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Robust Scheduling of Elective Patients under Block Booking by Chance Constrained Approaches.

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Abstract This paper considers an operation scheduling problem of elective patients: given an ordered list of patients, schedule them for surgery in the next time blocks previously booked during a specific duration of time, the *Daily Working Time* (DWT). We assume that the duration of surgeries and cleaning time after each surgery are random variables with normal probabilistic density function. Using real data from the *"Lozano Blesa" Hospital of Zaragoza* (LBHZ), their average and standard deviations are computed, and based on these values, we propose three optimization problems. (i) The first one is a simple Mixed Integer Linear Programming (MILP) problem that is based only on the average duration of surgeries and schedules the patients with the objective of obtaining a given *Daily Surgery Time* (DST) that obviously should be smaller than the DWT. (ii) By assuming some average and standard deviation of both, surgeries and cleaning times, a Mixed Integer Quadratic Constraint Programing (MIQCP) model is proposed that additionally of obtaining a given DST, allows to impose a *Minimum Confidence Level* (MCL) not exceeding the DWT by a chance constrain. (iii) The objective of obtaining a given DST is replaced by maximizing this DST in a New-MIQCP (N-MIQCP) that even if has a bigger complexity, can be used to estimate the "appropriate" target DST for a given MCL in the MIQCP model. To solve large instances of problems, a *Receding Horizon Strategy* (RHS) is proposed. Moreover, a discrete event simulation model of scheduling in the LBHZ is presented and the solutions obtained using realistic data with different approaches/models are compared. Finally, we propose a Decision Support System based on MIQCP and N-MIQCP that will help doctors in the scheduling of the LBHZ.

Keywords Scheduling · Operation planning · Discrete event systems

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1 Introduction

The *Operation Room* OR is one of the most expensive material resources of the hospitals. Approximately 60% of patients need it at some point during their hospital stay (NoA, 2005). Surgical costs typically account for approximately 40% of the hospital resource costs (Macario et al, 1995), while surgeries typically generate around 67% of hospital revenues(Jackson, 2002). Additionally, the demand, for surgical services is increasing due to the aging population. It is obvious that good planning and scheduling methods are necessary to improve the efficiency of the OR.

Researchers frequently differentiate between *strategic* (long term), *tactical* (medium term) and *operational* (short term) approaches to situate their planning or scheduling problems. Those works are furthermore categorized according to the decision level they address, i.e., to whom the particular decisions applies. Three classical levels are considered in bibliography (Abdelrasol et al, 2013):

1. *case mix planning* is a long term strategic planning that involves the hospital's mission and its translation into hospital resource capacity planning on the basis of highly aggregated information. Decision on the total supply of the most expensive and important resources are based on the hospital's mission.
2. *master surgery schedule* is a medium term tactical approach that determines how much operating room time is assigned to different surgeon groups on each weekday. These time allocations are commonly referred to as time block booking.
3. *scheduling of patients* is a short term operational approach to fix the patients that should be operated in the next time blocks.

In this paper, different mathematical programming models for scheduling of non-urgent surgeries (level 3 stated before) are proposed. These models are evaluated considering their computational complexity and quality of solution using a case study given by the Orthopedic Surgery Department of the Hospital *"Lozano Blesa" Hospital of Zaragoza* (LBHZ).

Due to the high computational complexity of the proposed models, a *Receding Horizon Strategy* (RHS) is used. This strategy is commonly used in the control of discrete event systems (Gokbayrak, 2011a,b), where a sliding time horizon window is fixed. In our approach RHS allows us to solve large instance of the scheduling problem by obtaining sequentially suboptimal solutions with a much lower computational time. A similar idea based on solving smaller subproblems is used in (Wu et al, 2013). They propose a progressive time-oriented decomposition heuristic framework for the capacity multi-level lot sizing problem.

The proposed scheduling problems will be used in the Orthopedic Surgery Department of the LBHZ. We propose a *Decision Support System* that helps doctors to perform a rapid, efficient and dynamic scheduling. It includes several features that enable to:

– *update the waiting list* by the inclusion of new arrival patients and by removing the patients operated when these events occurs.
– *performs a dynamic scheduling*. Once the first scheduling is computed and the patients confirm either their attendance or their absence, the DSS adds dynamically some constraints to the model and the scheduling is iterated.
– *update and customize internal data*. After each performed surgery, the data related with its duration and the doctor who has performed it are updated.

The operation planning and scheduling of elective patients is a problem studied in literature by many researchers. For a state of the art we can refer the reader to the survey (Cardoen et al, 2010) and the references herein. According to the descriptive fields proposed in (Cardoen et al, 2010), our paper can be classified as: "operation scheduling of elective inpatients by Mixed Integer Programing Problems with a multicriteria objective" (waiting time of patients and OR utilization). In addition, the surgery durations are not constant, but random variables and the problem is based on real data.

Some works combine planning and scheduling problem of elective patients with the urgent ones by using stochastic models (see, for example, (Lamiri et al, 2008a) and (Lamiri et al, 2008b)). The scheduling and planning of resources have been studied for other problems, as for example home care services as in (Lanzarone et al, 2010) and (Lanzarone et al, 2012). Petri net models have been used for modeling and management of healthcare systems (see, for example, (Amodio et al, 2009; Dotoli et al, 2009; Bernardi et al, 2014; Mahulea et al, 2017)). The contributions of this paper with respect to the previous results are: (1) the application of three different mathematical programming models to the particular problem in the studied hospital, (2) a chance-constrained approach considering both maximizing occupation rate of OR and respecting the order of patients on the waiting list; and (3) comparison, analysis and synthesis of the simulation results using realistic data from hospital.

This paper extends the results in (Clavel et al, September 2016) where an *Mixed Integer Linear Problem* (MILP) was proposed for operation scheduling of the elective patients. Considering the *daily surgery time* (DST) as the total time in a day that an OR is used for surgery, the MILP problem has the objectives of (a) to obtain a given DST and (b) to respect as much as possible the order of patients in the waiting list. The MILP obtains the scheduling based only on the average durations of each type of surgery that can be computed by using historical data. However, two problems may appear,

- P_1 - the obtained scheduling could be not robust enough if the surgery durations have large standard deviations. This uncertainty could result in uncomfortable situations for the medical management staff, either the doctors that usually may lengthen their working day either low utilization of the ORs is obtained.
- P_2 - a target DST is an input parameter in the optimization problem and in some cases it is difficult to select a good value for it in order to get solutions not exceeding the *Daily Working Time* (DWT) but having a good OR utilization.

In order to overcome P_1, we propose in this paper an *Mixed Integer Quadratic Constrained Problem* (MIQCP) that uses not only the average durations of the surgeries but also their standard deviations. In this way, each type of surgery has a pair of values (mean and standard deviation) that define its duration. Additionally, it considers the cleaning time between surgeries as random variables (with mean and standard deviation). These new assumptions allow us to introduce some chance constraints allowing to impose a *Minimum Confidence Level* (MCL) not exceeding the Daily Working Time.

To tackle the problem P_2 stated before we change the objective function. Instead of trying to obtain a given DST (as input parameter) we consider the objective of maximizing this DST (making it variable) keeping the chance constraints. A higher complexity New-MIQCP (N-MIQCP) is proposed.

The paper is organized as follows. In Sec. 2 related works proposed in literature are analyzed and comparing with our approach. Sec. 3 describes the problem statement and provides a motivation example. Sec. 4 shows the proposed mathematical programming problems (MILP, MIQCP, and N-MIQCP) to schedule the surgeries. In Sec. 5 heuristic approaches to reduce the computational times are presented. Using realistic data, in Sec. 6 some results obtained by implementing the problems in a machine with an Intel Core i5 and 8 GB of memory using a computer software (CPLEX) are analyzed and compared. A *Decision Support System* (DSS) for the daily scheduling in the studied department is explained in Sec. 7. Finally, in Sec. 8, we provide the conclusions and future works.

2 Related work

Different approaches have been proposed in the literature to address uncertain parameters in optimization problems. They can be divided in three main groups: stochastic programming, distributionally robust optimization, and robust optimization. In stochastic programming (Birge and Louveaux, 2011; Shapiro et al, 2009), uncertain parameters are modelled as random variables and their probability distribution is assumed to be know. In this way stochastic programming requires both a strong statistical background to manage the mathematical models and a thorough knowledge of the real problem to derive the probability distribution, which are not always easy to derive. The resulting optimization problems can be difficult to solve, in addition, if the used distributions are not reliable, the solutions produced may not prove to be robust. In other hand, distributionally robust optimization (Ben-Tal et al, 2010; Goh and Sim, 2010) and ambiguous chance-constrained approaches (Erdoğan and Iyengar, 2006) assume that the probability distribution is not known, but lies within a know family of distribution. The problem is difficult but computationally tractable approximations exists. Robust optimization approaches (Ben-Tal and Nemirovski, 1998; Bertsimas and Sim, 2003) assume that each uncertain parameter belongs to a given convex set, and no detailed knowledge of its probability distribution is required.

