☆ THE ART OF COMPUTER PROGRAMMING ☆

☆☆☆ ERRATA TO VOLUME 3 (2nd edition) ☆☆☆

This document is a transcript of the notes that I have been making in my personal copy of *The Art of Computer Programming*, Volume 3 (second edition) since it was first printed in 1998.

Four levels of updates — "errors," "amendments," "plans," and "improvements" — appear, indicated by four different typographic conventions:

▸**Page 666** line 1 _____ 04 Jul 1776

Technical or typographical errors (aka bugs) are the most critical items, so they are flagged with a '▸' preceding the page number. The date on which I first was told about the bug is shown; this is the effective date on which I paid the finder's fee. The necessary corrections are indicated in a straightforward way. If, for example, the book says 'n' where it should have said '$n + 1$', the change is shown thus:

$n \leadsto n + 1$

Page 666 line 2 _____ 14 Jul 1789

Amendments to the text appear in the same format as bugs, but without the '▸'. These are things I wish I had known about or thought of when I wrote the original text, so I added them later. The date is the date I drafted the new text.

Page 666 line 3 . . . . . 20 Nov 1917

Plans for the future represent a third kind of item. In such notes I sketched my intentions about things that I wasn't ready to flesh out further when I wrote them down. You can identify these items because they're written in slanted type, and preceded by a bunch of dots '. . . . . ' leading to the date on which I recorded the plan in my files.

Page 666 line 4 _____ 10 Jan 1938

The fourth and final category — indicated by page and line number in smaller, slanted type — consists of minor corrections or improvements that most readers don't want to know about, because they are so trivial. You wouldn't even be seeing these items if you hadn't specifically chosen to print the complete errata list in all its gory details. Are you sure you wanted to do that?

My shelves at home are bursting with preprints and reprints of significant research results that I want to digest and summarize, where appropriate, in the ultimate edition of Volume 3. I didn't do that in the second edition because I would surely have to do it over again later: New results continue to pour forth at a great rate, and I will have time to rewrite that volume only once. Volumes 4 and 5 need to be finished first. So I've put most of my effort so far into writing up those parts of the total picture that seem to have converged to their near-final form. It follows, somewhat paradoxically, that the updates in this document are most current in the areas where there has been least activity.

On the other hand I do believe that the changes listed here bring Volume 3 completely up to date in two respects: (1) All of the research problems in the previous edition — i.e., all exercises that were rated 46 and above — have received new ratings of 45 or less whenever I learned of a solution; and in such cases, the answer now refers to that solution. (2) All of the historical information about pioneering developments has been amended whenever new details have come to my attention.

The ultimate, glorious, future editions of Volumes 1–3 are works in progress. Please let me know of any improvements that you think I ought to make. Send your comments either by snail mail to D. E. Knuth, Computer Science, Gates Building 4B, Stanford University, Stanford CA 94305-9045, or by email to `taocp@cs.stanford.edu`. *(Use email for book suggestions only, please — all other correspondence is returned unread to the sender, or discarded, because I have no time to read ordinary email.) Although I'm working full time on Volume 4 these days, I will try to reply to all such messages within a year of receipt. Current news about The Art of Computer Programming is posted on*

`http://www-cs-faculty.stanford.edu/~knuth/taocp.html`

and updated regularly.

— Don Knuth, February 1998

Writing a series like The Art of Computer Programming
is similar to painting the Forth Rail Bridge.
No sooner is it finished than
the job must be started again.

— MALCOLM CLARK (1992)

The time when The Guardian *ceases to make mistakes altogether*
is not, at the moment, foreseeable.

— IAN MAYES (1998)

☆☆☆☆☆☆ SORTING AND SEARCHING ☆☆☆☆☆☆

Last updated 21 July 2000

Most of these corrections have already been made in recent printings.

▸**Page 1** line 7 — 05 May 1998

The Prince (1951) ↜↝ *The Prince* (1513)

Page 5 line 3 of exercise 3 — 16 Nov 1998

conditions (1) and (2) ↜↝ conditions (1) and (2)

▸**Page 7** line 2 — 03 Jan 2000

at most ten ↜↝ less than a dozen

Page 7 line −24 — 14 Sep 1998

Achtzehnhundert zwölf ↜↝ Achtzehnhundertzwölf

Page 7 line −7 — 11 Nov 1998

süssen Mädeln. ↜↝ langen Tag.

