On performance bounds for interval Time Petri Nets*

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Abstract

Interval time Petri Nets are Petri nets in which time intervals are associated to transitions. Their quantitative analysis basically consists in applying enumerative techniques that suffer the well known state space explosion problem. To overcome this problem several methods have been proposed in the literature, that either allow to obtain equivalent nets with a reduced state space or avoid the construction of the whole state space. The alternative method proposed here consists in computing performance bounds to partially characterize the quantitative behavior of interval time Petri Nets by exploiting their structural properties and/or by applying operational laws. The performance bound computation is not a new technique: it has been proposed for timed Petri nets. In this paper we present the results obtained from a preliminary investigation on the applicability of bounding techniques of timed Petri nets to interval time Petri Nets.

1. Introduction

Place/Transition (P/T) Petri nets [17] have been extended in the literature with suitable time interpretations for the modelling and analysis of real-time systems or with a performance evaluation perspective. Giving a time interpretation consists in specifying the behaviour in time in such a way that [9]: (1) the new model is compatible with the original P/T model, (2) part of the non-determinism present in the P/T model is reduced in order to take the timing constraints into account, and (3) the behaviour of the system is specified precisely enough to be able to check or compute the temporal properties under study.

Time Petri Nets (TPNs), as defined by Merlin and Faber in [16], reduce the non-determinism in the duration of activities of P/T Petri nets by associating a time interval with each transition. Interval limits define the earliest and the latest firing time of the transition, relative to the instant at which it was last enabled.

The same kind of reduction could be applied to the non-determinism involved in the choice among several conflicting transitions. Let us consider the case of a free-choice between two transitions t_1 and t_2. One might specify, as an additional interpretation of the net system, that transition t_1 cannot fire more than k_1 times per each firing of t_2 during a given observation period and vice-versa, that t_2 cannot fire more than k_2 times per each firing of t_1 during the same observation period. To the best of our knowledge, this possible interpretation was introduced in [9], with the purpose of illustration of basic concepts of time interpretation of P/T Petri net models, and it has not been elaborated later in the literature. We return to that interpretation in this paper, and we give to it the name "TPN with interval firing frequencies" (TPNF), as a particular case where new analytical techniques can be derived to compute temporal properties.

One more step in the reduction of the non-determinism in the duration of activities, in addition to the time interval approach proposed by Merlin and Faber, is to introduce a stochastic measure for the duration within the given interval. In this sense, Extended Time Petri Nets (XTPNs) of Juanole et al. [11, 21], reduce the non-determinism by associating a probability density function to each transition firing time that takes a non-null value only within a given firing interval for each transition.

Usually, TPNs are used to validate timing requirements, while the stochastic reduction of the non-determinism introduced in XTPNs as an extension of TPNs, allows to verify performance requirements. The quantitative analysis of such kind of nets basically consists in applying enumerative techniques, i.e., based on the construction of the graph of the state class [3, 4] or of the discrete reachability graph [18] for TPNs and of the randomized state graph [21] for XTPNs, that suffer the well known state space explosion problem even in case of bounded nets. To tackle this prob-

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lem, alternative methods have been proposed in the literature such as reduction methods [23] that allow to obtain nets with a reduced state space in which the timing and concurrency properties are preserved or parametric descriptions of transition firing sequences [19] that avoid the construction of the whole state space.

In this paper, we consider interval Time Petri Nets (i.e., TPNs, TPNFs, XTPNs) and structural subclasses of them, and we propose an efficient method to compute performance bounds for the throughput of transitions and for the mean marking of places by means of the solution of linear programming problems derived from the structure of the net, the initial marking, and the time interpretation.

The analysis method proposed here can be considered as a generalization of the existing well established performance bound computation techniques for timed Petri nets (in the sense of Ramchandani [20]) and stochastic Petri nets [7, 6, 8, 10, 14], since the Petri nets that we consider now preserve a higher level of non-determinism both in the duration of activities (interval time specification) and conflict resolution (interval firing frequencies).

The paper is organized as follows. In Section 2, basic definitions and notations for interval Time Petri Nets are given. Operational definitions of throughput of transitions and mean marking of places are also recalled. Section 3 includes the technique for the computation of bounds for interval Time Petri Nets by presenting a set of linear equations and inequalities that are used as constraints of linear programming problems stated to compute upper and lower bounds for the defined performance indices. In Section 4, specific techniques for the computation of performance bounds for structurally defined subclasses, like marked graphs or free choice nets, are introduced. An example of application to a communication protocol with ACK and with a time-out mechanism for controlling the message loss is presented in Section 5. Concluding remarks are summarized in Section 6.

2. Definition and notation

Time Petri Net (TPN) is a tuple $\mathcal{T} = (P, T, B, F, M_0, I)$ where P is the set of places, T is the set of transitions, $B : T \times P \to \mathbb{N}$ is the backward incidence function, $F : T \times P \to \mathbb{N}$ is the forward incidence function. The input sets of $p \in P$ and $t \in T$ will be denoted as ${}^\bullet p = \{t \in T : F(t,p) \geq 1\}$ and ${}^\bullet t = \{p \in P : B(t,p) \geq 1\}$ respectively; the output set of $p \in P$ will be denoted as $p^\bullet = \{t \in T : B(t,p) \geq 1\}$. $M_0 : P \to \mathbb{N}$ is the initial marking function, $I : T \to \mathbb{Q}_0^+ \times (\mathbb{Q}_0^+ \cup \infty)$ is the static interval function that assigns to each transition $\forall t \in T$ a time interval $I(t) = (a[t], b[t]), a[t] \leq b[t]$, where $a[t]$ is the *static earliest firing time* and $b[t]$ is the *static latest firing time*. $a[t]$ and $b[t]$ are relative to the instant at which the transition t was last enabled, so that if t has been last enabled at time τ then it may not fire before $\tau + a[t]$ and it must fire before or at $\tau + b[t]$ unless it is disabled before then by the firing of a conflicting transition. Firing itself is immediate. A state of a TPN is defined as a pair $S = (M, I_d)$ where M is the marking function and I_d is a firing interval function that associates to each transition the time (dynamic) interval $a_d[t], b_d[t]$ in which the transition is allowed to fire. Firing of a transition t at time θ from a state $S = (M, I_d)$ leads to a state $S' = (M', I'_d)$ where $M' = M + F(t, \cdot) - B(t, \cdot)$ and the new firing interval function I'_d assigns to each transition concurrent with t its remaining firing time interval, to each newly enabled transition its static firing time interval and to each disabled transition a null firing time interval.

In absence of multiple enabledness the memory policy associated to transitions of a TPN corresponds to the enabling policy [15] defined for Stochastic Petri Nets since only transitions concurrent with t take into account of their enabling time from their last enabling instant. In presence of multiple enabledness, we will assume that the *extended firing rule with non-deterministic strategy* [2] is adopted: the transitions of a TPN are then characterized by an infinite server semantics and their memory policy is still of type enabling.

