UNIVERSITY OF SALERNO

Department of Computer and Electrical Engineering and Mathematics Applied

Master Thesis in

COMPUTER ENGINEERING

MODELING AND ANALYSIS OF HEALTHCARE SYSTEMS BASED ON CLINICAL PATHWAYS

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Introduction

Public healthcare system is managed by governments, who define the purpose and targets of the service together with policies and financial resources. However, because of the complexity of understanding, planning and controlling the system behavior, good short-term and long-term decisions need to be made. This struggle is the same all through the management hierarchy, down to the daily work with patients. This means that the implementation of new legislation is often expensive, can have a long delay and is prone to fail.

For this reason, a method is proposed for the management of health systems, in particular hospital management, based on clinical pathways developed and used by medical personnel in hospitals.

In each department of the hospital, for each disease, treatment or surgery a clinical path is developed that provides recommendations to medical staff (doctors in general) about the different interventions that should follow.

Clinical pathways can also be used to promote effective and efficient health care through the introduction of new procedures or services, decreasing waiting lists for surgical interventions and so on.

When clinical pathways are distributed, resources must be available to perform the specified tasks and activities. The verification of the correctness of the use of resources and the evaluation of performances, however, is impossible to control without the use of models.

Generally, high-level models are preferred, if possible, graphic models. This is important because the analysis must be performed starting from the models created by the doctors and it is difficult for them to describe the clinical pathways using a formal mathematical model, which is however necessary to analyze the system. For this reason, it is necessary a transformation that allows to obtain the formal model starting from the graphic models.

Consequently, a new modeling method is introduced to describe health systems, particularly for hospital management and planning.

The main idea of the approach is to model different clinical pathways existing in a hospital for various medical problems and treatment of various diseases.

These paths can be defined by the interaction between activities (treatments and care), resources (medical personnel, medical equipment, operating rooms, etc.) and the requirements to be met.

The modeling and analysis workflow initiated by a hospital manager is as follows:

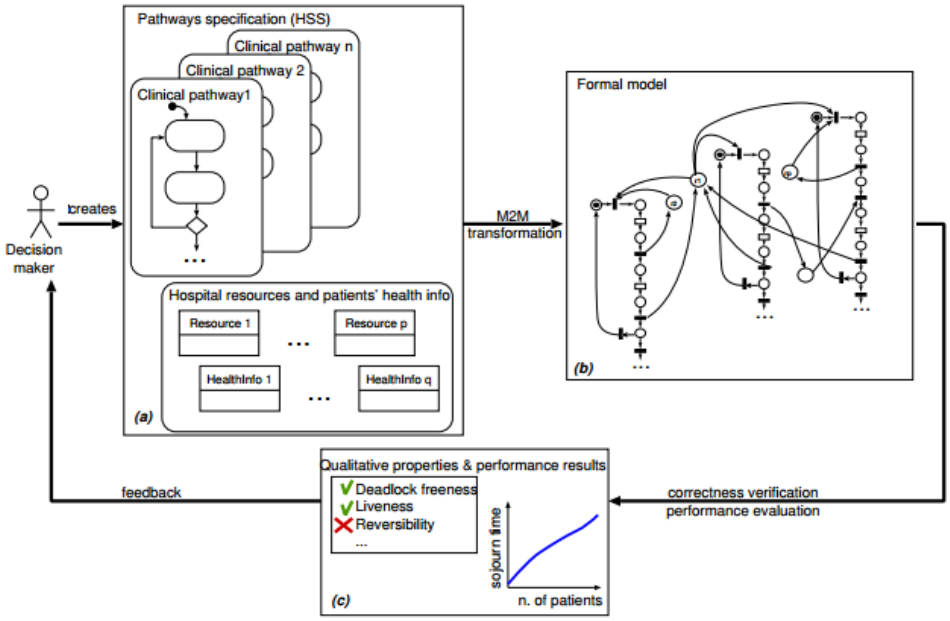

Figure 1 - Proposed methodology

The decision maker plans the care activities, considers the available resources and realizes the specifications of the clinical pathways of the health system (HSS). The goal is to define an intuitive language that does not require too much effort to be learned and conforms to standard modeling languages in order to exploit the available tools.

Of all the standard modeling languages, UML (Unified Modeling Language) is used, which is a standard for software system specifications.

The UML diagrams considered are: activity diagrams and class diagrams.

The former can be used to specify clinical pathways as workflows (one activity diagram for each clinical pathway), the other can be used to define overall hospital resources and patient health information (or medical records).

To support the evaluation of clinical pathways through qualitative and quantitative analyzes it is necessary to add semantics to both diagrams.

By exploiting the extension capabilities of the UML, it is possible to define the HSS as a profile. UML is a semi-formal modeling language, that is, it doesn't have a strict mathematical syntax, it can be used to specify the system but not for analysis or synthesis purposes. For this reason, we propose a set of rules for transforming a UML model enriched with HSS specifications into a formal model (M2M).

The formal model is a colored Petri net, concretely a Stochastic Wellformed Net (SWN) model, which can be used to estimate the performance measures, such as the time taken by the patient to complete the clinical pathway, using techniques based on the event-driven state or simulation, using currently available tools.

There are many software tools for analyzing patterns of colored Petri nets, such as GreatSPN, TimeNET, or CPN Tools.

Once the analysis and the synthesis are finished, the results are shown to the doctors in a form understandable to them, for example through the use of a monitor or directly mapped in the specification of the initial UML-HSS paths, this step is one of future developments to be achieved.

The problem addressed concerned the evaluation of the performance of two clinical pathways using CPN Tools software.
The paths under examination are the hip and the knee, which were represented in the unified modeling language (UML) with specifications (HSS) and transformed into formal models (SWN).
The formal model to be analyzed was obtained by making changes to HEAT, a hospital management software designed and implemented at the University of Zaragoza.

HEAT consists of two components:

- HEAT-Monitor, used to monitor patients admitted on a specific clinical path;
- HEAT-Designer, used to model and analyze clinical pathways in UML language and at the same time allows transformations in Petri net models for a formal analysis in the modeling of clinical paths.

This tool allows to obtain the SWN model compatible with the TimeNET analysis software, which, however, was not very robust in the case of large models.

Therefore, a transformation mechanism has been realized that allows obtaining the formal model in the format corresponding to CPN Tools.

Then, through the transformation made it was possible to analyze the network obtained with CPN Tools and to better evaluate the performance of the clinical pathways and consequently those of the management of the hospital system.

1 Healthcare system

1.1 Healthcare company

Who manages to run well a health company it is able of to manage well any company, but the opposite isn't true.

1.1.1 Company definition

The company is an economic institution that is founded with the finality to satisfy the needs of whom constitutes it and, to such purpose, it develops and manages operations of production of economic goods, that is the commodities and services.

The goals of those who set up a company aren't always and aren't only represented by financial profits; they can also include the help to live better (non-profit companies) and, in the case of public health companies, the satisfaction of the health needs of a population.

The activity of production of goods and services does not represent, therefore, the goal of the company but the mean by which the satisfaction of the corporate purpose is achieved.

With the recognition of the character of companies to health organizations intends:

- to get a greater separation of the powers of address that is assigned to the business organisms;
- to give greater flexibility and autonomy in responding to specific situations;
- to start evaluating productivity, understood as the relationship between costs and revenue;
- to introduce elements of competition between health organizations.

Corporatization has favored the rationalization of health activities, the initiatives of evaluation and improvement of the quality. However, it is necessary to consider some fundamental characteristics of the health system and of the political system which have made the process of corporatization partial and peculiar.

1.1.2 Peculiarities of healthcare companies

The most obvious peculiarities of public health companies compared to private industrial companies are:

- the results that are easily measurable and the revenues are health outcomes (improvement of health conditions or reduction of the pain);
- there is no real identification of the essential levels of assistance;
- the funding criteria are largely set by national and regional directives;
- it isn't possible to separate the responsibilities of address and management;
- the expertise of the region, municipalities, consortia of municipalities and companies are often not clear;
- the competition is strongly regulated;
- the activities carried out in the companies are largely of a professional nature, i.e. they are based on the skills of highly specialized professionals who understandably need greater autonomy. However, this can facilitate the prevailing of personal and group interests and objectives over those of the company;
- the areas of intervention and the types of services are numerous;
- users, except in relatively exceptional cases, can't evaluate the professional quality of the services received and therefore to give their objective feedback feed-back behaviour;
- it is more difficult to assess the effectiveness (of the professionals, of the organizational units and of the company) and it is also more difficult to understand what successes and failures depend on in achieving health outcomes.

It isn't surprising that the subsequent reorganizations of the system that have been introduced have only partially achieved their aims.

1.1.3 Management system

The management or operational system is defined as: the set of rules, procedures and programs that guide the behavior of people in companies.

In the public health sector, the need to introduce modern management tools has only been felt for about a decade. It is only in 1995 that, after some experiments conducted at regional level, the first applications of management control began.

And it was only in 1999 that management control began to have a certain diffusion at a national level, even though often its application was partial, limited to aspects of costs and productivity, and more formal than substantial.

The application of management tools in healthcare companies is more difficult than the other companies, for the peculiarities listed above.

Therefore, new organizational models have been introduced that leave a lot of autonomy to managers who are only asked to maximize the value of the organization.

The efforts made, the few successes and the many failures have led to the conclusion that:

- the size of the accounting control, however fundamental, is insufficient. The evolution of healthcare needs and demand, the innovations in available technologies, the lower standardization of health processes make the healthcare system unmanageable by means of accounting tools alone; often these can't even manage to contain spending. Furthermore, measuring only the costs incurred and the revenues obtained (such as real or virtual turnover of the services produced) ignores the essential problem of appropriateness and quality of services;
- but the professional dimension alone can't suffice if it ignores the efficiency aspects, the financial constraints and the need for economic rationality of the choices.

So, it would take that:

1. central government organisms leave greater management autonomy to health companies, but are at the same time clearer in the formulation of objectives and more attentive to the measurement of results;

2. continue to experiment with management methods to harmonize the accounting dimensions with the professional one and to correct the weaknesses of both;
3. informative tools are created to systematically measure the results produced not only in terms of costs, but also in terms of their appropriateness, health outcomes, user satisfaction and in relation to the organizational changes introduced;
4. in the company it is decided to move from *government* to *governance* and therefore to reconcile the need for a strong action of direction and verification with that of the autonomy of professionals.

We are oriented towards new organizational models that leave a lot of autonomy to the front-line managers who are asked only to maximize the value generated by the organization. This choice is accompanied by the provision of information tools that provide sufficient elements to guide financial incentive, career and to coordinate the various organizational units.

1.2 Management for processes

1.2.1 General information on process management

Process management means that the analysis, evaluation and planning of the business organization are focused on sets of activities that give rise to certain results. A supplier-customer chain is established, which closes with the end customer who receives the final product. In a large process each activity generates an output of products and information that represent the input of a subsequent activity.

According to the process approach, suppliers and customers must be identified in each activity; the head of each phase of the process must clarify the characteristics of the input and keep in mind both the needs of the immediately following customer and those of the final customer.

The process related to the diagnosis and treatment of a health problem in a health organization is rarely treated by a single professional or by professionals of a single

discipline. More professionals of the same discipline contribute, more disciplines, more professional categories, more organizational units and even more organizations. The greater the variety and specialization of contributions, the more interfaces between organizations, the greater the risk of continuity and integration, the process approach becomes more useful.

1.2.2 Advantages and criteria for evaluating management by processes

Process management is a fundamental tool for coordination and guidance towards acceptable results. It can be used to increase the coordination between the activities carried out and the company's goals.

The characteristics of the processes that can be considered are:

- the evaluability, which depends on how much during the process data are generated with which to build activity and result indicators;
- flexibility, that is the ease with which activities and times can be modified to adapt them to the changing needs of users, internal and external;
- constancy or reproducibility, that is the ability to supply a product in the period considered in the absence of causes of extraordinary variations;
- the effectiveness or degree of achievement of the objectives; for the processes represented by the treatment of a pathological condition it will concern the achievement of the best possible health outcomes starting from the existing knowledge and from the available resources;
- the efficiency or relationship between the results obtained and the resources used; in health, we can distinguish an economic efficiency, in which the resources used (costs) are put in relation with the health outcomes, and an operational efficiency, in which the resources used are put in relation with the volume of performance generated.

1.2.3 Process analysis

In the analysis of the process we try to answer these four questions:

- What benefits does the end customer receive from the process? If the benefits are poor or absent, the possibility of eliminating the process can be considered. In the clinical field it is fundamental to ask what the added value for the patient is at the end of a process, ie what the patient "takes home" as a modification of the state of health (for example a decrease in pain or an increase in survival) or a reduction annoyance (for example, decreased access for diagnosis).
- Are there any lack of homogeneity in how to implement the process? We must consider whether there are differences between the organizational units of the same company (if there are several organizational units that perform the same process or the same phase of a process), both between professionals and between different periods within the same organizational unit. It is especially useful to ask if there are occasions when things seem to work much better than usual. If there are, we can propose to describe the sequence of activities well and then generalize it.
- What added value do the individual activities have? Are they indispensable? The first problem to consider for professional services concerns their specific appropriateness, ie their potential effectiveness. We must also ask ourselves if there are other activities, for example other diagnostic tests, which are less expensive with equal effectiveness or for effectiveness only slightly lower.
- Can times be reduced?
- Is information available on what is being done by similar organizations?

An activity adds value if:

- complies with the user's expectations or requests;
- isn’t redundant or superfluous;
- isn’t duplicate;
- can’t be eliminated with the use of new technologies;
- it's cheap.

Regarding the possible reduction of the execution time of the process, it is important to consider if:

- the execution times of the individual activities can be reduced;
- you can reduce the total time of "crossing" the process by moving the moment of execution of some activities or by running them in parallel (at least partially at the same time) instead of in series (one after the other).

1.2.4 Two types of process improvement

Two types of process change can be distinguished:

1. the improvement or incremental redesign, which is based on the analysis of the process in question and is of the bottom up type, i.e. with high involvement of the person performing the process;
2. the innovative redesign or reengineering, which occurs when, either because the situation seems very unsatisfactory or because we have come to know different considerably better practices or because technological innovations have appeared, we decide to replace the old process with a new process radically different. In this case it isn't necessary to analyze the current process and the contribution of the person performing the process is less decisive. Some of the designers of the new process can also be external to the company, even if the involvement of those who then will have to perform the new process is obviously appropriate.

1.3 Clinical pathways

1.3.1 Definition

For professional processes we mean those in which the activities of health professionals are of primary importance.

A process, or better a macroprocess, is the whole management of a disease, from the first contact until the end of the follow-up.

It can be considered a process, or rather a microprocess, a small part of this (for example, as acceptance in the department); this process can in turn be divided into elementary activities or tasks.

Evaluation measures and the improvement of specific performances can be useful, but they are broader, systemic approaches, capable of simultaneously affecting more performances and more dimensions.

We define the clinical pathway as the macroprocess that corresponds to the entire management of a health problem that also includes assistance to the person for self-care, for any disability and psychological and social support.

Clinical pathways can be defined as multidisciplinary and inter-professional plans related to a specific category of patients in a specific context.

Clinical pathways aim to eliminate delays and waste as much as possible, retain unnecessary changes in treatments, ensure continuity and coordination of assistance, minimize patient risks and improve outcomes.

For a good clinical path, it is necessary that:

- there is an inter-professional, multidisciplinary and sometimes multi-agency approach;
- professional recommendations are based as much as possible on scientific evidence;
- there is adaptation and local sharing of the plan;
- the path is divided into phases of defined duration;
- specify the sequence of steps taken by the professionals involved (who must do what and when) in the different phases;
- the implementation of the path is evaluated by means of valid process indicators and possibly also of outcome, so that user involvement is promoted.

Clinical pathways can be considered the tools of coordination and integration to promote continuity of treatment, training of operators and identification of the best ways to use resources.

1.3.2 Brief history of clinical pathways and variability

Clinical pathways have appeared in a relatively recent period, with the affirmation of managed care. Managed care originated in the United States around the 1930s, when oil companies, public companies, the industries of extraction of mineral, etc. they began to have to negotiate pre-defined services for their employees with health organizations.

The first clinical paths were inspired by the technique of critical pathways used in the industrial world to optimize working time.
The critical path is the sequence of activities, from the order of materials to suppliers to the delivery of the product, which provides a minimum duration of the production process.

We realized that the clinical pathways couldn’t only improve efficiency, continuity of care and reduce variability, but also promote the application of scientific knowledge on the effectiveness of interventions and improve the general scenario.
The development of clinical pathways has made a significant contribution to the studies on the variability of health services.

The main factors of variability are:

- the different availability of health resources;
- the influence of "medical schools";
- professional uncertainty about the superiority of an intervention over another.

1.3.3 Advantages of clinical pathways

Clinical pathways have the advantage to favour the continuity of interventions and integration between organizational units, but also sometimes between different organizations. This reduces the problems between patients and health professionals.

Reconstruction and analysis of clinical pathways makes it possible to identify time and reduce waiting times, activities that aren't very useful or those that are too expensive, repetitions and avoidable risks.

The decomposition of the clinical path in phases allows to clarify the clinical criteria and organizational to insert the user in a phase and to transfer it to the next phase.

The following is the complete clinical pathway for a patient with arthrosis of the hip that contains different phases and different organizational units, clinics, departments and rehabilitation institutes.

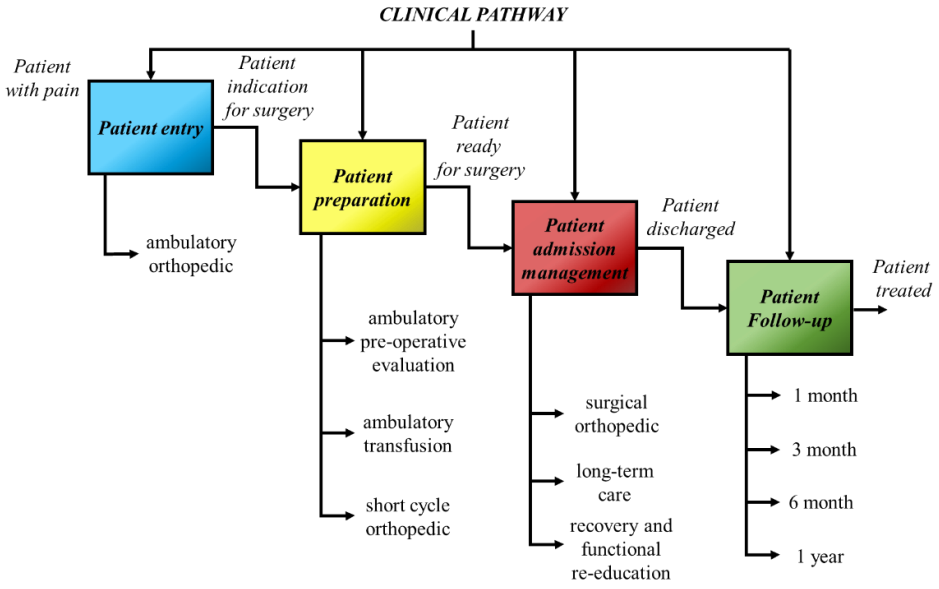

Figure 2 - Clinical Pathway arthrosis of the hip

Compared to other forms of management by processes, clinical pathways have the advantage of giving fundamental importance to the criteria of professional appropriateness of interventions and health outcomes. So, the true product of a health organization is not the performance (the products) but the outcomes.

Clinical pathways can influence clinical practice extensively for three reasons:

1. The most important is that the method of the paths involves the professionals who will have to apply them in their development, which favors their accession.
2. 2. Another reason is the multidisciplinary working groups that include people of all the professional skills involved in the treatment of the condition in question. The exchange of information and ideas among the participants leads to a greater understanding of the roles and responsibilities of each in the clinical process.

