High-quality Motion Deblurring from a Single Image

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The Problem
Blur Model

Blurred image $I$ = Sharp image $L$ + Blur kernel $f$ + Camera Noise $n$
Previous Work (1)

Hardware solutions:

[Levin et al. 2008]

[Ben-Ezra and Nayar 2004]

[Raskar et al. 2006]
Multi-frame solutions:

[Jia et al. 2004]

[Rav-Acha and Peleg 2005]

[Petschnigg et al. 2004]

[Yuan et al. 2007]
Previous Work (3)

Single image solutions:

- [Fergus et al. 2006]
- [Levin et al. 2007]
- [Jia 2007]
- [Yuan et al. 2008]
The Problems

Non-blind deconvolution

Blind deconvolution
An Example
Challenges (1)

Assuming no noise
Challenges (2)

With noise
An Example
Figure 3  A step signal constructed by finite Fourier basis functions. (a) The ground truth step signal. (b) The magnitude of the coefficients of the Fourier basis functions. (c) The phase of the Fourier basis coefficients. (d) The reconstructed signal from the 512 Fourier basis functions, with PSNR of 319.40. The loss is far below what humans can perceive.
An Example

\[ I = L \otimes f + n \]

To analyze the problems caused by image noise and kernel error, let us model the kernel and latent image as the sum of their current estimates \( f' \) and \( L' \) and the errors \( \Delta f \) and \( \Delta L \):

\[
I = (L' + \Delta L) \otimes (f' + \Delta f) + n \\
= L' \otimes f' + \Delta L \otimes f' + L' \otimes \Delta f + \Delta L \otimes \Delta f + n.
\]
An Example

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I = (L' + \Delta L) \otimes (f' + \Delta f) + n \\
= L' \otimes f' + \Delta L \otimes f' + L' \otimes \Delta f + \Delta L \otimes \Delta f + n.
\]
\[ p(L, f | I) \propto p(I | L, f)p(L)p(f), \]
\[ p(L, f|I) \propto p(I|L, f)p(L)p(f), \]

\[ n = L \otimes f - I. \]
Noise constraint
Noise
constraint

Possible noise models:

(1) \( \prod_i N(n_i \mid 0, \zeta_0) \)

(2) \( \prod_i N(\nabla n_i \mid 0, \zeta_1) \)

\( \prod_i N(n_i \mid 0, \zeta_0) \prod_i N(\nabla n_i \mid 0, \zeta_1) \)
Noise constraint

Possible noise models:

1. \( \prod_i N(n_i \mid 0, \zeta_0) \)
2. \( \prod_i N(\nabla n_i \mid 0, \zeta_1) \)

\[ \prod_i N(n_i \mid 0, \zeta_0) \prod_i N(\nabla n_i \mid 0, \zeta_1) \prod_i N(\nabla\nabla n_i \mid 0, \zeta_2) \]
A random variable following an independent Gaussian distribution also has its any order derivative following it. [Simon 2002]

\[ P(n) = \prod_i N(n_i \mid 0, \zeta_0) \prod_i N(\nabla n_i \mid 0, \zeta_1) \prod_i N(\nabla(\nabla n_i) \mid 0, \zeta_2) \]
\[ p(I|L, f) = \prod_{\partial^* \in \Theta} \prod_{i} N(\partial^* n_i | 0, \zeta_{\kappa}(\partial^*)) \]
\[
= \prod_{\partial^* \in \Theta} \prod_{i} N(\partial^* I_i | \partial^* I_i^c, \zeta_{\kappa}(\partial^*)) \tag{3}
\]

