Citation: D. Paesa, C. Franco, S. Llorente, G. López-Nicolás, and C. Sagüés. Reset Adaptive Observers and Stability Properties. Mediterranean Conference on Control and Automation, pp. 1435-1440. Marrakech-Morocco, June 23-25, 2010

Reset Adaptive Observers and Stability Properties

D. Paesa, C. Franco, S. Llorente, G. Lopez-Nicolas and C. Sagues

***Abstract*— This paper proposes a novel kind of adaptive observer called reset adaptive observer (ReO). A ReO is an adaptive observer consisting of an integrator and a reset law that resets the output of the integrator depending on a predefined condition. The main contribution of this paper is the application of the reset element theory to the adaptive observer LTI framework. The introduction of the reset element in the adaptive laws can decrease the overshooting and settling time of the estimation process without sacrificing the rising time. The stability and convergence LMI-based analysis of the proposed ReO is also addressed. Additionally, an easily computable method to determine the $\mathcal{L}_2$ gain of the ReO dealing with noise-corrupted systems is presented. A simulation example shows the potential benefit of the proposed reset adaptive observer.**

I. Introduction

Adaptive observers for linear time invariant systems (LTI) have been widely studied since 1970s. Initially, those works were characterized by having only a proportional feedback term in the adaptive laws and were known as proportional adaptive observers (PAO) [1], [2]. This approach guaranteed a zero steady-state estimation error assuming a persistent excitation condition. Nevertheless, PAO showed a poor robustness dealing with noise corrupted systems. The performance of PAOs was improved by adding an integral term to the adaptive laws [3], [4], [5]. This kind of adaptive observer is known as proportional integral adaptive observer (PIAO). This additional term can increase the steady state accuracy and improve the robustness against modeling errors and disturbances.

However, since the adaptive laws are still linear, they have the inherent limitations of linear feedback control. Namely, they cannot decrease the settling time and the overshoot of the estimation process simultaneously. Therefore, a trade-off between both requirements is needed. Nevertheless, this limitation can be solved by adding a reset element. A reset element consists of an integrator and a reset law which resets the output of the integrator as long as the reset condition holds. Reset elements were introduced by Clegg in 1958 [6], who proposed an integrator which was reset to zero when its input is zero. In 1974, Horowitz generalized that initial work substituting the Clegg integrator by a more general structure called the first order reset element (FORE) [7], [8]. The performance of those works were shown by simulations and it would take two decades to find the first stability analysis demonstrations [9], [10], [11]. The main contribution of those works was a stability test applicable to reset control systems called the H_β condition. Since the H_β condition can be expressed as a linear matrix inequality problem (LMIP), it can be easily solved.

A general analysis for reset control systems can be found in [12]. There, the authors modified the reset condition in such a manner that the system is reset when its input and output have different sign, rather than as long as its input is equal to zero. This is the main difference of [12], compared with other relevant works [13], [14]. Indeed, this approach addresses and solves the lack of robustness of the original formulation, which cannot be implemented in simulation packages (e.g. Simulink), since the integrator state is never reset due to the time discretization performed by the simulator.

Undoubtedly, during the last years there have been increasing research activities in the field of stability analysis and switching stabilization for reset systems [15], [16]. Nevertheless, this research has been mainly focused on control issues and, as a consequence, there are no results of reset elements applied to adaptive observers so far. We propose in this paper a new sort of adaptive observer called reset adaptive observer. As it has been pointed out, a ReO is an adaptive observer whose integral term has been substituted for a reset element. The reset condition of the ReO is based on the approach proposed by [12], that is, its integral term is reset as long as the estimation error and the integrated estimation error have opposite sign. The introduction of the reset element in the adaptive laws can improve the performance of the observer, due to the fact it is possible to decrease the overshoot and settling time of the estimation process simultaneously.

This paper is organized as follows. In Section II, the ReO formulation for LTI systems is presented. In Section III, a LMI-based stability condition which guarantees the convergence and stability of the estimation process is developed. Besides, an easily computable method to obtain the $\mathcal{L}_2$ gain of the ReO dealing with noise-corrupted system is presented. A simulation example is presented in Section IV in order to test the performance of our proposed ReO compared with traditional PIAO. Finally, concluding remarks are given in Section V.