In our approach, it is assumed that the uncertain parameters (surgery duration and cleaning time) follow a normal distribution and consequently, the expected total duration of an OR working day also follows a normal distribution. In this way, a resource capacity chance constraint can be introduced by requiring that the probability of overtime be no more than a given scalar α. The idea of using a chance constraint for the scheduling of ORs is also used in (Shylo et al, 2012; Hans et al, 2008). The authors in (Shylo et al, 2012) present an optimization framework for batch scheduling within a block booking system that maximizes the expected utilization of ORs resources subject to a set of probabilistic capacity constraints. They propose an algorithm that iteratively solves a series of mixed-integer programs that are based on a normal approximation of cumulative surgery duration. In (Hans et al, 2008) constructive and local search heuristics for maximization of ORs utilization and minimization of the overcoming risk is proposed. In their model, to address the randomness of surgery processing times, a planned time slack is reserved in each scheduling block, which is function of total mean and variance of surgeries assigned to the corresponding block. When determining an appropriate size of the planned slacks, the authors assume that the sum of surgery durations follows a normal distribution.

The previously explained approaches (Shylo et al, 2012; Hans et al, 2008) require to set in advance the patients that are going to be scheduled in the next blocks, therefore in these approaches all considered patients must be scheduled in one of the available blocks. For this, both approaches start with an initial scheduling obtained through the scheduling rule: *first-fit probabilistic*. Following this rule, sequentially each surgery is assigned to the first available block for which the probabilistic capacity constraint is satisfied after the assignment. Once the initial scheduling is obtained the expected occupation rate of the first blocks are improved by rescheduling the surgeries. In this way, the last blocks are totally or partially released.

In our case, an important criterion is to respect as much as possible the order of patients in the waiting list. That is, first patients should be scheduled in the first surgical block, while last patients should be preferably scheduled in the last block. This consideration is not taken into account in the previously explained approaches because:

1. When obtaining the initial solution, patients who are far behind on the waiting list, but are suitable to complete a surgical block, may be scheduled.
2. Once the initial scheduling has been obtained following the first-fit probabilistic rule, the patients are rescheduled, and in the final solution any patient can be assigned to any surgical block.

Unlike (Shylo et al, 2012; Hans et al, 2008), our approach does not require to know in advance the set of patients that should be scheduled in the next surgical blocks, since any patient on the waiting list may or may not be scheduled. Alternatively, we propose a linear cost function composed by two balanced terms that favors patients to be scheduled in an orderly manner at the same time that maximizes the expected occupation rate of the OR.

A realistic comparison between the approaches proposed in the related works (Shylo et al, 2012; Hans et al, 2008) and the one explained in this work is performed and analyzed in Sec.6. Realistic data of surgery duration and surgery arrival obtained from the studied department has been considered. For each one of the three approaches, the scheduling obtained for 50 scenarios and 2000 one-year replications has been analyzed. The average results shows that:

1. Similar occupation rate/confidence level is obtained using (Shylo et al, 2012) and our approach. However, using (Hans et al, 2008) a slightly worse occupation rate is obtained due to only 3 ORs are scheduled per week and (Hans et al, 2008) works better with a high volume of ORs.
2. According to the order of the patients, a far more ordered scheduling is obtained using our approach due to the fact that it is considered in the definition of the problem.

3 Problem Statement

Let $\mathcal{S} = \{s_1, s_2, \ldots, s_{|\mathcal{S}|}\}$ be the set of surgery types that can be performed in the considered hospital department and let $d : \mathcal{S} \to \mathbb{R}_{>0}$ be the *duration function*: $d(s_i)$ is the duration of the surgery s_i (the time from the moment when the patient enters in the OR until she/he leaves the OR). Similarly, let $c : \mathcal{S} \to \mathbb{R}_{>0}$ be the *cleaning time function* after surgeries: $c(s_i)$ is the cleaning time of the OR after surgery s_i.

Let us assume that the duration $d(s_i)$ of each type of surgery $s_i \in \mathcal{S}$ is a random variable with normal probability density function (pdf) $d(s_i) = N(\mu_{d(s_i)}, \sigma_{d(s_i)})$, where $\mu_{d(s_i)}$ is the average and $\sigma_{d(s_i)}$ is the standard deviation. The average and the standard deviation of each type of surgery are computed by using historical data from the hospital (for our case study we use the data of the last two years). However, if no historical data are available, these values can be initially assigned by the medical doctors based on their experience and external information. Moreover, both μ/σ are updated each time a new surgery is performed. In addition, we assume that the time to clean the OR after each type of surgery $s_i \in \mathcal{S}$ has been performed is also a random variables with normal pdf, i.e., $c(s_i) = N(\mu_{c(s_i)}, \sigma_{c(s_i)})$.

Furthermore, let us consider $\mathcal{W} = \{w_1, w_2, \ldots, w_{|\mathcal{W}|}\}$ an ordered list of patients such that if $w_j \in \mathcal{W}$, j is the order number of the patient w_j in the waiting list. Let $surg : \mathcal{W} \to \mathcal{S}$ be the function that for a given patient $w_j \in \mathcal{W}$ gives the surgery that should be performed. For example, if the surgery that should be performed on patient w_j is s_i, then $surg(w_j) = s_i$.

Finally let us assume an ordered set of time blocks $\mathcal{B} = \{b_1, ..., b_{|\mathcal{B}|}\}$, where $b_{|\mathcal{B}|}$ is the block corresponding to the latest date. Each block $b \in B$ has a fixed duration denoted by $l(b)$. For our case of study each block represent one OR working day, so we assume the same duration for each block $b \in B$. This duration is the DWT defined as X.

The parameters considered in this **Scheduling Problem** are as follows:

1. an ordered waiting list $\mathcal{W}$ composed by $|\mathcal{W}| = n$ patients defined as:
   - a row vector $\boldsymbol{P}_o = [1 \ \dots \ n]$ representing the preference order of the patients in the waiting list.
   - a row vector $\boldsymbol{\mu}_d = [\mu_{d(surg(w_1))} \ \cdots \ \mu_{d(surg(w_n))}]$ representing the average duration of the surgeries in the waiting list.
   - a row vector $\boldsymbol{\sigma}_d = [\sigma_{d(surg(w_1))} \ \cdots \ \sigma_{d(surg(w_n))}]$ representing the standard deviation of duration associated with the corresponding surgeries in the waiting list.
   - a row vector $\boldsymbol{\mu}_c = [\mu_{c(surg(w_1))} \ \cdots \ \mu_{c(surg(w_n))}]$ representing the average duration of cleaning times after surgeries.
   - a row vector $\boldsymbol{\sigma}_c = [\sigma_{c(surg(w_1))} \ \cdots \ \sigma_{c(surg(w_n))}]$ representing the standard deviation of the duration of cleaning times after surgeries.
2. a set $\mathcal{B}$ composed by $|\mathcal{B}| = m$ time blocks defined as:
   - a row vector $\boldsymbol{L}_{\boldsymbol{\mathcal{B}}} = [l(b_1) \ \dots \ l(b_m)]$ representing the duration of the time blocks.

For each time block to schedule $b_1, b_2 \cdots b_m$ there exist a binary decision vector $\boldsymbol{S}_1, \boldsymbol{S}_2 \cdots \boldsymbol{S}_m$ with a dimension equal to the number of the patients in the waiting list $n = |\mathcal{W}|$. If $\boldsymbol{S}_i[j] = 1$ then surgery of patient w_j should be performed in working day $i \leq m$.

The goal of this approach is the assignment of the patients from the waiting list $\mathcal{W}$ to the set of time blocks $\mathcal{B}$ (each patient being scheduled at most once) with the objectives,

- O_1 - maximize the DST (daily surgery time) of each block $b \in \mathcal{B}$;
- O_2 - respect as much as possible the order of the patients in $\mathcal{W}$.

Moreover a minimum guarantee of not expected overtime in each block time should be fixed.

■

Notice that for our particular case of study, we assume the same daily working time X for all days to schedule. However, this assumption can be easily relaxed assuming different values. Nevertheless, for sake of clarity, in this paper, we prefer to use the same X.

Let us introduce, by means of an small example, an illustration of a simplified hypothesis followed by the kind of desired result.

Example 1 Let us assume a waiting list $\mathcal{W}$ composed by $|\mathcal{W}| = n = 10$ patients defined by eps. (1) to (5). Moreover let us consider a set of three time blocks $\mathcal{B} = \{b_1, b_2, b_3\}$ to scheduled having the same duration of seven hours, i.e., $l(b_i) = X = 7$ [hours]$= 420$[minutes] $\forall i \in \{1...3\}$.

$$\boldsymbol{P}_o = [1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10] \tag{1}$$

$$\boldsymbol{\mu}_d = [75 \ 153 \ 90 \ 75 \ 202 \ 45 \ 97 \ 85 \ 111 \ 133] \tag{2}$$

$$\boldsymbol{\sigma}_d = [23 \ 23 \ 19 \ 23 \ 45 \ 12 \ 21 \ 24 \ 23 \ 24] \tag{3}$$

$$\boldsymbol{\mu}_c = [20 \ 20 \ 20 \ 20 \ 20 \ 20 \ 20 \ 20 \ 20 \ 20] \tag{4}$$

$$\boldsymbol{\sigma}_c = [10 \ 10 \ 10 \ 10 \ 10 \ 10 \ 10 \ 10 \ 10 \ 10] \tag{5}$$

One possibility of obtaining a computationally tractable solution for scheduling is to consider only the average duration of the surgeries, while cleaning times are ignored. So, instead of scheduling the patients in the available DWT of seven hours, the objective could be to impose a DST of, for example, 80% of the DWT, i.e., $0.8 \times 420 = 336$[minutes]. The rest of the time, i.e., $420 - 336 = 84$ minutes could be used for cleaning and to absorb any unexpected delay.