▸**Page 17** line 3 — 12 May 1998

$\big((a_1\,a_2\ldots a_n),\ (p_1,p_2,\ldots,p_n)\big)$ ↜↝ $\big((a_1,a_2,\ldots,a_n),\ (p_1,p_2,\ldots,p_n)\big)$

▸**Page 17** line 6 — 12 May 1998

$\mathrm{ind}(a_1,a_2,\ldots,a_n)$ ↜↝ $\mathrm{ind}(a_1\,a_2\ldots a_n)$

Page 18 line 1 — 23 May 1998

5 4 6 1 3 8 7 2; ↜↝ 5 4 6 1 3 8 7 2 (man 1 is 5th out, etc.);

▸**Page 20** line 2 of exercise 19 — 09 Aug 1999

$((n-1)m \bmod m)$ ↜↝ $((n-1)m \bmod n)$

Page 23 near the bottom — 20 Jul 2000

line −12: *Lílávatí* of Bháscara Áchárya ↜↝ *Līlāvatī* of Bhāskara
lines −11 and −8: Bháscara ↜↝ Bhāskara

▸**Page 23** bottom line — 01 May 1998

5–7.] ↜↝ 5–7].

▸**Page 32** line 3 of exercise 15 — 09 Aug 1998

$x_1 < x_2 < \cdots < x_n$ ↜↝ $x_1 < x_2 < \cdots < x_m$

▸**Page 40** lines 10 and 11 from the bottom — 18 May 2000

$+\sum_{j=0}^{k}$ ↜↝ $+\sum_{j=0}^{k-1}$ (twice)

▸**Page 41** replacement for the bottom line — 19 May 2000

$$R_m(z) = L(z) + \frac{2z}{z-z_0} + \frac{z}{z-z_1} + \frac{z}{z-\bar{z}_1} + \frac{z}{z-z_2} + \frac{z}{z-\bar{z}_2} + \cdots + \frac{z}{z-z_m} + \frac{z}{z-\bar{z}_m}$$

▸**Page 42** replacement for line 5 — 19 May 2000

$$\frac{2z}{1-z} + \frac{z/z_1}{1-z/z_1} + \frac{z/\bar{z}_1}{1-z/\bar{z}_1} + \cdots + \frac{z/z_m}{1-z/z_m} + \frac{z/\bar{z}_m}{1-z/\bar{z}_m} + R_m(z),$$

▸**Page 42** line 3 after (31) — 19 May 2000

$R_m(z) \to -z$ ↜↝ $R_m(z) \to cz$ for some constant c

Page 42 line 4 after (31) — 26 May 2000

when $n > 1$. ↜↝ when $n > 1$. (See also exercise 28.)

Page 45 replacement for line 3 — 18 May 2000

$$\sum_k \left\langle {n \atop k} \right\rangle \binom{k}{n-q} = \left\{ {n \atop q} \right\} q!, \qquad \text{integer } q \geq 0.$$

Page 47 new exercise — 26 May 2000

28. [*HM35*] Find the asymptotic value of the numbers z_m in Fig. 3 as $m \to \infty$, and prove that

$$\sum_{m=1}^{\infty} (z_m^{-1} + \bar{z}_m^{-1}) \;=\; e - \frac{5}{2}.$$

▸**Page 52** line 16 — 12 May 1998

poof ↜↝ proof

▸**Page 60** line 4 after the caption — 25 Nov 1998

(n_2+m-1) ↜↝ (n_2+m-2)

▸**Page 63** line 1 of (46) — 17 May 1999

$\frac{1}{2}\ln n$ ↜↝ $\frac{1}{2}n\ln n$

▸**Page 63** line 2 of (49) — 09 Aug 1999

$\dfrac{x^6}{n^2}$ ↜↝ $\dfrac{8}{9}\dfrac{x^6}{n^2}$

▸**Page 76** line −15 — 21 Jul 1998

of other ↜↝ of the other

▸**Page 77** top line — 07 Jun 1998

Table 2 ↜↝ **Table 1**

Page 77 line −4 — 03 Jun 1998

$K_1, \ldots, K_N$ ↜↝ $K_1 \ldots K_N$

▸**Page 82** replacement for line 11 — 01 May 1998

$$B = \big(\min 0,\ \text{ave}\ (N^2-N)/4,\ \max\ (N^2-N)/2,\ \text{dev}\ \sqrt{N(N-1)(N+2.5)/6}\big);$$

▸**Page 84** line 4 after the table — 23 May 1998

that w sort $\leadsto$ that we sort

▸**Page 90** line 11 — 09 Aug 1998

$h_t, \ldots, h_1 \leadsto h_{t-1}, \ldots, h_0$

▸**Page 92** in (9) — 22 May 2000

$\binom{r-1}{2} \le s \le \binom{r}{2} \leadsto \binom{r-1}{2} \le s < \binom{r}{2}$

▸**Page 92** line 1 of Theorem I — 09 Dec 1998

$O(Ne^{c\sqrt{\ln n}}) \leadsto O(Ne^{c\sqrt{\ln N}})$

▸**Page 102** line 2 of exercise 7 — 12 May 1999

$|a_2 - 1| \leadsto |a_2 - 2|$

▸**Page 105** line 4 — 22 May 2000

the running time $\leadsto$ the average running time

▸**Page 105** line 4 of exercise 42 — 22 May 2000

$N/g \leadsto N^{3/2}/g$

▸**Page 116** line 2 of step Q7 — 01 May 1998

$r + 1$. If $\leadsto$ $r + 1$.) If

Page 118 line 53 of the program — 27 Jul 1998

$j - N \leadsto j - N$.