Notation: In the following, we use the notation $(x, y) = \left[x^T \; y^T\right]^T$. Given a state variable x of a hybrid system with switches, we will denote its time derivative with respect to the time by $\dot{x}$. Furthermore, we will denote the value of the state variable after the switch by x^+. Finally, we omit its time argument and we write $x(t)$ as x.

This work was supported in part by DGA project PI065/09 and BSH Home Appliances Group.

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II. Reset Adaptive Observer Formulation

In this paper, we address the problem of the state estimation of linear time invariant systems which are described by

$$\begin{aligned} \dot{x} &= Ax + Bu + B_w w \\ y &= Cx \end{aligned} \quad (1)$$

where $x \in \mathbb{R}^n$ is the state vector, $u \in \mathbb{R}$ is the input vector, $w \in \mathbb{R}^n$ is the disturbance vector, $y \in \mathbb{R}$ is the output vector, $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times 1}$, $B_w \in \mathbb{R}^{n\times 1}$ and $C \in \mathbb{R}^{1\times n}$ are known constant matrices. We consider single-input single-output (SISO) systems only, since a suitable formulation of reset elements for multiple-input multiple-output (MIMO) systems is still an open research topic.

The structure of our proposed ReO applied to a LTI system (1) is given in Fig. 1. The ReO dynamics are described as follows:

$$\begin{aligned} \dot{\hat{x}} &= A\hat{x} + Bu + K_I\zeta + K_P\tilde{y} \\ \hat{y} &= C\hat{x} \end{aligned} \quad (2)$$

where $\hat{x}$ is the estimated state, K_I and K_P represent the integral and proportional gain respectively and $\tilde{y} = C\tilde{x} = C(x - \hat{x})$ is the output estimation error. In addition, ζ is the reset integral term which can be computed as

$$\begin{aligned} \dot{\zeta} &= A_\zeta\zeta + B_\zeta\tilde{y} & \tilde{y}\cdot\zeta \geq 0 \\ \zeta^+ &= A_r\zeta & \tilde{y}\cdot\zeta \leq 0 \end{aligned} \quad (3)$$

where $A_\zeta \in \mathbb{R}$ and $B_\zeta \in \mathbb{R}$ are two tuning scalars which regulate the transient response of ζ, and A_r is the reset matrix. Specifically, we define $A_r = 0$, since the reset integral term ζ is reset to zero when $\tilde{y}\cdot\zeta \leq 0$.

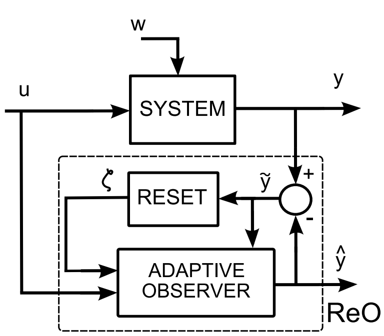

Fig. 1. Reset adaptive observer applied to a LTI system.

The reset observer can be regarded as a hybrid system with a *flow set* $\mathcal{F}$ and a *jump or reset set* $\mathcal{J}$. Regarding (3), the two conditions at the right side are the *flow* and the *jump* condition respectively. On one hand, as long as $(\tilde{y}, \zeta) \in \mathcal{F}$ the observer behaves as a proportional integral observer. On the other hand, if the pair $(\tilde{y}, \zeta)$ satisfies the *jump* condition, the integral term is reset according to the reset map A_r.

Thus, the observer flows whenever $\tilde{y}\cdot\zeta \geq 0$, that is, if $\tilde{y}$ and ζ have the same sign, whereas the observer jumps whenever $\tilde{y}\cdot\zeta \leq 0$, that is, if $\tilde{y}$ and ζ have different sign. According to this statement and since $\tilde{y} = C\tilde{x}$, we can formalize the definition of both sets by using the following representation:

$$\begin{aligned} \mathcal{F} &:= \left\{ (\tilde{x},\zeta) : \begin{bmatrix} \tilde{x} \\ \zeta \end{bmatrix}^T M \begin{bmatrix} \tilde{x} \\ \zeta \end{bmatrix} \geq 0 \right\} \\ \mathcal{J} &:= \left\{ (\tilde{x},\zeta) : \begin{bmatrix} \tilde{x} \\ \zeta \end{bmatrix}^T M \begin{bmatrix} \tilde{x} \\ \zeta \end{bmatrix} \leq 0 \right\} \end{aligned} \quad (4)$$

where M is defined as

$$M = \begin{bmatrix} 0 & C^T \\ C & 0 \end{bmatrix} \quad (5)$$

III. Stability and Convergence Analysis

In this section we state computable sufficient conditions for $\mathcal{L}_2$ stability via quadratic Lyapunov functions for reset observers defined by (2) and (3) applied to LTI systems described by (1).

