SPmodel.

3.2 The symmetries and perturbation model

The SPmodel associates a reference to every geometric element. The estimation of its location is represented by the estimated location vector relative to a base reference, and the estimation error by a differential location vector relative to the reference attached to the element:

$$\mathbf{x}_{AE} = \hat{\mathbf{x}}_{AE} \oplus \mathbf{d}_E$$

In $\mathbf{d}_E$, we assign a null value to the d.o.f. corresponding to the motion symmetries of the geometric element, because they do not represent an effective location error. Then, if B_E with dimension $e \times 6$ is the self-binding matrix of element E, there are $6 - e$ elements in $\mathbf{d}_E$ that must be zero. We call *perturbation vector* to a vector $\mathbf{p}_E$ of dimension e formed by the rest of elements of $\mathbf{d}_E$. Both vectors can be related by the self-binding matrix as follows:

$$\mathbf{d}_E = B_E^T \mathbf{p}_E \quad ; \quad \mathbf{p}_E = B_E \mathbf{d}_E$$

The SPmodel represents the information about the location of a geometric element by a triplet $(\hat{\mathbf{x}}_{AE}, \hat{\mathbf{p}}_E, C_E)$ where:

$$\mathbf{x}_{AE} = \hat{\mathbf{x}}_{AE} \oplus B_E^T \mathbf{p}_E$$
$$\hat{\mathbf{p}}_E = E[\mathbf{p}_E]$$
$$C_E = Cov(\mathbf{p}_E)$$

Transformation $\hat{\mathbf{x}}_{AE}$ is an estimation taken as base for perturbations, $\hat{\mathbf{p}}_E$ the estimated value of the perturbation vector, and C_E its covariance. When $\hat{\mathbf{p}}_E = 0$, we say that the estimation is *centered*.

The main advantage of this model is its generality: it is valid for any geometric feature or sensorial observation. Moreover, the representation of uncertainty using a perturbation vector does not depend on the base reference used, has a clear interpretation, and is not overparameterized. Problems related to singularities are also avoided: location vectors only have singularities in $\theta = \pm\pi/2$, and $\mathbf{p}_E$ has values near zero, being far from these singularities. Thus, covariance matrices are always well defined.

4 Integrating sensorial information

In this section we propose a general integration method based on extended Kalman filter theory, that uses the SPmodel and exploits the concept of binding matrices.

4.1 Extended Kalman filter

Under the Gaussianity hypothesis, the Kalman filter allows to obtain an optimal minimum variance estimation of the state of a linear dynamic system. In the nonlinear case, its extended version (EKF) obtains a suboptimal estimation, using a linear approximation of the system equations. This estimation can be improved by iteration.

The problem of estimating the location of a feature or of an object from a set of observations is nonlinear, due to the existence of orientation terms, and thus can be solved with EKF. We consider the estimation of static locations, and we only need a part of the filter equations (dropping the equations that predict the next state).

We use a measurement model with implicit function, similar to that proposed in [1]:

$$\mathbf{f}_k(\mathbf{x}, \mathbf{y}_k^t) = 0 \quad ; \quad \mathbf{y}_k^m = \mathbf{y}_k^t + \mathbf{u}_k \quad ; \quad \mathbf{u}_k \sim N(0, S_k)$$

where $\mathbf{x}$ is the state to be estimated, $\mathbf{y}_k^t$ is the theoretical value of k-th observation (value that should be obtained without measurement errors), and $\mathbf{y}_k^m$ is the measured value, that is affected by a white Gaussian noise $\mathbf{u}_k$, with zero mean and known covariance S_k. Linearizing $\mathbf{f}_k$ around $\hat{\mathbf{x}}_{k,j}$ and $\mathbf{y}_k^m$ we obtain:

$$-\mathbf{h}_{k,j} + H_{k,j}\hat{\mathbf{x}}_{k,j} = H_{k,j}\mathbf{x} + \mathbf{v}_{k,j}$$
$$\mathbf{v}_{k,j} \sim N(0, G_{k,j} S_k G_{k,j}^T)$$

where:

$$\mathbf{h}_{k,j} = \mathbf{f}_k(\hat{\mathbf{x}}_{k,j}, \mathbf{y}_k^m) \quad ; \quad H_{k,j} = \left.\frac{\partial \mathbf{f}_k}{\partial \mathbf{x}}\right|_{(\hat{\mathbf{x}}_{k,j}, \mathbf{y}_k^m)}$$
$$\mathbf{v}_{k,j} = -G_{k,j}\mathbf{u}_k \quad ; \quad G_{k,j} = \left.\frac{\partial \mathbf{f}_k}{\partial \mathbf{y}}\right|_{(\hat{\mathbf{x}}_{k,j}, \mathbf{y}_k^m)}$$