A TPN with interval firing frequencies (TPNF) is a TPN in which an interval of firing frequencies is associated to each transition in extended free-choice conflict. In a TPNF the extended free-choice conflicts are still not deterministic, but the non-determinism is reduced by assuring that, for each pair of conflicting transitions, the ratio between their throughputs during an observation period falls into a finite interval. Formally a TPNF is defined as $\mathcal{TF} = (\mathcal{T}, R)$, where $\mathcal{T}$ is the underlying TPN model and $R : T' \subseteq T \to \mathbb{Q}^+ \times \mathbb{Q}^+$ is the interval frequency function that assigns an interval $(r^i[t], r^s[t]), r^i[t] \leq r^s[t]$ to each transition $t \in T'$, where $T' = \bigcup_j ECS_j$ is the union of the equal conflict sets. For each equal conflict set ECS_j, there exists a transition $t' \in ECS_j$ such that $a[t'] \leq min_{t \in ECS_j}\{b[t]\}$ where $R(t') = (1, 1)$.

Extended Time Petri Net (XTPN) is the stochastic extension of TPN; formally, a XTPN is defined as a pair $\mathcal{XT} = (\mathcal{T}, F_0)$, where $\mathcal{T}$ is the underlying TPN model and F_0 is a functional that assigns to each transition $t \in T$ an initial firing probability density function defined over its static firing time interval $I(t)$. The state of a XTPN is a triplet $S = (M, I_d, F_d)$, where (M, I_d) is the state of the associated TPN model $\mathcal{T}$ and F_d is a functional that defines the firing probability density function to each transition with non-empty firing interval. A transition t, enabled in marking M, can fire at time θ if it is firable in the underlying TPN and its probability of firing before or at θ is not zero. The new state reached by the firing of t is a state $S' = (M', I'_d, F'_d)$, where (M', I'_d) is the state reached in the underlying TPN and F'_d is a functional that defines the new probability density functions of the transitions enabled in marking M' according to the enabling memory policy. Concretely, F'_d associates to the newly enabled transitions their initial probability density functions and to transitions concurrent with t it assigns the probability density functions of their remaining times to fire.

Let us introduce the basic quantities that can be collected during the period $(0,\Gamma), \Gamma \in \mathbb{R}^+$ by observing the behavior of an interval Time Petri Net[1]:

$$\bar{M}[p] = \frac{1}{\Gamma}\int_0^\Gamma M[p](\tau)d\tau \qquad (1)$$

the average marking of $p \in P$, where $M[p](\tau)$ is the number of tokens in p at time $\tau \in (0,\Gamma)$;

$$\bar{e}[t] = \frac{1}{\Gamma}\int_0^\Gamma e[t](\tau)d\tau \qquad (2)$$

the average enabling degree of $t \in T$, where $e[t](\tau) = min_{p\in{}^\bullet t}\{\frac{M[p](\tau)}{B(t,p)}\}$ is the number of instances of t enabled at time $\tau \in (0,\Gamma)$;

$$\theta_j[t] = \int_0^\Gamma e_j[t](\tau)d\tau \qquad (3)$$

the enabling time for the j-th instance of transition t during the experiment interval, where $e_j[t](\tau)$ is the characteristic function that evaluates to 1 iff the j-th instance is enabled at time $\tau \in (0,\Gamma)$ (i.e., $e[t](\tau) \geq j$);

$$x[t] = \frac{\Phi[t]}{\Gamma} \qquad (4)$$

the throughput of $t \in T$ in $(0,\Gamma)$, where $\Phi[t]$ is the number of firing of t during the experiment interval. $\Phi[t]$ can be expressed as $\Phi[t] = \sum_{j=1}^{\infty}\Phi_j[t]$ where $\Phi_j[t]$ is the number of firing of the j-th instance of t in $(0,\Gamma)$.

3. Bounds for interval Time Petri Nets

Given an interval Time Petri net, let $x[t]$ and $\bar{M}[p]$ be the throughput of transition $t \in T$ and the average marking of place $p \in P$, respectively, during an observation interval $(0,\Gamma), \Gamma \in \mathbb{R}^+$. The upper and lower bounds for $x[t^*]$ of a given transition t^* (or for $\bar{M}[p^*]$ of a given place p^*) can be computed by solving a linear programming problem (max-LP problem for the upper bound and min-LP problem for the lower bound) in which the objective function is $x[t^*]$ (or $\bar{M}[p^*]$) and subjects to a set of constraints that are derived from the Petri net structure and from the enabling operational law.

[1]To simplify the notation we omit the dependence of the basic quantities on Γ.

3.1. Structural constraints

Structural constraints are based on the net structure and, at most, on the initial marking. A first set of constraints is derived by considering that for all markings reachable at instant $\tau \in (0,\Gamma)$, denoted in vectorial form as $\mathbf{M_r}(\tau)$, we have that $\mathbf{M_r}(\tau) = \mathbf{M_0} + (\mathbf{F}-\mathbf{B})^\mathbf{T}\,\sigma_\mathbf{r}(\tau)$, where $\sigma_\mathbf{r}(\tau)$ is a feasible firing count vector until instant τ, $\mathbf{M_0}$ is the initial marking vector and $(\mathbf{F}-\mathbf{B})^\mathbf{T}$ is the incidence matrix. The average marking vector $\bar{\mathbf{M}}$ during the time interval $(0,\Gamma)$ has to satisfy the linear equality:

$$\bar{\mathbf{M}} = \mathbf{M_0} + (\mathbf{F}-\mathbf{B})^\mathbf{T}\sigma \qquad (5)$$

where σ is the average firing count vector during the experiment interval.

Proof. From definition (1) written in vectorial form and from the reachability equation:

$$\begin{aligned}\bar{\mathbf{M}} &= \frac{1}{\Gamma}\int_0^\Gamma \mathbf{M_r}(\tau)d\tau = \\ &\frac{1}{\Gamma}\int_0^\Gamma \mathbf{M_0} + (\mathbf{F}-\mathbf{B})^\mathbf{T}\,\sigma_\mathbf{r}(\tau)d\tau = \\ &\mathbf{M_0} + (\mathbf{F}-\mathbf{B})^\mathbf{T}\frac{\mathbf{1}}{\mathbf{\Gamma}}\int_\mathbf{0}^\mathbf{\Gamma}\sigma_\mathbf{r}(\tau)d\tau\end{aligned}$$

taking $\sigma = \frac{1}{\Gamma}\int_0^\Gamma \sigma_\mathbf{r}(\tau)d\tau$ the equality (5) follows. $\diamond$

A second set of constraints is derived from the token flow relations for places:

$$\sum_{t\in{}^\bullet p} x[t]F(t,p) \geq \sum_{t\in p^\bullet} x[t]B(t,p), \qquad \forall p \in P \qquad (6)$$

that become equalities if p is bounded. Obviously, the average marking vector, the average firing count vector and the transition throughputs are always non negative values:

$$\bar{\mathbf{M}}, \sigma \geq 0, \quad x[t] \geq 0 \quad \forall t \in T \qquad (7)$$

3.2. Enabling operational law constraints

This set of constraints is derived from the enabling operational law applied on Petri Nets (or *utilization law* with classical queueing systems terminology [13]); they take into account of the timing information of the net, that is the static interval function I. A first constraint is given by the following inequality:

Throughput upper bound inequality

$$\forall t \in T \text{ and } \forall p \in {}^\bullet t: \;\; x[t] \leq \frac{\bar{M}[p]}{a[t]B(t,p)} \qquad (8)$$

for all experiment intervals $(0,\Gamma), \Gamma \in \mathbb{R}^+$.