The strategy explained before has been used in the MILP problem explained in Sec. 4.1 and Tab. 1 shows a possible scheduling solution. Each row of this table represents the operation scheduling of one time block. The first column represents the ordinal number of the time blocks (or OR working day); the next four columns indicate the patients that should be operated ($\emptyset$ means no surgery). The sixth column is the DST rate of the solution (calculated as the sum of the average durations of the surgeries divided by the total time, i.e., $420[min]$); finally, the last column indicates the confidence of not exceeding the total time considered as independent sum of normal random variables. Notice that the surgery of patient w_6 has not been scheduled because others are more suitable to obtain DST closer to the objective (80%).

Table 1 Operation scheduling of the list of patients defined by eqs. (1) to (5) for an target DST rate of 80% and 3 time blocks of 7 hours.

Day	Sur. 1	Sur. 2	Sur. 3	Sur. 4	DST (%)	Conf. (%)
1	w_1	w_2	w_9	$\emptyset$	80.71	68.56
2	w_3	w_4	w_7	w_8	82.61	44.21
3	w_5	w_{10}	$\emptyset$	$\emptyset$	79.76	80.24

Since the MILP schedules the patients without a chance constraint imposing a confidence level not exceeding the total time, working days with high risks of exceeding the DWT of 420 minutes are obtained (e.g., day 2 has a probability of 65.79% of exceeding the DWT).

Previously to analyze the proposed solution obtained in the time block b_2 (see Tab. 1) let us recall some basic statistic concepts of the normal distribution (Patel and Read, 1996) that will be used to compute the confidence level not exceeding the working day in the optimization problems:

1. Let $x \sim N(\mu, \sigma)$ be a random variable with mean μ and standard deviation σ and let $z \sim N(1, 0)$ be a random variable with mean 1 and standard deviation 0. Then,
$$P(x \leq X_i) = P(z \leq Z_i) \tag{6}$$
where $Z_i = \frac{X_i - \mu}{\sigma}$.
2. Let $a, b, \ldots, z$ be independent random variables such that: $i \sim N(\mu_i, \sigma_i), \ \forall i = \{a, b, \ldots, z\}$ and let $U = a + b + ... + z$ be the sum of these variables, then U is a random variable with normal distribution $U \sim N(\mu_U, \sigma_U)$ where:
$$\begin{aligned} \mu_U &= \mu_a + \mu_b + ... + \mu_z \\ \sigma_U &= \sqrt{\sigma_a^2 + \sigma_b^2 + ... + \sigma_z^2} \end{aligned} \tag{7}$$

The time of using the OR in b_2, denoted T_{d2}, is the sum of,

1. individual durations of each surgery: $d(surg(w_i))$, $i = \{3, 4, 7, 8\}$;
2. corresponding cleaning time after each surgery: $c(surg(w_i))$, $i = \{3, 4, 7, 8\}$.

Since these variables ($d(surg(w_i))$ and $c(surg(w_i))$) are considered with normal pdf then, according to (7) , $T_{d2} \sim N(\mu_{T_{d2}}, \sigma_{T_{d2}})$ where:

- $\mu_{T_{d2}} = \sum_{i=\{3,4,7,8\}} (\mu_{d(surg(w_i))} + \mu_{c(surg(w_i))}) = 427$
- $\sigma_{T_{d2}} = \sqrt{\sum_{i=\{3,4,7,8\}} (\sigma^2_{d(surg(w_i))} + \sigma^2_{c(surg(w_i))})} = 48.03$

Therefore, $T_{d2} \sim N(427, 48.03)$. Since the total available time is 7 [hours] $\sim$ 420 [minutes], it is interesting to know the probability $P(T_{d2} \leq 420)$.

Normalizing T_{d2} according to (6),

$$P(T_{d2} \leq 420) = P(z \leq Z_i),$$

where $z \sim N(0,1)$.

Taking $X_i = 420$ then $Z_i = \frac{420-427}{48.03} = -0.145$. Therefore,

$$P(T_{d2} \leq 420) = P(z \leq -0.145),$$

and this probability is tabulated (Patel and Read, 1996): $P(z \leq -0.145) = 0.4421 \simeq 44.21\%$.

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In order to prevent time blocks with high risk of exceeding time, two mathematical models (MIQCP and N-MIQCP) including chance constraints are proposed.

4 Mathematical Programing Models

In this section, three alternative mathematical programming problems to solve the **Scheduling Problem** introduced in Sec. 3 are proposed (see Tab. 2). Objective O_2 (related with the preference order of the patients) is presented in the same form in all three problems. However, objective O_1 (related with the occupation rate) is considered in two different ways. In the first two problems, i.e., MILP and MIQCP, the objective O_1 is to obtain a given DST. However, the MIQCP problem provides in general better solutions because includes chance constraints to ensure a *minimum confidence level* (MCL) not exceeding the total DWT. The third problem considers the objective (O_1) of maximizing the DST, hence this DST is a variable in the N-MIQCP increasing the computational complexity of the problem (compared with the previous ones). Although, one can use the N-MICQP problem together with some statistical information on the patients to estimate which should be the target DST for a given confidence level not exceeding the working time and use it in the MIQCP problem. In fact, problem MIQCP is the one that should be used to schedule the surgeries while N-MICQP is used only once, at the beginning of the process, to compute the target DST.

Table 2 Comparison of the Mathematical Programming Problems

	Objectives			Constraints
	O_1		O_2	
	target DST	**maxim. DST**	**Order**	**MCL**
MILP	✓		✓	
MIQCP	✓		✓	✓
N-MIQCP		✓	✓	✓

In order to satisfy objectives O_1 and O_2, the following linear cost function composed by two balanced term (C_1 and C_2) is proposed:

$$\sum_{i=1}^{m}\left[\underbrace{\alpha_i \cdot (m-i+1)}_{C_1} + \beta \cdot \underbrace{\boldsymbol{P}_o \cdot \boldsymbol{S}_i \cdot (m-i+1)}_{C_2}\right], \tag{8}$$

where β is the relative weighting between C_1 and C_2, m is the number of time blocks to schedule and α a variable related with the occupation rate.

The terms C_1 and C_2 are two criteria related with O_1 and O_2 respectively:

- *criterion* C_1, related to O_1, consisting in the minimization of the absolute deviation between the target DST and the scheduled DST (in the first two problems) or maximizing the scheduled DST (in the third problem);
- *criterion* C_2, related to O_2, consisting in the minimization of the sum of the preference order of patients scheduled each day, giving more weights to the first days.

From a syntactic point of view, the three proposed problems (MILP, MIQCP, N-MIQCP) minimize the same objective function (8). However, depending on the definition of the variables α_i, the first term in the objective function is different. Variables α_i are defined by constraints in the problems.

For the first two problems (MILP and MIQCP), a variable α_i in the first term of (8) represents the absolute deviation (in minutes) of the scheduled DST of day i with respect to the target DST. For N-MIQCP problem, a variable α_i is the sum of durations of all scheduled surgeries in day i multiplied by -1 (since we want to maximize it). Moreover, α_i is multiplied by $(m-i+1)$ in order to get smaller deviations (or bigger utilization) in the first working days. This implies at the same time that if are not enough patients for all working days, the last days remain free.

The second term of (8) contains the binary decision vectors $\boldsymbol{S}_1, \boldsymbol{S}_2, \boldsymbol{S}_3, \ldots, \boldsymbol{S}_m$ and the row vector $\boldsymbol{P}_o$ (representing the order of patients in the waiting list). Multiplying $\boldsymbol{P}_o$ by $\boldsymbol{S}_i$, the sum of the preference order of surgeries scheduled in day i is obtained. This sum is minimized, therefore bigger preference is given to the first patients of the waiting list. Again we multiply the second term by $(m-i+1)$, implying that patients scheduled in the first days penalize more, forcing thus the scheduling of patients with lower preference order in the first days.

Regarding parameter β, it is known that exists a value such that the optimization problem with cost (8) returns the optimal solution of the corresponding multi-objective optimization problem (Athan and Papalambros, 1996). Furthermore, C_1 and C_2 in (8) have different units, so the choice of the β parameter will establish a compromise between O_1 and O_2. Hence, β is a design parameter and it is used to balance the importance of respecting the order of the patients in the waiting list against the one of maximization of the OR utilization.

In order to prevent solutions in which one patient is scheduled more than once the following set of constraints are required

$$\sum_{i=1}^{m} \boldsymbol{S}_i[j] \leq 1, \quad \forall j = 1, 2, \ldots, n. \tag{9}$$

Both the objective function (8) and the set of constraints (9) appear in all three mathematical programing problems.

4.1 MILP

In this first approach, objective O_1 is to obtain a desired DST that is given as a percentage p of DWT. Since the DST is not including the cleaning time, p should be such that $p < 100\%$. Let us assume that the DWT is denoted by X (in minutes), then

$$Obj = X \cdot \frac{p}{100} \tag{10}$$

is the target DST in minutes.