Page 121 replacement for line 2 of (25) — 15 Aug 1998

$$B_N = \tfrac{1}{6}(N+1)(2H_{N+1} - 2H_{M+2} + 1 - 6/(M+2)) + \tfrac{1}{2},$$

▸**Page 122** line 7 — 22 May 2000

Exercise 58 $\leadsto$ Exercise 42

▸**Page 125** line 1 of step R2 — 15 Aug 1998

$R_l \le \cdots \le R_r \leadsto R_l \ldots R_r$

▸**Page 125** line -3 — 15 Aug 1999

[rI4 $= l - j$] $\leadsto$ [rI4 $= j - l$]

▸**Page 126** lines *30* and *31* of the program — 15 Aug 1998

[rI4unknown] $\leadsto$ [rI4 unknown]
$b \leftarrow b - 1 \leadsto b \leftarrow b + 1$

Page 136 new sentence added to exercise 28 — 25 Oct 1998

Ignore the comparisons made when computing the median value s.

▸**Page 138** line 2 of exercise 55 — 05 Aug 1999

three keys (28). $\leadsto$ three keys (28), assuming that $M > 1$.

▸**Page 141** line −18 — 23 May 1998
{170, 175} ↜ {170, 275}

▸**Page 151** line −7 — 01 May 1998
heaps). ↜ heaps.)

▸**Page 152** line 9 — 09 Jan 2000
CACM **12** ↜ *CACM* **21**

Page 152 lines 24–25 — 20 Mar 2000
SODA **8** (1997), 83–92] ↜ *SICOMP* **28** (1999), 1326–1346]

▸**Page 166** line −12 — 29 Jul 1998
111.] ↜ 111].

▸**Page 166** line −8 — 13 Aug 1998
arrangement ↜ arrangements

▸**Page 166** line −3 — 31 Jul 1998
$K'_1 < \cdots K'_N$ ↜ $K'_1 < \cdots < K'_N$

Page 166 line −1 — 11 May 1999
key appears ↜ keys appear

Page 168 line 1 of exercise 18 — 16 Feb 2000
on N records ↜ of N records

Page 174 line 07 of the program — 05 Oct 1999
→ `TOP[`i`]` ↜ → `TOP[`i`]`.

▸**Page 177** line 1 — 20 Nov 1999
sort if ↜ sort it

Page 177 line −1 — 10 Aug 1998
signed-magnitude ↜ *signed magnitude*

▸**Page 183** line 23 — 09 Aug 1998
$\lceil \ln n! \rceil$ ↜ $\lceil \lg n! \rceil$

▸**Page 188** row b and column e of (24) — 16 Jun 1998
9 ↜ 7

▸**Page 204** line 2 — 05 Oct 1999
$M(m, n-2^t)$ ↜ $H(m, n-2^t)$

▸**Page 212** bottom line — 04 Jan 1999
acieves ↜ achieves

▸**Page 225** line −13 — 02 Jun 1998
needed in an ↜ needed in a

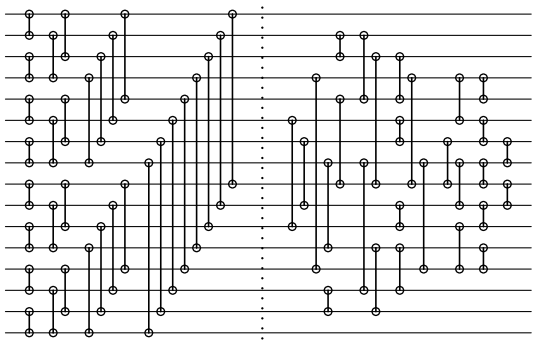
Page 225 line −8 — 21 Jan 2000

still unknown. ↛ known only in a very weak sense.