Proof. Let us consider a transition $t \in T$ where $a[t]$ is the earliest static firing time. Then, assuming the j-th instance becomes enabled at the instant $\tau \in (0,\Gamma)$, where

$\Gamma \in \mathbb{R}^+$, it cannot fire before $a[t] + \tau$: this means that the minimum firing waiting time is $a[t]$. Then maximum number of firing of the j-th instance of t during the experiment interval is given by $\left\lfloor \frac{\theta_j[t]}{a[t]} \right\rfloor$, so that:

$$\Phi_j[t] \leq \Phi_j^{max}[t] = \left\lfloor \frac{\theta_j[t]}{a[t]} \right\rfloor \leq \frac{\theta_j[t]}{a[t]}.$$

Then summing over all the instances of t and dividing by Γ the first and the last member we obtain:

$$x[t] \leq \frac{\sum_{j=1}^{\infty} \theta_j[t]}{\Gamma} \frac{1}{a[t]}$$

Replacing $\theta_j[t]$ with its definition and exchanging the integral and the sum signs we get:

$$x[t] \leq \frac{\int_0^{\Gamma} \sum_{j=0}^{\infty} e_j[t](\tau) d\tau}{\Gamma} \frac{1}{a[t]}$$

Now considering that the equalities $e[t](\tau) = \sum_{j=0}^{\infty} e_j[t](\tau)$ and $e[t](\tau) = min_{p \in {}^\bullet t} \{ \frac{M[p](\tau)}{B(t,p)} \}$ hold for all τ, we obtain:

$$x[t] \leq \frac{\int_0^{\Gamma} M[p](\tau) d\tau}{a[t] B(t,p) \Gamma} = \frac{\bar{M}[p]}{a[t] B(t,p)}, \forall p \in {}^\bullet t \qquad \diamond$$

Note that constraint (8) can be applied to transitions that are either persistent, i.e., once enabled they eventually fire, or in conflict. In case $a[t] = 0$ the inequality still holds: $x[t] \leq \infty$.

The following constraints hold, instead, only for persistent transitions:

Throughput lower bound inequalities Let us consider an observation interval $(0, \Gamma), \Gamma \in \mathbb{R}^+$ either large enough or such that $M[p](\Gamma) = 0$ and let $t \in T : {}^\bullet t = \{p\}$. Then:

$$x[t]b[t] \geq \frac{\bar{M}[p] - B(t,p) + 1}{B(t,p)} \qquad (9)$$

If $\forall \tau \in (0, \Gamma) : M[p](\tau) \leq N[p]$ we have the further constraint:

$$x[t]b[t] \geq k \frac{\bar{M}[p] - kB(t,p) + 1}{N[p] - kB(t,p) + 1} \qquad (10)$$

where $k \in \mathbb{N} : kB(t,p) \leq N[p] < (k+1)B(t,p)$.

Considering transitions with two input places, $t \in T : {}^\bullet t = \{p_1, p_2\}$, if $\forall \tau \in (0, \Gamma) : M[p_1](\tau) \leq N[p_1], M[p_2](\tau) \leq N[p_2]$ and $N[p_1] \leq N[p_2]$, then:

$$x[t]b[t]B(t,p_1) \geq \bar{M}[p_1] - B(t,p_1) + 1 - N[p_1] f_2 \qquad (11)$$

where $f_2 = \left(1 - \frac{\bar{M}[p_2] - B(t,p_2) + 1}{N[p_2] - B(t,p_2) + 1}\right)$. Finally, a generalization of the inequality (11) is the following:

$$x[t]b[t]B(t,p_1) \geq \bar{M}[p_1] - B(t,p_1) + 1 - N[p_1] \max_{1 < j \leq k} \{f_j\} \qquad (12)$$

where ${}^\bullet t = \{p_1, \ldots, p_k\}$, $\forall j = 1, ..., k$, $\forall \tau \in (0, \Gamma)$: $M[p_j](\tau) \leq N[p_j]$, $N[p_1] \leq N[p_j]$, and

$$f_j = \left(1 - \frac{\bar{M}[p_j] - B(t,p_j) + 1}{N[p_j] - B(t,p_j) + 1}\right).$$

Proof. The above constraints are derived from the ones proved in [10] by considering that:

- being t persistent, for each j-th instance of t once enabled at time instant τ_{ji} eventually fires at a time instant always less than or equal to $\tau_{ji} + b[t]$ so that its firing waiting times $S_{ji} \leq b[t], \forall i$;
- the enabling time for the j-th instance of t during the observation interval $(0, \Gamma)$ can be written as:

$$\theta_j[t] = \sum_{i=1}^{\Phi_j[t]} S_{ji} + \delta_j \leq \Phi_j[t] b[t] + \delta_j$$

  where S_{ji} represents the firing waiting time of the j-th instance enabled at a time instant $\tau_i : \tau_i + b[t] < \Gamma$, and $\delta_j \in [0, b[t])$ represents a possible not complete firing waiting time because of the choice of Γ.
- If Γ is chosen such as $M[p] = 0$ for a $p \in {}^\bullet t$ or $\Gamma \to \infty$ then $\delta_j = 0, \forall j$ and the following relation holds for the mean service time of t in $(0, \Gamma)$:

$$\bar{S}[t] \stackrel{def}{=} \frac{\sum_{j=1}^{\infty} \theta_j[t]}{\sum_{j=1}^{\infty} \Phi_j[t]} \leq b[t]. \qquad \diamond$$

In case of XTPNs the enabling law constraints given in this sub-section can be replaced with the ones defined in [10] for Stochastic Petri Nets where the mean service time $\bar{S}[t]$ of transition t is the expected value of the firing time distribution associated to t, i.e., $\bar{S}[t] = \int_{a[t]}^{b[t]} x f_t(x) dx$.

3.3. Routing constraints

Routing constraints for transitions that are in structural extended free-choice conflict can be defined only for TPNFs and, under certain restrictions, for XTPNs. Let us consider the equal conflict relation [24]: $t_i \; EQ \; t_j$ iff $B(t_i, \cdot) = B(t_j, \cdot) \neq \emptyset$; it is an equivalence relation and let ECS be an equal conflict set. Given an observation interval $(0, \Gamma), \Gamma \in \mathbb{R}^+$, for each pair of transitions $t_j, t_k \in ECS$ of a TPNF such that $a[t_k], a[t_j] \leq aLFT$, where $aLFT = min_{t \in ECS}\{b[t]\}$ we have:

$$r^i[t_j]x[t_k] \leq r^s[t_k]x[t_j], \;\; r^i[t_k]x[t_j] \leq r^s[t_j]x[t_k]. \qquad (13)$$

Proof. From definition of TPNF, the following inequalities hold for the number of firings of $t \in ECS, a[t] \leq aLFT$ during $(0, \Gamma)$:

$$r^i[t]\Phi[t_0] \leq \Phi[t] \leq r^s[t]\Phi[t_0]$$

where $t_0 \in ECS : a[t_0] \leq aLFT, R(t_0) = (1, 1)$. Dividing by Γ we obtain the same inequalities for the throughput. Then, $\forall t \in ECS, a[t] \leq aLFT$:

$$r^i[t] \leq \frac{x[t]}{x[t_0]} \leq r^s[t] \quad \text{and} \quad \frac{1}{r^s[t]} \leq \frac{x[t_0]}{x[t]} \leq \frac{1}{r^i[t]}$$