A variable α_i is defined as the absolute difference (in minutes) between the total scheduled time of day i and the Obj. This can be written as,

$$\alpha_i = |\boldsymbol{\mu}_d \cdot \boldsymbol{S}_i - Obj| \,, \tag{11}$$

where $\boldsymbol{\mu}_d$ is a vector containing the average durations of the surgeries in the waiting list while $\boldsymbol{S}_i$ is the binary decision vector defining the surgeries scheduled day i. In linear terms, the absolute value can be computed as the minimum α_i fulfilling

$$\begin{cases} \boldsymbol{\mu}_d \cdot \boldsymbol{S}_i - Obj \leq \alpha_i \\ \boldsymbol{\mu}_d \cdot \boldsymbol{S}_i - Obj \geq -\alpha_i, \end{cases} \quad \forall i = 1, 2, \ldots, m. \tag{12}$$

Putting all together, the following MILP is obtained.

$$\begin{array}{l} \min \sum_{i=1}^{m} \left(\alpha_i \cdot (m - i + 1) + \beta \cdot \boldsymbol{P}_o \cdot \boldsymbol{S}_i \cdot (m - i + 1) \right) \\ \text{Subject to:} \\ \begin{cases} \boldsymbol{\mu}_d \cdot \boldsymbol{S}_i - Obj & \leq \alpha_i, \ \forall i = 1, 2, \ldots, m \\ -\boldsymbol{\mu}_d \cdot \boldsymbol{S}_i + Obj & \leq \alpha_i, \ \forall i = 1, 2, \ldots, m \\ \sum_{i=1}^{m} \boldsymbol{S}_i[j] & \leq 1, \quad \forall j = 1, 2, \ldots, n \\ \boldsymbol{S}_i \in \{0, 1\}^n, \alpha_i \in \mathbb{R}, & \qquad \forall i = 1, 2, \ldots, m. \end{cases} \end{array} \tag{13}$$

Regarding the size of MILP (13), it has

- $(n + 1) \cdot m$ variables of which $\begin{cases} m \text{ real variables;} \\ n \cdot m \text{ binary variables;} \end{cases}$
- $2 \cdot m + n$ linear inequality constrains.

4.2 MIQCP

Let the surgery durations and cleaning times be assumed random variables with normal pdf. Therefore, the working time of day i (denoted by T_{di}) is also a random variable with normal pdf, i.e., $T_{di} \sim N(\mu_{T_{di}}, \sigma_{T_{di}})$ where,

- $\mu_{T_{di}} = (\boldsymbol{\mu}_d + \boldsymbol{\mu}_c) \cdot \boldsymbol{S}_i$
- $\sigma_{T_{di}} = \sqrt{(\bar{\boldsymbol{\sigma}}_d^2 + \bar{\boldsymbol{\sigma}}_c^2) \cdot \boldsymbol{S}_i}$ [1]

[1] In this paper $\bar{\boldsymbol{x}}^2$ is a vector such that $\bar{\boldsymbol{x}}^2(i) = \boldsymbol{x}(i) \cdot \boldsymbol{x}(i)$

This problem (MIQCP) improves the MILP (13) by including a set of chance constraints ensuring that the scheduled blocks have a confidence level not exceeding the total time (given by the DWT=X) greater than a threshold $0 \le Cl \le 1$, i.e., $P(T_{di} \le X) \ge Cl$. Some constraints of this set are quadratic, so the proposed model is a Mixed Integer Quadratic Constraint Programming (MIQCP) problem. By using the statistic concepts recalled in eqs. (6) and (7), this set of constraints is given by

$$\frac{X - \mu_{T_{di}}}{\sigma_{T_{di}}} \ge V_{Cl}, \quad \forall i = 1, 2, \dots, m, \tag{14}$$

where V_{Cl} is the value corresponding to a normal variable ($x \sim N(0,1)$) with an accumulative probability Cl, i.e., $P(x \le V_{Cl}) = Cl$.

Developing inequality (14),

$$\frac{X - \mu_{T_{di}}}{\sigma_{T_{di}}} \ge V_{Cl} \Rightarrow X - \mu_{T_{di}} \ge V_{Cl} \cdot \sigma_{T_{di}} \Rightarrow$$

$$X - (\boldsymbol{\mu}_d + \boldsymbol{\mu}_c) \cdot \boldsymbol{S}_i \ge V_{Cl} \cdot \sqrt{(\bar{\boldsymbol{\sigma}}_d^2 + \bar{\boldsymbol{\sigma}}_c^2) \cdot \boldsymbol{S}_i}.$$

Let $\boldsymbol{A} = \boldsymbol{\mu}_d + \boldsymbol{\mu}_c$ and $\boldsymbol{B} = \bar{\boldsymbol{\sigma}}_d^2 + \bar{\boldsymbol{\sigma}}_c^2$.

Therefore if $(X - \boldsymbol{A} \cdot \boldsymbol{S}_i > 0)$ then

$$[X - \boldsymbol{A} \cdot \boldsymbol{S}_i]^2 \ge \left[V_{Cl} \cdot \sqrt{\boldsymbol{B} \cdot \boldsymbol{S}_i}\,\right]^2 \Rightarrow$$

$$X^2 + [\boldsymbol{A} \cdot \boldsymbol{S}_i]^2 - 2 \cdot X \cdot \boldsymbol{A} \cdot \boldsymbol{S}_i \ge V_{Cl}^2 \cdot \boldsymbol{B} \cdot \boldsymbol{S}_i \Rightarrow$$

$$\left[V_{Cl}^2 \cdot \boldsymbol{B} + 2 \cdot X \cdot \boldsymbol{A}\right] \cdot \boldsymbol{S}_i - [\boldsymbol{A} \cdot \boldsymbol{S}_i]^2 \le X^2 \Rightarrow$$

Let $\boldsymbol{K} = V_{Cl}^2 \cdot \boldsymbol{B} + 2 \cdot X \cdot \boldsymbol{A}$ then the previous inequality becomes:

$$\boldsymbol{K} \cdot \boldsymbol{S}_i - [\boldsymbol{A} \cdot \boldsymbol{S}_i]^2 \le X^2.$$

Note that $X - \boldsymbol{A} \cdot \boldsymbol{S}_i > 0$ is a constraint imposing that the average working time of day i is lower than the total time X. In this way, the possible symmetric solutions obtained due to $[X - \boldsymbol{A} \cdot \boldsymbol{S}_i]^2$ are prevented. However, the model can only schedule working days with a confidence level not exceeding total time greater than 50%. In order to impose a confidence level lower than 50%, the constraint $X - \boldsymbol{A} \cdot \boldsymbol{S}_i > 0$ should be changed with $X - \boldsymbol{A} \cdot \boldsymbol{S}_i < 0$. In this paper, we consider $Cl \ge 50\%$.

Putting together, the set of chance-constraints that prevents the scheduling with a confidence level lower than Cl is showed in (15).

$$\begin{cases} \boldsymbol{K} \cdot \boldsymbol{S}_i - [\boldsymbol{A} \cdot \boldsymbol{S}_i]^2 \le X^2 \\ X - \boldsymbol{A} \cdot \boldsymbol{S}_i \ \ge 0, \end{cases} \qquad \forall i = 1, 2, \dots, m. \tag{15}$$

The full MIQCP problem is obtained as,

$$
\begin{aligned}
&\min \sum_{i=1}^{m} \left[\alpha_i \cdot (m-i+1) + \beta \cdot \boldsymbol{P}_o \cdot \boldsymbol{S}_i \cdot (m-i+1)\right] \\
&\text{Subject to:} \\
&\begin{cases}
\boldsymbol{\mu}_d \cdot \boldsymbol{S}_i - Obj & \leq \alpha_i, \quad \forall i = 1,2,\ldots,m \\
-\boldsymbol{\mu}_d \cdot \boldsymbol{S}_i + Obj & \leq \alpha_i, \quad \forall i = 1,2,\ldots,m \\
\sum_{i=1}^{m} \boldsymbol{S}_i[j] & \leq 1, \quad \forall j = 1,2,\ldots,n \\
\boldsymbol{K} \cdot \boldsymbol{S}_i - [\boldsymbol{A} \cdot \boldsymbol{S}_i]^2 & \leq X^2, \forall i = 1,2,\ldots,m \\
X - \boldsymbol{A} \cdot \boldsymbol{S}_i & \geq 0, \quad \forall i = 1,2,\ldots,m \\
\boldsymbol{S}_i \in \{0,1\}^n, \alpha_i \in \mathbb{R}, & \qquad \forall i = 1,2,\ldots,m.
\end{cases}
\end{aligned}
\tag{16}
$$

Regarding the size of MIQCP (16), the problem has

- $(n+1) \cdot m$ variables of which $\begin{cases} m \text{ real variables;} \\ n \cdot m \text{ binary variables;} \end{cases}$
- $3 \cdot m + n$ linear inequality constraints;
- m quadratic inequality constraints.

The MIQCP problem has two input parameters: the target DST percentage, i.e., p (it appear in Obj see (10)), and the MCL, i.e., Cl (by the value of V_{Cl}). However, these two parameters are dependent one by another, for example if we fix a confidence level Cl then the value of p is upper bounded, this bound depending on the chosen value of Cl. In the third problem, the target DST becomes a variable in the problem and the optimization problem will try to maximize it.

4.3 N-MIQCP

This problem improves the MIQCP by changing the input parameter DST to schedule (given as p) into a variable to be maximized. So, the N-MIQCP only has one input parameter: MCL not exceeding the DWT (given as Cl).