where \( I_i \in I \), denotes the pixel value, \( I_i^c \) is the pixel with coordinate \( i \) in the reconvolved image \( I^c = L \otimes f \), and \( \Theta \) represents a set consisting of all the partial derivative operators that we use (we set \( \Theta = \{ \partial^0, \partial_x, \partial_y, \partial_{xx}, \partial_{xy}, \partial_{yy} \} \) and define \( \partial^0 n_i = n_i \)). We compute derivatives with a maximum order of two because our experiments show that these derivatives are sufficient to produce good results. Note that using the likelihood defined in (3) does not increase computational difficulty compared to only using \( \prod_i N(n_i | 0, \zeta_0) \) in our optimization.
Figure 6 Effect of our likelihood model. (a) The ground truth latent image. (b) The blurred image $\mathbf{I}$. (c) The latent image $\mathbf{L}^a$ computed by our algorithm using the simple likelihood definition $\prod_i N(\mathbf{n}_i | 0, \zeta_0)$. (d) The computed image noise map $\mathbf{n}$ from (c) by $\mathbf{n} = \mathbf{L}^a \otimes \mathbf{f}^a - \mathbf{I}$. (e) The latent image result $\mathbf{L}^b$ recovered by using the complete likelihood definition (3). (f) The computed noise map from (e) by $\mathbf{n} = \mathbf{L}^b \otimes \mathbf{f}^b - \mathbf{I}$. $\mathbf{f}^a$ and $\mathbf{f}^b$ are estimated kernels, and (d) and (f) are normalized for visualization.
\[ p(L, f|I) \propto p(I|L, f)p(L)p(f), \]

\[ p(f) \]
Kernel Statistics

\[ \rho(f) = \prod_{j} e^{-\tau f_j}, \quad f_j \geq 0 \]
\[ p(L, f | I) \propto p(I | L, f)p(L)p(f), \]

\[ p(L) \]
Prior \( p(L) \) trabajando en dos escalas:

**Global**: para ayudar a mitigar la *ill-posedness* del problema (a nivel estadístico)

**Local**: para corregir los *ringing artifacts*
Image Global Statistics

Logarithmic density of image gradients
To illustrate, in Figure 7 we show the logarithmic image gradient distribution histogram from 10 natural images. In [Fergus et al. 2006], a mixture of $K$ Gaussian distributions are used to approximate the gradient distribution:

$$\prod_i \sum_{k=1}^{K} \omega_k N(\partial L_i|0, \varsigma_k),$$

where $i$ indexes over image pixels, $\omega_k$ and $\varsigma_k$ denote the weight and the standard deviation of the $k$'th Gaussian distribution. How-
Image Global Statistics
Image Global Statistics

\[
\log(P_1(\nabla L)) = \begin{cases} 
-k |\nabla L| & x \leq c \\
-(a(\nabla L)^2 + b) & x > c
\end{cases}
\]

\(\Phi(x)\) is central-symmetric, and \(k, a,\) and \(b\) are the curve fitting parameters, which are set as \(k = 2.7, a = 6.1 \times 10^{-4}\), and \(b = 5.0\).
Image Global Statistics

\[
\log(P_i(\nabla L)) = \begin{cases} 
-k |\nabla L| & x \leq c \\
-(a(\nabla L)^2 + b) & x > c
\end{cases}
\]

\[p_g(L) \propto \prod_i e^{\Phi(\partial L_i)}\]
Image Local Constraint

**Figure 8** Local prior demonstration. (a) A blurred image patch extracted from Figure 9(b). The yellow curve encloses a smooth region. (b) The corresponding unblurred image patch. (c) The restoration result by our method without incorporating $p_t$. The ringing artifact is propagated even to the originally smooth region. (d) We compute $\Omega$ containing all pixels shown in white to suppress ringing. (e) Our restoration result incorporating $p_t$, the ringing artifact is suppressed not only in smooth region but also in textured one.
Image Local Constraint

To formulate the local prior, for each pixel $i$ in blurred image $I$, we form a local window with the same size as the blur kernel and centered at it. One example is shown in Figure 8(a) where the window is represented by the green rectangle centered at pixel $i$ highlighted by the red dot. Then, we compute the standard deviation of pixel colors in each local window. If its value is smaller than a threshold $t$, which is set to 5 in our experiments, we regard the center pixel $i$ as in region $\Omega$, i.e., $i \in \Omega$. In Figure 8(d), $\Omega$ is shown as the set of all white pixels, each of which is at the center of a locally smooth window.
Image Local Constraint
Image Local Constraint