To obtain a suboptimal estimation of $\mathbf{x}$ we use the

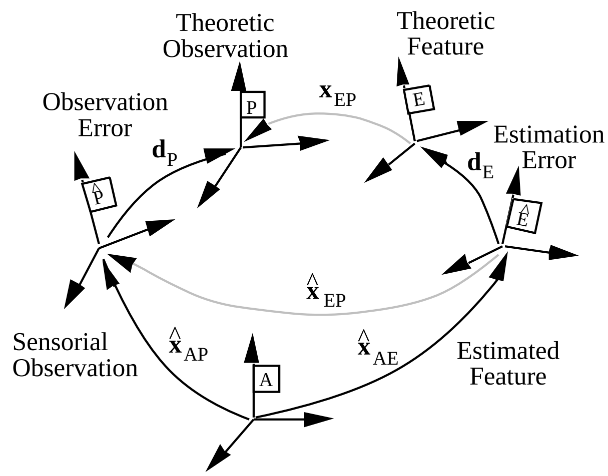


Figure 5: Observation-feature pairing.

extended Kalman filter with local iteration (EKFLI), whose equations are:

$$\hat{\mathbf{x}}_{k,0} = \hat{\mathbf{x}}_{k-1}$$
$$K_{k,j} = P_{k-1} H_{k,j}^T (H_{k,j} P_{k-1} H_{k,j}^T + G_{k,j} S_k G_{k,j}^T)^{-1}$$
$$\hat{\mathbf{x}}_{k,j+1} = \hat{\mathbf{x}}_{k-1} + K_{k,j}[-\mathbf{h}_{k,j} - H_{k,j}(\hat{\mathbf{x}}_{k-1} - \hat{\mathbf{x}}_{k,j})]$$
$$P_k = (I - K_{k,j} H_{k,j}) P_{k-1}$$

This filter relinearizes the last measurement equation $\mathbf{f}_k$ in the new estimation $\hat{\mathbf{x}}_{k,j}$, and iterates on j until $\hat{\mathbf{x}}_{k,j+1} \simeq \hat{\mathbf{x}}_{k,j}$. If nonlinearity problems are considerable, we use global iteration relinearizing all preceding measures around the last estimation.

4.2 Obtaining measurement equations

In this section we obtain and linearize the measurement equation applicable to estimate the location of a feature from several observations. The corresponding expressions for locating an object from several estimated features can be obtained in a similar way [7].

Consider a feature E (see figure 5) whose estimated location is given by $(\hat{\mathbf{x}}_{AE}, \hat{\mathbf{p}}_E, C_E)$, being its real location:

$$\mathbf{x}_{AE} = \hat{\mathbf{x}}_{AE} \oplus \mathbf{d}_E = \hat{\mathbf{x}}_{AE} \oplus B_E^T \mathbf{p}_E$$

The k-th measurement to be integrated corresponds to the location of an observation P, given by $(\hat{\mathbf{x}}_{AP}, \hat{\mathbf{p}}_P, C_P)$. We suppose it is centered: $\hat{\mathbf{p}}_P = 0$. The theoretic location of this observation is:

$$\mathbf{x}_{AP} = \hat{\mathbf{x}}_{AP} \oplus \mathbf{d}_P = \hat{\mathbf{x}}_{AP} \oplus B_P^T \mathbf{p}_P$$

To obtain an implicit measurement equation we make the following identifications:

$\mathbf{x} = \mathbf{p}_E$	state to be estimated
$\mathbf{y}_k^t = \mathbf{p}_P$	theoretic measure
$\mathbf{y}_k^m = \hat{\mathbf{p}}_P = 0$	actual measure
$S_k = C_P$	covariance of measurement error