Because the DST becomes a variable in the N-MIQCP problem that is maximized, a new definition of variables α_i different by the one in eq. (11) should be given. Instead of being the absolute difference of the scheduled time with respect to the desired value, in N-MIQCP problem the variables α_i are defined as the negative sum of the duration of the surgeries scheduled in day i.

$$
\alpha_i = -\boldsymbol{\mu}_d \cdot \boldsymbol{S}_i, \forall i = 1,2,\ldots,m. \tag{17}
$$

Notice that the variables α_i can be removed from the problem while in the objective function can be used its definition given by (17). The full N-MIQCP problem is obtained as,

$$
\begin{aligned}
&\min \sum_{i=1}^{m} \left[-\boldsymbol{\mu}_d \cdot \boldsymbol{S}_i \cdot (m-i+1) + \right. \\
&\qquad \left. + \beta \cdot \boldsymbol{P}_o \cdot \boldsymbol{S}_i \cdot (m-i+1)\right] \\
&\text{Subject to:} \\
&\begin{cases}
\sum_{i=1}^{m} \boldsymbol{S}_i[j] & \leq 1, \quad \forall j = 1,2,\ldots,n \\
\boldsymbol{K} \cdot \boldsymbol{S}_i - [\boldsymbol{A} \cdot \boldsymbol{S}_i]^2 & \leq X^2, \forall i = 1,2,\ldots,m \\
X - \boldsymbol{A} \cdot \boldsymbol{S}_i & \geq 0, \quad \forall i = 1,2,\ldots,m \\
\boldsymbol{S}_i \in \{0,1\}^n, & \qquad \forall i = 1,2,\ldots,m.
\end{cases}
\end{aligned}
\tag{18}
$$

The size of N-MIQCP (18) is given by

– $n \cdot m$ binary variables;
– $n + m$ linear inequality constraints;
– m quadratic inequality constraints.

Even if the number of variables and of the constraints is smaller than the MIQCP (16), the computational complexity of N-MIQCP (18) is in general higher. This is due to the fact that in MIQCP (16) the DST of the solution belong in general to a symmetric interval around the target DST while in the case of N-MIQCP (18) the DST is maximized.

Finally, let us notice that N-MIQCP (18) can be used to estimate the DST achievable for a given confidence level. The pair of values DST confidence level can be used as input parameters in the MIQCP (16) that is computationally more efficient.

5 Heuristic Approaches

In order to reduce the computational complexity, this section introduces first a *receding horizon strategy* (RHS) to obtain suboptimal scheduling for a large number of surgical blocks. Second, a methodology using N-MIQCP and MIQCP allowing to obtain the operation scheduling with reduced computation time is proposed.

5.1 Suboptimal Solution Using a Receding Horizon Strategy

The optimization problems presented in the previous section can be optimality solved by using IBM ILOG CPLEX Optimization Studio which is often referred as CPLEX (IBM, 2016), one of the fastest software solution for integer problems (Gearhart et al, 2013). Although CPLEX is quite fast, due to the large size of the problems, the computational time and memory usage to solve the optimizations problems increase exponentially with the number of patients in the waiting list and time blocks (ORs working days) to schedule.

After some simulations with different number of patients in the waiting lists (n) and with different number of time blocks to schedule (m), we observed that the variable that more influence the computational time is m. Moreover, the computational time depends also on the value of the design parameter β. It has been observed that, the greater is β (more importance is given to the order of the patients O_2), the smaller is the computational time. The fact that more importance is given to the order in which patients are scheduled, allows a lower combinatorial of patients and consequently the solution converge before in the optimal one. For example, for a value of β enough large the scheduling obtained has the patients perfectly ordered.

In order to reduce the computational complexity, in (Clavel et al, September 2016) we proposed to solve the optimization problem of m blocks iteratively (similar with the *Receding Horizon Strategy* (RHS) (Camacho and Bordons, 2004)). The idea is to schedule initially $N \leq m$ time blocks by using the optimization problem and the full waiting list of patients. From the obtained solution, only the first $N_1 \leq N$ time blocks are considered and the corresponding patients are removed from the waiting list. After that, another N time blocks are scheduled by using the same optimization problem and the updated waiting list. Again, from the solution only the first N_1 blocks are considered and the procedure is repeated until all m blocks are scheduled.

In our case of study, the scheduling is performed weekly: on Monday of week “x” the scheduling of the time blocks available in week “x+2” is performed. Normally there are $m = 3$ time blocks available each week for each surgical team, so in this case is not necessary to use the RHS. However RHS approach could be interesting in other hospitals or departments which greater number of time blocks to scheduled. However, RHS approach could help to estimate an approximate surgery date for a new arrived patient by scheduling all others patients in the waiting list. This possibility also allows hospital manager to know the number of time blocks necessary to schedule all patients in the waiting list and increase or decrease the hospital resources to a surgical department.

5.2 Towards Computational Tractable Scheduling

This subsection discusses an approach to obtain a computational tractable solution for operation scheduling using MIQCP (16) and N-MIQCP (18) that include chance constrains (see Fig. 1).

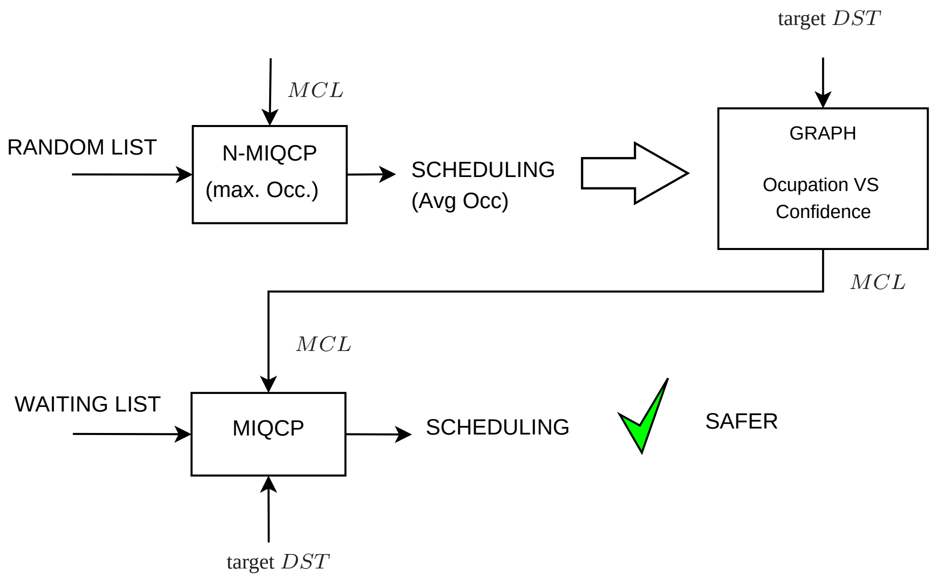

Fig. 1 Approach for Computational Tractable Scheduling.

The difference between these two optimization problems is that MIQCP (16) requires a target DST as input parameter and the N-MIQCP (18) maximizes this DST. However the computational time for the N-MIQCP is much higher than for MIQCP.

Step 1. Approximate DSTs corresponding to MCLs using N-MIQCP (18)**.** The objective of this step is to know the appropriate target DST that should be introduce as input parameter in the MIQCP for a given MCL. We consider that a DST is appropriated for a given MCL if it is achievable and it is close to maximum. Besides of the MCL, the approximation of DST also depends on the type of surgeries that may appear in the waiting list (durations and probability of appearing) and the total time available (DWT=X). For this, the approximated DSTs should be updated by performed this first step once every two months. Fig. 2 shows the relation between average DST and minimum confidence level obtained

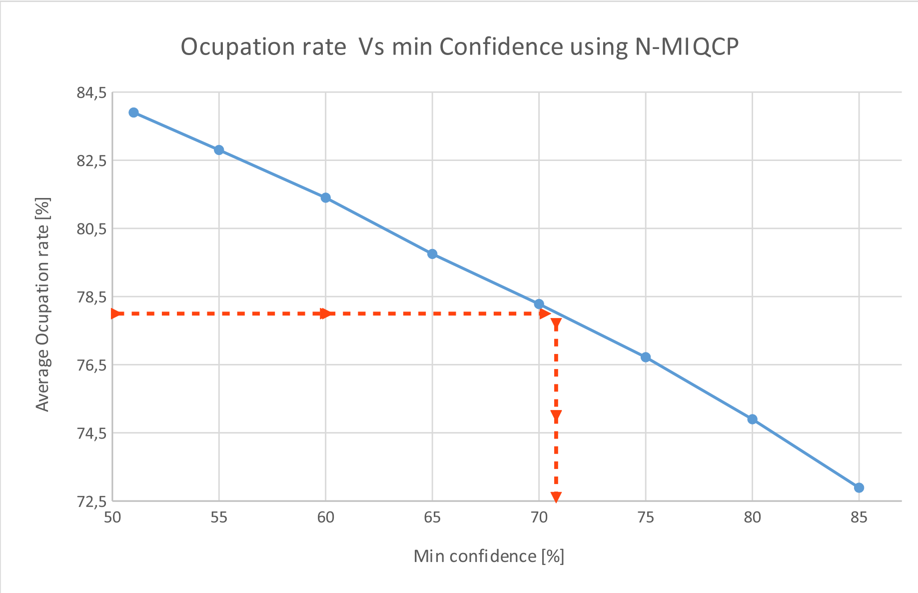

Fig. 2 Average occupation rate obtained using N-MIQCP depending on the minimal percentage of confidence required (blue). The input parameters corresponds to the studied Surgery Department.

in step 1 for the particular parameters of the Orthopedic Surgery Department in Zaragoza. Notice that for example, if the objective is to obtain a target DST equal to 78%, then the minimum confidence level is 71%. Using historical data on the performed surgeries, compute first the probability of appearance of each surgery as the total number of patients with the same surgery divided by the total number of surgeries. Based on these probabilities, a large random waiting list can be generated. Using N-MIQCP (18) with the receding horizon strategy (e.g., $N_1 = N = 3$) the patients on the waiting list are scheduled with a large number of time blocks m (ensuring that all patients are scheduled). Different scheduling should be obtained fixing different confidence levels. Each obtained solution corresponding to a given confidence level is used to compute the expected average DST. A representation of the average DST vs. a given minimum confidence level can be obtained.