A. State stability analysis

Let us begin analyzing the error system dynamics which can be obtained subtracting (2) from (1). Then, the state error dynamic $\tilde{x} = x - \hat{x}$ is defined by

$$\dot{\tilde{x}} = (A - K_PC)\tilde{x} - K_I\zeta + B_w w \quad (6)$$

This dynamic can be augmented by connecting (6) to (3) as follows:

$$\begin{aligned} \dot{\eta} &= A_\eta\,\eta + B_\eta\,w & \eta \in \mathcal{F} \\ \eta^+ &= A_R\,\eta & \eta \in \mathcal{J} \\ \xi &= C_\eta\,\eta \end{aligned} \quad (7)$$

where $\eta = [\tilde{x}\ \ \zeta]^T$,

$$A_\eta = \begin{bmatrix} A - K_PC & -K_I \\ B_\zeta C & A_\zeta \end{bmatrix}, \quad (8)$$

$$B_\eta = \begin{bmatrix} B_w \\ 0 \end{bmatrix}, \quad (9)$$

$$C_\eta = \begin{bmatrix} C & 0 \end{bmatrix}, \quad (10)$$

and

$$A_R = \begin{bmatrix} I & 0 \\ 0 & A_r \end{bmatrix}. \quad (11)$$

Additionally, we assume in the following that the reset observer (2)-(3) holds the following assumptions

Assumption 1. *The reset observer described by (2)-(3) is such that* $\eta \in \mathcal{J} \Rightarrow A_R\,\eta \in \mathcal{F}$.

This condition guarantees that after each reset, the solution will be mapped to the flow set $\mathcal{F}$ and, as a consequence, it is possible flowing after resets.

Assumption 2. *The reset observer described by (2)-(3) is such that the reset times* $t_{i+1} - t_i \geq \rho\ \forall i \in \mathbb{N}$, $\rho > 0$.

This assumption ensures that the reset observer uses time regularization to avoid Zeno solutions. It guarantees that the time interval between any two consecutive resets is not smaller than $\rho \in \mathbb{R}$ which is a positive constant called the dwell time.

Remark 1. It is important to note that both assumptions are quite natural to assume for hybrid system [17], and consequently, these conditions are commonly used in most of current reset system formulations available in literature [12], [14], [18].

Let us now state a sufficient condition for the existence of a quadratically stable ReO based on a LMI approach.

Theorem 1. *For given A_η, B_η and A_R the augmented error dynamic shown in (7) with $B_w = 0$ is quadratically stable, if there exist a matrix $P = P^T > 0$ and scalars $\tau_F \geq 0$ and $\tau_J \geq 0$ subject to*

$$A_\eta^T P + P A_\eta + \tau_F M < 0, \tag{12}$$

$$A_R^T P A_R - P - \tau_J M \leq 0 \tag{13}$$

which is a linear matrix inequality problem in the variables P, τ_F and τ_J.

Proof. Let us begin considering the following quadratic Lyapunov function for the augmented error dynamics described by (7):

$$V(\eta) = \eta^T \, P \, \eta \tag{14}$$

To prove the quadratically stability of our proposed reset adaptive observer, we have to check that:

$$\begin{array}{ll} \dot{V}(\eta) < 0 & \eta \in \mathcal{F} \\ V(\eta^+) \leq V(\eta) & \eta \in \mathcal{J} \end{array} \tag{15}$$

According to (4), since $\mathcal{F} := \left\{\eta : \eta^T \, M \, \eta \geq 0\right\}$ and employing the S-procedure [19], the first term of (15) is equivalent to the existence of $\tau_F \geq 0$ such that

$$\dot{V}(\eta) < -\eta^T \tau_F M \eta \tag{16}$$

Then, let us take derivative of (14) to obtain

$$\begin{aligned} \dot{V}(\eta) &= \dot{\eta}^T P \eta + \eta^T P \dot{\eta} \\ &= \eta^T (A_\eta^T P + P A_\eta) \eta \end{aligned} \tag{17}$$