▸**Page 226** line −7 — 30 Jun 1999

Hughes ↛ Hugues

Page 227 replacement for bottom part of Fig. 49 — 10 Aug 1998



$n = 16$ 60 modules, delay 10

▸**Page 240** line −5 — 27 Aug 1998

$r \le 4n^2 + \sqrt{n}\lg n$ ↛ $r \le 4n^2 + O(n^{3/2}\log n)$

Page 242 lines −3, −13, −17 — 17 Aug 1998

Fig. 61 ↛ Fig. 60
Fig. 61 ↛ Fig. 60

▸**Page 246** line 2 of exercise 66 — 05 Oct 1999

exercise 63 ↛ exercise 64

Page 246 lines −1, −3 — 17 Aug 1998

Fig. 60 ↛ Fig. 61
Fig. 60 ↛ Fig. 61

▸**Page 247** line 4 — 29 Oct 1999

$1 \le i \le N$ ↛ $1 < i \le N$

Page 264 line 2 of exercise 23 — 11 May 1999

the same order that ↛ the same order as

▸**Page 271** line 1 of step D1 — 06 May 1998

`D[`j`]` -1 and `TAPE[`k`]` $\leftarrow j$ ↛ `D[`j`]` $\leftarrow 1$ and `TAPE[`j`]` $\leftarrow j$

Page 275 equation number in wrong font — 14 Nov 1999

(12) ↛ (12)

▸**Page 282** line −10 — 14 Nov 1999

$v_1 + u_0 + v_0$ ↛ v_1

Page 286 line 3 of exercise 14 — 14 Nov 1999

$T_{(n(k)+1),k}$ ↜ $T_{(n(k)+1)k}$

▸**Page 299** line −18 — 16 Jan 2000

Programmer ↜ Programmers

Page 348 line 3 of exercise 1 — 10 Aug 1998

mixed radix system ↜ mixed-radix number system

▸**Page 348** line −13 — 18 May 1998

preform ↜ perform

Page 389 line −15 — 03 Jul 1999

SODA **8** (1997), 370–379 ↜ *J. Algorithms* **31** (1999), 66–104.

▸**Page 400** equation (12) — 23 Jun 1998

$c_1 = 1/N^\theta$ ↜ $c = 1/N^\theta$

▸**Page 403** line 17 — 02 Jun 1998

a interesting ↜ an interesting

Page 406 line 14 — 14 Nov 1999

$P_{N-1,m-1}$ ↜ $P_{(N-1)(m-1)}$

Page 412 equation number in wrong font — 14 Nov 1999

(1) ↜ (1)

Page 414 line −9 — 14 Nov 1999

number of ↜ number of

▸**Page 416** lines 5 and 6 after the program — 14 Nov 1999

C1 is weighted more heavily than C2 ↜ $C1$ is weighted more heavily than $C2$

▸**Page 435** line 3 — 19 Jan 2000

about n^2 ↜ about N^2

Page 443 equation numbers in wrong font — 12 Dec 1999

(20) ↜ (20)
(21) ↜ (21)

▸**Page 446** line −8 — 13 May 1998

in a several ↜ in several

Page 447 near the top — 03 Dec 1998

line 4: cost c ↜ weight w
line 6: $-c$ ↜ $-w$
line 7: it has been ↜ an optimum tree has been
lines 8 and 9: a sequence of such transformations will make $l_k \le l_{k+1}$ ↜ we have found an optimum tree in which $l_k = l_{k+1}$

▸**Page 447** at the end of the proof of Lemma X — 03 Dec 1998

line −4 of the proof: $\boxed{j{+}1}$ ↜↝ $\boxed{j{-}1}$ (twice)

line −3 of the proof: $\boxed{i{+}1}$ ↜↝ $\boxed{i{-}1}$

line −2 of the proof: $\boxed{k{+}1}$ ↜↝ $\boxed{k{-}1}$

▸**Page 449** near the top — 03 Dec 1998

line 4: $\boxed{k{-}2}$ ↜↝ $\boxed{s{-}1}$

line 6: $j < i < k - 1$ ↜↝ $j < i < s$.

▸**Page 453** line 22 — 11 May 1999

It it is smaller, ↜↝ If it is smaller,

▸**Page 455** line 2 of exercise 11 — 11 May 1999

empirical data ↜↝

Page 457 line 1 of exercise 28 — 11 May 1999

a "optimum binary search" ↜↝ an "optimum binary search"

▸**Page 459** line 3 — 28 Jul 1999

Doklady Akademiĭa Nauk ↜↝ *Doklady Akademii Nauk*

▸**Page 460** line 2 of Theorem A — 08 Dec 1998

1.4404 ↜↝ 1.4405

Page 465 line −2 — 28 Nov 1999

$-a$, 0 or 0 ↜↝ $-a$, 0, or 0

Page 466 line 17 — 28 Nov 1999

$-a$, 0 or 0 ↜↝ $-a$, 0, or 0

▸**Page 469** lines −3 and −2 — 02 Jun 1998

$.143 + .153 + .143 + .143 = .582$ ↜↝ $.143 + .152 + .143 + .143 = .581$

▸**Page 475** lines 3 and 4 after the caption — 23 Oct 1998

both (although ↜↝ both, although

▸**Page 477** line −5 — 28 Nov 1999

(1989) ↜↝ (1990)