3. The third reason is the multi-axial orientation, which simultaneously considers the requirements of appropriateness, effectiveness, efficiency, continuity, timeliness, fairness, integration and user satisfaction.

The clinical pathways also promote the development of information systems for the detection of indicators related not only to activity volumes and costs, but also to professional processes and outcomes.

The advantages of clinical paths for users aren't few, even if they are usually not completely realized:

- Clinical pathways allow healthcare professionals to provide users with the same information about what they need to do, without differences or contradictions.
- It is possible to develop clinical pathways versions for patients that allow them to know in detail the content and timing of the interventions and to be aware of the outcomes that can be expected.
- The patients can be themselves an important source of information of the measure in which the path is applied.

From the application of clinical pathways, we can reasonably expect a significant reduction in unjustified variability in the behavior of health professionals, an increase in productivity and also an improvement in patient safety and outcomes.

Clinical pathways can be considered a tool to improve efficiency in the use of scarce resources without compromising the professional quality of care, which in this way can improve.

1.3.4 Disadvantages of clinical pathways

The main disadvantage is that healthcare professionals consider that clinical pathways can lead to a loss of flexibility and autonomy.

The professional recommendations of a good clinical pathway should be the most confirmed but:

a) they are for patients presenting the condition under examination and not for all. The individual professional isn't only allowed to deviate from the recommendations, but rather must do so, if he thinks that the recommendations aren't the best for the case in exam. He is enough that you declare it and explain its reasons, also to contribute to the updating and the greater articulation of the path;
b) the paths must always be considered provisional, subject to verification and continuous updates, depending on the results of their application and any new knowledge or technological acquisitions.

The reference to a single specific health problem can be considered a limitation of the clinical pathways because many patients have more diseases and disorders; moreover, this type of patients is destined to increase because of the ageing of the population.

1.4 Construction and representation of clinical pathways

1.4.1 Main clinical categories

In describing or building a clinical path it is very useful to consider some categories of activities that are always present.

These categories have been defined as main clinical categories and each could also be considered a process.

They are the following:

- direct clinical-social evaluation;
- laboratory and instrumental checks;
- documentation, including both the medical record and forms and certificates;
- requests for medical advice;

- various treatments and treatments;
- nutrition;
- education and information of the patient and the family;
- discharge planning;
- the activities that the patient must perform;
- patient safety;
- course monitoring.

1.4.2 Temporal phases or sub-processes of a care path

A clinical path can be divided not only into the main clinical categories, but also, depending on the moments and the locations in which the patient is located, in temporal sub-processes, also called care episodes or phases.

The phases are as follows:

- entry into the process or taking charge;
- the initial phase, called that pre-operational phase;
- the intermediate phase, the surgical intervention;
- the final phase, postoperative phase;
- any transfer to another organizational unit or other organization;
- discharge from the hospital phase;
- the follow-up, is also considered in a clinical course, which should be long enough to ascertain the stability of the outcomes and to exclude complications;
- the exit from the clinical pathway.

The same main clinical category can be performed in different phases and can include activities performed in different organizational units during the same phase.

So, it is possible to talk about clinical-organizational phases. For each phase, it is necessary to specify, in addition to the different main categories that are performed, the patient's condition at the beginning and end of a phase from the phase and the ways to evaluate these conditions, so that the clinical path can be judged.

1.4.3 Times

When considering the main clinical categories and the phases of a clinical pathways, it is important to give great importance to the duration of the activities because it is measurable in an easy and objective way.

The times taken from the beginning to the end of the entire route or phase are also called times of crossing the path or phase.

If the times of crossing the different phases of the paths are added, the time of crossing the clinical path is obtained.

The starting point for the different activities must also be considered.
It must be indicated, when the surgery should be performed after admission of the patient, how long it should last, for how many days a drug to be administered, after how many days the discharge should take place.

The durations naturally depend not only on the professional skills, but also on the evolution of the clinical picture and even more on the organizational conditions.

2 Healthcare system modeling

2.1 General modeling

Healthcare systems divided into clinical pathways are defined through well-structured processes by physicians who, using graphic models, determine the series of steps that must be performed for the treatment of a specific disease.

The methodology behind the process of building an integrated clinical pathway is the PDCA model (Plan – Do – Check – Act).

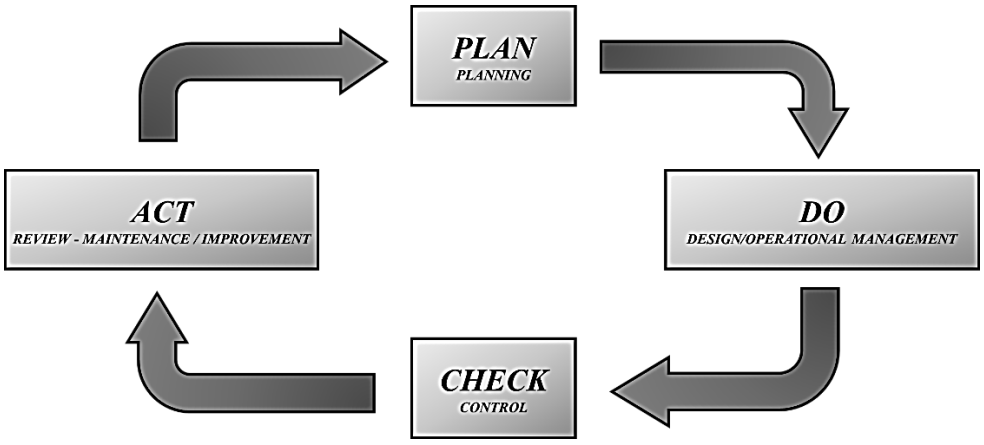

Figure 3 – PLAN: The analysis of the problem to be treated and the planning of the clinical pathway

The PDCA is a model studied for the continuous improvement of the quality that also having a strong impact in the brief period but it has full development in the middle long period.

The choice of the administrative and / or health problems can be made based on:

- a punctual mapping of processes and lines of activity;
- a need identified in the territory and contextualised in the organizational reality;
- of a priority of health;
- a qualifying, excellent activity that distinguishes the company or the service involved;
- the will of a professional, a service, a group of operators who want to experience a path with respect to another because they are considered strategic and / or in need of re-visitation and reorganization.

In the health sector, after making the choice, the collection and analysis phase of the needs begins.

After having carried out the analysis and the definition of the pathway to be treated, it is necessary to design it.

The motivation for choosing the problem is very important for the construction of the pathway.

Therefore, it is necessary to define:

- the best clinical practice (appropriateness);
- the goal to be achieved;
- elimination of unnecessary or redundant actions, with the aim of obtaining the best result (effectiveness);
- temporal-spatial succession of necessary and achievable actions;
- strategies to optimize resources and time (efficiency);
- the recipients of the pathway;
- the expected goals and outcomes.

From the literature and experience of those who have already carried out the process, it can be said that the best tool to develop and represent a path is the flowchart, understood as a sequence of activities and decisions, made in a standard way that make it simple and immediate description, communication and understanding of the process to all the people involved.

It is a graphical tool that allows:

- to identify the components of a structure and the persons responsible for managing each single episode or activity;
- to obtain a global vision of the entire pathway and of the individual activities;
- to schematically define the steps of a procedure;
- to scan the chronology of the process.

At the graphic level, the elements that contribute to the construction of the diagram and then to the description of a business process are represented by geometric shapes as shown in the following figure.

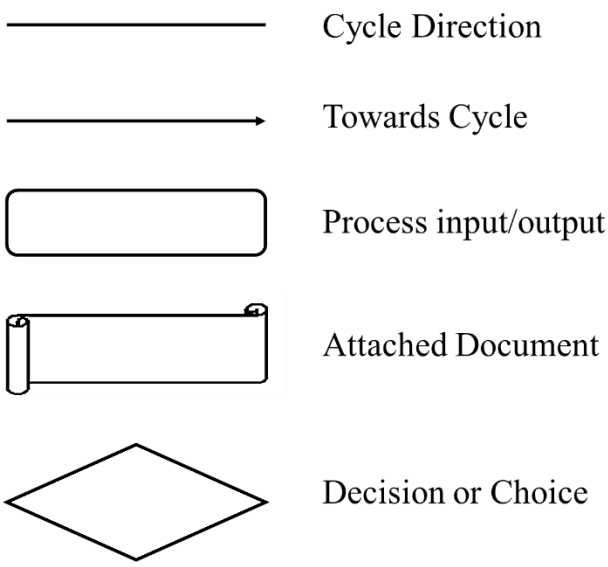

Figure 4 - Flowchart components

In clinical reasoning we define the moments of the path oriented towards the achievement of a goal, of a final result, the operative modalities, the places and times in which a determined action is developed, also highlighting the interconnections between the different structures and professionals involved.

Therefore, analysis techniques are required that require models with well-defined structures. There are several standard modeling languages for this purpose such as: Unified Modeling Language (UML), Business Process Model e Notation (BPMN), Integration DEFinition (IDEF), Extended Enterprise Modeling Language (EEML), etc.

Given that computer systems are defined for management processes, it is convenient to use the activity diagrams and class diagrams that are used in the UML model.

2.2 Modeling using UML

Unified Modeling Language (UML) includes several types of diagrams that allow you to specify a system from different points of view.

Two types of UML diagrams are considered:

- activity diagrams, which can be used to specify clinical pathways as workflows, an activity diagram is used for each clinical pathway;
- class diagrams, which can be used to define overall hospital resources and patient health information or medical records.

Unified Modeling Language (UML) is a generic modeling language for system specifications. The semantics of UML diagrams are expressed in natural language, while their abstract syntax is provided in terms of UML meta-models, which are class diagrams (Class Diagrams - CD) that define modeling constructs.

The Class Diagram are used in this case both in terms of modeling of meta-models and modeling.

The level of meta-modeling was used to define:

- the domain model;
- UML extensions of the HSS profile.

At the modeling level, the CD is used to specify hospital resources and patient health information.

The main elements of a CD model are classes and associations.

A class is the base unit that encapsulates all the information of an object (an object is an instance of a class).

Through the classes we can model the environment in question (a resource, a cure phase, etc.).

A class is graphically represented by a rectangle containing several vertically stacked compartments: the upper part shows the name of the class and the lower part lists the class attributes.

Two classes can be linked to one another with association, aggregation, generalization, or extension relationships.

In UML associations they are bidirectional and are represented graphically as a line between the two classes. Cardinals can be assigned to both sides of the association.

The cardinality of an association indicates the number of instances of a class associated with an instance of another class and the following syntax is used: "1 :: *" for one or many and "*" for zero or many. If cardinality is not shown, it is assumed to be one. In addition to cardinality, association ends may also be characterized by role names and other properties such as the constraints shown in braces.

Aggregation, represented as an empty diamond shape, is a special type of association that is used to specify that a class instance is a collection of instances of another class. For example, the aggregation between clinical pathway and a specific care phase that a clinical pathway instance is a collection of one or more assistance phase instances.

Generalization is a direct binary relationship related to the classification between a more general (super-class) and a more specific class (subclass).

Finally, the extension relationship is used, at a meta-modeling level, to indicate that the properties of a meta-class are extended through a stereotype.

UML includes several diagrams to specify the behaviour of the system.

In this case a UML personalization for the health sector is proposed through the profiles.

The UML profile is a meta-modeling technique for extending UML, since the standard semantics of UML model elements can be refined in a strictly additive way.

Stereotypes and tags are the main extension mechanisms used to define a UML profile.

In particular, a stereotype extends one or more UML meta-classes and can be applied to UML model elements that are instances of extended meta-classes.

Just like classes, a stereotype can have properties that are called tags. When a stereotype is applied to a model element, the value assigned to a stereotype property is called a tag-value.

2.3 HSS: a domain-specific modeling language for clinical pathways

In this section we define a specific domain model for modeling and analyzing clinical pathways, namely the Healthcare System Specification (HSS).

For the HSS definition, we apply a consolidated systematic approach to model-based systems engineering.

The approach consists of two main phases: first a domain model is defined for the domain of interest and then the domain concepts are mapped to the UML extensions, i.e. stereotypes and tags that make up the profile.

2.3.1 The domain model

A domain model is a visual glossary, represented by a class diagram, which includes a set of conceptual classes and their relationships in the domain of interest.

The class diagram presented in the following figure shows the domain model that was constructed with the aim of capturing those concepts in the health domain that are relevant for the analysis of clinical pathways, based on the needs of hospital managers.

The domain model specifies the requirements for the domain-specific language that will be defined as the UML profile in the second step.

In addition, it describes the type of information that must be modeled by hospital managers to allow analysis.

In particular, a health analysis context includes two modeling views: a structural and a behavioral view.

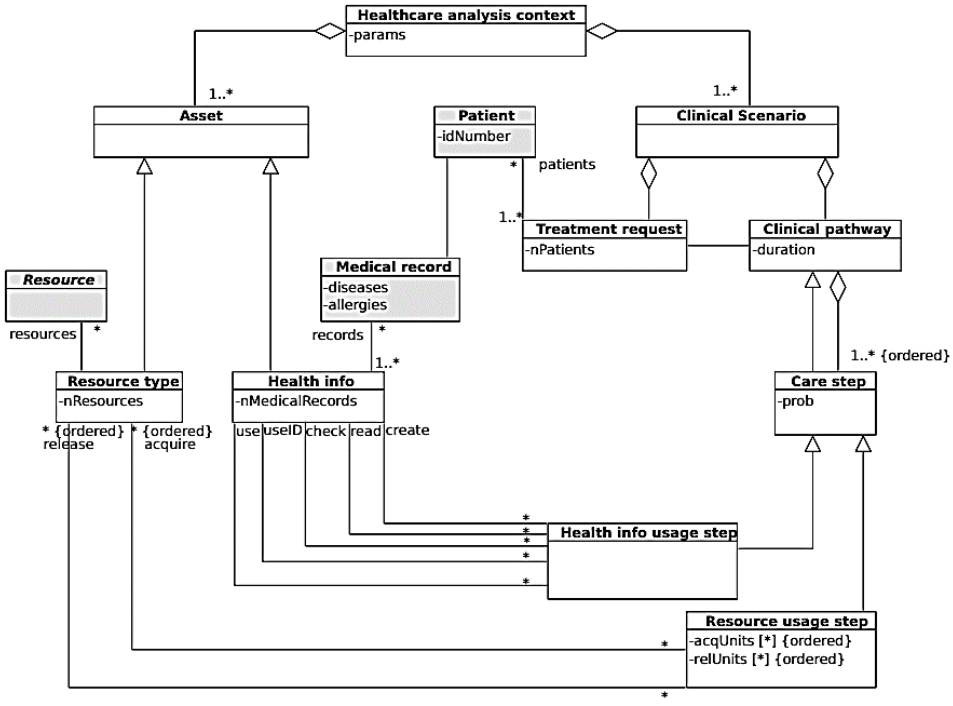

Figure 5 - Domain model for the modeling and analysis of clinical pathways

The structural view (left part of the figure) is concerned with the specification of the hospital's assets, that are the resource types (e.g., beds, operation rooms, personnel) and the patients' health information (e.g., patients included in auto-transfusion programs). The grey classes in the figure, related to the Resource type and Health info concepts respectively, represent fine-grained data about the resources and the patients' medical records (e.g., patients' allergies and suffered diseases). In particular, the Resource class is an abstract concept that can be further refined according to the type of resource.

The behavioral view (right part) is concerned with the specification of one or more clinical scenarios. A clinical scenario includes a clinical pathway to be followed for the treatment of a disease and the treatment requests.

The treatment request concept represents the workload in the healthcare domain, in terms of number of patients ($nPatients$) that need to be treated according to a given clinical pathway.

Data for patients who have requested treatment are represented by the $Patient$ class.

A clinical pathway includes a set of care steps that need to be carried out in a given temporal order. A care step represents a task (e.g., provide rules for antibiotics) to be performed by the personnel in charge of the patient that may require the usage of resources (e.g., the doctor) or patient's health information (e.g., the patients' allergies) to be carried out.

The Resource usage step and Health info usage step are particular types of care steps. The former enables to specify the type and number of hospital resources to be assigned to a (set of) step(s) through the acquire and release concepts (association end names of the associations between the Resource usage step and Resource type classes).

In particular, in a single step it is possible to allocate (acquire) – or deallocate (release) – as many resources for each type as indicated by the $acqUnits$ – or $relUnits$ – attribute, which represent a list of numbers ordered according to the allocated/deallocated resource types. The Health info usage step allows one to specify the type of operation to be performed on a given patients' health information: create (e.g., add the patient in the auto-transfusion program list), read (i.e., read the patient's data in the list), check (i.e., check if the patient is included in the list), useID and use (i.e., use the information of the patient's status either considering or not considering, respectively, her/his identity).

It is worth to observe that, from the analysis perspective the proposed approach is based on the availability of summary information on the number of patient treatment requests ($nPatients$), number of resources by type ($nResources$) and of the patients' health information ($nMedicalRecords$).

Therefore, the user doesn't need to specify explicitly fine-grained data in the structural view (grey classes in the figure).

On the other hand, in order to support the performance analysis of clinical pathways, the domain model includes also timing and stochastic concepts. The duration property represents the time spent to carry out a clinical pathway typically a performance measure to be estimated with the analysis, as well as the time needed to carry out a care step, i.e. a parameter usually provided as input to the analysis.

2.3.2 The HSS profile

The HSS language is defined as an UML profile that includes a set of stereotypes and imports the model library of the MARTE profile – which is a standard – in order to reuse the predefined data-types.

The domain model is mapped to UML extensions by applying a set of patterns, which are aimed at defining a small set of new extensions to facilitate the fast learning of the language to the final users.

First, a stereotype is created for those white classes of the domain model that have explicitly named attributes or association terminations, that is, all the Asset and Clinical Scenario classes in the previous figure.

To define the correct extension for each stereotype, the UML meta-classes were considered whose instances are elements of the UML class diagram or activity model.

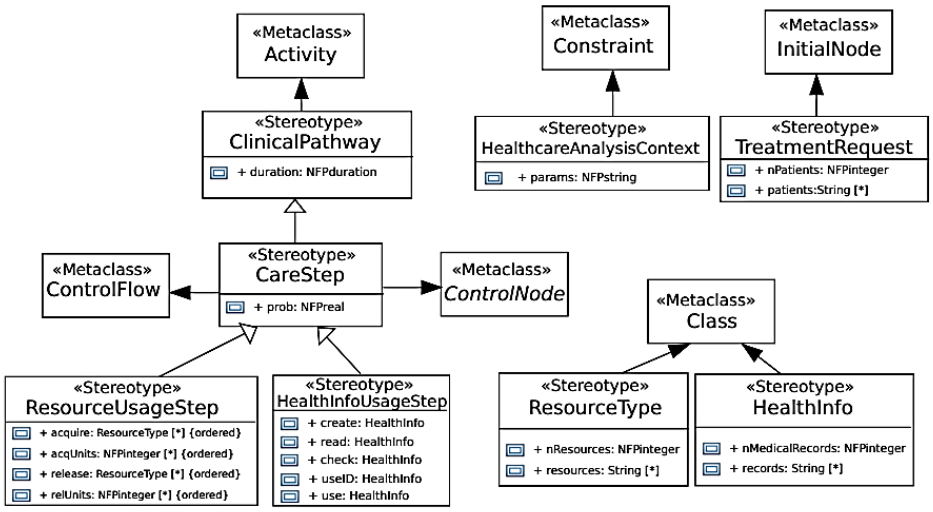

Figure 6 - UML extensions for healthcare modeling and analysis

For example, the $ClinicalGuideline$ stereotype extends the Activity meta-class, so the stereotype can be used in the activity diagram of a clinical pathway. On the other hand, the $ResourceType$ and $HealthInfo$ stereotypes extend the Class meta-class so the stereotypes can be used in the class diagrams.

The generalization relation of the domain model is maintained in the mapping.