Step 2. Compute the scheduling by using MIQCP (16)**.** Using the obtained relation, for a given minimum confidence level, the corresponding DST is taken and used in MIQCP (16) for scheduling of patients. In Sec. 6 will be shown that the computational time necessary to solve this problem is much smaller than solving N-MIQCP (18) while the obtained solution is *safer*, in general, than the solution obtained by using MILP (13).

6 Results

In this section experimental setting of the heuristics approaches presented together with some simulation results are analyzed and compared. In particular a comparison between different models/approaches is analyzed through a realistic simulation of one year scheduling in the studied hospital department.

6.1 Receding Horizon Strategy

In order to fix the values of N and N_1 used in the RHS, different simulation using CPLEX (version 12.6.2) in a computer with an Intel Core i3 and 4 GB of memory has been performed.

First, the influence of the number of days considered (N_1) in each iteration is checked. For this, from a waiting list composed by 120 patients and with the same receiding time horizon ($N = 7$), the MILP (13) is used to schedule 30 working days with different values of N_1. Tab. 3 shows the cost obtained for the different instances. Unlike what usually happens in control theory, increasing the value of N_1, a trend of cost improvement is observed. For this, the following simulations are performed with $N = N_1$. In Tab. 4 a comparison between the optimal solution and the one obtained by using the RHS is showed. Moreover, the computational costs are given. In particular, the first column represent the particular mathematical programing problem being used, the next two columns fix the size of the problem (m and n). Notice that each instance has a different size (depending on the complexity of the model) in order to be able to obtain the optimal solution. The fourth one indicates the optimal cost and the fifth one the time necessary to obtain optimal solutions for the particular instance (other instance with the same size can not be solved optimally). Column sixth shows the parameter $N = N_1$ of the RHS, while the seventh and eight columns represent the cost obtained using RHS and the relative error to the optimal cost respectively. Finally the last column indicate the computational time required to solve the different instances using RHS.

Table 3 Influence of N_1 in MILP problem solutions ($n = 120$, $m = 30$, $N = 7$, $\beta = 2$)

N	N_1	Cost
7	1	88668
	2	88171
	3	88295
	4	88382
	5	88181
	6	86824
	7	86937

As expected, increasing the horizon N, solutions with better costs are usually obtained. Unfortunately, increasing the horizon N, the computational time is increased also. However, increasing N does not implies that always a better cost is obtained. For example, solving the particular instance in Tab. 4 of N-MIQCP using the RHS, it can be seen that the cost obtained with $N = N_1 = 5$ and $N = N_1 = 4$ is the same (-4902) and moreover, it is worse than the one obtained for $N = N_1 = 3$ (-4940).

6.2 Computational Tractable Scheduling

In this subsection are shown the computational time improvements, as well as the results obtained using the heuristic approach with N-MIQCP and MIQCP. Using again the same

Table 4 Cost and computational time for instances of the three proposed problems solved optimally and solved by using the RHS with different parameters of $N = N_1$

Model	# days (m)	# patients (n)	Op.Cost	Time [s]	$N = N_1$	RHS Cost	ε_r [%]	Time [s]
MILP	12	40	5552	150	6	5850	5.36	4.76
					7	5754	3.63	6.05
					8	5693	2.53	10.07
MIQCP	9	30	2600	548	3	2860	10	0.63
					4	2720	4.61	1.19
					5	2654	2.07	4.47
N-MIQCP	6	25	-4951	373	3	-4940	0.22	11.05
					4	-4902	0.99	289
					5	-4902	0.99	1558

computer (Intel Core i3 and 4 GB of memory), the average computational times to schedule $m = 42$ working days (for lists generated random based on the probabilities) using N-MIQCP and MIQCP (both with receding horizon strategy with $N_1 = N = 3$) are shown in Tab. 5 and 6 and discussed in Ex. 2.

Example 2 Let us compare the computational time for solving N-MIQCP (18) and MIQCP (16) using in both cases the RHS. For this, we consider different list of $n = 150$ patients generated random and $m = 42$ time blocks to schedule. Let us assume also that the DWT is 6.5 hours.

First, the N-MIQCP (18) is solved assuming different MCL not exceeding DWT, namely $Cl = 68\%$, $Cl = 72.5\%$ and $Cl = 78\%$. For each Cl, 50 replications with different scenarios (waiting list) has been scheduled. In Tab. 5 the average computational times to solve the different instances and the average DST (in percentage respect to the DWT) of the obtained solutions are presented. Then, the MIQCP (16) is solved with exactly the same

Table 5 Average Computational time and average DST obtained by solving different instances of N-MIQCP (18) using RHS ($N = N_1 = 3$).

MCL (%)	Avg. Time[s] N-MIQCP	Avg. DST (% DWT)
68	71.6	78.9
72.5	130	77.59
78	285	75.57

MCL not exceeding DWT. Since MIQCP (16) requires as input parameter a target DST, a value 0.4 greater than the average DST obtained from the solutions of N-MIQCP (18) (Tab. 5) is used. In this way similar DST will be obtained.

Tab. 6 shows the average computational times and the average DST obtained by solving 50 replications of the different instances using MIQCP (16).

It can be seen that for a same MCL the average computational times to solve instances using MIQCP (16) (Tab. 5) decreases ($24 < 71$, $36 < 130$ and $47 < 285$) with respect to the obtained by using N-MIQCP (18) (Tab. 6). Moreover, the average DST obtained with the both problems are really similar ($78.9 \sim 78.75$, $77.59 \sim 77.31$ and $75.57 \sim 75.34$). ■

Table 6 Average computational time and average DST obtained by solving different instances of MIQCP (16) using RHS ($N = N_1 = 3$).

MCL (%)	Target DST	Avg. Time[s] N-MIQCP	Avg. DST (% DWT)
68	79.3	24	78.75
72.5	78	36	77.31
78	76	47	75.34

6.3 Influence of parameter β in the proposed model

In this subsection, the values of the parameter β in each model (MILP MIQCP and N-MIQCP) are fixed.

In order to be able to compare two different scheduling from the point of view of the order of the patients, we define the indicator Ω. This indicator measures the disorder of the patients in the obtained scheduling, so the smaller it is, the more orderly are the patients in the scheduling. To compute this value, for each time block scheduled b_i we define an interval $[f_i, l_i]$. If the preference order of the surgeries scheduled in the time block b_i belong to the interval $[f_i, l_i]$ do not increase the value of Ω. On contrary, each patient with a preference order outside the interval, increases the value of Ω. The formal calculation of Ω is given in Algorithm 1 where Np is the total number of patients scheduled and Pd is the average number of patients scheduled per time block.

Algorithm 1: Calculation of Ω parameter in a scheduling of m time blocks

$\Omega := 0$
$Np := \sum_{i=1}^{m} (sum(S_i))$
$Pd := \frac{Np}{m}$
for all $b_i \in \mathcal{B}$ **do**
  $f_i :=$min$(1, \lfloor Pd \cdot (i-1) \rfloor - 3)$
  $l_i := \lceil Pd \cdot i \rceil + 4$
  for all w_j scheduled the day b_i **do**
    if $j \notin [f_i, l_i]$ **then**
      $\Omega := \Omega +$min$(|j - f_i|, |j - l_i|)$
    end if
  end for
end for

Now the influence of parameter β in the MILP problem is shown in Tab. 7. Scheduling of 35 time blocks have been obtained (with RHS) for 50 different scenarios (waiting list) composed by 120 patients. The target DST is fixed to 78% and different values of β has been used. The waiting lists have been generated randomly as is explained in Step 1 of Sec. 5.2. The DST obtained (average and standard deviation) and the average value of parameter Ω have been analyzed.

It can be seen that decreasing β, better results of DST are obtained: The average DST are closer to the target and the standard deviation decreases. This means that the data are more concentrated around the average value. Unfortunately, this improvement is achieved by allowing a greater disorder of the patient in the operations scheduling: decreasing the value of β, the value of Ω is increased. According to these results, $\beta = 2$ is fixed for MILP (13). Moreover, performing similar simulations using MIQCP (16) and N-MIQCP (18) a value of

Table 7 Influence of β in MILP: Small β gives more weight to obtain the target DST ($m = 30$, $n = 100$, $N = N_1 = 7$ and Target $DST = 78\%$).

β	**Avg. DST**	**Std. Dev. DST**	Ω
1	77.85	0.97	86.4
2	77.58	1.37	64.52
3	77.30	1.71	55.4

$\beta = 2$ is fixed for MIQCP (16) while a value of $\beta = 4$ is fixed for N-MIQCP (18). Using these values, a good compromise between the DST (O_1) and the order of the patients (O_2) are obtained from the medical point of view. Note that a greater value of β is necessary in N-MIQCP problem compared with the other two problems because of the different definition of α_i.

6.4 One year realistic simulation in a Orthopedic Department of the LBHZ

In this subsection, in order to test the proposed approach and to compare it with the approaches proposed in (Shylo et al, 2012; Hans et al, 2008), we implement a discrete event simulation model of the scheduling. It is used to simulate scheduling decision for each team in the Orthopedic department at the LBHZ. One year length (52 weeks) is set for each simulation run. The new patients needing a surgery are assumed to arrive according to a Poisson distribution with a mean of 9 per week. Moreover for each set of simulations, 50 replications are performed (each replications is a schedule for one year). The block schedule used in the simulation is identical to the one that is used by the studied department: each team have 3 blocks per week from 8:30 A.M. until 3 P.M.