Page 478 bottom line — 19 Aug 1998

Lecture Notes in Comp. Sci. **1136** (1996), 91–106 ↜↝ *JACM* **45** (1998), 288–323

▸**Page 493** at left of the table — 28 Nov 1999

between I and J: Θ ↜↝ Δ

between R and Π: Φ ↜↝ Σ

▸**Page 500** line 9 — 11 Sep 1998

See n ↜↝ Set n

Page 505 line −5 — 28 Nov 1999

node.) ↜↝ node).

Page 512 new exercise — 08 Mar 1999

▸**45.** [*M25*] If the seven keys of Fig. 33 are inserted in random order by the algorithm of exercise 15, what is the probability of obtaining the tree shown?

▸**Page 513** line −15 — 25 Feb 2000

Mecmuasi ↜↝ *Mecmuası*

▸**Page 514** line 10 — 15 Oct 1999

`K(3,3)` ↜↝ `K(3:3)`

▸**Page 515** line −14 — 17 Jul 1998

good good spread ↜↝ good spread

Page 523 line 2 after (13) — 19 Dec 1999

rA ≡ K. ↜↝ rA ≡ K; rI2 ≡ `LINK[`i`]` and/or R.

▸**Page 534** line 6 — 17 Dec 1998

to step R2. ↜↝ to step R1.

▸**Page 548** line −5 — 07 Jul 1998

Witold Lipski ↜↝ Witold Litwin

▸**Page 550** line 3 — 20 Nov 1999

construct polynomial ↜↝ construct a polynomial

▸**Page 550** line −4 of exercise 8 — 18 Jun 1998

$\{(-1)^k q_k \theta\}$ ↜↝ $\{(-1)^{k+1} q_k \theta\}$

▸**Page 550** line −3 of exercise 8 — 24 Jul 1998

$\theta\}$ ↜↝ $\theta\}^+$

▸**Page 555** line 7 of exercise 55 — 06 Apr 2000

Eq. 2.3.4.4–(9) ↜↝ Eq. 2.3.4.4–(21)

▸**Page 557** line 6 of exercise 72 — 11 May 1999

than the expected ↜↝ then the expected

▸**Page 561** line −12 — 12 Dec 1999

for examples ↜↝ for example

Page 571 first two lines of (11) — 20 Nov 1999

[semicolons are missing after the numeric values]

▸**Page 573** line −15 — 17 Nov 1999

IEEE/ ↜↝ *IEEE*

▸**Page 574** line −2 — 04 Aug 1998

1, 2, 5, 7, and 8 ↜↝ 0, 1, 4, 6, and 7

Page 575 line 18 — 20 Mar 2000

In 1997, A. E. Brouwer found $\leadsto$ A. E. Brouwer [*SICOMP* **28** (1999), 1970–1971] has found

Page 578 line 22 — 12 Dec 1999

Proc.ACM $\leadsto$ *Proc. ACM*

▸**Page 584** line 6 — 06 Jul 1998

(1862) $\leadsto$ (1864)

▸**Page 584** line −13 — 06 Jul 1998

$R_{p(n)} \leadsto R_{p(1)}$

▸**Page 587** last line of answer 12 — 03 Jan 2000

$\lceil \lg n \rceil \leadsto \lceil \lg n \rceil + 1$

Page 588 line 17 — 14 Sep 1998

`ACHTZEHNHUNDERT␣8ZWOLF␣8E` $\leadsto$ `ACHTZEHNHUNDERTZWOLF␣8EIN`

Page 588 line 34 — 11 Nov 1998

`SUSSEN␣8MADE` $\leadsto$ `LANGEN␣8TAG␣`

▸**Page 589** answer 19 — 09 Jul 1998

line 1: $(x_i, x_j) \leadsto \{x_i, x_j\}$
line 7: $(x_i, x_i) \leadsto \{x_i, x_i\}$
lines 7 and 8: those of exercises 18 and 19 $\leadsto$ the method of exercise 18

▸**Page 589** line 2 of answer 21 — 05 Mar 1999

`ERS,` $\leadsto$ `-ERS,`

▸**Page 594** line 1 of answer 20 — 07 Jan 2000

Zolnowski $\leadsto$ Zolnowsky

▸**Page 595** line 3 — 18 May 2000

$(uv)^{\binom{k}{2}}(-u^{-n}v^{1-n})^k \leadsto (uv)^{\binom{j}{2}}(-u^{-n}v^{1-n})^j$