Then, the $CareStep$ is a sub-stereotype of $ClinicalGuideline$ and inherits the tags of the stereotype as well as the extended meta-class: a care step can be applied to activity diagrams -by inheritance- activity nodes (i.e. actions) and activity edges (i.e. transitions) within an activity diagram.

The class attributes are mapped to stereotype tags: when defining a tag, a type needs to be associated to it.

Indeed, at user model level, the stereotyped model element will be characterized by tagged-values, where the values should conform to the type assigned to the tag.

We have applied the reference association pattern to the associations of the domain model whose association-ends are explicitly named.

In particular, acquire and release association-ends of the associations between Resource usage step and Resource type classes are mapped to homonyms tags of the $ResourceUsageStep$ stereotype.

The type defined for the tags is the stereotype corresponding to the referenced class, i.e., $ResourceType$. The association-end multiplicity and the order constraint is preserved in the mapping.

The same pattern applies to create, read, check, useID and use association-ends (associations between $HealthInfoUsageStep$ and $HealthInfo$ classes), resulting in the homonyms tags of the HealthInfoUsageStep stereotype.

Finally, the tags resources ($ResourceType$ stereotype), records ($HealthInfo$) and patients ($TreatmentRequest$) trace back to the homonym association ends related to the grey classes in the domain model.

The purpose of these tags is to keep trace of the fine-grained data by referencing external sources, so a general string type is associated to them.

2.4 UML–HSS activity diagrams

The UML profile with HSS specifications described identifies the language to be used and allows to obtain models for a healthcare context to define available resources, health information and a series of clinical paths.

At this point it is necessary to define the UML-HSS model which, starting from the UML-HSS specifications, allows to obtain:

- a UML class diagram that models hospital resources (available resources and health information);
- a set of UML activity diagrams that model clinical pathways.

Clinical pathways are defined as activity diagrams that capture the flow of activity control (sequential execution, alternative paths).

Thus, the UML activity diagram elements that guarantee parallel execution of tasks aren't included. Since the pathways describe the flow of patients and patients can't perform concomitant activities i.e. treatments, but this isn't a limitation of the proposed methodology. The motivation of not using these elements is to provide a modeling language with few primitives to be easy to use by physicians. However, this doesn't mean that the models are sequential, the competition lies in the parallel execution of the clinical pathways corresponding to the different patients competing resources for the hospital.

Therefore, the nodes of the UML-HSS activity diagram are:

- Action node (represented as rounded rectangles), e.g., Evaluate before hospitalization;
- Choice node (represented as a diamond) models decision with alternate paths;
- Merge node (also represented as a diamond) is used to unify the alternate paths;
- Initial node (shown as a solid circle) represents the beginning of a clinical pathway;
- Final node (shown as a solid circle with a hollow circle outside) models the end of a clinical pathway

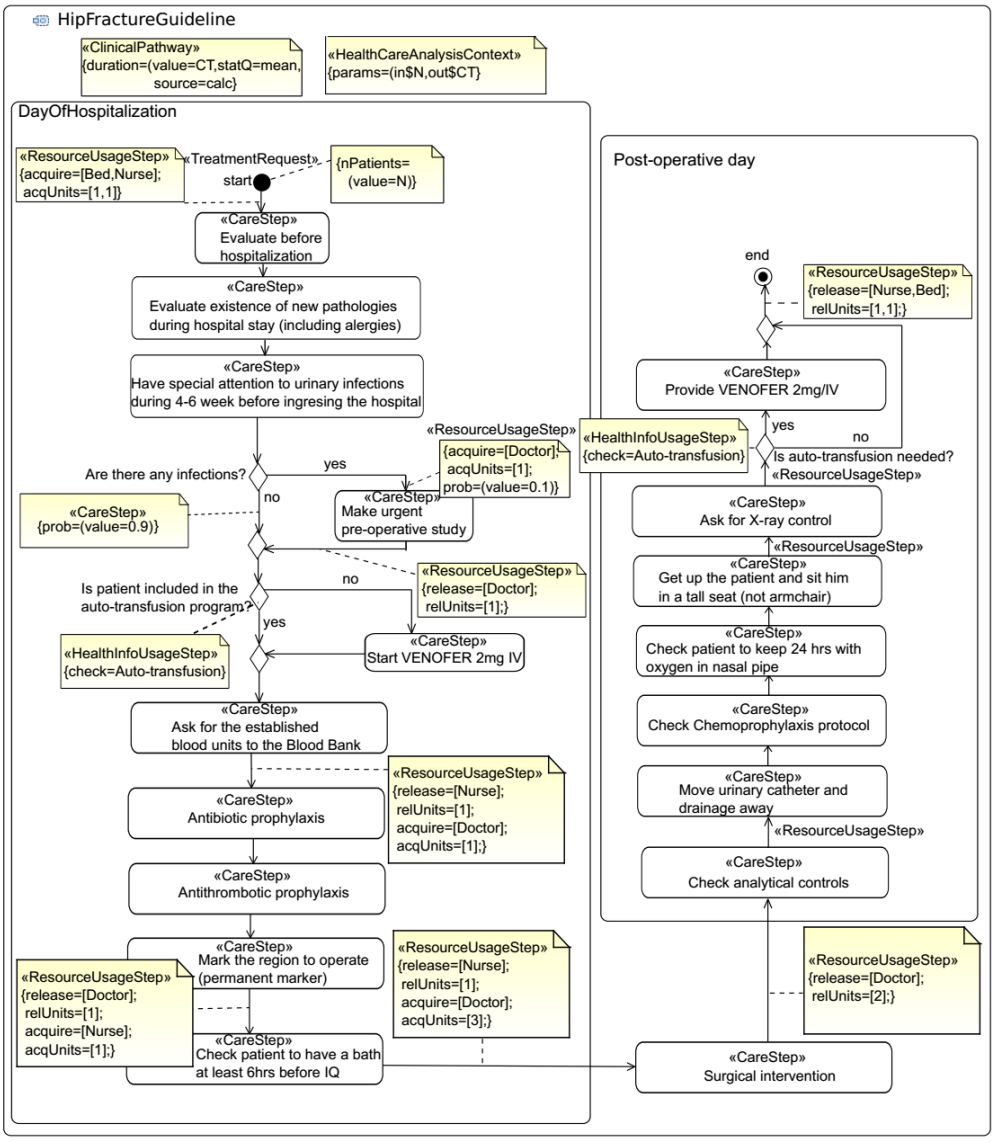

Figure 7 - Activity diagram UML-HSS clinical pathway

3 Model to model transformation

3.1 Method of transformation

The models obtained can't be analyzed directly to evaluate qualitative and quantitative properties using formal techniques.

For this reason, a model-to-model (M2M) transformation technique is required to automatically obtain a Stochastic Well-formed Net (SWN) model.

M2M transformation techniques are at the base of the Model Driven Engineering (MDE) paradigm, in which the target models are generated automatically by the source models applying the transformation rules defined on the source and destination meta-models.

The input parameters of the transformation are:

1. A specification language: the UML and the HSS profile described as specification language, ie the UML class diagram;
2. UML models: built according to the specification language consisting of a UML class diagram that models hospital resources (available resources and health information) and a (set of) UML activity diagrams that model clinical pathways.

The output to be obtained is a formal model that is represented by the SWN useful for performance analysis.

To obtain the desired output from the input parameters, it is necessary to apply some rules that define the transformation from a model to another.

The Stochastic Well-formed Net (SWN) model represents stochastic timed colored Petri Nets and it is used to formally represent clinical pathways.

In particular, Petri Nets are a very powerful and compact tool that allow to better describe systems to discrete events as are the clinical pathways of the health system.

The introduction of colors is necessary to define the different types of people that make up the system (patients, doctors, nurses, assistants, etc.) and to define some additional information (eg autotransfusion programs).

The timing of the networks, associated with the transitions, indicates the time necessary for treatments and treatments present within a clinical path.

Finally, we talk about stochastic networks because the time intervals are distributed through random variables and are added to the model probabilistic time distributions. In the case of the health care system the probability is associated with certain choices or decisions that aren't deterministically defined, such as if the patient has infections, if he is included in the autotransfusion program, if he is allergic to some medicine, if he can be discharged etc.

3.2 SWN: Stochastic Well-formed Net

A Stochastic Well-formed Net (SWN) [16] is a high-level Petri net:

$$N = < P, T, C, D, W^-, W^+, W^h, \Phi, \Pi, \Omega, M_0 >$$

P is the set of places;

T is the set of transitions;

$C = \{C_1, \dots, C_n\}$ is the set of basic color classes.

Basic color classes are finite and disjoint sets, and each class C_i can be partitioned into several static (disjoint) subclasses $C_i = C_i^1 \cup \dots \cup C_i^{Ki}$ when it is necessary to make a distinction among groups of colors of the class.

D is a function that associates a color domain to each place and transition of the net.

Color domains are expressed as Cartesian product of basic color classes (repetition of the same class is allowed): tokens in a place $p \in P$ incorporate information and they be instances of a data structure whose type is the color domain of p.

SWN transitions can be considered as procedures with formal parameters, where the latter range in the transition color domain: the classes in the color domain define the types associated with the transition parameters.

The color domain of $t \in T$ is implicitly defined by the color domains of its input, output and inhibitor places, and the relation between transition and place color

domains is defined through the input, output and inhibitor arc functions W^-, W^+, W^h.

A transition t whose formal parameters have been instantiated to actual values is called transition instance, denoted as $[t;\ c]$, where the assignment c is a color tuple belonging to the transition color domain of t. Only transition instances can fire and their enabling and firing depend on the expression of the arcs connected to the transitions. An arc expression is a sum of weighted tuples of elementary functions defined on the basic color classes.

When the same variable appears in many arc expressions related to the same transition, the different occurrences actually denote the same object. On the other hand, if the same variable is used in several arc expressions, each related to different transitions, there is no relation between the objects represented by the different variable occurrences.

Φ is a function that associates to each transition $t \in T$ a guard expression: guards are used to restrict the set of admissible color instances of a transition to those satisfying a given predicate. A predicate is expressed in terms of standard predicates and it is a boolean expression. By default, $\Phi(t) = true$ is assumed.

Π is the priority function that assigns a priority level to each transition.

Timed transitions are graphically represented by white tick boxes, such as transition task A and they are characterized by zero priority. Priority levels greater than zero are reserved, instead, for immediate transitions, graphically represented as black thin boxes, such as transitions yes and no.

Ω is a function that associates to each timed transition a (mean) firing rate, that is the parameter of the negative exponential probability distribution function characterizing the random firing delay of the transition, and to each immediate transition a weight. Transition weights are used for the probabilistic resolution of conflicts among immediate transitions with the same priority.

Finally, M_0 is the initial marking function that assigns to each place either a multiset over its color domain or a parameter.

3.3 Rules of transformation

The SWN model is obtained through the application of transformation techniques that are composed of a transformation in three steps that are formed by precise rules.

The set of transformation rules is shown in the following figures and the first column provides the identifier of the rule and its description, the second column depicts the elements of the UML–HSS model of the source model and the third column shows the elements of the model SWN of the target model. In the second column the bold-italic font is used to indicate the identifiers of the elements, while in the third column the bold-italic font is used to indicate the SWN positions associated with the labels.

3.3.1 Step 1

This step contains the main transformation rules.

Transformation rules that are divided into two groups depending on the type of diagrams on which they are applied:

- Rules applied to the activity nodes of the AD modeling the clinical pathways. The identifiers associated to the activity nodes are used in the transformation to label the places of the SWN model.

  The SWN models obtained from the application of the rules will be composed via place merging in the second step of the transformation to get the global model. All the SWN places produced by the rules R1-R5 have the set of patients *P* as color domain.
- Rules applied to the classes of the class diagram modeling the available resources and the healthcare information.

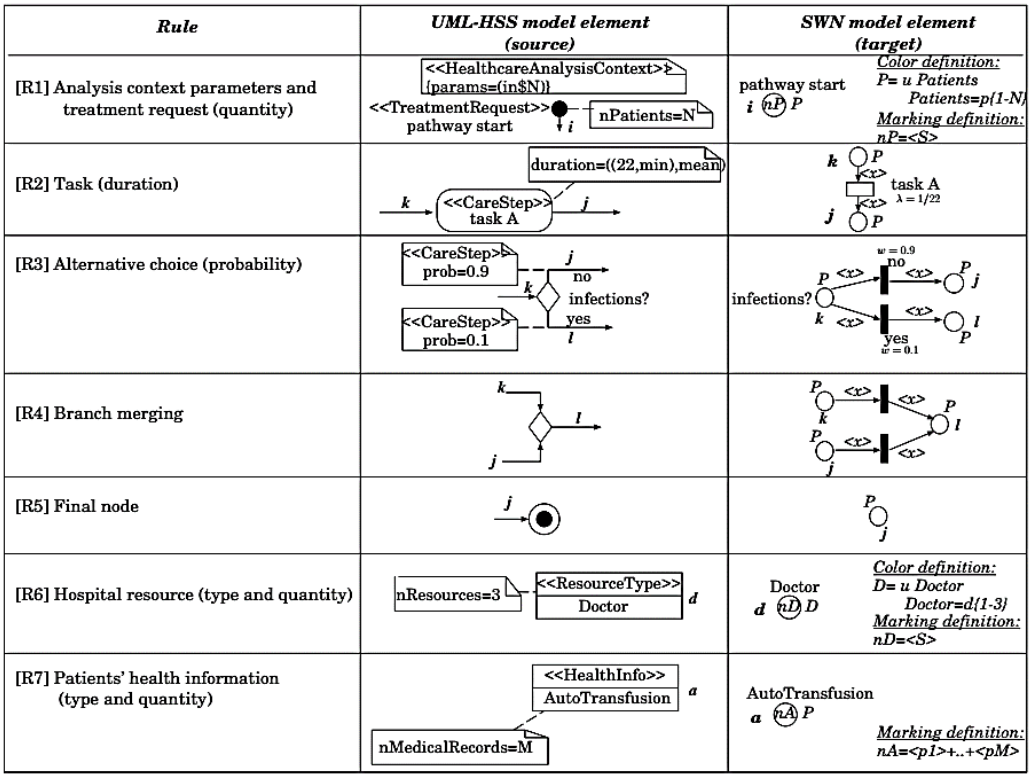

Figure 8 - Application of transformation rules (first step). Rules R1-R5 are applied to the nodes of the activity diagram modeling the clinical pathway, whereas rules R6-R7 are applied to the classes of the class diagram modeling the available resources and healthcare information.

Rule R1 maps the initial node of the activity diagram (stereotyped $TreatmentRequest$ in figure) into a single SWN place with a label i, that is the identifier of the activity edge leaving the initial node. A color domain P and an initial marking nP is also set to the SWN place, according to the $nPatients$ tagged value. In particular, P is the color class consisting of a unique static color subclass Patients, which includes N different colors $p_1 \ldots p_N$ – each color corresponds to a patient identifier- and the initial marking nP corresponds to a set of N tokens, one per each color p_i. Observe that N can be a natural number or a parameter.

Rule R2 transforms an action node task A into a SWN subnet consisting of a timed transition with an input place and an output place. The input and output places are labeled with the identifiers of the task A incoming and outgoing edges, respectively (denoted k and j in figure). Since the task A is a $CareStep$ with an associated mean duration (e.g., 22 minutes), the firing rate of the corresponding SWN transition is set to the inverse of the duration value.

The expression assigned to the input and output arcs of the transition task A corresponds to the projection function $h\,x\,i$, so the firing of the transition removes a colored token from the input place and adds it to the output place.

Rule R3 transforms a choice node to a pair of conflicting SWN immediate transitions. The rule represents a probabilistic choice and the $prob$ tagged–values, associated to the outgoing edges of the choice node, are used to set the weights of the SWN immediate transitions. The label of the SWN places are identical to the labels of the input/output activity edge (k, j and l, respectively).

Rule R4 transforms a merge node to a SWN subnet that unifies alternative flows: in particular, the places of the subnet are labeled with the identifiers of the incoming and outgoing activity edges (k, j and l, respectively).

Rule R5 is applied to final nodes of an activity diagram. Each final node is mapped to a single SWN place labeled with the identifier of the incoming activity edge.

Rule R6 maps a class, stereotyped $ResourceType$, to a single SWN place: the place name is set to the class name (e.g., Doctor) and the place label is set to the class identifier (e.g., d). The $nResources$ tagged–value is used to define the color domain D and the initial marking nD of the place, where D is the color class consisting of a unique static color subclass Doctor which includes 3 different colors $d1$; $d2$ and $d3$ – each color corresponds to a doctor identifierand the initial marking nD corresponds to a set of three colored tokens, one per each color di. For each resource type, a different color class is defined.

Rule R7 is similar to *R6* but applied to the classes stereotyped $HealthInfo$ to map the type of patients' health information. However, in this case, the color domain of the SWN place represents the set of patients, thus it is equal to the color domain P assigned to the SWN places created with the rules *R1–R5*. The initial marking of the place nA is set according to the $nMedicalRecord$ tagged-value. For example, in figure, the initial marking corresponds to the subset of patients included in the auto-transfusion program.

3.3.2 Step 2

Step 2 is related to the composition of the SWN elements from Step 1.

The resulting SWN subnets are composed by merging the places with common labels. The resulted SWN model represents the control flow of the patients according to the clinical pathway modeled by the activity diagram. Additionally, the SWN model includes isolated places (derived from rules *R6* and *R7*) that represent the hospital resources and patients' health information.

3.3.3 Step 3

Step 3 deals with the allocation of resources.

Resource assignment and healthcare information manipulation by rules in the following figure.

The composed SWN model is expanded using the rules to consider resource acquisition/release and patients' healthcare information manipulation/usage (i.e., create, read, check, useID or use).

The rules are applied to all activity edges (*rules R8* to *R12*) or activity decision nodes (*rule R13*), in the activity diagram modeling a clinical pathway, that are stereotyped $ResourceUsageStep$ or $HealthInfoStep$.

Remember that these stereotypes are defined in the HSS profile, and they are applied to the activity diagram to specify the resource utilization and information interchanged.

The identifier of the activity edge, in *rules R8* to *R12*, is denoted by j, and the source node of the edge can be an initial, decision or merge node, whereas the target node of the edge can be a decision, merge or final node.

The graphical representation of these nodes represents all these possibilities.

Finally, *rule R13* is applied only to decision nodes stereotyped $HealthInfoStep$.