Rearranging terms of equations (16) and (17), the first term of (15) holds if the following inequality is satisfied

$$\eta^T (A_\eta^T P + P A_\eta) \eta + \eta^T \tau_F M \eta < 0 \tag{18}$$

which can be rearranged as an equivalent LMI problem in the variables $P > 0$ and $\tau_F \geq 0$

$$A_\eta^T P + P A_\eta + \tau_F M < 0, \tag{19}$$

which is analogous to (12) and consequently, proves the first equation of (15).

Similarly, employing again the S-procedure, the second term of (15) holds if there exits $\tau_J \geq 0$ such that

$$V(\eta^+) \leq V(\eta) + \eta^T \tau_J M \eta \tag{20}$$

which is equivalent to

$$\eta^T A_R^T P A_R \eta - \eta^T P \eta - \eta^T \tau_J M \eta \leq 0 \tag{21}$$

Rearranging terms, (20) can be also rewritten as an equivalent LMI problem in the variables $P > 0$ and $\tau_J \geq 0$ as follows

$$A_R^T P A_R - P - \tau_J M \leq 0 \tag{22}$$

which is analogous to (13) and proves the second equation of (15) and, as a consequence, completes the proof of the theorem. □

B. Input-output stability analysis

Now, we present our results on the input-output properties of the ReO. For this reason, let us define the $\mathcal{L}_2$ o Root Mean Square (RMS) gain of the system (7) as the following quantity

$$\mathcal{L}_2 = \sup_{\|w\|_2 \neq 0} \frac{\|\xi\|_2}{\|w\|_2} \tag{23}$$

where the $\mathcal{L}_2$ norm $\|u\|_2^2$ of a signal u is defined as follows

$$\|u\|_2^2 = \int_0^\infty u^T u \, dt \tag{24}$$

and sup means *supremum* which is taken over all non-zero trajectories of (7).

Additionally, we present the following lemma that will be used in the sequel [19].

Lemma 1. *The $\mathcal{L}_2$ gain of a LTI system with an input signal w and an output signal ξ is less than γ, if there exists a quadratic function $V(x) = x^T P x$, $P > 0$ and $\gamma > 0$ such that*

$$\dot{V}(x) < \gamma^2 w^T w - \xi^T \xi \tag{25}$$

Theorem 2. *For given A_η, B_η, C_η and A_R the augmented error dynamic shown in (7) is quadratically stable and has a $\mathcal{L}_2$ gain from w to ξ which is smaller than γ, if there exist a matrix $P = P^T > 0$ and scalars $\tau_F \geq 0$, $\tau_J \geq 0$ and $\gamma > 0$ subject to*

$$\begin{bmatrix} A_\eta^T P + P A_\eta + C_\eta^T C_\eta + \tau_F M & P B_\eta \\ B_\eta^T P & -\gamma^2 I \end{bmatrix} < 0, \tag{26}$$

$$A_R^T P A_R - P - \tau_J M \leq 0 \tag{27}$$

which is a linear matrix inequality problem in the variables P, τ_F, τ_J and γ.

Proof. To prove the stability of our proposed reset adaptive observer and that the $\mathcal{L}_2$ gain from w to ξ is smaller than γ, we have to check that:

$$\begin{array}{ll} \dot{V}(\eta) < \gamma^2 w^T w - \xi^T \xi & \eta \in \mathcal{F} \\ V(\eta^+) \leq V(\eta) & \eta \in \mathcal{J} \end{array} \tag{28}$$

The first equation of (28) relays on (25) and the second equation of (28) is equal to the second equation of (15) which has been already proved. Then, let us concentrate on the first equation of (28). Again, since $\mathcal{F} := \left\{\eta : \eta^T\, M\, \eta \geq 0\right\}$ and employing the S-procedure, the first term of (28) is equivalent to the existence of $\tau_F \geq 0$ such that

$$\dot{V}(\eta) < \gamma^2 w^T w - \xi^T \xi - \eta^T \tau_F M \eta \qquad (29)$$