▸**Page 595** line 6 of answer 23 — 29 Jun 1999

$(q_n^{h_1+k_1}p_n) \leadsto (q_n^{h_1+k_n}p_n)$

▸**Page 596** line 3 of answer 27 — 03 Jan 2000

$$\sum_n \frac{H_n(w,z)}{(1-z)\dots(1-z^n)} \leadsto \frac{H_n(w,z)}{(1-z)\dots(1-z^n)}$$

▸**Page 599** line 11 of answer 14 — 02 Jun 2000

$(l \cdot a,\ m \cdot b,\ n \cdot c) \leadsto \{l \cdot a,\ m \cdot b,\ n \cdot c\}$

▸**Page 603** line 1 of answer 11 — 09 Aug 1999

$\sum_{t_1 \ge 1, \dots, t_k \ge 1} \leadsto \sum_{t_1 \ge 1, \dots, t_{k-1} \ge 1}$

Page 607 new answer — 26 May 2000

tree function
Corless
Gonnet
Hare
Jeffrey
Knuth

28. The poles of $L(z)$ are the values of $T(1/e)$, where $T(z)$ is the (multivalued) tree function defined by $T(z) = ze^{T(z)}$. Thus for $m > 0$ we have the convergent series

$$z_m = -\sigma_m + \sum_{n\ge 0} \frac{1}{\sigma_m^n} \sum_k (-1)^k \begin{bmatrix} n \\ k \end{bmatrix} \frac{(\ln \sigma_m)^{n+1-k}}{(n+1-k)!}, \qquad \sigma_m = -1 - (2m+1)\pi i$$

[Corless, Gonnet, Hare, Jeffrey, and Knuth, *Advances in Computational Mathematics* **5** (1996), 329–359, formula (4.18)]; in particular, we have $z_m = (2m+\frac12)\pi i + \ln(2\pi em) + (\frac14 - \frac{i}{2\pi}\ln(2\pi em))/m + O((\log m)^2/m^2)$.

Let $P(z) = \sum_{m=0}^{\infty}(z/(z-z_m) + z/(z-\bar z_m))$. It follows that $P(x) - P(-x) = \sum_{m=0}^{\infty} 4\Re(xz_m/(x^2-z_m^2)) = \sum_{m=1}^{\infty} O((x\log m)/(x^2+m^2)) = \sum_{m=1}^{x} O((x\log x)/x^2) + \sum_{m=x+1}^{\infty} O((x\log m)/m^2) = O(\log x)$ for $x > 1$. But we know that $L(x) + P(x) = cx$ for some c; hence $2cx = L(x) - L(-x) + O(\log x)$, and by letting $x \to \infty$ in (25) we find $c = -1/2$. Hence $L_1 = \sum_{m=0}^{\infty} 2r_m^{-1}\cos\theta_m - 1/2$. (This result is due to Svante Janson.)

Page 611 lines 3–5 of answer 28 — 23 Mar 2000

[M. Talagrand ... $\Theta(n^{1/6})$.] ↜ [J. Baik, P. Deift, and K. Johansson, *J. Amer. Math. Soc.* **12** (1999), 1119–1178, showed that the standard deviation is $\Theta(n^{1/6})$; moreover, the probability that the length is less than $2\sqrt{n} + tn^{1/6}$ approaches $\exp(-\int_t^{\infty}(x-t)u^2(x)\,dx)$, where $u''(x) = 2u^3(x) + xu(x)$ and $u(x)$ is asymptotic to the Airy function $\mathrm{Ai}(x)$ as $x \to \infty$.]

Page 615 line 2 — 04 Jul 1999

STOC **27** (1995), 178–189 ↜ *JACM* **46** (1999), 1–27

Page 620 line 12 — 02 Aug 1998

$NT - S - C$ ↜ $NT - S - C$ ▮

Page 624 line 22 of answer 31 — 15 Aug 1998

[center the 'T' in the frequency column]

▸**Page 624** line 23 of answer 31 — 15 Aug 1998

is the ↜ is

Page 625 line −2 of answer 33 — 02 Aug 1998

$N - 1$ ↜ $N - 1$ ▮

Page 625 bottom line — 02 Aug 1998

M ↜ M ▮

▸**Page 626** last line of answer 36 — 15 Aug 1998

slow!) ↜ slow!

▸**Page 626** replacement for line 13 of answer 37 — 22 May 2000

$$\sum_{N\ge 0} g''_{NM}(1)\frac{M^N w^N}{N!} = M(M-1)e^{(M-2)w}\left(\frac{w^2}{4}e^w\right)^2 + Me^{(M-1)w}\left(\frac{w^4}{16} + \frac{5w^3}{18}\right)e^w.$$

[Also change $g'_{MN}(1)$ ↜ $g'_{NM}(1)$ on line 11.]