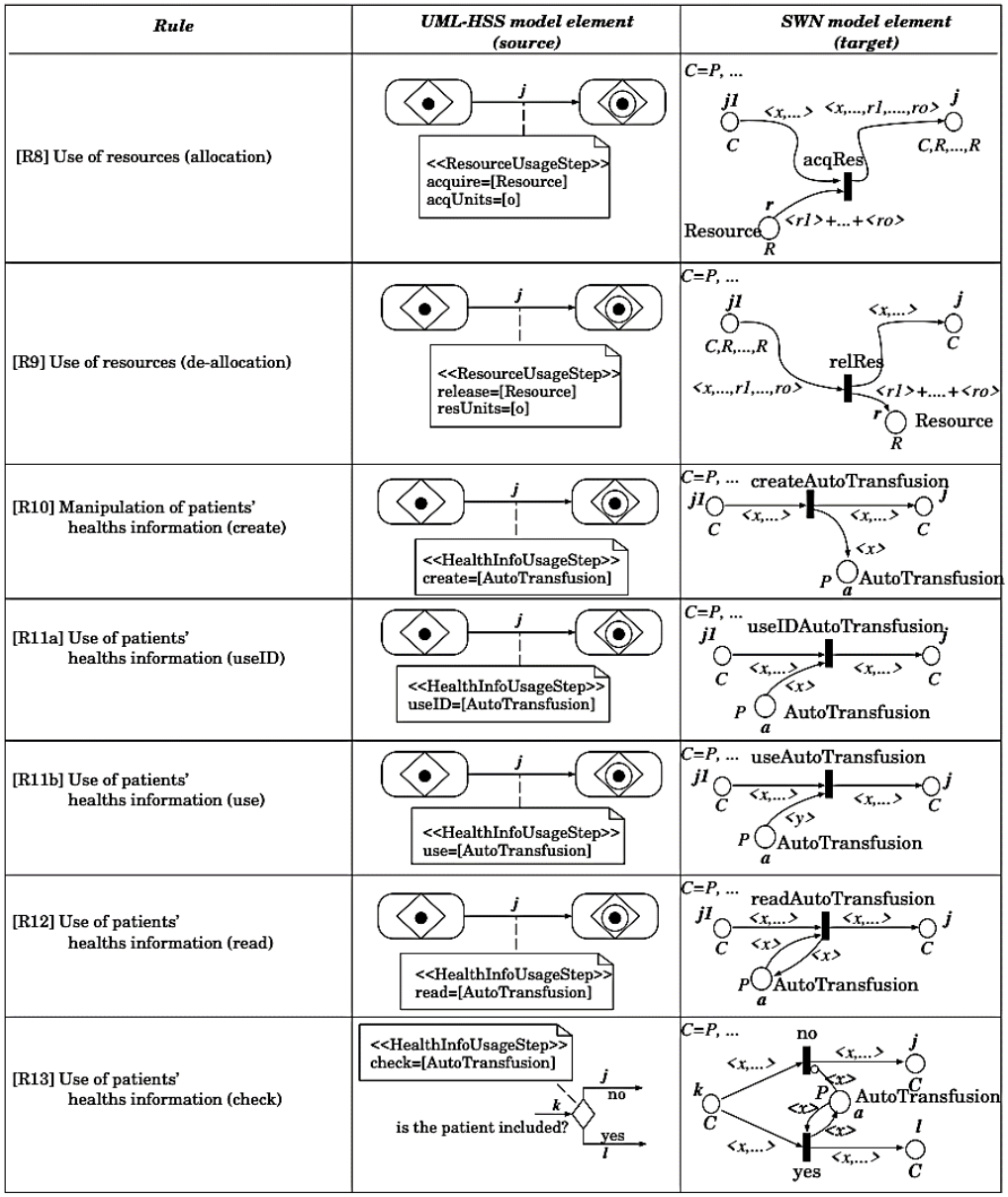

Figure 9 - Application of transformation rules

Rule R8 is related to an activity edge, stereotyped $ResourceUsageStep$, with acquire and $acqUnits$ tagged-values. This rule allows the allocation of a number of o instances of a resource (arc expression $< r1 > + \cdots + < ro >$) and consists in adding before place j in the input SWN model (i.e., the SWN model before the application of this rule) an immediate transition and a place. The immediate transition removes from the corresponding resource place as many tokens as specified by the $acqUnit$s tagged-value, i.e., o.

In figure, place $j1$ and transition $acqRes$ are added for the allocation of resource $Resource$. The place $j1$ inherits the input arc and the color domain of the place j before the application of this rule. We denote such color domain C, a shorthand notation for a Cartesian product of color classes that includes the basic color class P (the set of patients). The color domain of place j and all the places that follow in the path will be replaced with the Cartesian product of the color domain C and the o color domains of place $Resource$.

Rule R9 is the complementary of *rule R8* that considers the resource release ($ResourceUsageStep$ stereotype, release and $relUnits$ tagged-values). Again, *rule R9* modifies the initial definition of the color domain associated to the places of the control flow, in order to keep track of the resources allocated to each patient.

Rule R10 is related to an activity edge stereotyped $HealthInfoUsageStep$ with create tagged-value. This tagged-value is used to create a new medical record. Before the original place j, a new place called $j1$ and an immediate transition are added to the SWN model. The input arc of the original node j will be the input arc of the new node $j1$. Moreover, the immediate transition is connected to the place corresponding to the create tagged-value (an $HealthInfo$ place, see *rule R7*) such that the firing of this transition will produce a token in such place with a color representing the patient in place $j1$. This rule permits for example to create a medical record containing the patients following the clinical pathway of auto-transfusion program.

Rules R11a and *R11b* – usedID and use operations – permit the use of the existent information in the medical records. Both rules require the introduction of a new place j1 and an immediate transition with an input arc coming from the $HealthInfo$ place specified by the useID (or use) tagged-value.

The difference between the rules *R11a* and *R11b* is that, in the former, the identity of the patient is considered while in the latter is not. This is reflected in the expression of the arc between the patients' health information place ($AutoTransfusion$) and the immediate transition: the *rule R11a* defines the synchronization based on the token color (i.e., the other input arc of the transition

includes the same variable x in the arc expression), whereas the *rule R11b* defines the synchronization based on the presence of a token (i.e., a different variable y is used in the arc expression).

Rule R12 is related to a read operation and maps the $HealthInfoUsageStep$ annotation to an immediate transition with a test arc (i.e., input-output arc) to the health information place specified by the read tagged-value. The read operation may appear in the specification of a clinical pathway when it is necessary to check if a patient followed other clinical pathway (in this case the auto-transfusion pathway).

Rule R13 is related to a check operation and specifies a boolean condition (e.g., whether the patient is included in the list of the auto-transfusion program or not). It is mapped to a pair of non-free choice conflicting transitions. Both transitions are connected to the place representing the patients' health information: one transition models the true value of the condition and it is connected with a test arc, the other transition models the false value and it is connected with an inhibitor arc.

3.4 Final representation

The SWN model, automatically generated from the UML-HSS specification of the clinical pathways via M2M transformation, has the capacity to represent the multi-faceted nature of a complex healthcare system. This model represents the perspective based on the deployment and operation of a concurrent set of clinical pathways interacting throughout the competition for shared resources and interchanging information to impose causality relations.

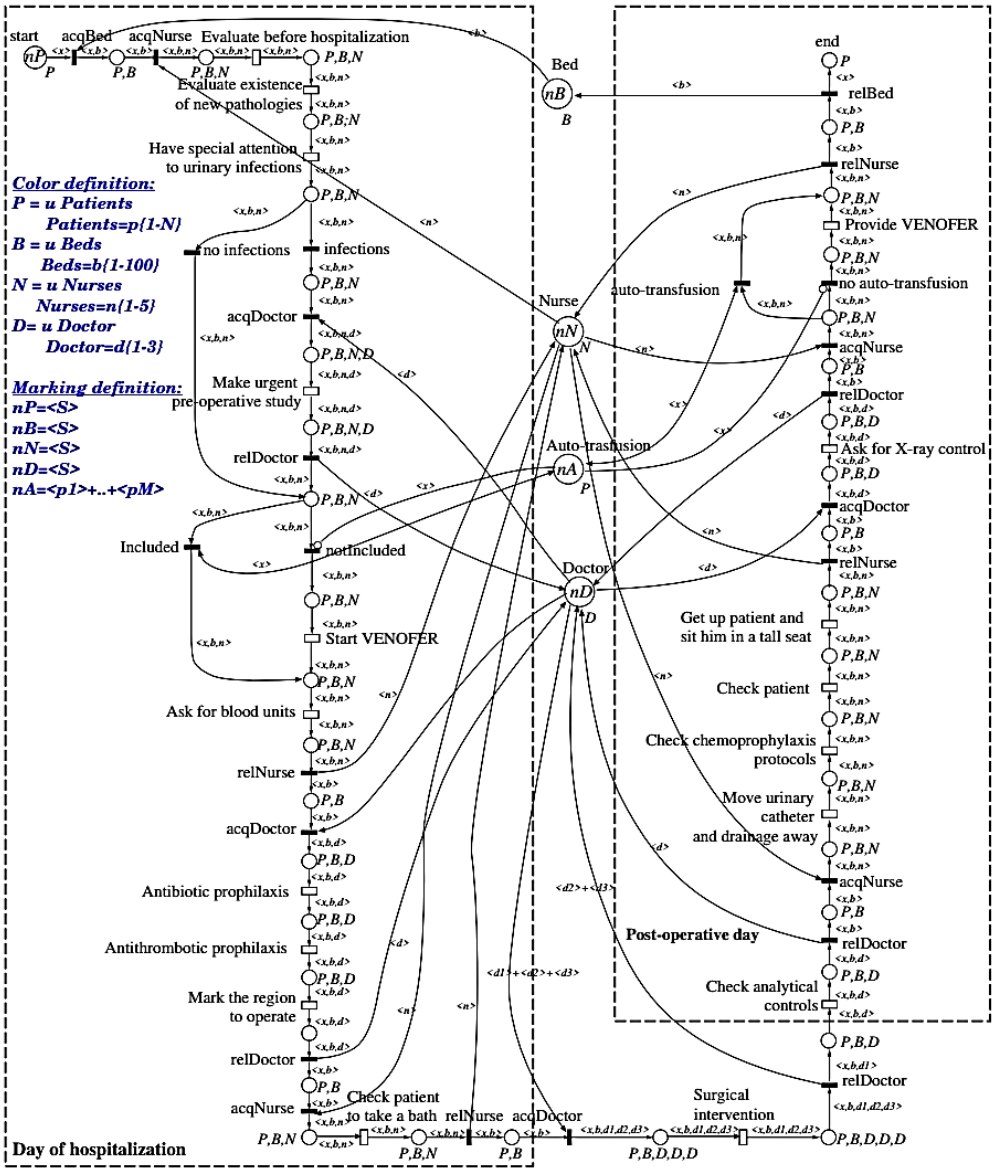

Figure 10 - SWN model of the clinical pathway

4 Implementation and validation of the real healthcare system

4.1 Introduction of HEAT

In order to optimize the health management of clinical pathways by reducing time, costs and improving the use of treatments, it is necessary to introduce new advances in technology.

But, before introducing a new technology, it is very important to have a tool to evaluate its impact before implementation.

HEAT is designed to monitor the efficiency and effectiveness of clinical pathways and to help their development and modification.

HEAT (Healthcare Efficiency and Effectiveness Analysis Tool) is an open source software application developed to support the design and analysis of clinical pathways.

There are other software for the development and analysis of clinical pathways, but as these tools are to be used by staff - doctors and nurses - a simpler model with a friendly graphic language is needed.

There are also other different modeling languages for clinical guidelines and clinical pathways for decision support systems. These models have user friendly interfaces for doctors and can be used to simulate the system in different scenarios, but it isn’t possible to formally analyze the system.

There are other proposed models in which the clinical pathways are modeled by colored Petri Nets. These are formal models that can be used for analysis, but the models are more difficult for doctors to understand.

For this reason, the HEAT software has been developed, which provides a friendly interface for doctors but at the same time supports the transformation into a formal model, which allows us to study the qualitative and quantitative properties of the system, such as the time needed for the hospitalization of a patient, the duration of the resources in use or to understand where are the bottlenecks of the system and how to solve them.

The petri net, obtained with HEAT, can be used to evaluate the performance of clinical scenarios with some external tools such as GreatSPN, TimeNET and CPN Tools.

4.2 Description of HEAT

HEAT is designed to be used by hospital staff and assumes that the user is unaware of the use of new technologies or tools. For this reason, the resulting software has a friendly user interface.

HEAT is a multiplatform software developed with Qt Creator (using the C ++ language) and MySQL.

Qt Creator is a cross-platform framework for developing graphical user interfaces that can be run on various hardware platforms.

MySQL is an open source relational database management system that can also be run on different hardware platforms. It is used for Web applications and can be useful in future updates.

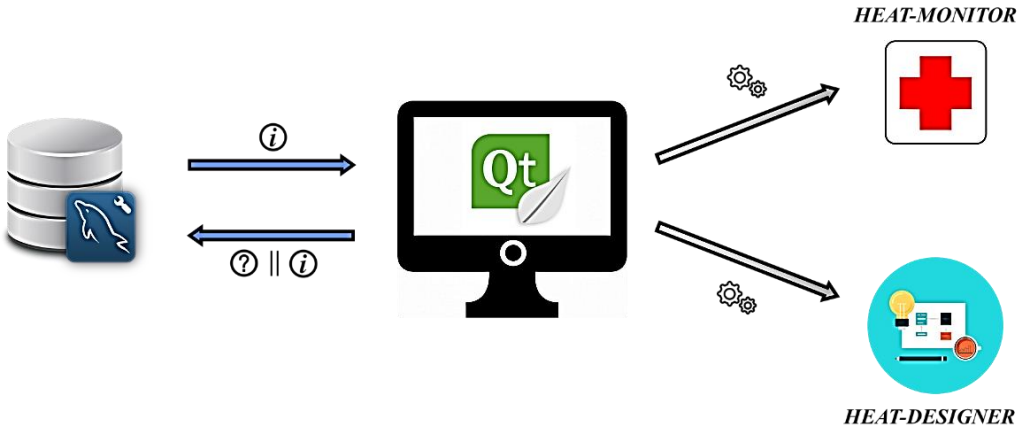

Figure 11 – Flow creation GUI

For HEAT Qt Creator is used for software development and for creating GUI interfaces.

Instead, MySQL is used as a data manager for clinical path information.

HEAT consists of two different tools.

The first, called HEAT - Designer, which can be used to design new clinical scenarios, verify the correct use of resources and verify the correct communication strategies between different clinical pathways.

To model clinical pathways, HEAT uses the specific domain modeling language Healthcare System Specification (HSS), defined as UML (Unified Modeling Language).

This language allows medical personnel to describe the system without any knowledge of UML or other mathematical theory.

Furthermore, the tool can also be used to transform clinical pathways into Stochastic Well-formed Nets (SWN) which represents a formal model that can be used for final analysis through available packages.

The second HEAT tool, called HEAT-Monitor, can be used to monitor the flow of patients following the clinical pathways modeled within the hospital, providing clinicians with useful information on the behavior of clinical pathways. This tool updates the process statistics to check the correct functionality of current pathways and seeks better solutions when needed.

The main purpose of HEAT is the development of clinical scenarios in order to control the correct use of resources, the time required for each resource, to check if the clinical paths are faithfully followed and whether the clinical pathways and the system work properly or if it is necessary to improve them.

All this information will help the decision maker to reduce the costs and time of patient treatment, to avoid unnecessary resources and to plan better programs in order to decrease the queue of patients on the waiting list.

This section is an overview of the main features of HEAT which consists of two HEAT-Designer and HEAT-Monitor programs.

4.2.1 HEAT-Designer

It has been developed to allow the medical doctors to create or update the clinical pathways.

Moreover, it allows the decision maker to administrate the clinical scenarios. A clinical scenario is composed by a set of clinical pathways, hospital's available resources, medical staff and the relevant patients' health information (i.e., medical records) for the clinical pathways.

The Graphical User Interface (GUI) of this tool provides:

(a) a menu bar with three drop-down menus:
- Escenario (Clinical scenario);
- Administrar (Settings);
- Vias clinicas (Clinical pathways);

(b) a tool bar;

(c) properties area;

(d) design area.

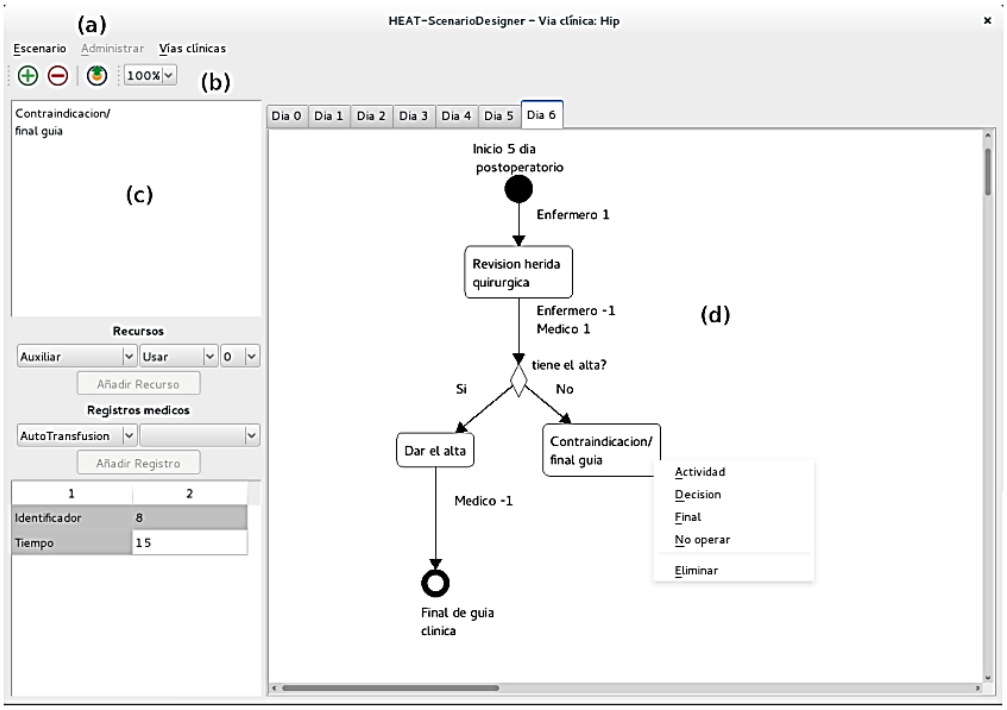

Figure 12 - GUI HEAT-Designer

The *Scenario* menu, allows to select one of the available clinical scenarios, create new scenarios, modify the properties of an existing one or delete them.

Additionally, provides the possibility to transform (by the transformation tool described below) a clinical scenario in order to analyze the system.

Once a clinical Scenario is selected, the second menu called Settings becomes selectable allowing to configure the selected scenario by using four submenus:

(1) Doctors submenu permits to configure the medical staff working in the selected clinical scenario;
(2) Resources submenu allows to add/modify/delete the available resources in the system. Medical records are the relevant patient's information used by the clinical pathways in order to set the steps to follow and make a good decision to each patient (e.g., Is the patient included in the "Auto Transfusion" program?);
(3) Medical Records, submenu that allows you to edit information on medical records;
(4) Clinical pathways, lets the user to select the clinical pathways used for the medication of the different pathologies treated on the selected clinical scenario.

The third menu of the menu bar, also called Clinical pathways, allows to create a new clinical pathway, save the current clinical pathway in the database and open/delete an existing one from the database.

Also allows to import/export the clinical pathways from/in an xml file.

The development of clinical pathways is the main feature of this program.

As defined a clinical pathway is composed by a set of activity diagrams, one for each day of hospitalization. On the other hand, an activity diagram is a set of linked steps to be followed in a specific order helping the doctors on the decision-making for a specific medical condition.

Furthermore, this activity diagrams include the use of resources needed in each step.

The diagrams are developed in the Design area with a friendly interface, where UML-HSS' specifications knowledge is needless.

A new diagram starts with the initial node, and from it, the next diagram's component can be easily added by a pop-up menu. On the top of this area there is a set of tabs to select the day of the pathway to be developed.

On the toolbar we find the buttons + and −, to add and remove respectively the pathway's days and some other useful tools, i.e., zoom of the Design area and export button.

The properties area shows the properties for the selected item of the diagram.
This area, divided into three parts, allows us to introduce the different elements of the HSS.

On the top a text area allows to introduce the description of each activity of the clinical pathway. The area in the middle contains a menu used to allocate and/or release resources and the different actions to perform with the medical records. On bottom (iii), there is a table with all properties of the selected item. Using it, we can modify other features of the diagram, such as task's average timing or the probability of a decision to be made.

Once the clinical scenario is introduced, we can transform it into a formal model through the transformation tool.

Depending on the property to be analyzed we can obtain an SWN containing all the details of the scenario.

In this transformation, each day of a care map is analyzed checking if there exist incongruences in the use of assets: to ensure that no more than the available resources are requested and all of them are released before the end of the day.

If there is no inconsistence, the transformation is performed and the resulting net is exported to different formats to be analyzed by different tools (i.e., GreatSPN's file format and TimeNet's xml format).

4.2.2 HEAT-Monitor

This second tool of HEAT will be used by medical doctors on the daily work.