In this case, the time derivative of (15) is

$$\begin{aligned} \dot{V}(\eta) &= \dot{\eta}^T P \eta + \eta^T P \dot{\eta} \\ &= \eta^T A_\eta^T P \eta + w^T B_\eta^T P \eta + \eta^T P A_\eta \eta + \eta^T P B_\eta w \\ &= \eta^T (A_\eta^T P + P A_\eta) \eta + w^T B_\eta^T P \eta + \eta^T P B_\eta w \end{aligned} \qquad (30)$$

Rearranging terms of equations (29) and (30), the first term of (28) holds if the following inequality is satisfied

$$\begin{aligned} &\eta^T (A_\eta^T P + P A_\eta) \eta + w^T B_\eta^T P \eta + \eta^T P B_\eta w \\ &\quad + \xi^T \xi + \eta^T \tau_F M \eta - \gamma^2 w^T w < 0 \end{aligned} \qquad (31)$$

Since $\xi^T \xi = \eta^T C_\eta^T C_\eta \eta$, (31) can also be rearranged as an equivalent LMI problem in the variables $P > 0$ and $\tau_F \geq 0$ as follows

$$\begin{bmatrix} A_\eta^T P + P A_\eta + C_\eta^T C_\eta + \tau_F M & P B_\eta \\ B_\eta^T P & -\gamma^2 I \end{bmatrix} < 0, \qquad (32)$$

which is analogous to (26) and proves the first equation of (28) and, as a consequence, completes the proof of the theorem. □

C. Tuning guidelines

The proposed Reo mainly relays on four tuning gains. Namely, the proportional gain K_P and the integral gain K_I, which modify the convergence speed of the state estimation error, and the reset term gains A_ζ and B_ζ, which regulate the transient response of the reset term.

Before tuning the algorithm on a real system, it is strongly recommended to first perform some simulations, and tune the observer gains following the next guidelines. Firstly, the gains of the reset term A_ζ and B_ζ have to be chosen. Analyzing (3), it is evident that the reset term ζ stands for a low-pass filter whose cutoff frequency relays on A_ζ and whose gain depends on B_ζ. Typically, it is selected $B_\zeta = 1$, because the effect of the integral term can be increased by tuning the integral gain K_I, therefore the transient response of the integral term only relays on A_ζ. To guarantee a proper integration of the error dynamic, A_ζ should be chosen to be Hurwitch with $|A_\zeta|$ at least 5 times lower than the minimum absolute value of the eigenvalues of A.

The second step is to find suitable K_P and K_I in such a way that the response of the state estimation error is fast enough but without overshooting. Since the pair (A_η, C_η) is constant, it can be done by using any pole placement method. Once both gains have been computed, it is time to exploit the potential benefit of the reset element. The aim is to increase the integral gain in order to obtain a quicker and oscillating response due to the fact that most of the overshoots will be removed by resetting the integral term. Consequently, we will obtain a state estimation error response quicker than before but without overshooting. This fact underlines the benefit of the reset adaptive observers, which are mainly nonlinear and, as a consequence, it can achieve some specifications that cannot be achieved by pure linear observers.

IV. Simulation Results

In this section, an example is presented in order to show the effectiveness of our proposed reset adaptive observer. Consequently, we compare the simulation results obtained by our proposed ReO with two PIAOs. On the one hand, the first PIAO will be tuned to minimize the overshooting and, as a consequence, it provides a smooth response. On the other hand, the latter PIAO will be designed to minimize the rising time, and hence, it gives an oscillating and faster response. The next simulation example will show that our proposed ReO can achieve both requirements (i.e. a smooth and quick response) simultaneously. These simulation results have been obtained by using Simulink with the ode45 solver.

Let us consider the following third-order noise-corrupted LTI system:

$$\begin{aligned} \dot{x}_1 &= -4.5x_1 - 4x_2 + 0.6x_3 + 0.1u + 0.5w \\ \dot{x}_2 &= 0.4x_1 - 2x_2 - 1.1u + 0.5w \\ \dot{x}_3 &= -0.5x_1 - 3x_3 - 0.5u + 0.5w \\ y &= x_1 \end{aligned} \qquad (33)$$

with $x(t = 0) = [-2.3; 1.5; 1.8]^T$, $u(t) = sin(4t)$ and $w(t) = sin(15t)$. The aim is to develop an adaptive observer for the system described by (33) which satisfies that the state estimation error tends to zero without overshooting as fast as possible. According to (1), (33) has the following parameters:

$$A = \begin{bmatrix} -4.5 & -4 & 0.6 \\ 0.4 & -2 & 0 \\ -0.5 & 0 & -3 \end{bmatrix},$$

$$B = \begin{bmatrix} 0.1 \\ -1.1 \\ -0.5 \end{bmatrix},$$

$$B_w = \begin{bmatrix} 0.5 \\ 0.5 \\ 0.5 \end{bmatrix},$$

$$C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}.$$

Additionally, let us outline the tuning parameter for each adaptive observer. Notice that when it is possible, the tuning parameters are equal for each adaptive observer in order to make the results more comparable.

PIAO for SISO LTI systems are described by:

$$\begin{aligned} \dot{\hat{x}} &= A\hat{x} + Bu + K_I z + K_P \tilde{y} \\ \hat{y} &= C\hat{x} \\ \dot{z} &= A_z z + B_z \tilde{y} \end{aligned} \qquad (34)$$

where $A_z \in \mathbb{R}$ and $B_z \in \mathbb{R}$ are two tuning scalars which regulate the transient response of the integral term z. As it is said above, we have designed a conservative and non-oscillating PIAO and a quicker and oscillating PIAO for the system (33). Since the tuning process of each observer involves several parameters, let us outline how all these tuning parameters have been determined. Firstly, we have designed the conservative PIAO in such a manner that its rising time is equal to 0.6 seconds without overshooting. After that, to design the oscillating PIAO we have increased the K_I gain until its rising time is equal to 0.2 seconds, that implies an oscillating estimation process. Finally, to make the results more comparable, the ReO has the same K_I and K_P than the oscillating PIAO.

Specifically, the parameters of the conservative PIAO are $\hat{x}(t = 0) = [0; 0; 0]^T$, $z(t = 0) = 0$, $A_z = -0.5$, $B_z = 1$, $K_P = [2.65; -1.7525; -2.425]^T$, and $K_I = [0.53; -0.35; -0.485]^T$, whereas the tuning parameters of the oscillating PIAO are $\hat{x}(t = 0) = [0; 0; 0]^T$, $z(t = 0) = 0$, $A_z = -0.5$, $B_z = 1$, $K_P = [2.65; -1.7525; -2.425]^T$, and $K_I = [26.5; -12.2675; -19.4]^T$.

On the other hand, the ReO for the system (33) has been designed according to (2)-(3) and it has the following tuning parameters: $\hat{x}(t = 0) = [0; 0; 0]^T$, $\zeta(t = 0) = 0$, $A_\zeta = -0.5$, $B_\zeta = 1$, $K_P = [2.65; -1.7525; -2.425]^T$, $K_I = [26.5; -12.2675; -19.4]^T$ and $A_r = 0$. Notice that the K_P and K_I gains of the ReO are equal to the gains of the oscillating PIAO.

The state estimation error $\tilde{x}(t) = [\tilde{x}_1(t); \tilde{x}_2(t); \tilde{x}_3(t)]^T$ of all adaptive observers is shown in Fig. 2. It is evident that our proposed ReO has a better performance compared with traditional PIAOs, since it has a response as quick as the oscillating PIAO but without overshooting. Notice that if we decrease the integral gain K_I of the oscillating PIAO to avoid overshooting it will behave as the conservative PIAO and, thus, its rising time will be higher than the obtained by the ReO. On the other hand, if we increase the integral gain K_I of the conservative PIAO to reduce its rising time, it will behave as the oscillating PIAO and, as a consequence, its response will be oscillating.

Additionally, our proposed ReO also obtains the best performance as long as we compare the bound of the $\mathcal{L}_2$ gain of each adaptive observer. Indeed, the ReO has a γ = 0.2675, whereas the oscillating PIAO has a γ = 0.3379 and the conservative PIAO has a γ = 0.3305. These values have been obtained minimizing the value of γ^2 according to Theorem 2. Namely, the matrices obtained for the ReO were

$$P = \begin{bmatrix} 0.146 & 0.014 & -0.012 & -0.003 \\ 0.014 & 0.350 & -0.285 & -0.000 \\ -0.012 & -0.285 & 0.270 & -0.000 \\ -0.003 & -0.000 & -0.000 & 4.086 \end{bmatrix},$$