The measurement equation can be obtained in a systematic way, using the concept of binding matrix of a pairing. Let us consider the case of the *direct constraint*. From the cycle of transformations shown in figure 5 we obtain:

$$\begin{aligned} \mathbf{x}_{EP} &= \ominus \mathbf{d}_E \ominus \hat{\mathbf{x}}_{AE} \oplus \hat{\mathbf{x}}_{AP} \oplus \mathbf{d}_P \\ &= \ominus B_E^T \mathbf{p}_E \oplus \hat{\mathbf{x}}_{EP} \oplus B_P^T \mathbf{p}_P \end{aligned}$$

The measurement equation is:

$$\begin{aligned} \mathbf{f}_k(\mathbf{x}, \mathbf{y}_k^t) &= B_{EP}\mathbf{x}_{EP} = 0 \\ &= B_{EP}(\ominus B_E^T \mathbf{p}_E \oplus \hat{\mathbf{x}}_{EP} \oplus B_P^T \mathbf{p}_P) \\ &= B_{EP}(\ominus B_E^T \mathbf{x} \oplus \hat{\mathbf{x}}_{EP} \oplus B_P^T \mathbf{y}_k^t) = 0 \end{aligned}$$

Linearizing this equation around an estimation $\hat{\mathbf{x}}_{k,j}$ and the actual measure $\mathbf{y}_k^m = \hat{\mathbf{p}}_P = 0$ we obtain:

$$\begin{aligned} \mathbf{h}_{k,j} &= \mathbf{f}_k(\hat{\mathbf{x}}_{k,j}, \mathbf{y}_k^m) = B_{EP}(\ominus B_E^T \hat{\mathbf{x}}_{k,j} \oplus \hat{\mathbf{x}}_{EP}) \\ H_{k,j} &= \left.\frac{\partial \mathbf{f}_k}{\partial \mathbf{x}}\right|_{(\hat{\mathbf{x}}_{k,j}\ \mathbf{y}_k^m)} \\ &= B_{EP}\ J_{1\oplus}\{\ominus B_E^T \hat{\mathbf{x}}_{k,j}, \hat{\mathbf{x}}_{EP}\}\ J_{\ominus}\{B_E^T \hat{\mathbf{x}}_{k,j}\}\ B_E^T \\ G_{k,j} &= \left.\frac{\partial \mathbf{f}_k}{\partial \mathbf{y}}\right|_{(\hat{\mathbf{x}}_{k,j}\ \mathbf{y}_k^m)} \\ &= B_{EP}\ J_{2\oplus}\{\ominus B_E^T \hat{\mathbf{x}}_{k,j} \oplus \hat{\mathbf{x}}_{EP}, 0\}\ B_P^T \end{aligned}$$

where $J_{1\oplus}$ and $J_{2\oplus}$ are the jacobians of the composition of location vectors with respect to its first and second operand, and $J_{\ominus}$ the jacobian of inversion (their detailed calculation can be found in [6]). Similar expressions can be obtained in the case of using the inverse constraint.

4.3 Integration mechanism

The equations of EKFLI, together with the measurement equations developed above, form a completely general method for integrating sensorial information. In order to apply it to different integration problems we only need the self-binding matrices of the observed and estimated geometric elements, and the binding matrix of their pairing.

Convergence of the filter equations is accelerated by using a good initial estimation to make the first linearization. This estimation can be obtained by algebraic methods, from a minimal set of observations. For example, if we want to estimate the location of an edge from a set of points observed with a proximity sensor, we may use the edge defined by the first two points.

5 Conclusions

We have proposed a new model to represent sensorial information and its uncertainty, based on the theory of symmetries. This model is completely general, allows an easy interpretation of uncertainty values, and avoids the problems of singularities. Based on it, we have defined an integration mechanism whose main advantage is generality. It allows to obtain a suboptimal estimation of the location of a feature or of an object from a set of partial and uncertain observations.

The model and the integration method have been tested using simulated sensorial data [7]. Experimentation in real environments is being carried out using a PUMA robot, static and camera-in-hand vision systems, and several laser and infrared proximity sensors mounted on the robot hand.

References

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