With this second tool, the doctors can manage the patients and their information (e.g., clinical history number, name, doctor in charge of the patient, patient's health information – Medical Records – and diseases). As well as monitoring the activities carried out for the treatment.

It has a similar GUI as the previous one with some differences.

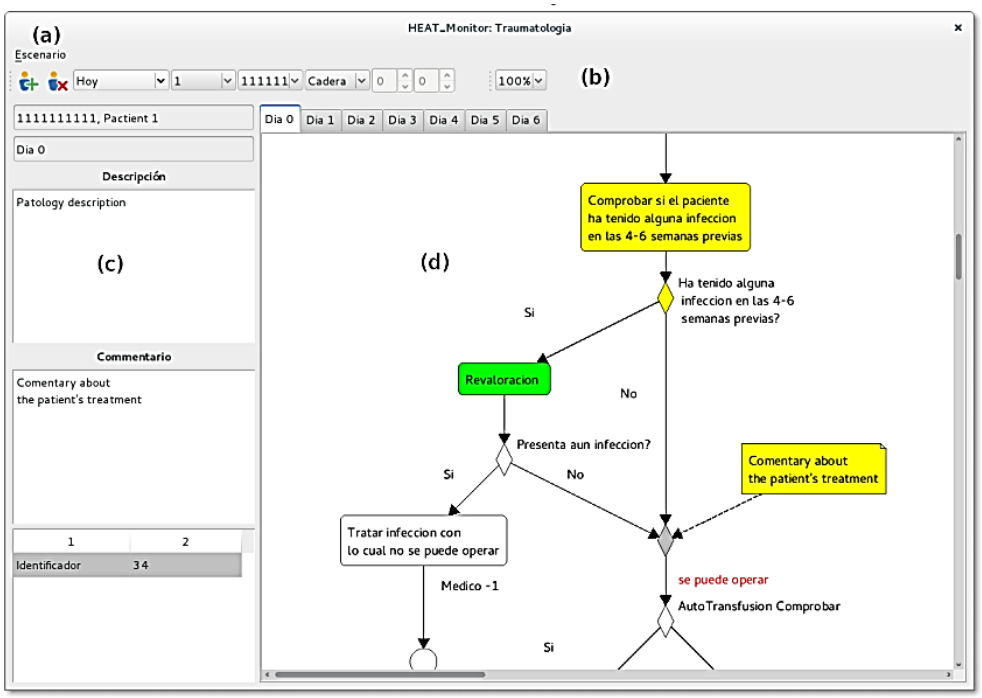

Figure 13 – GUI Heat Monitor

In the menu bar of HEAT-Monitor, there is only the scenario menu used to select a clinical scenario. The tool bar contains the buttons to introduce new patients and delete them as well as the three lists to select the patients.

The first list (a) can be used to choose which patients will be selected:

(1) patients with activities to be carried out during the current day;

(2) all hospitalized patients;

(3) all patients that have been treated.

The second list (b) chooses the section of the clinical scenario to which the patients belong.

The third (c) can be used to select the patient to be monitored and the fourth list (d) is used, in the case that the patient follows different treatments, to display the pathway.

Last two boxes (e) determine a range of age to display the patients.

The main area shows the pathway for the disease of the selected patient. On it, the current activity is shown in green while the already performed activities are shown in yellow.

The result of every decision and the time required on each activity are used to update average duration of the activity.

The properties area of HEAT-Patients Monitor is also divided in three parts.

On the top is shown the information on the current patient and the day of treatment to be followed.

In the middle there are two text areas: the first one shows the description of the patient's pathology and the second allows to introduce some relevant reports or commentaries about the patient treatment.

Comments that will be shown on the diagram as yellow tags linked to the corresponding item. In the bottom, some characteristics of the items are shown.

The usage of HEAT-Patients Monitor is based in the same methodology as the development of a pathway with the previous tool. A pop-up menu allows the user to set the activity as done, while, in case of choice, solve the decision.

4.3 CPN Tools

CPN Tools is a tool for editing, simulating and analyzing colored Petri nets.

The tool offers control of incremental syntax and code generation, which occur during the construction of a network.

A fast simulator efficiently manages unplanned and timed networks. Full and partial state spaces can be generated and analyzed, and a standard status space report contains information.

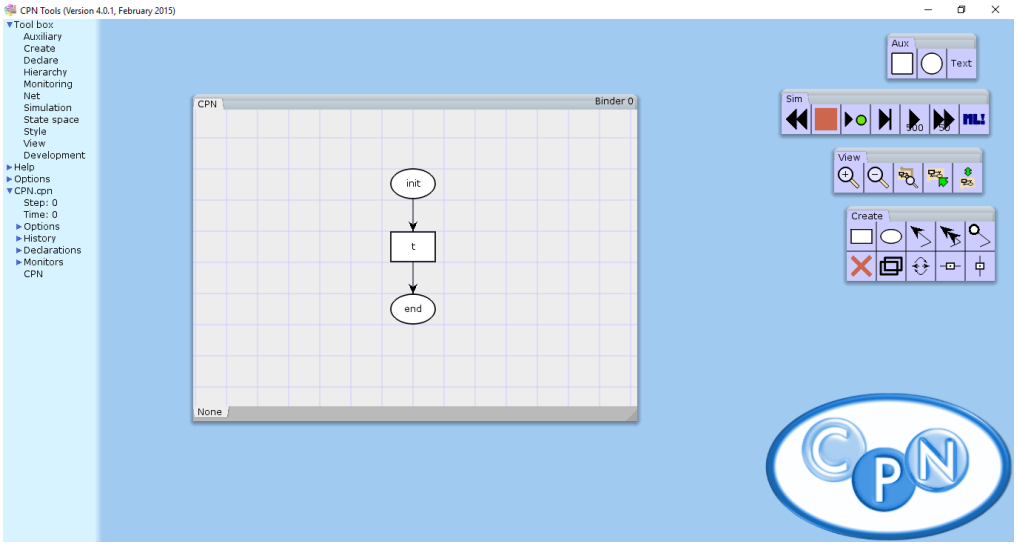

Figure 14 - CPN Tools

This tool allows to simulate large colored Petri nets.

It allows to define modular Petri nets in which it is possible to define high level networks, in which the transitions represent lower level Petri nets.

In this way it is possible to simulate models with complex discrete events that are flexible and easily reusable thanks to the presence of the modules.

CPN Tools allows to define the colors to associate with the Petri net, simples or compounds colors, that is colors that are made up of the combination of several simple colors.

Each color is associated with a place that will then assume its type.

The transitions that are available can be simple, timed or stochastic transitions in which the temporal variable is expressed by simply associating functions related to the defined timing even with a random probability distribution.

As for the arcs, there are three different types: input, output, inhibitor. At each arch it is possible to associate an expression that identifies the value that must be checked.

Finally, we have tokens, that is, the brands of the network, which obviously assume the color and the time defined by the place they are in and the transition through them.

It is also possible to implement other functions, which allow to carry out checks or inhibit certain choices to the detriment of others in the network.

CPN Tools allows you to simply simulate networks and perform parallel simulations of the same network so you can provide average results on multiple simulations.

The results obtained are saved in log files and identify the network performance as the path is reported, ie the places and transitions, carried out to complete the simulation.

4.4 Transformation mechanism for CPN Tools

HEAT, as previously mentioned, provides the transformation of the formal SWN model into a format compatible for GreatSPN and TimeNET.

It is necessary for a better performance evaluation to introduce a transformation mechanism that provides the SWN model in the xml format compatible with CPN Tools.

This is essential, because GreatSPN and TimeNET aren’t very robust tools in evaluating the performance of large colored Petri Nets, as they appear to be those of the clinical pathways of the health system.

Therefore, a preliminary analysis of the format of the xml file compatible with CPN Tools is performed, evaluating which are the main tags to report for obtaining the final network.

After evaluating the static components that represent the software interface, we have gone on to identify the dynamic ones.

The static xml tags are those related to the working environment and to the definition of the main components that allow the realization and the analysis of the networks in CPN Tools.

While, the dynamic tags represent the part related to the modular Petri net that represents the SWN model that needs to be obtained in output.

Therefore, the dynamic tags that were found to be necessary are those relative to the places, transitions and arcs, but also those relating to the definition of the colors and the functions to be realized.

Depending on the attributes associated with the xml tags it is possible to determine the different types of places, transitions and arcs.

In order to define the modules, it was necessary to associate the communication ports provided by CPN Tools to the seats and transitions, which allow to determine the connection points between the high-level Petri net and the lower-level ones (modules) and the transitions that identify the modules.

Therefore, the tags that allowed to determine these components were taken into account in the implementation.

Another aspect that we considered was that of associating the ID to each tag. Each tag has been associated with a unique identifier that allowed to define the membership of the place, the transition and the arc, i.e. if this component belonged to a high-level network (modular) or a low-level network (module). The id relative to high-level networks is set to 1 while the low-level ones to 0, in addition to places the value 0 has been associated, to transitions 1 to arcs 2.

In addition, the clinical pathways are composed of several days, in the case in question we must say that they are one week, then the attributes of each place have been inserted within the ID the value corresponding to the day from 1 (day0) to 7 (day6).

Once the fundamental aspects of the xml tags have been taken into consideration, I have moved on to the implementation of functions that have enabled the use of the model-to-model transformation rules (M2M) to obtain a file containing the timed and stochastic colored Petri nets defined through the xml format compatible with CPN Tools.

This functionality should be added to the GUI graphical interface for HEAT-Designer, by associating it with the Transformaciones button.

The image below shows how to perform the transformation using the HEAT-Designer interface:

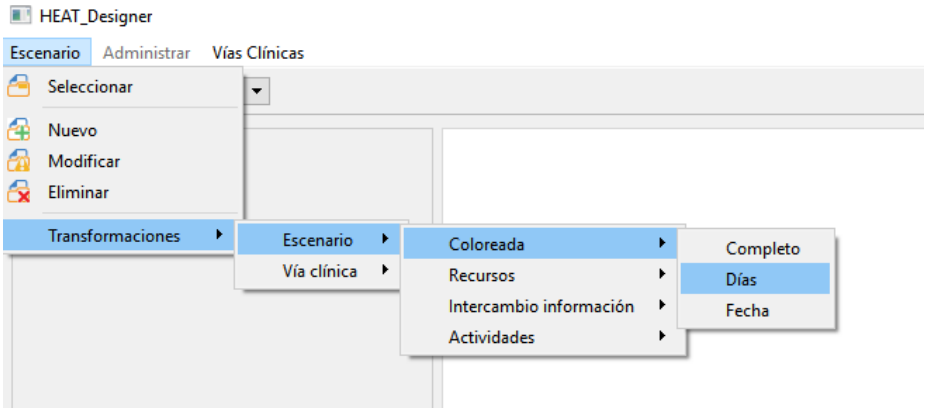

Figure 15 - HEAT-Designer: how to make the SWN transformation

After the changes made, it is possible to perform three types of transformation:

- The transformation respect to the days, in which the Petri nets are created relative to every single day of the clinical way; this can be applied both to a single clinical pathway and to a scenario.
- The transformation respect to a clinical way, in which a modular Petri net is created (high level Petri net) in which each module is related to a day of the clinical path. Each module will contain the corresponding Petri net.
- The transformation respect to a clinical scenario, which contains more clinical pathways, in this case more modular Petri nets are created, depending on the number of different clinical pathways considered in the scenario, which are subdivided by days. Also in this case every day (module) will contain the corresponding Petri net.

CPN Tools allows to define modular timed colored Petri nets.

In the case under examination, each module represents a day of the clinical course. The defined color sets are related to the resources present: patients, doctors, nurses, auxiliaries.

The patient resource is timed because it is necessary to evaluate the time it takes to carry out the treatment and/or treatment during the hospital journey.

In addition to the simple color set, compound color sets have also been defined which represent the product of the various resources, for example in the place relative to the surgical operation the associated color will be given by patient and doctors, given that these will be the entities to carry out the activity.

Regarding the timing of the transitions, the time is represented with an exponential distribution, made available by CPN Tools, in which the input parameter corresponds to the mean.

```
fun expTime (mean: int) =
  let
    val realMean = Real.fromInt mean
    val rv = exponential((1.0/realMean))
  in
    floor (rv+0.5)
  end;
```

Figure 16 - Exponential function CPN Tools

While, the probability associated with choices in Petri nets has been set to 0.5 since this is the value present in the UML-HSS used.

Below in the figure you can see the different processing modes:

- Transformation in days

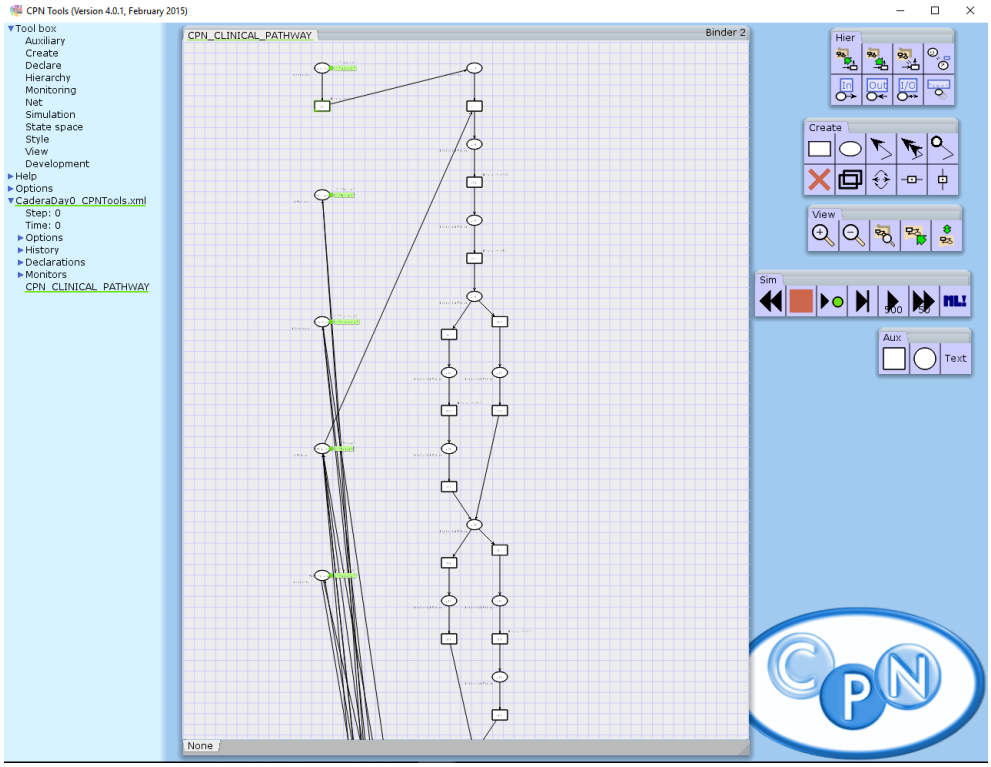

Figure 17 – Transformation day

In this case, a folder is created containing several xml files that contain the Petri nets related to the chosen clinical path.

Loading the file in CPN Tools you get the result presented in the figure, the network presented is that of the first day of the hospital CADERA.

- Clinical pathways transformation

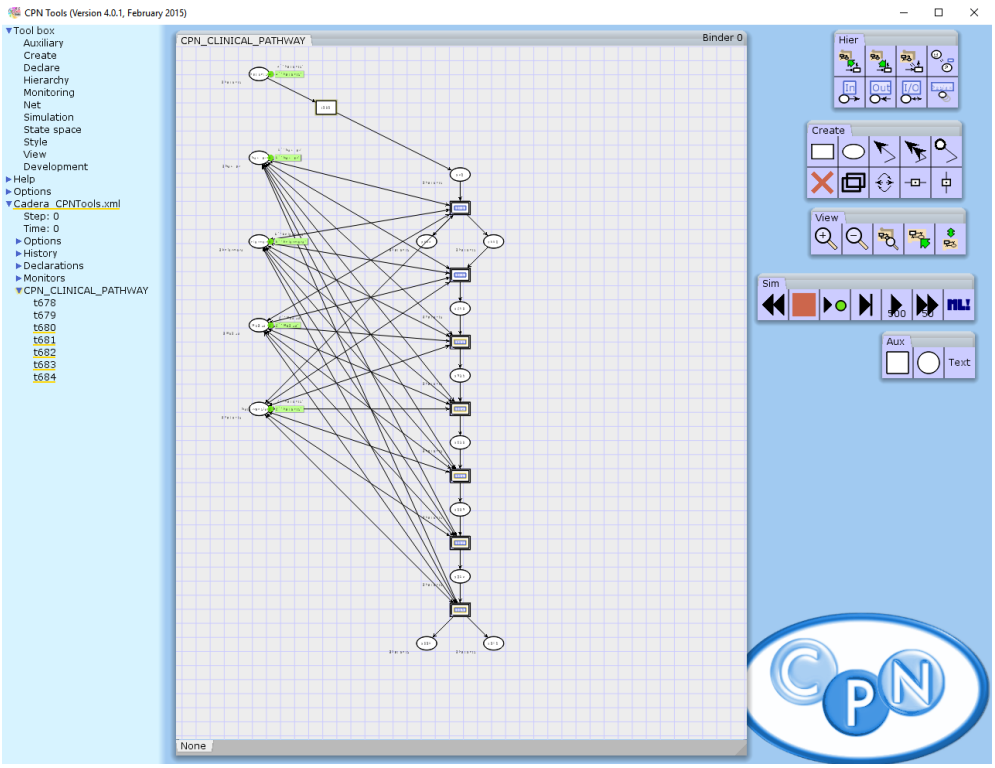

Figure 18 - Clinical pathway transformation

In this case, a folder is created containing an XML file that contains the modular Petri net relative to the chosen clinical path. Loading the file in CPN Tools you get the result presented in the figure, the network presented is that of the hospital CADERA.

- Hospital scenario transformation

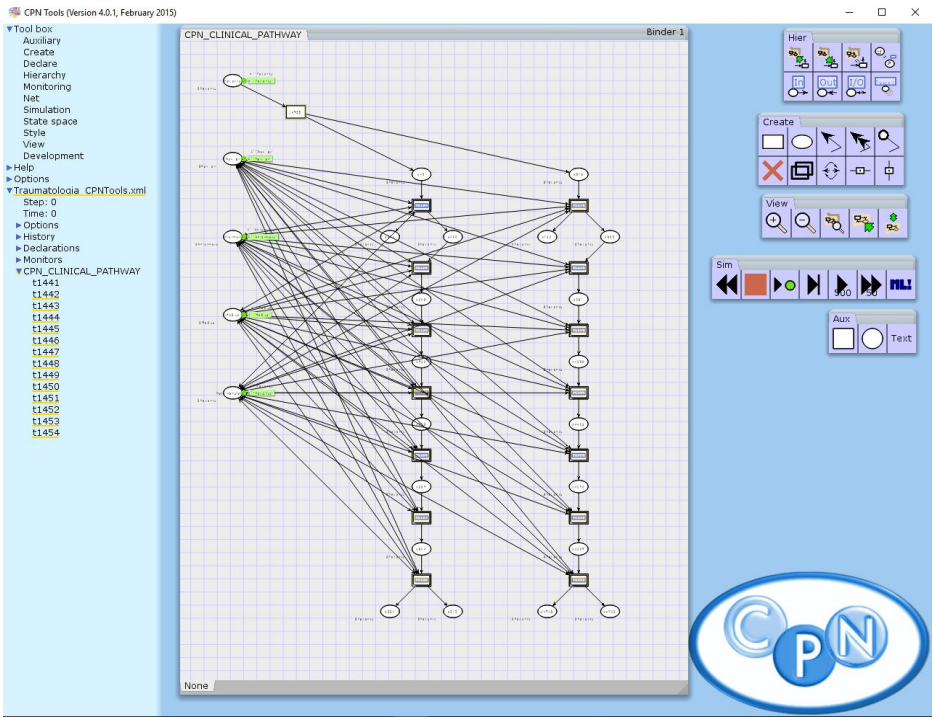

Figure 19 - Hospital scenario transformation

In this case, a folder is created containing an xml file that contains the modular Petri net for the chosen clinical path scenario. Loading the file in CPN Tools you get the result presented in the figure, the network presented is that of the scenario Traumatology that contains the clinical paths CADERA and RODILLA.