Let x be the current simulation week, the steps in the simulation algorithm of the scheduling process are described as follow:

Step 1. Generate the initial waiting list and initialize the current simulation week. An initial waiting list composed by 100 patients is generated randomly using realistic data of the studied department. Moreover the current week “x” is initialized to x = 1

Step 2. Scheduling the time blocks. Surgeries from the current waiting list are assigned to time blocks booking in the week “x+2”. In order to perform the assignment, besides of the three proposed models in this work, the approaches in (Shylo et al, 2012; Hans et al, 2008) have been implemented. These approaches need a base scheduling of the blocks to obtain the final assignment. The *first-fit probabilistic rule* (Π_{FFP}) under the probabilistic constraints (14) is used to obtain this base scheduling. By using Π_{FFP} sequentially each surgery is assigned to the first available block for which the probabilistic capacity constraint (14) is satisfied after the assignment.

Step 3. Generate a new set of arrival patients. A number a of new arriving surgeries is generated based on Poison distribution with a fixed arrival rate considering realistic data from the historical data.

Step 4. Process all scheduled blocks for the current week. We process 2000 replications of each block available in the current week obtaining different average metrics (overtime, utilization, confidence level).

Step 5. Update the waiting list. The patients scheduled in the current week “x” (they should be surgically operated week“x+2”) are removed from the waiting list. Moreover the new arrival patients (Step 3) are included at the end of the waiting list.

Step 6. Increment the current week (next week) and stop the simulation if the current week exceeds the end week of the simulation, otherwise proceed to Step 2.

Some events as changes in surgery dates, cancellation of surgeries or the addition of emergency/urgent surgeries are not considered in our simulation model because are difficult to predict and they are managed in real time based on expert opinions.

First, the scheduling obtained by using problems (13), (16) and (18) are tested and compared by the simulation algorithm and represented in Tab. 8.

The input parameters of the problems are shown in the first three columns of the Tab. 8 (INPUT PARAMETERS). The last 10 columns of Tab. 8 show the SIMULATION RESULTS in the operation scheduling related to: a) Occupation rate (DST), b) confidence level and c) order of the patients (Ω). A minimum confidence level of $Cl = 70\%$ is considered for N-MIQCP and MIQCP problems.

Table 8 Comparing of one year scheduling using MILP, MIQCP and N-MIQCP

	INPUT PARAMETERS			SIMULATION RESULTS								
	Objectives		Constr.	DST (% of DWT)				Confidence level				Order
MODEL	**target DST**	**maxim. DST**	**min** C_l	**Ave.**	**Max**	**Min**	**Std.**	**Ave.**	**Max**	**Min**	**Std.**	$\boldsymbol{\Omega}$
MILP	78.3			77.95	82.26	72.22	1.49	78.34	96.72	56.02	7.43	368
MIQCP	78.7		70	78.02	81.61	72.47	1.48	78.4	95.87	70.13	5.68	466
N-MIQCP		✓	70	78.29	85.19	68.76	2.67	77.31	97.52	70.04	6.37	436

Let us analyze the results obtained:

1. From the DST point of view, the solutions of the three problems are very similar, the average values are closer than 0.4% of its target (78.3%) and the greater standard deviation is obtained by using N-MIQCP. This happens because MILP and MIQCP problem try to obtain a target DST and consequently the values of occupation are concentrated around this target DST.
2. Taking into account the confidence level not exceeding the DWT, the average values obtained by using the three problems are really close. However, by using MILP, days with higher probabilities of exceed the DWT are obtained (minimum confidence level of 56.02 is obtained). This problem (MILP) does not impose a minimum confidence level.
3. In relation with the order of the patients, the solution with the lower coefficient Ω is obtained by solving MILP, however the three models obtains similar and acceptable values of Ω from a medical point of view results.

As expected, for the same target DST rate (for example 78.3%), the solutions obtained with MIQCP and N-MIQCP are safer than the ones obtained by using MILP. This happens because MIQCP and N-MIQCP prevent by chain constraint time blocks with low confidence level. In addition the solution obtained using MIQCP (Avg. DST=78.02%) and N-MIQCP (Avg. DST=78.29%) are very similar from the occupation rate point of view. Furthermore, like it was shown in Tab. 5 and 6, the computational time necessary to solve MIQCP is lower than the one of the N-MIQCP.

Tab. 9 shows a comparison between the scheduling obtained by using: (1) the Π_{FFP} rule (commonly used in hospitals), (2) the batch scheduling approach in (Shylo et al, 2012), (3) the constructive algorithm proposed in (Hans et al, 2008) and (4) our approach based on the MIQCP. The scheduling are obtained fixing a minimum confidence level of $Cl = 70\%$ for the 4 approaches. The average annual values of overtime probability, occupation rate, and order of the patients (Ω) are analyzed. Moreover the total overtime and the total number of treated patients per year are considered.

Table 9 Comparison of the one year scheduling using different chain-constrained approaches with a minimun confidence level of 70%

Approach	OR Conf. Level	Overtime (Year) [min]	OR utilization DST (% DWT)	Surgeries (Year)	Ω (Year)
Π_{FFP} rule (commonly used)	81.98	806.41	76.12	429.9	1935.9
Constructive Alg. (Hans et al, 2008)	80.43	922	76.69	432.02	2840
Batch Scheduling (Shylo et al, 2012)	75	1183	79.06	447.78	3993.1
MIQCP (proposed here)	77.31	1059	78.28	438.3	395.4

The 3 approaches analyzed in Tab. 9 improve the occupation rate of the time blocks with respect to the obtained by using the Π_{FFP} scheduling rule. However, the improvement in the occupation rate of the time block implies a decreasing in the confidence level. For example, the Batch Scheduling approach achieves the highest occupation rate (79.06 %) and the highest number of treated patients (447.78), and consequently the lower confidence level (75%) and the highest total overtime (1183[min]) is obtained. Taking into account the pairs of values occupation rate and confidence level, the Batch Scheduling approach obtain the better solution with: 1) the highest occupation rate and 2) a confidence level within the allowed.

Our MIQCP approach obtain a little worse occupation rate (78.28 %) than the Batch Scheduling (79.06 %). However, considering the order of the patients by the value of parameter Ω, it can be check that our MIQCP approach obtain the best scheduling. Doctors in the studied hospital department consider that the scheduling obtained using the other approaches are not suitable from a medical point of view because of the great disorder of the patients. So, using our MIQCP approach and imposing a MCL of 70%, the improving in the occupation rate with respect to use the Π_{FFP} rule (commonly used in hospitals) is of 2.16 %. Only considering the Orthopedic department implies an increment Δ_{Oc} of 109 hours per year in the use of the ORs (19):

$$\begin{aligned} \Delta_{Oc} &= 0.0216 \cdot 6.5 \left[\frac{hours}{block}\right] \cdot 3 \left[\frac{block}{team \cdot week}\right] \cdot 52 \left[\frac{week}{year}\right] \cdot 5[team] \\ &= 109.512 \left[\frac{hour}{year}\right] \end{aligned} \tag{19}$$

Unfortunately, it also implies an increment Δ_{Ov} of 21.05 hours per year in the overtime (20), but always without comprise the minimum confidence level established.

$$\begin{aligned} \Delta_{Ov} &= (1059 - 806.4) \left[\frac{min}{year \cdot team}\right] \cdot 5[team] \cdot \frac{1}{60} \left[\frac{hour}{min}\right] \\ &= 21.05 \left[\frac{hour}{year}\right] \end{aligned} \tag{20}$$

In Tab. 10 we show the results obtained for a one year scheduling using our MIQCP approach for different MCL Cl. It can be seen that imposing a little lower MCL in MIQCP than in the Batch scheduling approach, similar occupation rates and total overtime are obtained. For example, using Batch Scheduling imposing a MCL of $Cl = 70$ (Tab. 9) and using the MIQCP approach with a MCL of $Cl = 67.7$ (Tab. 10) the average occupation rates are

79.06[%] and 79.08[%] respectively and the total overtime are 1183[min] and 1190[min] respectively. However, the scheduling are much more ordered using the MIQCP approach ($\Omega = 386.64$) than using the Batch Scheduling approach ($\Omega = 3993.1$).

Table 10 Comparison of one year scheduling using MIQCP approach with different MCL Cl [%]

MCL Cl [%]	OR Conf. Level	Overtime (Year) [min]	OR utilization DST (% DWT)	Surgeries (Year)	Ω (Year)
51	59.49	2425.1	83.9	469.64	446.58
55	63.14	2093.2	82.80	463.92	411.78
60	67.75	1712.0	81.4	455.56	426.52
65	72.99	1340.8	79.75	446.56	410.44
67.7	75.18	1190.07	79.08	441.42	386.64
70	77.31	1059	78.28	438.3	395.38
75	81.57	805.86	76.72	428.08	380.62
80	86.1	562.33	74.76	418.962	378.82
85	89.82	376.84	72.89	408.14	366.42

7 Decision Support System for Scheduling

The Orthopedic Department in which the DSS will be used is composed by medical doctors divided in 5 medical teams. Each doctor has assigned his own patients and the waiting list of a team is composed by the patients of the doctor belonging to the team. Each team must operate the patients from his waiting list during the time blocks previously booked. So each manager team should schedule the patient from his waiting list in the time blocks available to his team. In order to perform a rapid, efficient and dynamic operation scheduling we propose a decision support system (DSS). The core of the DSS is the MIQCP presented in Sec. 4 for operation scheduling, but also it includes other features that enable a) updating the waiting list, b) dynamic planning and, c) improving the input data by updating the surgeries durations.