$$\tau_F = 0.0368,$$

$$\tau_J = 0.03,$$

while the optimal matrix obtained for the oscillating PIAO

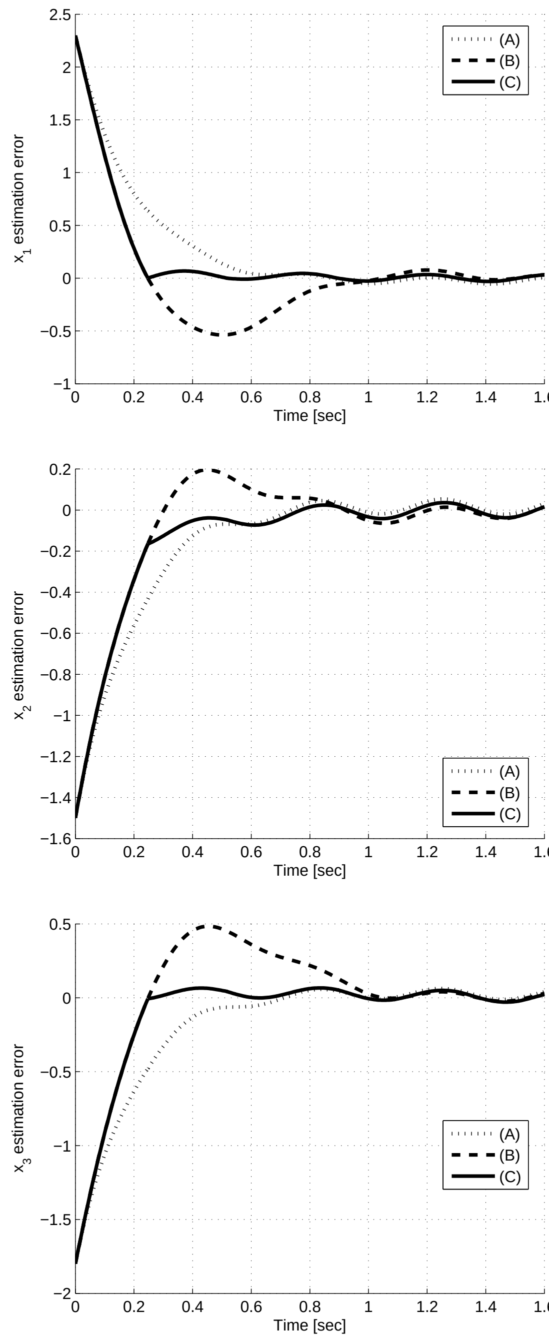

Fig. 2. State estimation error $\tilde{x}(t)$ for each adaptive observer. (A) Dotted lines have been obtained by using the conservative proportional integral adaptive observer. (B) Dashed lines have been obtained by using the oscillating proportional integral adaptive observer. (C) Solid lines have been obtained by using the reset adaptive observer.

was

$$P = \begin{bmatrix} 0.635 & 0.145 & -0.166 & 0.321 \\ 0.144 & 4.681 & -5.595 & 13.423 \\ -0.166 & -5.595 & 6.982 & -17.659 \\ 0.321 & 13.423 & -17.659 & 73.288 \end{bmatrix},$$

and finally, the optimal matrix obtained for the conservative PIAO was

$$P = \begin{bmatrix} 3.416 & 7.689 & -9.831 & 3.6732 \\ 7.689 & 22.180 & -27.957 & 10.601 \\ -9.831 & -27.957 & 35.537 & -13.793 \\ 3.673 & 10.601 & -13.793 & 10.972 \end{bmatrix}.$$

V. Conclusion

This paper has presented a new adaptive observer called reset adaptive observer (ReO). The stability and convergence analysis of this novel proposal has been proved by using quadratic Lyapunov functions. Additionally, a method to determine the $\mathcal{L}_2$ gain of the proposed reset adaptive observer has also been developed. This method is based on a linear matrix approach which is easily computable.

Simulation results have been given to underline the potential benefit of including a reset element in the adaptive laws. Namely, the reset element can decrease the overshoot and settling time of the estimation process without sacrificing the rise time for some kind of systems. Besides, a lower bound of the $\mathcal{L}_2$ gain can also be achieved dealing with noise corrupted systems.

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