Considering that:

$$\frac{x[t_j]}{x[t_k]} = \frac{x[t_j]}{x[t_0]} \frac{x[t_0]}{x[t_k]}$$

it is trivial to obtain inequalities (13). ◇

Let us consider now a XTPN and denote as $f_t(y)$ the initial firing probability density function of $t \in ECS$. If 1) no other transition $t' \notin ECS$ can become enabled concurrently with $t \in ECS$ and 2) there exists $p \in {}^{\bullet}t : M[p](\tau) \leq B(t,p), \forall \tau \in (0, \Gamma)$ then the probability that t fires first is marking and time independent, and it is given by:

$$P[t] = \int_{a[t]}^{aLFT} f_t(y) \Big[\prod_{t' \in ECS, t' \neq t} \int_y^{b[t']} f_{t'}(z) dz \Big] dy.$$

We can then define the following routing constraints for all $t_i, t_j \in ECS$, such that $a[t_i], a[t_j] < aLFT$:

$$\frac{x[t_i]}{P[t_i]} = \frac{x[t_j]}{P[t_j]} \tag{14}$$

Moreover, if there exists $t \in ECS : a[t] \geq aLFT$, then transitions $t' \in ECS$ with $a[t'] > aLFT$ have null firing probabilities and we can deduce that $x[t_j] = 0$. Although restrictions 1) and 2) given above are quite strong, sufficient structural conditions can be applied in order to verify them, i.e., structural mutual exclusion condition based on P-invariants for the verification of 1) and the structural marking bound computation for the verification of 2).

3.4. Application of preselection policy

The bounds for transition throughputs and for mean number of tokens in places of an interval Time Petri net can be possibly improved by applying a preselection policy to timed transitions in free-choice conflict to make them persistent so that inequalities (9) and (10) can be included in the set of constraints of the LP problem.

Let us assume to have a free-choice conflict between n transitions $t_1, .., t_n$ of a TPN, where ${}^{\bullet}t_k = \{p\}, \forall k = 1, .., n$, characterized by firing intervals $I(t_k) = [a_k, b_k]$ as depicted in the upper part of Figure 1. Let $aLFT = min_{k=1,...,n}\{b_k\}$ be the actual latest firing time of the conflicting transitions: when the conflicting transitions are enabled, each transition t_k can fire at a time instant $\tau_k \in [a_k, aLFT]$. Without loss of generality we can assume that $a_k \leq aLFT$ for the first L transitions and $a_k > aLFT$ for the remaining ones. Then, there is an actual conflict only between the first L transitions and the behavior of the free-choice conflict is *equivalent* to the net depicted in the lower part of Figure 1.

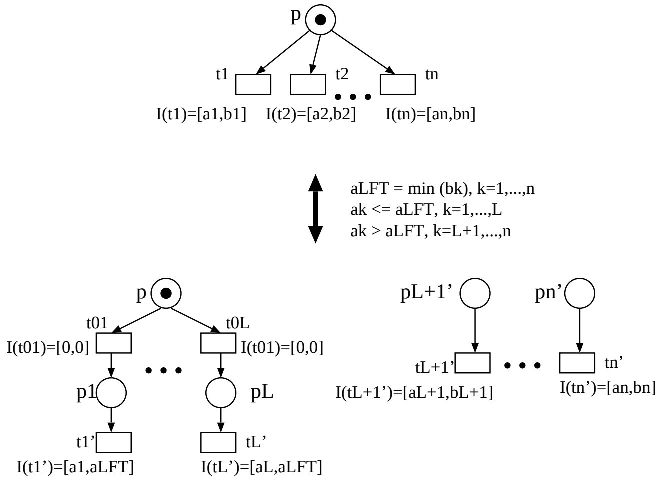

Figure 1. Preselection policy in free-choice TPNs.

Our equivalence notion is basically a timed trace equivalence that preserves the maximum and the minimum throughputs of timed transitions.

Let us consider the integer reachability graph (RG) [18] associated to a TPN $\mathcal{T}$. The node of the RG are integer states of the net, that is states where the current local time for each enabled transition is an integer; this RG includes only a discrete part of all the possible net behaviors, but it has been proved that this knowledge is sufficient to determine the min/max durations of a given feasible firing sequence.

A path of the RG from the initial state s_0 to a state s_n represents a feasible firing schedule and it is characterized by transitions, whose firing happens timeless, and by time durations, and it can be denoted as $\sigma : s_0 \xrightarrow{\sigma_0} s_1 \xrightarrow{\sigma_1} ... \xrightarrow{\sigma_{n-2}} s_{n-1} \xrightarrow{\sigma_{n-1}} s_n$ where either $\sigma_i \equiv (t, 0), t \in T$ or $\sigma_i \equiv (\tau, n_i), n_i \in \mathbb{Q}^+$.

Let us consider the set $\Sigma = \{\sigma : s_0 \xrightarrow{\sigma_0} s_1 \xrightarrow{\sigma_1} ... \xrightarrow{\sigma_{n-2}} s_{n-1} \xrightarrow{\sigma_{n-1}} s_n, n \in \mathbb{N}\}$ of the feasible finite firing schedules of $\mathcal{T}$. From the set Σ we construct the set Σ^c containing the finite firing schedules of Σ reduced in a canonical form. As canonical form of a feasible firing schedule we mean a firing schedule where the immediate transitions are considered not observable, and hence eliminated, and the subsequences representing only time elapsing are collapsed into a unique transition from the initial state to the final state of the subsequence representing the global time duration of the subsequence.

Let us denote as π_1 and π_2 the first and the second projection functions, respectively; $T_{imm} = \{t \in T : a[t] = b[t] = 0\}$ and $T_{timed} = T \setminus T_{imm}$ the sets of immediate transitions and of timed transitions of $\mathcal{T}$. The canonical

$$\phi(0) = 0;$$

$$\phi(i) = \begin{cases} \phi(i-1)+1 & \text{if } \pi_1(\sigma_{\phi(i-1)}) \in T_{timed} \\ \phi(i-1)+k & \text{if } \exists k \in \mathbb{N}^+ : \forall l \in [\phi(i-1), \phi(i-1)+k-1]\ \pi_1(\sigma_l) \notin T_{timed} \wedge \\ & \quad \exists l \in [\phi(i-1), \phi(i-1)+k-1]\ \pi_1(\sigma_l) = \tau \\ \phi(i-1)+k+1 & \text{if } \exists k \in \mathbb{N}^+ : \forall l \in [\phi(i-1), \phi(i-1)+k-1]\ \pi_1(\sigma_l) \in T_{imm} \wedge \\ & \quad \phi(i-1)+k+1 \leq n \\ \# & \text{otherwise} \end{cases}$$

Table 1. Index function

form can be constructed in a recursive way and it is based on the definition of the index function given in Table 1:

Definition 1 *Let $\sigma : s_0 \xrightarrow{\sigma_0} s_1 \xrightarrow{\sigma_1} \ldots \xrightarrow{\sigma_{n-2}} s_{n-1} \xrightarrow{\sigma_{n-1}} s_n \in \Sigma$ be a feasible firing schedule, the canonical form of σ is an observable feasible firing schedule $\sigma' : s'_0 \xrightarrow{\sigma'_0} s'_1 \xrightarrow{\sigma'_1} \ldots \xrightarrow{\sigma'_{n'-2}} s_{n'-1} \xrightarrow{\sigma'_{n'-1}} s_{n'}$ where:*

$$n' = \max_{\phi(i) \neq \#} \{i\},$$

$$s'_i = s_{\phi(i)},$$

$$\sigma'_i = \begin{cases} \left(\pi_1(\sigma_{\phi(i+1)-1}), 0\right) & \textit{if } \pi_1(\sigma_{\phi(i+1)-1}) \in T_{timed} \\ \left(\tau, \sum_{l=\phi(i)}^{\phi(i+1)-1} \pi_2(\sigma_l)\right) & \textit{otherwise} \end{cases}$$

With the previous definition we can finally define our notion of equivalence between two TPNs:

Definition 2 *Let Σ_1^c and Σ_2^c be the sets containing the finite firing schedules of $\mathcal{T}_1$ and $\mathcal{T}_2$, respectively, reduced in their canonical form. Then $\mathcal{T}_1$ and $\mathcal{T}_2$ are equivalent ($\mathcal{T}_1 \equiv \mathcal{T}_2$) iff there exists a bijection $\beta : T^1_{timed} \to T^2_{timed}$, defined between the sets of timed transitions of the two models, such that:*

$$\forall\, \sigma = s_0 \xrightarrow{\sigma_0} s_1 \xrightarrow{\sigma_1} \ldots \xrightarrow{\sigma_{n-2}} s_{n-1} \xrightarrow{\sigma_{n-1}} s_n \in \Sigma_1^c :$$

$$\exists \mid \sigma' = s'_0 \xrightarrow{\sigma'_0} s'_1 \xrightarrow{\sigma'_1} \ldots \xrightarrow{\sigma'_{n-2}} s'_{n-1} \xrightarrow{\sigma'_{n-1}} s'_n \in \Sigma_2^c,$$

where:

$$\sigma'_i = \begin{cases} (\beta(t_i), 0) & \textit{if } t_i \in T^1_{timed} \\ \sigma_i & \textit{otherwise} \end{cases}$$

and

$$\forall\, \sigma' = s'_0 \xrightarrow{\sigma'_0} s'_1 \xrightarrow{\sigma'_1} \ldots \xrightarrow{\sigma'_{n-2}} s'_{n-1} \xrightarrow{\sigma'_{n-1}} s'_n \in \Sigma_2^c :$$

$$\exists \mid \sigma = s_0 \xrightarrow{\sigma_0} s_1 \xrightarrow{\sigma_1} \ldots \xrightarrow{\sigma_{n-2}} s_{n-1} \xrightarrow{\sigma_{n-1}} s_n \in \Sigma_1^c,$$

where:

$$\sigma_i = \begin{cases} (\beta^{-1}(t_i), 0) & \textit{if } t_i \in T^2_{timed} \\ \sigma'_i & \textit{otherwise} \end{cases}$$

Property 1 *The transformation of a timed free-choice net into an immediate free-choice net as depicted in Figure 1 leads to a $\equiv$ equivalent net.*

Proof sketch. Let us consider a TPN $\mathcal{T}_1$ with a timed free-choice conflict $FCS = \{t_1, ..., t_n\}$ and let $\mathcal{T}_2$ be the TPN derived from the former by applying the transformation of the timed free-choice conflict into an immediate one as depicted in Figure 1. The proof consists in the following steps:

1. define a bijection β over the sets of timed transitions of the two models. β is equal to the identity function for transitions $t \in T^1_{timed} \setminus FCS$ and it assigns to each transition of $t_k \in FCS$ the corresponding timed transition t'_k as depicted in Figure 1.
2. Verify, by induction on the length n of the canonical firing schedules, that for each canonical firing schedule $\sigma \in \Sigma^{1c}$ of $\mathcal{T}_1$ there exists a unique firing schedule $\sigma' = \beta^*(\sigma) \in \Sigma^{2c}$ and vice versa, where β^* is the bijection induced by β.

Property 2 *The equivalence $\equiv$ preserves the minimum and the maximum throughputs of timed transitions.*

Proof (for the minimum throughput). Let us consider two TPN models $\mathcal{T}_1$ and $\mathcal{T}_2$ such that $\mathcal{T}_1 \equiv \mathcal{T}_2$, and let Σ^{1c} and Σ^{2c} be the sets of the canonical firing schedules of $\mathcal{T}_1$ and of $\mathcal{T}_2$, respectively. We first have to define the minimum throughput of a transition $t \in T^1_{timed}$ as function of canonical firing schedules belonging to the set Σ^{1c}. Given $\Gamma \in \mathbb{Q}^+$, let us consider the following subset of canonical firing schedules of $\mathcal{T}_1$:

$$\Sigma^{1c}(\Gamma) = \{s_0 \xrightarrow{\sigma_0} \ldots \xrightarrow{\sigma_{n-1}} s_n \in \Sigma_1^c \mid \sum_{i=0}^{n-1} \pi_2(\sigma_i) = \Gamma$$
$$\text{or} \Big(\sum_{i=0}^{n-2} \pi_2(\sigma_i) < \Gamma \text{ and} \sum_{i=0}^{n-1} \pi_2(\sigma_i) > \Gamma \Big) \quad (15)$$

that is the subset of canonical firing schedules of time length equal to Γ or strictly between $\Gamma - \pi_2(\sigma_n)$ and Γ, where σ_n is the last event of the schedule causing the state change. Let us define as $n_t(\sigma)$ the number of occurrences of t in $\sigma \in \Sigma^{1c}(\Gamma)$ then the minimum throughput of t in the time interval $(0, \Gamma)$ is given by:

$$X_{min}[t](\Gamma) = \frac{min_{\sigma \in \Sigma^{1c}(\Gamma)} \Big(n_t(\sigma) \Big)}{\Gamma}.$$

We prove now that the minimum throughput of t is equal to the minimum throughput of $\beta(t)$, where β is the bijection defined by the equivalence $\equiv$. We denote as $\Sigma^{2c}(\Gamma)$ the subset of canonical firing schedules of $\mathcal{T}^2$ analogous to $\Sigma^{1c}(\Gamma)$, then β induces a bijection between the subsets $\Sigma^{1c}(\Gamma)$ and $\Sigma^{2c}(\Gamma)$ such that:

$$\forall \sigma \in \Sigma^{1c}(\Gamma) \;\; \exists \mid \sigma' \in \Sigma^{2c}(\Gamma) : \; n_t(\sigma) = n_{\beta(t)}(\sigma')$$

and vice versa

$$\forall \sigma' \in \Sigma^{2c}(\Gamma) \;\; \exists \mid \sigma \in \Sigma^{1c}(\Gamma) : \; n_{\beta(t)}(\sigma') = n_t(\sigma).$$

So that:

$$\min_{\sigma' \in \Sigma^{2c}(\Gamma)} \Big(n_{\beta(t)}(\sigma') \Big) = \min_{\sigma \in \Sigma^{1c}(\Gamma)} \Big(n_t(\sigma) \Big) = n(\Gamma).$$

We can conclude that:

$$X_{min}[\beta(t)](\Gamma) = \frac{n(\Gamma)}{\Gamma} = X_{min}[t](\Gamma).$$

Analogous consideration can be done for the maximum throughput. $\diamond$

Similar transformations of timed free-choice conflicts into immediate ones can be carried out in case of TPNFs and, under the restrictions 1) and 2) stated in sub-section 3.3, of XTPNs. In particular, the transformation of a free-choice conflict among n transitions of a TPNF is illustrated in Figure 2. The replacement of a timed free-choice conflict of a XTPN with an immediate one leads to a general Stochastic Petri Net with finite support probability distribution functions (PdFs) defining the service time of transitions (i.e., the PdFs used in the original XTPN), in which the immediate free-choice conflict is characterized by immediate transitions t_{0i} with weights equal to the probabilities $P[t_i]$, defined in sub-section 3.3, and by persistent timed transitions t'_i with a PdF of the firing times that is the minimum of the PdFs associated to the timed transitions t_i.