5 Performance of the health system

5.1 Performance evaluation

After the transformation of the UML into the Petri net (SWN) through the use of the HEAT software, the analysis of the performance of the clinical pathways has been carried out using CPN Tools.

Moreover, starting from the results obtained in previous works, simulating the networks with TimeNET it is possible to verify the goodness of the transformation carried out in CPN Tools.

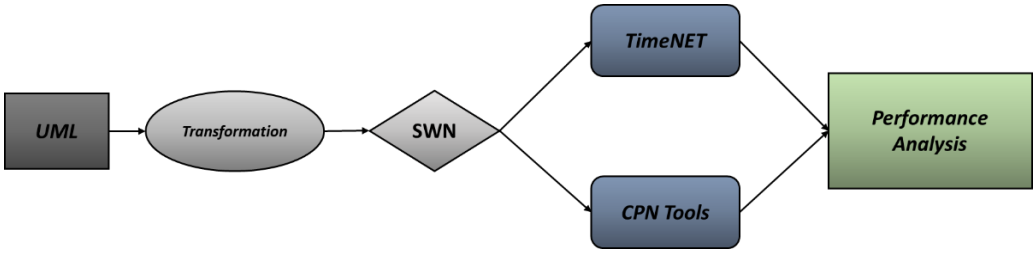

Figure 20 - Complete scheme

The expected results will not be the same since the simulated Petri net in TimeNET represents an approximation of the real model while the network obtained from the transformation in CPN Tools faithfully represents the real model of the system.

In fact, the transformation carried out in CPN Tools also considers information related to the patient's health (Autotransfusion).

The tests performed are related to the calculation of the cycle time of the clinical path, ie the time taken by the patient to complete the hospital journey.

In particular, the cycle time of the clinical pathways was obtained by varying the number of available resources and the number of patients.

The clinical pathways are performed in parallel and each clinical path, composed of 7 clinical days, is performed in series (in chronological order it starts with day0 and ends day6).

A schematic representation is as follows:

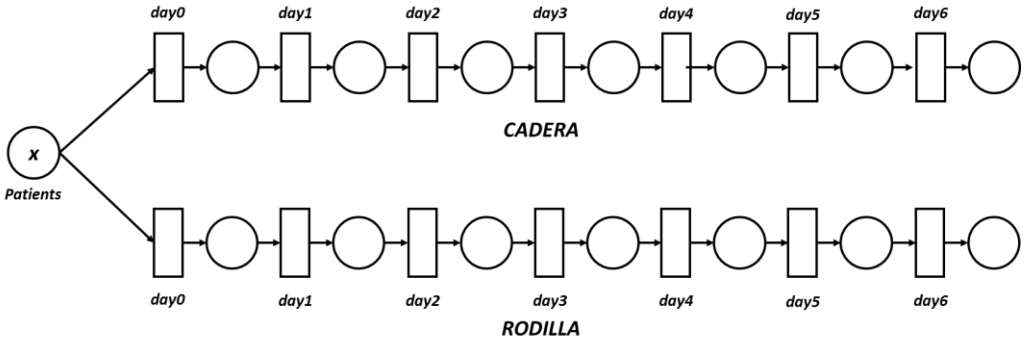

Figure 21 - Schematic representation of clinical scenario

Initially, the clinical pathways are individually tested and then passed to parallel simulations that allowed to test a clinical scenario of the hospital system.

Once the simulation was performed in CPN Tools, the results obtained were analysed using a C ++ script.

The input values are:

- #Patients;
- #Doctors;
- #Nurses;
- #Auxiliaries;
- #Autotransfusion.

The values in output are:

- cycle timeT_c;
- average system throughput X;
- average number of patients terminating the clinical course $\#token_{AVG}$;
- probability that the tokens reach the desired place $\mathbb{P}$.

5.1.1 Cycle time

The *cycle time* T_c is the time taken by a system to process resources (patients) in an observation period T (*Lead Time*).

The *Lead Time* is set at $T = 480\,\text{min}\ \ (8\,ore)$ and is chosen in according to the simulations carried out in TimeNET (obtained from the HEAT-Designer analysis report).

Therefore, the *cycle time* can be calculated with the following formula:

$$T_c = \frac{T}{WIP}$$

The Work In Progress (*WIP*) identifies the number of resources processed in the observation period T:

$$WIP = X \cdot T$$

where X is the system throughput.

The *throughput* of a system represents the number of resources processed in the unit of time, which in terms of Petri nets is represented by the number of tokens that are processed by the network respect to the time taken to process these tokens:

Il *throughput* di un sistema rappresenta il numero di risorse processate nell'unità di tempo, che in termini di reti di Petri è rappresentato del numero di token che vengono processate dalla rete rispetto al tempo impiegato affinché tali token vengano processati:

$$X = \frac{\#Token_{endPlace}}{\max(Time_{endPlace})}$$

where:

- $\#Token_{endPlace}$ is the number of tokens that are in the terminal place of the network;
- $\max(Time_{endPlace})$ is the maximum time, the maximum time is considered because it identifies the useful time to process all the tokens $\#Token_{endPlace}$.

In this case, the *throughput* identifies the number of patients who terminate the clinical pathway at a given time.

Considering these aspects, the *cycle time* is:

$$T_c = \frac{T}{WIP} = \frac{T}{X \cdot T} = \frac{1}{X}$$

To simulate and evaluate the networks, different simulations were considered.

Since the aspects considered are to be evaluated during the observation period, it is necessary to respect this relationship:

$$T_c < T$$

Furthermore, the simulations considered are those in which the relationship is valid and which allow to study the case, i.e. those in which the patient's clinical path has been successfully completed.

In fact, in the modular networks in CPN Tools there are some terminal places (in day0 and in day6) in which there is the cancellation of the operation to potential problems of the patient. Therefore, the simulations that aren't included in the test cases are appropriately discarded.

In conclusion, the *cycle time* formula for different simulations ($\#Sim$) is:

$$T_c = \frac{1}{\#Sim} \sum_{i=1}^{\#Sim} \frac{1}{X_i}$$

5.1.2 Decision function

For the purposes of the tests, a further modification was made to the network.

In fact, to be able to test the network in the desired conditions it is necessary that the number of tokens reaching the final place is almost the total number.

For this reason, a decision function has been added to the Petri net, which allows to choose the branch that leads to the final place of the clinical path.

This function was added only if the choice could lead to places related to the cancellation of the operation (day0) or if the patient could not be discharged (dia6).

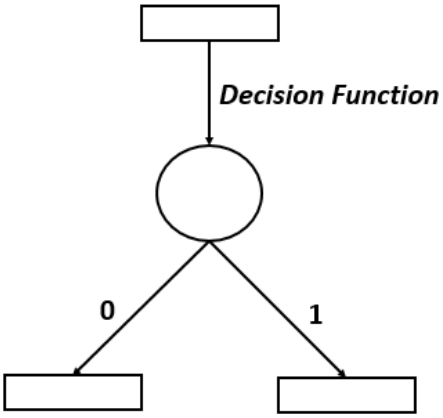

Figure 22 - Decision function application

This result is obtained using the function made available by CPN Tools in which given a number between 1 and 100 (which can be seen in probabilistic terms) we define a threshold that determines the choice.

In the case in question the threshold is set at 90, this means that about 90% of the token among those considered will end the clinical course.

5.2 Evaluation of single clinical path performance

The first test case considers a single clinical course.

The SWN model obtained by HEAT-Designer has been simulated in CPN Tools and the network obtained is as follows:

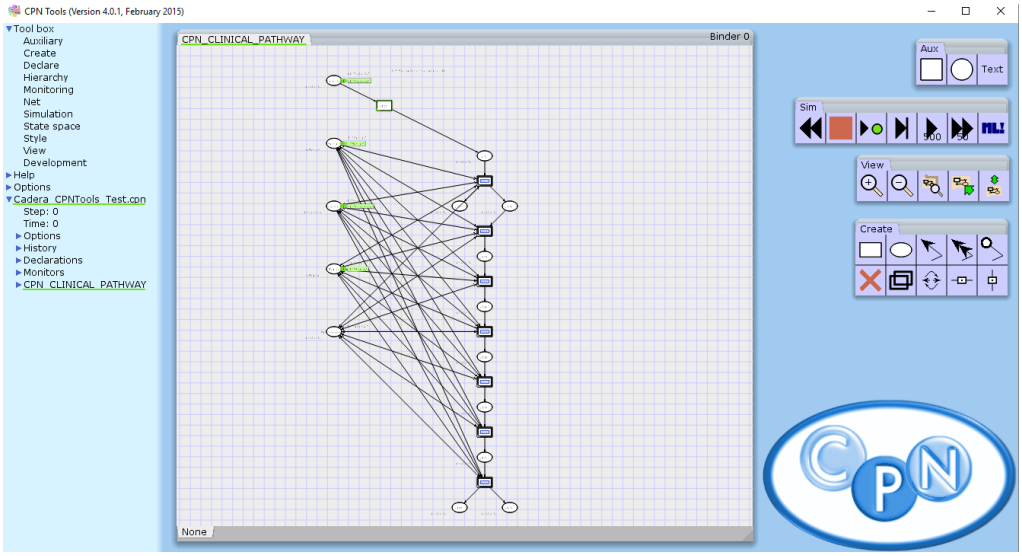

Figure 23 - SWN model of a clinical path in CPN Tools

In the case contemplated are considered $\#Sim = 10$, $\#Sim = 20$, $\#Sim = 50$ simulations.

In particular, 100 simulations of the network were carried out in CPN Tools and some simulations were considered for the analysis.

For comparison with the results in TimeNET, initially no patient was considered in the Autotransfusion program.

The input parameters corresponding to the first 4 columns are:

- Aut – Autotransfusion
- A – Auxiliaries
- E – Nurses
- M – Doctors
- P – Patients

The output parameter is the cycle time T_c.

The results obtained from the tests are the following:

$$\#Sim = 10$$

Aut	A	E	M	P	T_c	X	$\#token_{AVG}$	$\mathbb{P}$
0	1	1	3	1	309.7	0.0033474	1	1
0	1	1	3	5	227.44	0.00497541	5	1
0	1	1	3	10	185.859	0.00574204	9.5	0.95
0	1	1	3	15	196.474	0.00542392	14.1	0.94
0	1	1	3	20	177.008	0.00574455	19.4	0.946667
0	1	1	3	25	179.061	0.00567293	23.3	0.965
0	1	1	3	30	185.682	0.00543875	28.8	0.912
0	1	1	5	1	365.9	0.00289151	1	1
0	1	1	5	5	183.18	0.00564783	4.6	0.92
0	1	1	5	10	163.457	0.00656445	9.3	0.93
0	1	1	5	15	158.093	0.00669841	13.9	0.926667
0	1	1	5	20	144.195	0.0069946	18.9	0.945
0	1	1	5	25	140.339	0.00741586	23.8	0.952
0	1	1	5	30	152.385	0.00673757	27.1	0.903333
0	1	1	10	1	337.7	0.00310919	1	1
0	1	1	10	5	138.995	0.0073338	4.7	0.94
0	1	1	10	10	110.123	0.00924085	9.5	0.95
0	1	1	10	15	107.224	0.00939454	13.7	0.913333
0	1	1	10	20	101.366	0.00991287	18.5	0.925
0	1	1	10	25	97.4125	0.0103714	23.1	0.924
0	1	1	10	30	95.1883	0.010537	27.4	0.913333
0	1	1	15	1	350.1	0.00290893	1	1
0	1	1	15	5	141.59	0.00717934	4.9	0.98
0	1	1	15	10	114.887	0.00881906	9.4	0.94
0	1	1	15	15	101.768	0.00988281	13.8	0.92
0	1	1	15	20	100.133	0.0100202	18.6	0.93
0	1	1	15	25	99.0453	0.0102118	22.9	0.916
0	1	1	15	30	94.6862	0.0106236	28.2	0.94
0	1	2	3	1	336.4	0.00301978	1	1
0	1	2	3	5	235.895	0.00465231	4.7	0.94
0	1	2	3	10	219.669	0.00484751	9.1	0.91
0	1	2	3	15	190.351	0.00535086	13.9	0.926667
0	1	2	3	20	187.25	0.00548236	18.9	0.945
0	1	2	3	25	189.417	0.00533734	23.7	0.948
0	1	2	3	30	207.206	0.00495801	27.7	0.923333
0	1	2	5	1	332.7	0.00316406	1	1
0	1	2	5	5	180.098	0.00628774	9.4	0.94
0	1	2	5	10	179.612	0.00578867	4.6	0.92
0	1	2	5	15	149.382	0.00680594	14	0.933333

Aut	A	E	M	P	T_c	X	$\#token_{AVG}$	$\mathbb{P}$
0	1	2	5	20	138.827	0.00732338	18.7	0.935
0	1	2	5	25	152.494	0.00674402	23.6	0.944
0	1	2	5	30	162.304	0.00636555	28.2	0.94
0	1	2	10	1	350.1	0.00290893	1	1
0	1	2	10	5	139.69	0.00765366	4.2	0.84
0	1	2	10	10	97.6611	0.0105409	9.2	0.92
0	1	2	10	15	87.9402	0.0115541	13.5	0.9
0	1	2	10	20	78.9755	0.0128477	19.2	0.96
0	1	2	10	25	79.3128	0.0126849	23.3	0.932
0	1	2	10	30	80.2642	0.0126264	28.3	0.943333
0	1	2	15	1	370.4	0.00277922	1	1
0	1	2	15	5	129.955	0.00801772	4.7	0.94
0	1	2	15	10	94.3736	0.0106665	9	0.9
0	1	2	15	15	86.666	0.0118223	13.6	0.906667
0	1	2	15	20	82.915	0.0122421	17.2	0.928
0	1	2	15	25	80.7453	0.0124579	23.8	0.94
0	1	2	15	30	80.1066	0.012627	27.5	0.916667
0	1	5	3	1	298.4	0.00359147	1	1
0	1	5	3	5	210.745	0.0053945	4.8	0.96
0	1	5	3	10	200.606	0.00509218	9.3	0.93
0	1	5	3	15	191.801	0.00529512	13.8	0.92
0	1	5	3	20	181.807	0.00559986	19.1	0.955
0	1	5	3	25	186.142	0.00542807	23.2	0.928
0	1	5	3	30	186.718	0.00550121	27.2	0.906667
0	1	5	5	1	297.6	0.00362774	1	1
0	1	5	5	5	207.33	0.0053825	4.4	0.88
0	1	5	5	10	154.106	0.00667904	9.3	0.93
0	1	5	5	15	147.034	0.00696271	14	0.933333
0	1	5	5	20	145.355	0.00726923	18.6	0.93
0	1	5	5	25	145.435	0.00698368	22.6	0.92
0	1	5	5	30	155.165	0.00674389	27.6	0.904
0	1	5	10	1	329.1	0.00316227	1	1
0	1	5	10	5	139.005	0.00743945	4.7	0.94
0	1	5	10	10	94.48	0.0107833	9.3	0.93
0	1	5	10	15	88.638	0.0115944	13.5	0.9
0	1	5	10	20	82.4818	0.0123068	19	0.95
0	1	5	10	25	79.9138	0.0126186	22.2	0.888
0	1	5	10	30	78.0156	0.0128685	27.7	0.923333
0	1	5	15	1	402	0.00251449	1	1
0	1	5	15	5	140	0.00765807	4.7	0.94
0	1	5	15	10	102.774	0.0101572	9	0.9
0	1	5	15	15	82.8769	0.012189	14	0.933333

Aut	A	E	M	P	T_c	X	$\#token_{AVG}$	$\mathbb{P}$
0	1	5	15	20	77.9399	0.0129301	18.3	0.915
0	1	5	15	25	76.9567	0.0131209	23.4	0.936
0	1	5	15	30	75.8287	0.0132983	27.3	0.91
0	1	10	3	1	343.5	0.00312713	1	1
0	1	10	3	5	231.72	0.00457822	4.3	0.86
0	1	10	3	10	202.854	0.00512505	9.1	0.91
0	1	10	3	15	188.421	0.00555332	14.1	0.94
0	1	10	3	20	186.509	0.00539457	18.5	0.925
0	1	10	3	25	193.083	0.00528131	22.9	0.916
0	1	10	3	30	199.2	0.00518825	27.5	0.916667
0	1	10	5	1	369.7	0.00289338	1	1
0	1	10	5	5	215.822	0.00508162	4.5	0.9
0	1	10	5	10	136.455	0.00762895	8.9	0.89
0	1	10	5	15	164.364	0.00641327	14.2	0.946667
0	1	10	5	20	150.552	0.00679714	18.7	0.935
0	1	10	5	25	151.021	0.00681869	23.1	0.924
0	1	10	5	30	155.418	0.00669341	27.5	0.916667
0	1	10	10	1	330.8	0.00319986	1	1
0	1	10	10	5	135.085	0.0079445	4.7	0.94
0	1	10	10	10	102.134	0.0100018	9.3	0.93
0	1	10	10	15	87.4083	0.0115963	14.2	0.946667
0	1	10	10	20	83.085	0.0120607	18.3	0.915
0	1	10	10	25	81.9123	0.0123164	23.1	0.924
0	1	10	10	30	79.9434	0.0126571	28.1	0.936667
0	1	10	15	1	334.5	0.00319127	1	1
0	1	10	15	5	126.33	0.00801436	4.5	0.9
0	1	10	15	10	85.5036	0.0119155	9.4	0.94
0	1	10	15	15	81.9827	0.0123085	14.4	0.96
0	1	10	15	20	80.4357	0.0125744	18.2	0.91
0	1	10	15	25	80.3492	0.0125453	22.1	0.884
0	1	10	15	30	78.4896	0.0129026	27.1	0.903333
0	1	15	3	1	344.9	0.00307065	1	1
0	1	15	3	5	200.17	0.00528805	4.6	0.92
0	1	15	3	10	201.753	0.0052007	9	0.9
0	1	15	3	15	180.891	0.0057062	14.1	0.94
0	1	15	3	20	186.596	0.00541765	18	0.9
0	1	15	3	25	190.741	0.00534717	23.2	0.928
0	1	15	3	30	196.576	0.00529865	27.9	0.93
0	1	15	5	1	333.6	0.00319339	1	1
0	1	15	5	5	180.35	0.00569526	4.7	0.94
0	1	15	5	10	161.47	0.00640131	9.4	0.94
0	1	15	5	15	149.204	0.00702858	13.8	0.92