▸**Page 626** line 2 of answer 38 — 22 May 2000

converges to $\leadsto$ is asymptotic to

▸**Page 627** answer 41 — 22 May 2000

line 2: $\rho^{k+1}/(k+1) - \rho^k/k \leadsto (\rho^{k+1}/(k+1) - \rho^k/k)/\ln\rho$
line 6: $(k-1)^2 \leadsto (k-2)^2$
line 8: $(\log\log N)^{-3} \leadsto (\log\log N)^{-2}$

▸**Page 627** line 10 of answer 42 — 22 May 2000

$N/gh \leadsto N/gh + 1$

Page 630 line 2 — 02 Aug 1998

$p \neq 0. \leadsto p \neq 0.$ ∎

▸**Page 632** last line of answer 23 — 05 Jun 2000

the form (20) $\leadsto$ the form (19)

▸**Page 636** last line of answer 41 — 15 Aug 1999

Chapter 11 $\leadsto$ Chapter 14

Page 636 lines 7 and 8 of answer 44 — 15 Aug 1999

[insert a bit of space between these lines]

▸**Page 636** line 5 of answer 48 — 05 Jun 2000

$-\delta_0(n) \leadsto -\delta_0(n) + O(n^{-100})$

▸**Page 636** replacement for answer 49 — 05 Jun 2000

49. The right-hand side of Eq. (40) can be improved to the estimate $e^{-x}(1 - \frac{1}{2}x^2/n + O((x^3+x^4)n^{-2}))$. The effect is to subtract half the sum in exercise 47, replacing $O(1)$ in (50) by $2 - \frac{1}{2}(1/\ln 2 + \delta_1(n)) + O(n^{-1})$. (The "2" comes from the "$2/n$" in (46).)

▸**Page 637** lines 3 and 4 — 05 Jun 2000

.0000001725, 00041227, ..., .341 $\leadsto$
.000000172501, .000041227, .0002963, .0008501433, .0062704, .06797, .1525, .348

▸**Page 637** line 2 of answer 53 — 15 Aug 1999

$(p^k q^{n-k} + q^k p^{n-k})x_n \leadsto (p^k q^{n-k} + q^k p^{n-k})x_k$

Page 638 last line of answer 54 — 15 Aug 1999

Über $\leadsto$ *über*

▸**Page 638** line −10 of answer 55 — 04 Aug 1999

$R_{i+1} \leadsto R_{l+1}$

▸**Page 638** last two lines of answer 55 — 05 Aug 1999

does not look at ... fast. $\leadsto$ does not look at K_{N+1}, but it still might examine K_0 in step Q9.

▸**Page 642** line 9 — 02 Aug 1998

return to (b). $\leadsto$ return to P2. ∎
[Also change **Step1**, ..., **Step5** to **P1**, ..., **P5**, respectively, in this answer.]

Page 642 answer 16 — 02 Aug 1998

line 1: **Step 1** ↝ **I1**
line 2: **Step 2** ↝ **I2**
line 3: **Step 3** ↝ **I3**
line 4: **Step 4** ↝ **I4**
line 4: return to step 2. ↝ return to I2. ∎

▸**Page 644** line −2 — 15 Aug 1999

h_n^{-1} ↝ h_N^{-1}

Page 650 line 12 of answer 5 — 02 Aug 1998

`DEC1  1` ↝ `DEC1  1` ∎

▸**Page 652** replacement for line 4 of answer 18 — 05 Oct 1999

$\frac{1}{2}\sum_{k=0}^{CN-1}\sum_j \binom{N}{j}p_k^j(1-p_k)^{N-j}\binom{j}{2} = \frac{1}{2}\sum_{k=0}^{CN-1}\binom{N}{2}p_k^2 \le \frac{N-1}{4}\sum_{k=0}^{CN-1}p_kB/C$, because $p_k \le B/CN$.

▸**Page 653** line 3 of answer 3 — 05 Oct 1999

Eq. 1.2.9–(10).) ↝ Eq. 1.2.9–(10).

Page 655 line 2 of and 7 — 05 Oct 1999

6 ↝ $2\cdot 3$

▸**Page 657** line −5 — 05 Oct 1999

noninteger, ↝ noninteger.