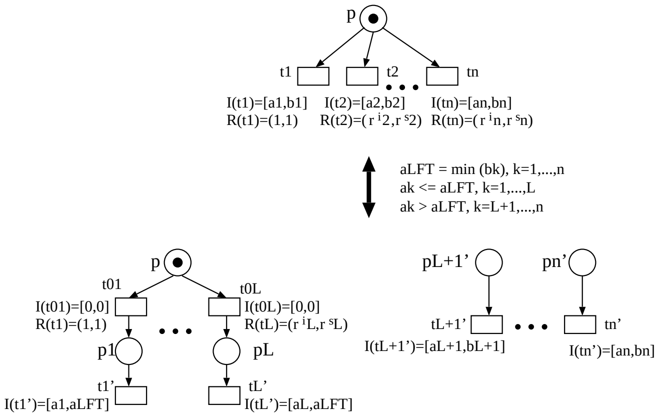

Figure 2. Preselection policy in free-choice TPNFs.

4. Bounds for special classes of interval Time Petri Nets

In case of live and bounded systems similar techniques as the ones defined for Stochastic Petri Nets which make use of P-semiflows [5, 7, 6, 8] can be applied for the computation of bounds of operational performance measures for interval Time Petri Nets. In this section, we analyze the simplest structural classes of Petri Nets that is Marked Graphs (MGs) and Free Choice nets (FCs). Interval Time Petri Nets belonging either to MG class or to FC class satisfy the "performance monotonicity property", that is *local pessimistic transformation, such as incrementing the average firing times, leads to a slower transformed net system*, so the we can use the extreme points of the transition interval firing times for the computation of their performance bounds.

4.1. Marked Graphs

A Marked Graph is an ordinary Petri Net such that $\forall p \in P : \; |^{\bullet}p| = |p^{\bullet}| = 1$. Transitions belonging to a MG are characterized by the same visit ratios and hence by the same throughput $x[t] = x, \forall t \in T, \forall (0, \Gamma), \Gamma \in \mathbb{R}^+$.

The steady state average cycle time of the MG, defined as $T_C = \frac{1}{x}$, can be bounded by solving the following LP problem[6]:

$$\begin{aligned} T_C \geq \quad & \text{maximum} \quad && \mathbf{y} \cdot \mathbf{B^T} \cdot \bar{\mathbf{s}} \\ & \text{subject to} && \mathbf{y} \cdot (\mathbf{F} - \mathbf{B})^{\mathbf{T}} = 0 \\ & && \mathbf{y} \cdot \mathbf{M_0} = 1 \\ & && \mathbf{y} \geq 0 \end{aligned} \quad (16)$$

where $\mathbf{y}$ is a vector of variables, $\mathbf{B^T}$ and $(\mathbf{F} - \mathbf{B})^{\mathbf{T}}$ are the pre-incidence and the incidence matrices, respectively, $\mathbf{M_0}$ is the initial marking vector and $\bar{\mathbf{s}}$ is the mean service time vector.

In case of TPNs (or TPNFs[2]) we can also use this LP problem to compute the transition throughput bounds by replacing the unknown vector $\bar{\mathbf{s}}$ with the available timing information.

Upper bound We consider the best performance case that is the case in which the mean firing time of each transition is equal to the minimum of its static firing interval, i.e., $\forall t \in T : \bar{s}[t] = a[t]$. Let $T_C^{\mathbf{a}}$ be the solution of the LP

[2]Note that marked graph TPNFs reduce to TPNs.

problem (16) in which the vector $\bar{\mathbf{s}}$ is replaced by the vector of the transition earliest firing times $\mathbf{a} = [a[t]]_{t\in T}$, then the throughput upper bound of each transition $t \in T$ is given by:

$$UB[x_t] = \frac{1}{T_C^{\mathbf{a}}},\ \forall t \in T.$$

Lower bound We consider the worst performance case that is the case in which the mean firing time of each transition is equal to the maximum of its static firing interval, i.e., $\forall t \in T : \bar{s}[t] = b[t]$. Let $T_C^{\mathbf{b}}$ be the solution of the LP problem (16) in which the vector $\bar{\mathbf{s}}$ is replaced by the vector of the transition latest firing times $\mathbf{b} = [b[t]]_{t\in T}$. In case of a deterministic Petri Net the value $T_C^{\mathbf{b}}$ corresponds to the exact cycle time value. Being the deterministic Petri Net the worst performance approximation of the given interval Time Petri Net we can take this value as a throughput lower bound for the transitions of the interval Time Petri Net:

$$LB[x_t] = \frac{1}{T_C^{\mathbf{b}}},\ \forall t \in T.$$

In case of XTPNs, being a subclass of Stochastic Petri nets, the LP problem (16) can be used to compute the transition throughput bounds of marked graph XTPNs by taking $\bar{s} = \left[\int_{a[t]}^{b[t]} x f_t(x)dx\right]$.

4.2. Free Choice nets (FCs)

A FC net is an ordinary Petri Net such that $\forall p \in P : |p^\bullet| > 1 \Rightarrow {}^\bullet(p^\bullet) = \{p\}$. Relative throughput of transitions $t \in T$ belonging to a FC net are characterized by visit ratios $v[t]$ that depend exclusively on the net structure and on the routing rates of conflicting transitions. Under the assumption that preselection policy is adopted for conflicts among timed transitions then it is possible to compute a lower bound for the average interfiring time of a transition $t \in T$ (inverse of its throughput) by solving the LP problem (16) in which vector $\bar{\mathbf{s}}$ of mean service times is replaced by the vector $\mathbf{D} = \mathbf{v}\cdot\bar{\mathbf{s}}$ of average service demands, normalized for the transition t whose average interfiring time is under study (i.e., $v[t] = 1$) [5].

In TPNs and in TPNFs conflicts among transitions are not deterministic, hence the routing rates of conflicting transitions cannot be calculated. However, it is always possible to apply the preselection policy to timed conflicting transitions and to compute the transition throughput bounds by solving the general LP problem for the transformed net in which also inequalities (8) and (9) can be included in the set of constraints.