Aut	A	E	M	P	T_c	X	$\#token_{AVG}$	$\mathbb{P}$
0	1	15	5	20	148.287	0.00691853	18.7	0.935
0	1	15	5	25	143.266	0.00708568	22.8	0.912
0	1	15	5	30	152.274	0.00684329	27.7	0.923333
0	1	15	10	1	312.6	0.00343933	1	1
0	1	15	10	5	120.56	0.00870763	4.8	0.96
0	1	15	10	10	97.3306	0.0103716	9.1	0.91
0	1	15	10	15	87.5114	0.0117174	13.6	0.906667
0	1	15	10	20	81.7022	0.0123659	18.9	0.945
0	1	15	10	25	78.2354	0.0128814	23.2	0.928
0	1	15	10	30	79.7115	0.0126874	27.8	0.926667
0	1	15	15	1	312.7	0.00331676	1	1
0	1	15	15	5	128.73	0.00803892	4.7	0.94
0	1	15	15	10	96.9833	0.0104666	9.5	0.95
0	1	15	15	15	82.9531	0.0122589	14.2	0.946667
0	1	15	15	20	80.0014	0.0125956	17.8	0.89
0	1	15	15	25	74.5543	0.013565	23.9	0.956
0	1	15	15	30	73.578	0.0136747	27.9	0.93
0	5	1	3	1	330.1	0.00340135	1	1
0	5	1	3	5	232.16	0.00470489	4.5	0.9
0	5	1	3	10	195.646	0.00531983	9.5	0.95
0	5	1	3	15	191.782	0.00540789	14.3	0.953333
0	5	1	3	20	175.158	0.00580337	19	0.95
0	5	1	3	25	188.167	0.0054072	23.2	0.928
0	5	1	3	30	188.378	0.00537574	27.7	0.923333
0	5	1	5	1	317.4	0.00329541	1	1
0	5	1	5	5	186.62	0.00608876	5	1
0	5	1	5	10	173.35	0.00642103	9.5	0.95
0	5	1	5	15	149.518	0.00686598	13.7	0.913333
0	5	1	5	20	157.953	0.00642979	18.5	0.925
0	5	1	5	25	138.124	0.00752377	23.5	0.94
0	5	1	5	30	150.143	0.00690508	27.4	0.913333
0	5	1	10	1	313.3	0.00330444	1	1
0	5	1	10	5	153.055	0.00697106	4.8	0.96
0	5	1	10	10	111.526	0.00947237	9	0.9
0	5	1	10	15	93.8887	0.0107577	14.2	0.946667
0	5	1	10	20	91.6882	0.0109814	19	0.95
0	5	1	10	25	86.4701	0.0116614	23.7	0.948
0	5	1	10	30	89.6464	0.0111934	27.5	0.916667
0	5	1	15	1	319	0.00326047	1	1
0	5	1	15	5	145.58	0.0076306	4.5	0.9
0	5	1	15	10	96.6039	0.0104011	9.5	0.95
0	5	1	15	15	93.6963	0.0107805	13.7	0.913333

Aut	A	E	M	P	T_c	X	$\#token_{AVG}$	$\mathbb{P}$
0	5	1	15	20	92.2341	0.0108892	18.2	0.91
0	5	1	15	25	88.4165	0.011395	23.2	0.928
0	5	1	15	30	88.2248	0.0113765	27.8	0.926667
0	5	2	3	1	331.4	0.00306962	1	1
0	5	2	3	5	206.887	0.00544595	4.6	0.92
0	5	2	3	10	202.986	0.00511141	8.9	0.89
0	5	2	3	15	204.489	0.00510899	13.8	0.92
0	5	2	3	20	183.409	0.00564967	18.6	0.93
0	5	2	3	25	175.515	0.00586902	22.6	0.904
0	5	2	3	30	197.276	0.00519664	27.5	0.916667
0	5	2	5	1	324.4	0.00320022	1	1
0	5	2	5	5	184.855	0.00593973	4.8	0.96
0	5	2	5	10	176.369	0.00631126	9.2	0.92
0	5	2	5	15	128.656	0.00809959	13.9	0.926667
0	5	2	5	20	138.559	0.00743222	18.6	0.93
0	5	2	5	25	146.94	0.0069822	23.6	0.944
0	5	2	5	30	148.316	0.00687588	27.9	0.93
0	5	2	10	1	320	0.00329074	1	1
0	5	2	10	5	129.765	0.00857916	4.6	0.92
0	5	2	10	10	92.2169	0.0112962	9.5	0.95
0	5	2	10	15	80.0481	0.0132019	14.1	0.94
0	5	2	10	20	70.7156	0.0147792	18.6	0.93
0	5	2	10	25	64.1473	0.0158731	24.1	0.964
0	5	2	10	30	62.738	0.0162719	28.2	0.94
0	5	2	15	1	350.1	0.00290893	1	1
0	5	2	15	5	112.59	0.0092192	4.9	0.98
0	5	2	15	10	78.2358	0.0128955	8.9	0.89
0	5	2	15	15	64.3625	0.015961	13.5	0.9
0	5	2	15	20	59.9895	0.0174947	18.3	0.915
0	5	2	15	25	55.0815	0.0183034	23.3	0.932
0	5	2	15	30	51.7662	0.0194771	27.6	0.92
0	5	5	3	1	345.1	0.00297744	1	1
0	5	5	3	5	211.363	0.00538	4.5	0.9
0	5	5	3	10	179.808	0.00574333	9.1	0.91
0	5	5	3	15	206.04	0.00493716	13.8	0.92
0	5	5	3	20	169.584	0.00606256	18	0.9
0	5	5	3	25	183.163	0.00551558	23.5	0.94
0	5	5	3	30	185.036	0.00553439	28.2	0.94
0	5	5	5	1	353.3	0.00291126	1	1
0	5	5	5	5	180.6	0.0058849	4.5	0.9
0	5	5	5	10	148.035	0.0074869	9.3	0.93
0	5	5	5	15	135.943	0.00766684	14.3	0.953333

Aut	A	E	M	P	T_c	X	$\#token_{AVG}$	$\mathbb{P}$
0	5	5	5	20	139.248	0.0076163	18.7	0.935
0	5	5	5	25	139.511	0.00731791	22.9	0.916
0	5	5	5	30	147.257	0.00687395	27.9	0.93
0	5	5	10	1	362	0.0028084	1	1
0	5	5	10	5	120.095	0.00872617	4.7	0.94
0	5	5	10	10	81.6683	0.0130443	9.4	0.94
0	5	5	10	15	77.755	0.013538	13.5	0.9
0	5	5	10	20	71.0666	0.0143884	18.7	0.935
0	5	5	10	25	69.3897	0.0149583	22.5	0.9
0	5	5	10	30	58.7602	0.0173566	28.2	0.94
0	5	5	15	1	306.9	0.00348062	1	1
0	5	5	15	5	110.405	0.00978591	4.8	0.96
0	5	5	15	10	73.4569	0.0145402	9.1	0.91
0	5	5	15	15	50.7692	0.0207643	13.6	0.906667
0	5	5	15	20	48.5145	0.0212372	18.3	0.915
0	5	5	15	25	47.534	0.0214389	23.2	0.928
0	5	5	15	30	41.8927	0.0242075	27.5	0.916667
0	5	10	3	1	388.4	0.00264664	1	1
0	5	10	3	5	200.262	0.00526046	4.5	0.9
0	5	10	3	10	185.465	0.00568417	9.1	0.91
0	5	10	3	15	181.026	0.00561128	14.1	0.94
0	5	10	3	20	187.611	0.00558911	18.2	0.91
0	5	10	3	25	192.049	0.00539694	23.2	0.928
0	5	10	3	30	193.572	0.00524558	27.6	0.92
0	5	10	5	1	374.7	0.00274668	1	1
0	5	10	5	5	183.992	0.00598521	4.4	0.88
0	5	10	5	10	153.012	0.0069717	9.4	0.94
0	5	10	5	15	152.394	0.00715899	13.7	0.913333
0	5	10	5	20	149.667	0.0070607	18.2	0.91
0	5	10	5	25	149.916	0.00690442	23.1	0.924
0	5	10	5	30	123.965	0.00810795	28.4	0.946667
0	5	10	10	1	362.3	0.00283005	1	1
0	5	10	10	5	124.505	0.00840657	4.9	0.98
0	5	10	10	10	72.804	0.0139535	9.6	0.96
0	5	10	10	15	69.2606	0.0149098	14	0.933333
0	5	10	10	20	65.125	0.0156487	18.5	0.925
0	5	10	10	25	56.7103	0.0179086	23.3	0.932
0	5	10	10	30	58.3474	0.017509	27.8	0.926667
0	5	10	15	1	320.8	0.0032859	1	1
0	5	10	15	5	115.218	0.00922268	4.5	0.9
0	5	10	15	10	68.2025	0.0152688	9.4	0.94
0	5	10	15	15	59.6824	0.0179129	13.8	0.92

Aut	A	E	M	P	T_c	X	$\#token_{AVG}$	$\mathbb{P}$
0	5	10	15	20	53.5258	0.0194937	18.8	0.94
0	5	10	15	25	47.7242	0.021331	23.6	0.944
0	5	10	15	30	47.4881	0.0215459	27.1	0.903333
0	5	15	3	1	327.5	0.00324122	1	1
0	5	15	3	5	189.22	0.0056371	4.9	0.98
0	5	15	3	10	161.62	0.00641096	9.4	0.94
0	5	15	3	15	153.382	0.00672614	14	0.933333
0	5	15	3	20	138.196	0.00738711	18.5	0.925
0	5	15	3	25	145.356	0.00716285	23.2	0.928
0	5	15	3	30	166.956	0.00624061	27.6	0.92
0	5	15	5	1	354.9	0.00295172	1	1
0	5	15	5	5	182.97	0.00622534	4.6	0.92
0	5	15	5	10	154.999	0.00668043	9.6	0.96
0	5	15	5	15	149.554	0.0070622	14	0.933333
0	5	15	5	20	141.386	0.00738146	18.2	0.91
0	5	15	5	25	148.776	0.00724663	23.5	0.94
0	5	15	5	30	149.679	0.00685516	27.8	0.926667
0	5	15	10	1	322	0.00317638	1	1
0	5	15	10	5	121.285	0.0084516	4.8	0.96
0	5	15	10	10	76.6433	0.0138069	9.2	0.92
0	5	15	10	15	74.4485	0.0137068	13.8	0.92
0	5	15	10	20	59.8099	0.0169431	18.9	0.945
0	5	15	10	25	69.4507	0.0148228	23.1	0.924
0	5	15	10	30	64.5123	0.0156594	27.3	0.91
0	5	15	15	1	336.1	0.00312165	1	1
0	5	15	15	5	114.715	0.00914306	4.6	0.92
0	5	15	15	10	67.9678	0.0157537	9.6	0.96
0	5	15	15	15	59.8274	0.0176164	13.8	0.92
0	5	15	15	20	46.7439	0.0220402	18.8	0.94
0	5	15	15	25	48.1888	0.0210765	22.5	0.9
0	5	15	15	30	44.508	0.0229874	28.2	0.94

Table 1 - Results for 10 simulations

$$\#Sim = 20$$

Aut	A	E	M	P	T_c	X	$\#token_{AVG}$	$\mathbb{P}$
0	1	1	3	1	347.8	0.00299545	1	1
0	1	1	3	5	240.155	0.00472374	4.85	0.97
0	1	1	3	10	199.3	0.00529508	9.35	0.935
0	1	1	3	15	191.063	0.00546444	13.9	0.926667
0	1	1	3	20	178.654	0.00572652	18.55	0.9275

Aut	A	E	M	P	T_c	X	$\#token_{AVG}$	$\mathbb{P}$
0	1	1	3	25	189.21	0.00539135	23	0.92
0	1	1	3	30	188.986	0.00540034	27.95	0.931667
0	1	1	5	1	360.65	0.00294231	1	1
0	1	1	5	5	188.543	0.00548079	4.65	0.93
0	1	1	5	10	168.907	0.00642147	9.25	0.925
0	1	1	5	15	160.9	0.00648042	13.85	0.923333
0	1	1	5	20	142.894	0.0071755	18.85	0.9425
0	1	1	5	25	149.926	0.00679252	23.55	0.942
0	1	1	5	30	151.273	0.00677521	27.65	0.921667
0	1	1	10	1	349.6	0.00302179	1	1
0	1	1	10	5	143.35	0.0071596	4.75	0.95
0	1	1	10	10	113.129	0.00895991	9.3	0.93
0	1	1	10	15	103.819	0.00970791	13.7	0.913333
0	1	1	10	20	97.9202	0.010274	18.4	0.92
0	1	1	10	25	96.7921	0.0104348	23.05	0.922
0	1	1	10	30	95.8916	0.0104783	27.45	0.915
0	1	1	15	1	362.35	0.00283754	1	1
0	1	1	15	5	141.39	0.00725546	4.8	0.96
0	1	1	15	10	110.267	0.009186	9.45	0.945
0	1	1	15	15	97.4186	0.0103579	13.85	0.923333
0	1	1	15	20	97.2388	0.01039	18.6	0.93
0	1	1	15	25	96.2762	0.0104544	22.95	0.918
0	1	1	15	30	94.0235	0.0107022	27.85	0.928333
0	1	2	3	1	337.85	0.00304437	1	1
0	1	2	3	5	220.873	0.0049817	4.7	0.94
0	1	2	3	10	209.175	0.00501141	9.35	0.935
0	1	2	3	15	187.864	0.00543013	13.8	0.92
0	1	2	3	20	186.086	0.00547484	18.6	0.93
0	1	2	3	25	192.987	0.00525394	23.7	0.948
0	1	2	3	30	197.037	0.00518364	27.55	0.918333
0	1	2	5	1	328.55	0.00315491	1	1
0	1	2	5	5	193.116	0.00552316	4.55	0.91
0	1	2	5	10	172.449	0.00635108	9.4	0.94
0	1	2	5	15	153.702	0.00671534	14	0.933333
0	1	2	5	20	139.715	0.00734055	18.75	0.9375
0	1	2	5	25	156.315	0.0065972	23.25	0.93
0	1	2	5	30	159.434	0.00644341	28.3	0.943333
0	1	2	10	1	362.35	0.00283754	1	1
0	1	2	10	5	142.697	0.00760608	4.4	0.88
0	1	2	10	10	97.7697	0.0105221	9.25	0.925
0	1	2	10	15	89.6625	0.0113236	13.4	0.893333
0	1	2	10	20	80.1615	0.0126786	19	0.95

Aut	A	E	M	P	T_c	X	$\#token_{AVG}$	$\mathbb{P}$
0	1	2	10	25	80.3536	0.0126026	23.45	0.938
0	1	2	10	30	75.8702	0.0132916	28.15	0.938333
0	1	2	15	1	357.45	0.0029103	1	1
0	1	2	15	5	125.598	0.00830553	4.85	0.97
0	1	2	15	10	96.6553	0.0104771	9.2	0.92
0	1	2	15	15	89.9433	0.0113751	13.5	0.9
0	1	2	15	20	81.0561	0.0124379	18.65	0.9325
0	1	2	15	25	79.7616	0.012677	23.25	0.93
0	1	2	15	30	79.2832	0.012746	27.85	0.928333
0	1	5	3	1	316.2	0.00346468	1	1
0	1	5	3	5	224.478	0.00495611	4.6	0.92
0	1	5	3	10	203.174	0.00509476	9.1	0.91
0	1	5	3	15	177.45	0.0057614	14.05	0.936667
0	1	5	3	20	185.202	0.00547919	18.5	0.925
0	1	5	3	25	190.443	0.00538419	23.2	0.928
0	1	5	3	30	194.412	0.00524088	27.45	0.915
0	1	5	5	1	311.15	0.00343589	1	1
0	1	5	5	5	198.447	0.00569714	4.45	0.89
0	1	5	5	10	152.858	0.00680832	9.35	0.935
0	1	5	5	15	146.366	0.00705974	14.1	0.94
0	1	5	5	20	141.006	0.00736054	18.5	0.925
0	1	5	5	25	142.989	0.00717446	22.85	0.914
0	1	5	5	30	153.217	0.0068169	27.85	0.928333
0	1	5	10	1	340.45	0.00303369	1	1
0	1	5	10	5	124.353	0.00833243	4.85	0.97
0	1	5	10	10	98.5872	0.0105756	9.5	0.95
0	1	5	10	15	88.9134	0.0115264	13.45	0.896667
0	1	5	10	20	80.1692	0.0126243	19	0.95
0	1	5	10	25	82.2778	0.0122761	22.65	0.906
0	1	5	10	30	78.2731	0.0129093	27.9	0.93
0	1	5	15	1	383.45	0.0026459	1	1
0	1	5	15	5	139.168	0.00762167	4.7	0.94
0	1	5	15	10	97.5947	0.0106434	9.15	0.915
0	1	5	15	15	84.4612	0.0119723	13.85	0.923333
0	1	5	15	20	80.7269	0.0125402	18.05	0.9025
0	1	5	15	25	77.4512	0.0130187	23.15	0.926
0	1	5	15	30	76.7249	0.0131418	27.35	0.911667
0	1	10	3	1	325.65	0.00326748	1	1
0	1	10	3	5	226.255	0.00461951	4.5	0.9
0	1	10	3	10	203.799	0.00513804	9	0.9
0	1	10	3	15	189.027	0.00546926	14.05	0.936667
0	1	10	3	20	188.162	0.00544817	18.65	0.9325

Aut	A	E	M	P	T_c	X	$\#token_{AVG}$	$\mathbb{P}$
0	1	10	3	25	191.269	0.00533552	23.35	0.934
0	1	10	3	30	191.87	0.00526524	27.75	0.925
0	1	10	5	1	347.55	0.00302939	1	1
0	1	10	5	5	206.181	0.00534029	4.65	0.93
0	1	10	5	10	144.82	0.0072441	8.85	0.885
0	1	10	5	15	143.955	0.00715687	14.25	0.95
0	1	10	5	20	157.466	0.00665182	18.55	0.9275
0	1	10	5	25	154.959	0.00673461	23.2	0.928
0	1	10	5	30	153.563	0.00669664	27.9	0.93
0	1	10	10	1	334.45	0.0032144	1	1
0	1	10	10	5	129.533	0.00811769	4.7	0.94
0	1	10	10	10	97.3389	0.0104437	9.35	0.935
0	1	10	10	15	85.8464	0.0118392	14.15	0.943333
0	1	10	10	20	83.7801	0.0120546	18.3	0.915
0	1	10	10	25	82.3689	0.0122172	22.95	0.918
0	1	10	10	30	79.105	0.0127395	28.15	0.938333
0	1	10	15	1	328.05	0.00320448	1	1
0	1	10	15	5	125.857	0.00806628	4.65	0.93
0	1	10	15	10	88.2309	0.0115149	9.3	0.93
0	1	10	15	15	81.2856	0.0124186	14	0.933333
0	1	10	15	20	81.8002	0.0123709	18.35	0.9175
0	1	10	15	25	80.731	0.0124857	22.6	0.904
0	1	10	15	30	78.0185	0.0129588	27.4	0.913333
0	1	15	3	1	348.8	0.00301089	1	1
0	1	15	3	5	221.725	0.00487667	4.6	0.92
0	1	15	3	10	203.499	0.00511939	9.15	0.915
0	1	15	3	15	183.474	0.00564753	14.15	0.943333
0	1	15	3	20	188.611	0.00541506	18.45	0.9225
0	1	15	3	25	188.53	0.00535986	23.45	0.938
0	1	15	3	30	190.248	0.00542651	28.3	0.943333
0	1	15	5	1	350.9	0.00299362	1	1
0	1	15	5	5	184.942	0.00559207	4.65	0.93
0	1	15	5	10	159.418	0.00644431	9.35	0.935
0	1	15	5	15	135.086	0.00758694	13.8	0.92
0	1	15	5	20	149.879	0.00688908	18.35	0.9175
0	1	15	5	25	153.958	0.00683218	22.75	0.91
0	1	15	5	30	160.334	0.00651444	27.65	0.921667
0	1	15	10	1	310.05	0.00343079	1	1
0	1	15	10	5	124.15	0.00848437	4.8	0.96
0	1	15	10	10	93.7836	0.0108587	9.05	0.905
0	1	15	10	15	89.3302	0.0113779	13.7	0.913333
0	1	15	10	20	80.8822	0.0124658	18.9	0.945