7.1 Updating the waiting list

In general, a new patient is added at the end of the waiting list but the surgeon, depending on the priority of the pathology of the patient, could decide to put him in a higher position of the waiting list. The DSS, based on medical criteria, automatically creates the ordered waiting list of patients. Each patient have 2 parameters that influence directly in his/her position in the waiting list.

1. The first and the most important one is the time waiting for surgery. This time is calculated as the difference in days between the actual day and the day that the patient was introduced in the list. The patient with highest number of waiting days, have a score of 10 while the newest patient has a score of 0. The other patients have a proportional score between 10 and 0. This score denoted as S_1 have a weight in the calculation of total score (denoted S_T) of p_1.

2. The second parameter has to do with the priority of the surgeries. Although the DSS schedules non-urgent surgeries, there exist 3 levels of priority 1, 2 and 3 with a corresponding score (S_2) of 0, 5 and 10, respectively. The weight of S_2 in the computation of S_T is p_2.

Assuming $p_1 + p_2 = 1$, the final score that allows to order the waiting list is obtained as follow:

$$S_T = p_1 \cdot S_1 + p_2 \cdot S_2 \tag{21}$$

Finally the patients are ordered according to their total score. The patient who has the highest total score will be the first in the waiting list, while the patient who has the lowest punctuation will be the last one in the waiting list.

7.2 Iterative planning

The manager of each medical team perform the operation scheduling for the next m time blocks ensuring a MCL (this is done by solving MIQCP (16)). Next, the time blocks scheduled are assigned to the available dates for the corresponding team. Then the secretary calls the patients scheduled in the following m days. Once all patients have been called, the secretary give back to the team manager the list of patients that have been confirmed and the ones that cannot be contacted (or they cannot be hospitalized in the following days due to external reasons). In this moment, the team coordinator should schedule again the empty gaps. This process is repeated until the next m time blocks are completely scheduled.

Once the first scheduling has been computed and secretary confirms the attendance or the absence of patients, constraints will be added to the MIQCP (16) and the scheduling will be iterated. This addition of constraints can be seen also as a reduction of number of variables with respect to the initial problem since the new constraints fix the values of some variables. If a patient with preference order j confirms the attendance in time block i, then the following constraint is added:

$$\boldsymbol{S}_i[j] = 1. \tag{22}$$

However, in case that a patient with preference order j cannot be contacted or he/she cannot be hospitalized, then the following m constraints are added:

$$\boldsymbol{S}_i[j] = 0, \forall i = 1, \ldots, m. \tag{23}$$

7.3 Updating and customizing the average durations

The average duration and standard deviation of each type of surgery has been computed using historical data obtained during last two years in the hospital department. During a period of two years, it is possible to obtain a sufficiently high number of surgeries and the durations are representative. However, there exist significant differences between the different surgeons. Moreover, for each surgeon, these average durations are continuously improved because after performing the same surgery several times the surgeon has more experience. Therefore, it is very important to dynamic update also these input values.

After each surgery is performed, the time spent and the surgeon who performed the surgery will be registered in a database. The DSS updates the average durations of the surgeries depending on the surgeon.

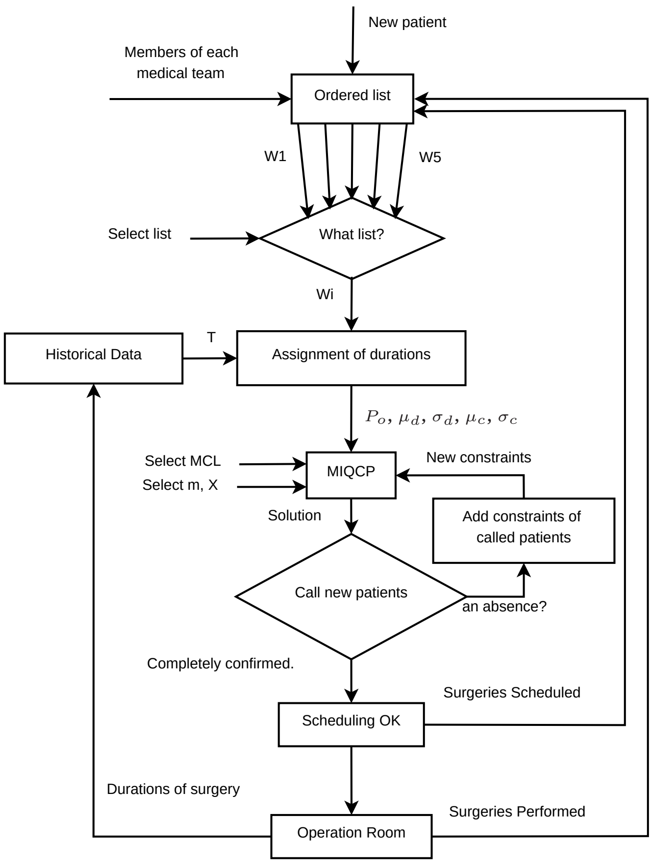

Fig. 3 Flowchart of the DSS for operation planning.

7.4 Overview of the DSS

The flowchart of the DSS is given in Fig. 3 and starts by adding a new patient to the waiting list. Each surgeon has his own waiting list while the waiting list of the medical team is composed by the fusion of the lists of the surgeons that compose the team. Each surgeon is responsible for introducing their patients in the DSS. The method to add a patient belonging to a determinate surgeon is as follows: the DSS recognizes the surgeon (using a personal password) and he/she enters the name of the patient, the pathology and the priority of the surgery. Additionally, the DSS saves the information of the actual date in order to compute the waiting time in the list and the surgeon that have to perform the surgery. Medical teams are not always composed by the same surgeons, so they should be periodically updated. When a team coordinator decides to plan the next m time blocks, he selects in the DSS the waiting list of his team and automatically the tool assigns average theoretical durations and standard deviation to each surgery based on the pathology and on the surgeon. In this way, the vectors $\boldsymbol{\mu}_d$, $\boldsymbol{\sigma}_d$, $\boldsymbol{\mu}_c$ and $\boldsymbol{\sigma}_c$ are generated and the DSS performs an operation scheduling in an iterative way (as is described in 7.2). The input data that the team's manager have to introduce in the DSS to schedule the next time blocks are: i) the corresponding team, ii) the number of time blocks to schedule m and its duration X and iii) the MCL. Depending on the MCL that the manager team impose, the DSS automatically sets the "appropriated" target DST to solve the MIQCP. These "appropriated" values of target DST for each MCL (Fig. 2) are saved in the DSS and their are updated periodically by solving a large instance of N-MIQCP. The states of patients that have been scheduled change from pending to schedule. Once a specific surgery is performed, the surgeon introduces the operating time in the tool. This new input data is used to update the average duration (as is described in 7.3). Additionally, the tool removes the patients that have been operated from the waiting list. If finally a scheduled surgery is not performed, the DSS changes the state of this surgery from scheduled to pending.

8 Conclusions

By modeling and solving MILP (13) it is possible to perform surgical operations scheduling of elective patients with a given *Daily Surgery Time* (DST) of *Operation Room* (OR), respecting as much as possible the order of the patients in the waiting list. However, it has been shown that high DST rates of OR lead to unsafe scheduling from the probability of exceeding the total *Daily Working Time* (DWT). Considering the duration of the surgeries and the duration associated with the cleaning time as random variables with normal pfd and using some statistics concepts, MIQCP (16) has been developed. By solving this model it is possible to perform surgical operations scheduling of elective patients with a given DST but, at the same time, ensuring a *Minimum Confidence Level* (MCL) of not exceeding DWT. Of course, this model also respects as much as possible the order of the patients in the waiting list. MIQCP has two input parameters: target DST (given as a percentage of DWT) and a MCL not exceeding the DWT. These two parameters should be consistent. In order to know the suitable target DST rate for a given MCL, the N-MIQCP problem is developed. N-MIQCP perform surgical scheduling of elective patients maximizing the occupation rate of ORs, but at the same time, ensuring a MCL not exceeding the DWT. For the set of pathologies in the studied department and a DWT of 6.5 hours, several solutions using N-MIQCP have been obtained for different MCL. For each solution, the resulting average occupation rates have been computed and use it in MIQCP to get a smaller computational time (Sec. 5.1). Using MILP (13) scheduling is obtained faster, however there exists a high risk of exceeding the DWT. In order to obtain safer scheduling, we are going to use MIQCP (16). The target DST used in MIQCP (16) is computed by using N-MIQCP (18).

The problems have been tested and compared using realistic data from the Orthopedic Surgery Department of the "Lozano Blesa"Hospital of Zaragoza (LBHZ), Spain. Moreover, a receding horizon strategy to schedule a large number of working days is proposed (Sec. 5.2). It has been observed that using MIQCP (16) (MCL=70% and target DST=78.7%) is possible to obtain an average confidence level of 77.31% and an average DST of 78.28%. These values are considered appropriated from a medical point of view. Moreover, the scheduling obtained using our approach respect the order of the patients in the waiting list, making it the only suitable for the LBHZ. Using our MIQCP approach instead of the *first fit probabilistic* (Π_{FFP}) rule (commonly used) it is possible increase in 109.5 hours per year (see eq. (19)) the use of the ORs only considering the Orthopedic Department. However, the overtime is also increased in 21,05 hours per year (see eq. (20)) but always without compromising the minimum level of confidence allowed.

Currently, we are developing a software tool based on a proposed DSS that includes the MIQCP for the daily scheduling, and the N-MIQCP for update from time to time the approximated target DST for each MCL.

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