▸**Page 658** answer 1 — 09 Jul 1998

$S(n)+S(n)$ ↝ $S(m)+S(n)$

Page 664 line 4 of answer 23 — 20 Mar 2000

[*SODA* **6** (1995), 28–37] ↝ [*SICOMP* **28** (1999), 1722–1758]

▸**Page 669** line 4 of answer 32 — 05 Oct 1999

G_T ↝ G_t

▸**Page 669** answer 35 — 05 Oct 1999

line 4: and$[n-1{:}n]$ ↝ and $[n-1{:}n]$
line 5: $(k-1)D_{k-1}$ ↝ $(k-1)D_{k+1}$

▸**Page 671** line 3 of answer 40 — 27 Oct 1998

$t = 4n$ ↝ $t = 4n + \sqrt{n}\ln n$

▸**Page 671** replacement for lines 4–7 of answer 40 — 28 Oct 1998

Experiments show that the expected time to reach *any* primitive sorting network — not necessarily the bubble sort — is very nearly $2n^2$. Curiously, R. P. Stanley and S. V. Fomin have proved that if the comparators $[i_k : i_k{+}1]$ are chosen nonuniformly in such a way that $i_k = j$ occurs with probability $j/\binom{n}{2}$, the corresponding expected time comes to exactly $\binom{n}{2}H_{\binom{n}{2}}$.

▸**Page 671** line 4 of answer 49 — 05 Oct 1999

$\{0,0,0,1,1,1\})$ ↝ $\{0,0,0,1,1,1\}$

Page 672 line −11 — 11 May 1999

consists of two ↝ consist of two

Page 690 line 5 of answer 23 — 14 Nov 1999

$u^{(1)}, \ldots .$ ↝ $u^{(1)}, \ldots, u^{(q-1)}.$

▸**Page 704** answer 18 — 14 Nov 1999

first line: $q_j < r_1$ ↝ $q_j < r_j$
last line: 10).] ↝ 10.]

▸**Page 712** lines 5 and 10 of answer 40 — 03 Dec 1998

$q_{k-2}(l_{k-2} - l_{k-4})$ ↝ $q_{k-2}(l_{k-4} - l_{k-2})$

▸**Page 715** line 2 of answer 9 — 29 Jul 1998

$\log_{(\sqrt{10}+2)/3} n$ ↝ $\log_{(\sqrt{10}+2)/3} n$

▸**Page 715** last line of answer 11 — 28 Nov 1999

6.2.4–8. ↝ 6.2.4–8.]

▸**Page 722** line 9 — 28 Nov 1999

(1991) ↝ (1992)

Page 722 bottom line — 02 Aug 1998

rA and rX. ↝ rA and rX. ∎

▸**Page 726** line 2 of answer 31 — 29 Jul 1998

W. Prodinger ↝ H. Prodinger

▸**Page 727** lines 7 and 8 of answer 34 — 28 Nov 1999

$+\cdots$ ↝ $+\cdots$) (twice)

Page 728 new answer — 08 Mar 1999

45. The probability of {THAT, THE, THIS} before {BUILT, HOUSE, IS, JACK}, {HOUSE, IS, JACK} before {BUILT}, {HOUSE, IS} before {JACK}, {IS} before {HOUSE}, {THIS} before {THAT, THE}, and {THE} before {THAT} is $\frac{3}{7} \cdot \frac{3}{4} \cdot \frac{2}{3} \cdot \frac{1}{2} \cdot \frac{1}{3} \cdot \frac{1}{2} = \frac{1}{56}$.

Page 732 line 4 of answer 29 — 25 Feb 1999

1274.] ↝ 1274; see also R. Pyke, *Annals of Math. Stat.* **30** (1959), 568–576, Lemma 1.]

Page 740 line 2 of answer 56 — 29 May 1998

to appear ↝ 37–71

Page 742 lines 7 and 8 of answer 68 — 23 Nov 1998

D. E. Knuth, P. Flajolet, ...to appear. ↝ P. Flajolet, P. V. Poblete, and A. Viola, *Algorithmica* **22** (1998), 490–515; D. E. Knuth, *Algorithmica* **22** (1998), 561–568.

Page 742 line 2 of answer 73 — 19 Dec 1999

characters. ↝ characters. [It was invented as early as 1970 by Alfred L. Zobrist, whose original technical report has been reprinted in *ICCA Journal* **13** (1990), 69–73.]

▸**Page 744** line −7 — 12 Dec 1999

And If ↝ And if

Page 749 Table 2 ..... 06 May 1999

In the next edition I plan to give these constants to 36 hexadecimal places, instead of 45 octal places.

▸**Page 753** entry for Kronecker delta ——— 14 Aug 1998

1.2.6 ↜↝ 1.2.3

▸**Page 755** entry for $\exp x$ ——— 09 Aug 1999

1.2.2 ↜↝ 1.2.9

Page 757 and following ——— 16 Feb 1998

Miscellaneous changes to the existing index of Volume 3 are collected here, including corrections and amendments to the old entries as well as new entries that are occasioned by the new material. Thus, the lines of the full index that have changed serve also as an index to the present document. However, when a correction or amendment has caused an old index entry to be deleted, the deletion is usually not indicated.