Timed transitions of a XTPN, under the restrictions 1) and 2) stated in sub-section 3.3, can be made persistent and the XPTN can be transformed into a general Stochastic Petri net. Then it is possible to apply the LP problem (16) in which vector $\bar{\mathbf{s}}$ is replaced by $\mathbf{D} = \mathbf{v} \cdot \bar{\mathbf{s}}$. The vector of

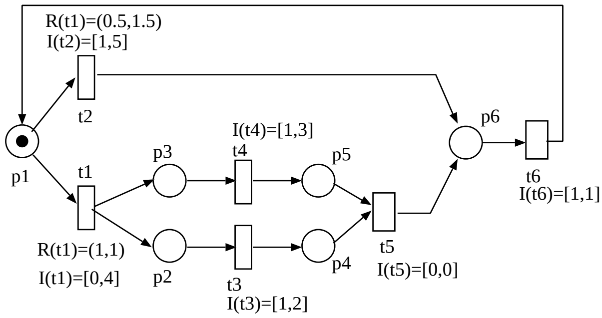

Figure 3. Free choice TPNF with transitions covered by all T-semiflows.

visit ratios can be calculated by solving the linear system of equations:

$$(\mathbf{F}-\mathbf{B})^{\mathbf{T}} \cdot \mathbf{v} = 0 \tag{17}$$
$$\frac{v[t_i]}{P[t_i]} = \frac{v[t_j]}{P[t_j]}, \quad \forall t_i, t_j \text{ s.t. } P[t_i] \neq 0, P[t_j] \neq 0$$
$$v[t_0] = 1, \qquad \text{for a } t_0 \text{ s.t.} P[t_0] \neq 0$$

Upper bound for TPNF Another possibility of computing throughput upper bound for a transition t_i of a free-choice TPNF is to solve the following non linear problem after the preselection policy has been applied to each timed conflicting transition:

$$\begin{aligned} T_{C_i} \geq \quad & \text{maximum} \quad \mathbf{y}\cdot\mathbf{B^T}\cdot\mathbf{v}\cdot\bar{\mathbf{a}} && (18)\\ & \text{subject to} \quad \mathbf{y}\cdot(\mathbf{F}-\mathbf{B})^{\mathbf{T}} = 0 \\ & (\mathbf{F}-\mathbf{B})^{\mathbf{T}}\cdot\mathbf{v} = 0 \\ & \mathbf{y}\cdot\mathbf{M_0} = 1 \\ & r^i[t_j]v[t_k] \leq r^s[t_k]v[t_j] && (19)\\ & r^i[t_k]v[t_j] \leq r^s[t_j]v[t_k] && (20)\\ & v[t_i] = 1 \\ & \mathbf{y} \geq 0, \mathbf{v} \geq 0 \end{aligned}$$

where constraints (19) and (20) have to be applied for each pair of transitions in conflict.

Lower bound for TPNF The set of transitions T of a FC net can be partitioned into the set of transitions that are covered by all T-semiflows, T_{all}, from the rest of transitions, $T \setminus T_{all}$. Then, due to the fact that free-choice conflicts are not deterministic, transitions in $T \setminus T_{all}$ have throughput lower bounds equal to zero. Transitions belonging to T_{all} may have instead non null throughput lower bounds, such as for example transition t_6 of Figure 3.

Throughput lower bounds of transitions $t \in T_{all}$ of a TPNF can be computed by transforming the initial net $\mathcal{TF}$ into the TPNF $\mathcal{TF}'$ that results from the replacement of each timed free-choice conflict into an immediate one, and by solving the following LP problem on $\mathcal{TF}'$:

$$\begin{aligned} T_C^{ub} = \quad & \text{maximum} \quad \sum_{j=1}^{m} \frac{b[t_j]v[t_j]}{SE(t_j)} && (21) \\ & \text{subject to} \quad (\mathbf{F}-\mathbf{B})^{\mathbf{T}} \cdot \mathbf{v} = 0 \\ & \qquad r^i[t_j]v[t_k] \leq r^s[t_k]v[t_j] && (22) \\ & \qquad r^i[t_k]v[t_j] \leq r^s[t_j]v[t_k] && (23) \\ & \qquad v[t^*] = 1 \\ & \qquad \mathbf{v} \geq 0 \end{aligned}$$

where m is the cardinality of the set of transitions T' of $\mathcal{TF}'$, $b[t_j]$ is the latest firing time of transition $t_j \in T'$, $SE(t_j)$ represents the structural enabling bound [5] for t_j and can be computed a priori (so it is not a variable), and transition $t^* \in T'$ is covered by all T-semiflows. Constraints (22) and (23) apply to transitions of $\mathcal{TF}'$ in conflict. The inverse of the solution of the above LP problem, $\frac{1}{T_C^{ub}}$, is a lower bound for transition throughput of $t^* \in T_{all}$ of the original FC net $\mathcal{TF}$.

The LP problem (21) is actually a modification of the one stated in [5] for deterministic and Stochastic free-choice Petri nets in which a set of constraint related to the visit ratios of transitions have been added.

5. Example

The example presented in this section is taken from [12], where an untimed Petri net model is used to represent the behavior of a communication protocol. First, we have added timing information to the untimed Petri net, obtaining the TPN model depicted in Figure 4: the communication between two entities (source and sink) is started by the source that sends a message to the sink (transition T_1) and waits for an ack from the latter. When the message transmission has been executed (transition T_2), the sink receives the message (transition T_5) and sends the ack (transition T_6) to the sender. After the reception of the ack (transition T_8) the source entity is re-initialized. The source entity is provided with a time-out mechanism that allows to detect the message loss during the transmission (transition T_3) and to resend it (transition T_4).

Let us consider transition T_1 (the send action): the min/max throughputs, for Γ sufficiently large, have been computed on the class state graph, generated by the TINA tool [1], and they are equal to $X_{min}[T_1] = 0$ and to $X_{max}[T_1] = 0.2$. The lower and upper bounds have been calculated by solving the general LP problem on the TPN derived from the application of the preselection policy to transitions T_2 and T_3, and they are equal to the min/max throughputs, respectively.

Then, we have added stochastic information to the TPN model obtaining a XTPN in which each timed transition t

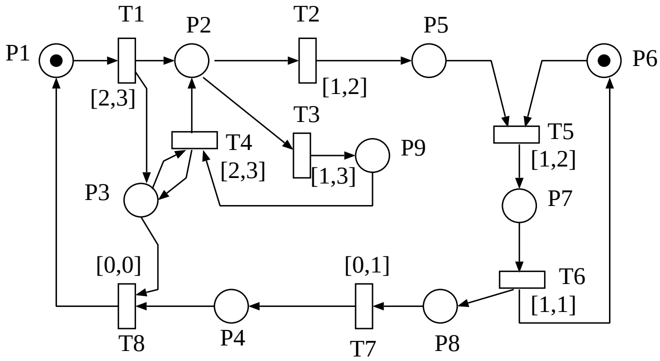

Figure 4. The TPN model of the transmission protocol

such that $a[t] < b[t]$ is characterized by a uniform distribution over the firing interval $(a[t], b[t])$ and the timed transition T_6 is characterized by a deterministic distribution with delay 1 (T_8 is immediate).