$I \rightarrow L$

$L$
Image Local Constraint
Image Local Constraint

\[ p_2(L) = \prod_{i \in \text{white}} N(\nabla L_i - \nabla I_i | 0, \sigma_1) \]
Image Local Constraint

\[ p_2(L) = \prod_{i \in \text{white}} N(\nabla L_i - \nabla I_i | 0, \sigma_1) \]
Combining All constraints

\[ \min E(L, f) = \min \log [p(n)p_1(\nabla L)p_2(L)p(f)] \]

Two-step iterative optimization

- Optimize \( L \)
- Optimize \( f \)
Optimize $L$

$$E(L) = \log[p(n)p_1(\nabla L)p_2(L)]$$

$$E(L) = \sum_{\nabla^*} w_{\nabla^*} \left( \left\| \nabla^* L \otimes f - \nabla^* I \right\|^2_2 \right) + \log p(n)$$

$$\lambda_1 \left\| \log p_1(\nabla L) \right\|_1 + \lambda_2 \left( \sum_{i \in \text{white}} \left\| \nabla L_i - \nabla I_i \right\|^2_2 \right)$$

$$\frac{\log p_1(\nabla L)}{\log p_2(L)}$$

Convolution is time-consuming
Optimize $L$

Idea: separate convolution

$$E(L) = \left( \sum_{\nabla^*} w_{\nabla^*} \left\| \nabla^* L \otimes f - \nabla^* I \right\|_2^2 \right) + \log p(n)$$

$$\lambda_1 \left\| \log p_1(\nabla L) \right\|_1 + \lambda_2 \left( \sum_{i \in \text{white}} \left\| \nabla L_i - \nabla I_i \right\|_2^2 \right) \over \log p_1(\nabla L)$$

$$\over \log p_2(L)$$
Optimize $L$

Idea: separate convolution

$$E'(L) = \left( \sum_{V^*} w_{V^*} \| \nabla^* L \otimes f - \nabla^* I \|^2 \right) + \frac{\log p(n)}{\log p_1(\nabla L)}$$

$$\lambda_1 \| \log p_1(\Psi) \|_1 + \lambda_2 \left( \sum_{i \in \text{white}} \| \nabla L_i - \nabla I_i \|^2 \right) \frac{\log p_2(L)}{\log p_2(\nabla L)}$$
Optimize $L$

Idea: separate convolution

$$E'(L) = \left( \sum_{\nabla^*} w_{\nabla^*} \left\| \nabla^* L \otimes f - \nabla^* I \right\|_2^2 \right) + \log p(n)$$

$$\lambda_1 \left\| \log p_1(\Psi) \right\|_1 + \lambda_2 \left( \sum_{i \in \text{white}} \left\| \Psi_i - \nabla I_i \right\|_2^2 \right)$$

$$\log p_1(\nabla L) + \log p_2(L)$$
Optimize $L$

Idea: separate convolution

$$E'(L) = \left( \sum_{\nabla^*} w_{\nabla^*} \left\| \nabla^* L \otimes f - \nabla^* I \right\|_2^2 \right) + \frac{\log p(n)}{\log p_1(\nabla L)}$$

$$\lambda_1 \left\| \log p_1(\Psi) \right\|_1 + \lambda_2 \left( \sum_{i \in \text{white}} \left\| \Psi_i - \nabla I_i \right\|_2^2 \right) + \frac{\lambda_2 \left( \sum_{i \in \text{white}} \left\| \Psi_i - \nabla I_i \right\|_2^2 \right) \nabla L \right\|_2^2}{\log p_2(L)}$$
Iteratively optimize $L$:
Update $L$
Update $\Psi$

\[
E'(L) = \left( \sum_{\nabla^*} w_{\nabla^*} \| \nabla^* L \otimes f - \nabla^* I \|_2^2 \right) + \\
\lambda_1 \| \log p_1(\Psi) \|_1 + \lambda_2 \left( \sum_{i \in \text{white}} \| \Psi_i - \nabla I_i \|_2^2 \right) + \gamma \left( \| \Psi - \nabla L \|_2^2 \right)
\]
Iteratively optimize $L$:

**Update** $L$

**Update** $\Psi$

\[
E'(L) = \left( \sum_{\nabla^*} w_{\nabla^*} \left\| \nabla^* L \otimes f - \nabla^* I \right\|_2^2 \right) + \gamma \left( \left\| \Psi - \nabla L \right\|_2^2 \right)
\]

Plancherel’s theorem implies:

the sum of the square of a function equals the sum of the square of its Fourier transform
Iteratively optimize $L$:

- Update $L$
- Update $\Psi$

We minimize $E'(F(L))$ instead, with a closed-form solution.

$$E'(L) = \left( \sum_{v^*} w_{v^*} \left\| \nabla^* L \otimes f - \nabla^* I \right\|_2^2 \right) + \gamma \left( \left\| \Psi - \nabla L \right\|_2^2 \right)$$

$$E'(F(L)) = \left( \sum_{v^*} w_{v^*} \left\| F(\nabla^* L) \cdot F(f) - F(\nabla^* I) \right\|_2^2 \right) + \gamma \left( \left\| F(\Psi) - F(\nabla L) \right\|_2^2 \right)$$
Iteratively optimize $L$:

Update $L$

Update $\Psi$

\[
E'(\Psi) = \left( \sum_{\nabla^*} w_{\nabla^*} \left\| \nabla^* L \otimes f - \nabla^* I \right\|_2^2 \right) + \\
\lambda_1 \left\| \log p_1(\Psi) \right\|_1 + \lambda_2 \left( \sum_{i \in \text{white}} \left\| \Psi_i - \nabla I_i \right\|_2^2 \right) + \gamma(\left\| \Psi - \nabla L \right\|_2^2)
\]
Iteratively optimize $L$:

- Update $L$
- Update $\Psi$

$$E'(\Psi) = \lambda_1 \| \log p_1(\Psi) \|_1 + \lambda_2 \left( \sum_{i \in \text{white}} \| \Psi_i - \nabla I_i \|_2^2 \right) + \gamma \left( \| \Psi - \nabla L \|_2^2 \right)$$

$$= \sum_i E'_{\Psi_i}$$

Each $E'_{\Psi_i}$ only contains a single variable $\Psi_i$.
Iteratively optimize $L$:
Update $L$
Update $\Psi$

Iteration $i$ (converge)
Time: about 30 seconds for an 800x600 image

Iteration 8 (converge)
A comparison

RL deconvolution
A comparison

Our deconvolution
Two-step iterative optimization

- Optimize $L$
- Optimize $f$

$$
\min E(L, f) = \min \log[p(n)p_1(\nabla L)p_2(L)p(f)]
$$

$$
E(f) = \left( \sum_{\nabla^*} w_{\nabla^*} \| \nabla^* L \otimes f - \nabla^* I \|^2_2 \right) + \| f \|_1
$$

A form of L1-norm regularized problem and is solved using an interior point method
Iteration 1
Iteration 2
Iteration 6
Iteration 8
Time: about 350 seconds for an 800x600 image

Convergence
Results
Results
Results
Results
More results
More results
More results
More results
More results
More results
More results
More results
Running Time

• Non-blind deconvolution (an 800 x 600 image)

  Approximately 30 seconds in our Matlab implementation

  Less than 10 seconds in our C++ implementation

• Blind deconvolution
  – Approximately 10 minutes for an 800 x 600 image
Program Available
Conclusion

• A well-constrained single-image deblurring model
• Advanced optimization

• Improvement is possible for
  – Spatially-variant blur
  – Saturated images
General Blur Images
General Blur Images
Thank You
Initialization

• User specified initialization

• Multi-Scale approach
  – Initialize the clear image in each scale by the one recovered in a upper layer
Color channel

• For a blur images with a large PSF, we take all the three channels (RGB) into computation

• For a blur image with a small PSF, only the luminance channel are taken into computation