The throughput of T_1 calculated by using analytical approximation technique [22] is equal to $X(T_1) = 0.12$. The computation of throughput bounds of T_1 has been carried out by transforming the timed free-choice conflict into an immediate one as stated at the end of Section 4 and by solving, first, the general LP problem subject to constraints (5,6,7,8, 9,11,14) and, then, by solving the general LP problem stated for Stochastic Petri Nets [5], with the following results: $X_1^{UB}[T_1] = 0.167$ and $X_1^{LB}[T_1] = 0.067$, in the first case, and $X_2^{UB}[T_1] = 0.122$ and $X_2^{LB}[T_1] = 0.085$, in the second case. Concerning upper bound throughput, the same result $X_2^{UB}[T_1]$ is obtained by applying the LP problem (16) where vector $\bar{\mathbf{s}}$ has been replaced by $\mathbf{D} = \mathbf{v} \cdot \bar{\mathbf{s}}$ and the vector of visit ratios has been computed by solving the set of linear equations (17).

6. Conclusion

A first step for the development of structural performance analysis techniques for interval Time Petri Nets has been achieved in this paper.

We have shown that it is possible to compute upper and lower bounds for the throughput of transitions and for the mean marking of places in linear time on the net size, by solving proper linear programming problems stated from the net structure, the initial marking, and the parameters that define the time interpretation.

The technique presented here is an extension of a previous linear programming based bound computation technique developed for timed and stochastic Petri nets. In the case of the net interpretations considered in this paper, the firing of transitions is restricted within an interval that defines per each transition the earliest and the latest firing time relative to the instant at which it was enabled. A similar interval based definition is possible for the conflict resolution policy at free choice conflicts that leads to the introduction of the TPNFs. TPNFs have a practical interest, for example, in the modelling of flexible manufacturing systems (FMS). Usually, to model a production plan with Petri Nets it is necessary to establish the proportion of parts that must be produced for each class of parts during a period of time and, in many cases, this is carried out by fixing firing frequencies of transitions that represent the starting of the production of each class of parts. The possibility of modelling the production plan with an interval frequency increases the expression power of the model, making the FMS even more flexible (i.e., it is possible to define the production plan with a "fairness constraint" instead of with fixed ratios).

Additionally, if the probability density function over the firing interval of each transition is also given, the mean value of these variables can be also introduced in the derived linear programming problems to improve the quality of the computed bounds.

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References

[1] B. Berthomieu. TIme petri Net Analyzer (TINA) toolbox. http://www.laas.fr/tina/.

[2] B. Berthomieu. La méthode des Classes d'États pour l'Analyse des Réseaux Temporels - Mise en Oeuvre, Extension à la multi-sensibilisation, Modélisation des Systèmes Réactifs. In *Proc. of MSR'2001*, pages 254–263, Toulouse, France, 2001. Hermes Sciences.

[3] B. Berthomieu and M. Diaz. Modeling and Verification of Time Dependent Systems Using Time Petri Nets. *IEEE Trans. on Soft. Eng.*, 12(3):259–273, March 1991.

[4] B. Berthomieu and M. Menasche. An enumerative approach for analysing Time Petri Nets. In *IFIP congress*, Paris, France, 1983.

[5] J. Campos. Performance Bounds. In Balbo, G. and Silva, M., editor, *Performance Models for Discrete Event Systems with Synchronizations: Formalisms and Analysis Techniques*, pages 587–635, Editorial KRONOS, Zaragoza, Spain, September 1998. Chapter 17.

[6] J. Campos, G. Chiola, J. Colom, and M. Silva. Properties and Performance Bounds for Timed Marked Graphs. *IEEE Trans. on CIrcuits and Systems I: Fundamental Theory and Applications*, 39(5):386–401, May 1992.

[7] J. Campos, G. Chiola, and M. Silva. Properties and Performance Bounds for Closed Free Choice Synchronized Monoclass Queueing Networks. *IEEE Trans. on Automatic Control*, 36(12):352–391, December 1991.

[8] J. Campos and M. Silva. Structural Techniques and Performance Bounds of Stochastic Petri Net Models. In G. Rozenberg, editor, *Advances in Petri Nets*, volume 609 of *LNCS*, pages 352–391. Springer Verlag, Berlin, 1992.

[9] G. Chiola. Timed Petri Nets. In G. Balbo and M. Silva, editors, *Performance Models for Discrete Event Systems with Synchronizations: Formalisms and Analysis Techniques*, volume 1. Editorial KRONOS, Zaragoza, Spain, September 1998.

[10] G. Chiola, C. Anglano, J. Campos, J. M. Colom, and M. Silva. Operational Analysis of Timed Petri Nets and Application to the Computation of Performance Bounds. In IEEE-CS, editor, *Proc. of the* 5^th *Int. Workshop on Petri Nets and Performance Models*, pages 128–137, Toulouse, France, October 1993.

[11] G. Juanole and Y. Atamna. Dealing with Arbitrary Time Distributions with the Stochastic Timed Petri Net Model - Application to Queueing Systems. In *Proc. of PNPM*, pages 32–41, 1991.

[12] G. Juanole, M. Diaz, and F. Vernadat. Reseaux de Petri etendus et methodologie pour l'analyse de performances. Technical report, LAAS, Toulouse (France), 2003. n.03480.

[13] E. Lazowska, J. Zahorjan, G. Scott Graham, and C. Sevcik. *Quantitative System Performance: Computer System Analysis Using Queueing Network models*. Prentice-Hall, 1984.

[14] Z. Liu. Performance Analysis of Stochastic Timed Petri Nets Using Linear Programming Approach. *IEEE Trans. on Soft. Eng.*, 24(11), November 1998.

[15] M. A. Marsan and et al. The Effect of Execution Policies on the Semantics and Analysis of Stochastic Petri Nets. *IEEE Trans. on Soft. Eng.*, 15(7):832–846, 1989.

[16] P. Merlin and D. Faber. Recoverability of communication protocols. *IEEE Trans Commun.*, COM-24(9), 1976.

[17] T. Murata. Petri Nets: Properties, Analysis and Applications. *Proceedings of the IEEE*, 77(4):541–580, April 1989.

[18] L. Popova. On Time Petri Nets. *Journal Inform. Process. Cybern., EIK (formerly: Elektron. Inform. verarb. Kybern.)*, 27(4):227–244, 1991.

[19] L. Popova-Zeugmann and D. Schlatter. Analyzing paths in time Petri nets. *Fundam. Inf.*, 37(3):311–327, 1999.

[20] C. Ramchandani. *Analysis of asynchronous concurrent systems by Timed Petri Nets*. PhD thesis, MIT Cambridge, MA, United Kingdom, 1974.

[21] J. Roux and G. Juanole. Functional and Performance Analysis Using Extended Time Petri Nets. In *Proceedings of PNPM*, pages 14–23, 1987.

[22] M. Scarpa and A. Bobbio. Kronecker representation of Stochastic Petri nets with discrete PH distributions. In *Int. Computer Performance and Dependability Symposium - IPDS98*, pages 52–61. IEEE CS Press, 1998.

[23] R. H. Sloan and U. A. Buy. Reduction Rules for Time Petri Nets. *Acta Informatica*, 33(7):687–706, 1996.

[24] E. Teruel, G. Franceschinis, and M. Silva. Untimed Petri Nets. In Balbo, G. and Silva, M., editor, *Performance Models for Discrete Event Systems with Synchronizations: Formalisms and Analysis Techniques*, pages 27–76, Editorial KRONOS, Zaragoza, Spain, September 1998. Chapter 2.