Aut	A	E	M	P	T_c	X	$\#token_{AVG}$	$\mathbb{P}$
0	1	15	10	25	80.5249	0.0126122	23	0.92
0	1	15	10	30	80.0432	0.0126231	27.8	0.926667
0	1	15	15	1	320.05	0.00325309	1	1
0	1	15	15	5	137.048	0.00757163	4.7	0.94
0	1	15	15	10	94.7206	0.0107126	9.35	0.935
0	1	15	15	15	82.4375	0.0122751	13.95	0.93
0	1	15	15	20	81.4366	0.0124363	18.5	0.925
0	1	15	15	25	76.0388	0.0132458	23.6	0.944
0	1	15	15	30	75.6917	0.0134923	28.1	0.936667
0	5	1	3	1	349.85	0.00310331	1	1
0	5	1	3	5	216.73	0.00495638	4.5	0.9
0	5	1	3	10	204.296	0.00511531	9.15	0.915
0	5	1	3	15	196.628	0.00525501	13.9	0.926667
0	5	1	3	20	187.757	0.00549495	18.95	0.9475
0	5	1	3	25	186.224	0.00546061	23.2	0.928
0	5	1	3	30	190.429	0.00530036	27.75	0.925
0	5	1	5	1	333.7	0.00314235	1	1
0	5	1	5	5	200.482	0.00556438	4.85	0.97
0	5	1	5	10	161.612	0.00672649	9.4	0.94
0	5	1	5	15	149.408	0.00693733	13.9	0.926667
0	5	1	5	20	146.274	0.00707108	18.8	0.94
0	5	1	5	25	144.825	0.00713387	23.05	0.922
0	5	1	5	30	151.461	0.00677599	27.75	0.925
0	5	1	10	1	317.3	0.00327936	1	1
0	5	1	10	5	154.688	0.00681254	4.8	0.96
0	5	1	10	10	110.068	0.00944379	9.4	0.94
0	5	1	10	15	95.4354	0.0105724	13.95	0.93
0	5	1	10	20	90.0875	0.0111738	19.15	0.9575
0	5	1	10	25	86.4613	0.0116242	23.85	0.954
0	5	1	10	30	90.859	0.0110444	27.4	0.913333
0	5	1	15	1	318.4	0.00330935	1	1
0	5	1	15	5	139.516	0.00771718	4.6	0.92
0	5	1	15	10	102.241	0.00993408	9.25	0.925
0	5	1	15	15	95.8124	0.0104905	13.55	0.903333
0	5	1	15	20	92.3489	0.0109365	18.55	0.9275
0	5	1	15	25	89.2033	0.0112825	23.45	0.938
0	5	1	15	30	88.3661	0.0113775	27.5	0.916667
0	5	2	3	1	341.35	0.00302568	1	1
0	5	2	3	5	198.002	0.0057723	4.6	0.92
0	5	2	3	10	193.589	0.00533991	9.15	0.915
0	5	2	3	15	191.202	0.00543858	13.9	0.926667
0	5	2	3	20	181.877	0.00565148	18.3	0.915

Aut	A	E	M	P	T_c	X	$\#token_{AVG}$	$\mathbb{P}$
0	5	2	3	25	177.488	0.00575876	23.25	0.93
0	5	2	3	30	199.58	0.00511913	27.65	0.921667
0	5	2	5	1	331.1	0.00315123	1	1
0	5	2	5	5	172.755	0.00630626	4.6	0.92
0	5	2	5	10	172.265	0.00653181	9.15	0.915
0	5	2	5	15	147.138	0.00728998	13.85	0.923333
0	5	2	5	20	141.536	0.00733639	18.55	0.9275
0	5	2	5	25	143.93	0.00713319	23.3	0.932
0	5	2	5	30	146.477	0.00704921	27.75	0.925
0	5	2	10	1	320.45	0.00330917	1	1
0	5	2	10	5	128.804	0.00852954	4.5	0.9
0	5	2	10	10	86.7896	0.0119337	9.45	0.945
0	5	2	10	15	78.7291	0.0133819	13.9	0.926667
0	5	2	10	20	71.0701	0.0145761	18.6	0.93
0	5	2	10	25	62.7945	0.0161123	24	0.96
0	5	2	10	30	64.1906	0.0158221	28.2	0.94
0	5	2	15	1	362.35	0.00283754	1	1
0	5	2	15	5	120.683	0.00870919	4.55	0.91
0	5	2	15	10	74.5128	0.0136048	8.85	0.885
0	5	2	15	15	60.3543	0.0169483	13.65	0.91
0	5	2	15	20	58.0644	0.0178235	18	0.9
0	5	2	15	25	52.6208	0.0192299	23.55	0.942
0	5	2	15	30	50.8346	0.0199311	27.35	0.911667
0	5	5	3	1	344.85	0.00299389	1	1
0	5	5	3	5	204.576	0.0053706	4.6	0.92
0	5	5	3	10	203.652	0.0050896	9.1	0.91
0	5	5	3	15	182.265	0.00573009	13.55	0.903333
0	5	5	3	20	176.766	0.00585646	18.2	0.91
0	5	5	3	25	187.099	0.00545704	23.55	0.942
0	5	5	3	30	189.513	0.00536651	28	0.933333
0	5	5	5	1	344.3	0.00305891	1	1
0	5	5	5	5	182.991	0.00612751	4.45	0.89
0	5	5	5	10	156.388	0.00708968	9.1	0.91
0	5	5	5	15	146.185	0.00718464	14.3	0.953333
0	5	5	5	20	137.937	0.00760664	18.5	0.925
0	5	5	5	25	140.808	0.00723982	22.7	0.908
0	5	5	5	30	152.999	0.00671222	27.9	0.93
0	5	5	10	1	345.15	0.00305951	1	1
0	5	5	10	5	110.898	0.00952196	4.85	0.97
0	5	5	10	10	83.8301	0.0127925	9.45	0.945
0	5	5	10	15	73.5434	0.0140932	13.75	0.916667
0	5	5	10	20	66.5067	0.0156135	18.45	0.9225

Aut	A	E	M	P	T_c	X	$\#token_{AVG}$	$\mathbb{P}$
0	5	5	10	25	66.4406	0.0156834	23.15	0.926
0	5	5	10	30	61.2522	0.016685	27.6	0.92
0	5	5	15	1	315.85	0.00335976	1	1
0	5	5	15	5	117.3	0.00917416	4.75	0.95
0	5	5	15	10	72.1624	0.0147291	9.2	0.92
0	5	5	15	15	55.4476	0.0190039	13.85	0.923333
0	5	5	15	20	50.2792	0.0205358	18.75	0.9375
0	5	5	15	25	48.0126	0.0212606	23.3	0.932
0	5	5	15	30	44.0902	0.0231656	27.75	0.925
0	5	10	3	1	363.2	0.00285885	1	1
0	5	10	3	5	201.988	0.00535581	4.3	0.86
0	5	10	3	10	204.337	0.00512767	9.1	0.91
0	5	10	3	15	190.479	0.0054414	14.2	0.946667
0	5	10	3	20	184.28	0.00553111	18.25	0.9125
0	5	10	3	25	184.196	0.00551166	23.25	0.93
0	5	10	3	30	196.149	0.00527221	27.35	0.911667
0	5	10	5	1	329.85	0.00322021	1	1
0	5	10	5	5	174.432	0.00616546	4.4	0.88
0	5	10	5	10	153.532	0.00695424	9.2	0.92
0	5	10	5	15	153.467	0.0068177	13.9	0.926667
0	5	10	5	20	150.262	0.00694347	18.5	0.925
0	5	10	5	25	151.878	0.00673729	23.35	0.934
0	5	10	5	30	132.096	0.00768239	28.05	0.935
0	5	10	10	1	362.75	0.00284619	1	1
0	5	10	10	5	118.363	0.0088817	4.85	0.97
0	5	10	10	10	74.9653	0.0137393	9.55	0.955
0	5	10	10	15	73.0415	0.0143951	13.9	0.926667
0	5	10	10	20	65.9803	0.0153954	18.6	0.93
0	5	10	10	25	60.6824	0.0170646	23.1	0.924
0	5	10	10	30	59.241	0.0173699	28.05	0.935
0	5	10	15	1	322.7	0.00329217	1	1
0	5	10	15	5	123.631	0.00883414	4.5	0.9
0	5	10	15	10	69.0826	0.0153149	9.05	0.905
0	5	10	15	15	61.5388	0.017133	13.95	0.93
0	5	10	15	20	52.5178	0.0195585	18.65	0.9325
0	5	10	15	25	48.1284	0.0212107	23.55	0.942
0	5	10	15	30	47.1152	0.0218811	27.05	0.901667
0	5	15	3	1	335.35	0.00312172	1	1
0	5	15	3	5	192.655	0.00547824	4.7	0.94
0	5	15	3	10	162.228	0.00648387	9.45	0.945
0	5	15	3	15	160.243	0.00648682	13.95	0.93
0	5	15	3	20	154.494	0.00672119	18.45	0.9225

Aut	A	E	M	P	T_c	X	$\#token_{AVG}$	$\mathbb{P}$
0	5	15	3	25	142.481	0.00731401	23.25	0.93
0	5	15	3	30	136.396	0.00750638	27.8	0.926667
0	5	15	5	1	354.15	0.00298009	1	1
0	5	15	5	5	174.642	0.00640631	4.6	0.92
0	5	15	5	10	147.813	0.00728359	9.55	0.955
0	5	15	5	15	147.203	0.00715252	13.95	0.93
0	5	15	5	20	146.735	0.00719241	18.5	0.925
0	5	15	5	25	144.906	0.00717617	23.5	0.94
0	5	15	5	30	143.383	0.00714752	27.95	0.931667
0	5	15	10	1	338.05	0.0030581	1	1
0	5	15	10	5	123.838	0.00845948	4.6	0.92
0	5	15	10	10	79.6447	0.0133874	9.2	0.92
0	5	15	10	15	74.6376	0.0138319	13.55	0.903333
0	5	15	10	20	71.075	0.0143906	18.7	0.935
0	5	15	10	25	67.0472	0.0152951	23.2	0.928
0	5	15	10	30	61.1412	0.0165033	27.65	0.921667
0	5	15	15	1	341.05	0.00305405	1	1
0	5	15	15	5	116.387	0.00908288	4.55	0.91
0	5	15	15	10	68.6545	0.0153133	9.35	0.935
0	5	15	15	15	59.0067	0.0180853	13.7	0.913333
0	5	15	15	20	48.1995	0.0212749	18.75	0.9375
0	5	15	15	25	47.7031	0.021339	22.9	0.916
0	5	15	15	30	45.0184	0.0225796	27.7	0.923333

Table 2 - Results for 20 simulations

$$\#Sim = 50$$

Aut	A	E	M	P	T_c	X	$\#token_{AVG}$	$\mathbb{P}$
0	1	1	3	1	359.32	0.00290627	1	1
0	1	1	3	5	234.238	0.00466864	4.76	0.952
0	1	1	3	10	204.128	0.00514179	9.32	0.932
0	1	1	3	15	200.868	0.00517821	13.88	0.925333
0	1	1	3	20	183.96	0.00553809	18.68	0.934
0	1	1	3	25	183.472	0.00556577	23.06	0.9224
0	1	1	3	30	187.857	0.00546823	27.78	0.926
0	1	1	5	1	346.74	0.00305507	1	1
0	1	1	5	5	197.693	0.00534627	4.6	0.92
0	1	1	5	10	169.043	0.00624308	9.3	0.93
0	1	1	5	15	160.802	0.00648138	13.7	0.913333
0	1	1	5	20	151.098	0.0067663	18.58	0.929
0	1	1	5	25	145.168	0.00704486	23.4	0.936

Aut	A	E	M	P	T_c	X	$\#token_{AVG}$	$\mathbb{P}$
0	1	1	5	30	152.432	0.00672425	27.68	0.922667
0	1	1	10	1	342.46	0.00311008	1	1
0	1	1	10	5	150.736	0.00687179	4.62	0.924
0	1	1	10	10	117.5	0.0086382	9.06	0.906
0	1	1	10	15	102.42	0.00985104	13.92	0.928
0	1	1	10	20	98.3491	0.0102343	18.62	0.931
0	1	1	10	25	96.3172	0.0104604	23.26	0.9304
0	1	1	10	30	95.4525	0.0105339	27.52	0.917333
0	1	1	15	1	334.44	0.00318014	1	1
0	1	1	15	5	148.718	0.00700489	4.62	0.924
0	1	1	15	10	110.408	0.0092023	9.44	0.944
0	1	1	15	15	101.335	0.0099822	13.98	0.932
0	1	1	15	20	98.685	0.0102127	18.52	0.926
0	1	1	15	25	95.302	0.0105674	23.38	0.9352
0	1	1	15	30	93.1203	0.0108009	28.14	0.938
0	1	2	3	1	353.16	0.00291354	1	1
0	1	2	3	5	233.919	0.00469156	4.56	0.912
0	1	2	3	10	201.316	0.00521166	9.32	0.932
0	1	2	3	15	190.455	0.00538853	13.96	0.930667
0	1	2	3	20	186.472	0.00551146	18.66	0.933
0	1	2	3	25	193.261	0.00525965	23.26	0.9304
0	1	2	3	30	197.556	0.00514831	27.64	0.921333
0	1	2	5	1	329.08	0.00318785	1	1
0	1	2	5	5	197.934	0.00554012	4.64	0.928
0	1	2	5	10	163.859	0.00665893	9.38	0.938
0	1	2	5	15	149.671	0.00694141	13.78	0.918667
0	1	2	5	20	148.841	0.00691725	18.54	0.927
0	1	2	5	25	148.108	0.00696471	23.28	0.9312
0	1	2	5	30	153.028	0.00673552	28.14	0.938
0	1	2	10	1	334.44	0.00318014	1	1
0	1	2	10	5	138.239	0.00773643	4.58	0.916
0	1	2	10	10	99.3057	0.0103196	9.28	0.928
0	1	2	10	15	91.1283	0.0111531	13.58	0.905333
0	1	2	10	20	85.298	0.0119035	18.62	0.931
0	1	2	10	25	82.3258	0.0122905	23.06	0.9224
0	1	2	10	30	79.2875	0.0127268	27.72	0.924
0	1	2	15	1	345.38	0.00303337	1	1
0	1	2	15	5	134.64	0.00785899	4.64	0.928
0	1	2	15	10	95.9696	0.0105938	9.22	0.922
0	1	2	15	15	87.5646	0.0116402	13.72	0.914667
0	1	2	15	20	80.2964	0.012589	18.82	0.941
0	1	2	15	25	79.2724	0.0127256	23.16	0.9264

Aut	A	E	M	P	T_c	X	$\#token_{AVG}$	$\mathbb{P}$
0	**1**	**2**	**15**	**30**	**77.6158**	**0.0129913**	**27.96**	**0.932**
0	**1**	**5**	**3**	**1**	**348.28**	**0.00306667**	**1**	**1**
0	**1**	**5**	**3**	**5**	**227.874**	**0.00481061**	**4.54**	**0.908**
0	**1**	**5**	**3**	**10**	**194.504**	**0.00537007**	**9.16**	**0.916**
0	**1**	**5**	**3**	**15**	**181.76**	**0.00563254**	**14.02**	**0.934667**
0	**1**	**5**	**3**	**20**	**188.023**	**0.00540356**	**18.64**	**0.932**
0	**1**	**5**	**3**	**25**	**190.894**	**0.00531496**	**23.26**	**0.9304**
0	**1**	**5**	**3**	**30**	**192.548**	**0.00529686**	**27.5**	**0.916667**
0	**1**	**5**	**5**	**1**	**347.98**	**0.00307506**	**1**	**1**
0	**1**	**5**	**5**	**5**	**187.069**	**0.00594135**	**4.54**	**0.908**
0	**1**	**5**	**5**	**10**	**159.225**	**0.00661333**	**9.32**	**0.932**
0	**1**	**5**	**5**	**15**	**153.631**	**0.00677134**	**14.06**	**0.937333**
0	**1**	**5**	**5**	**20**	**143.865**	**0.00716**	**18.62**	**0.931**
0	**1**	**5**	**5**	**25**	**148.848**	**0.00695029**	**23.08**	**0.9232**
0	**1**	**5**	**5**	**30**	**150.491**	**0.00687711**	**27.76**	**0.925333**
0	**1**	**5**	**10**	**1**	**339.84**	**0.00309569**	**1**	**1**
0	**1**	**5**	**10**	**5**	**131.638**	**0.00797884**	**4.7**	**0.94**
0	**1**	**5**	**10**	**10**	**98.7546**	**0.0105273**	**9.42**	**0.942**
0	**1**	**5**	**10**	**15**	**85.8241**	**0.0118446**	**13.86**	**0.924**
0	**1**	**5**	**10**	**20**	**81.868**	**0.0123678**	**18.86**	**0.943**
0	**1**	**5**	**10**	**25**	**81.5294**	**0.012433**	**22.94**	**0.9176**
0	**1**	**5**	**10**	**30**	**78.9264**	**0.012773**	**27.8**	**0.926667**
0	**1**	**5**	**15**	**1**	**352.48**	**0.00301551**	**1**	**1**
0	**1**	**5**	**15**	**5**	**129.513**	**0.00807791**	**4.72**	**0.944**
0	**1**	**5**	**15**	**10**	**94.4331**	**0.0109266**	**9.26**	**0.926**
0	**1**	**5**	**15**	**15**	**83.9491**	**0.012167**	**13.76**	**0.917333**
0	**1**	**5**	**15**	**20**	**82.0146**	**0.01233**	**18.46**	**0.923**
0	**1**	**5**	**15**	**25**	**79.013**	**0.0127831**	**23.2**	**0.928**
0	**1**	**5**	**15**	**30**	**78.1249**	**0.01293**	**27.36**	**0.912**
0	**1**	**10**	**3**	**1**	**330.72**	**0.0032094**	**1**	**1**
0	**1**	**10**	**3**	**5**	**232.719**	**0.00458824**	**4.52**	**0.904**
0	**1**	**10**	**3**	**10**	**197.392**	**0.0052689**	**9.22**	**0.922**
0	**1**	**10**	**3**	**15**	**190.488**	**0.00543064**	**13.96**	**0.930667**
0	**1**	**10**	**3**	**20**	**187.628**	**0.00540106**	**18.52**	**0.926**
0	**1**	**10**	**3**	**25**	**189.639**	**0.00541288**	**23.14**	**0.9256**
0	**1**	**10**	**3**	**30**	**193.954**	**0.00524971**	**27.86**	**0.928667**
0	**1**	**10**	**5**	**1**	**342.92**	**0.00311182**	**1**	**1**
0	**1**	**10**	**5**	**5**	**195.76**	**0.00567333**	**4.74**	**0.948**
0	**1**	**10**	**5**	**10**	**154.965**	**0.00687317**	**9.08**	**0.908**
0	**1**	**10**	**5**	**15**	**155.955**	**0.00668322**	**14.1**	**0.94**
0	**1**	**10**	**5**	**20**	**147.787**	**0.00706601**	**18.9**	**0.945**
0	**1**	**10**	**5**	**25**	**144.745**	**0.00709804**	**23.06**	**0.9224**

Aut	A	E	M	P	T_c	X	$\#token_{AVG}$	$\mathbb{P}$
0	1	10	5	30	152.115	0.00676387	27.64	0.921333
0	1	10	10	1	324.78	0.00325296	1	1
0	1	10	10	5	134.664	0.00776517	4.7	0.94
0	1	10	10	10	94.4622	0.0108699	9.34	0.934
0	1	10	10	15	86.5654	0.0117012	13.9	0.926667
0	1	10	10	20	83.6872	0.0120574	18.58	0.929
0	1	10	10	25	82.6494	0.0122121	23.02	0.9208
0	1	10	10	30	79.8712	0.012662	27.76	0.925333
0	1	10	15	1	330.18	0.00318916	1	1
0	1	10	15	5	127.649	0.0080379	4.58	0.916
0	1	10	15	10	92.3843	0.0110501	9.16	0.916
0	1	10	15	15	81.4948	0.0124487	13.86	0.924
0	1	10	15	20	82.1514	0.0123363	18.4	0.92
0	1	10	15	25	79.8595	0.0126623	22.82	0.9128
0	1	10	15	30	78.2458	0.0128913	27.64	0.921333
0	1	15	3	1	362.8	0.00287323	1	1
0	1	15	3	5	217.795	0.00489577	4.58	0.916
0	1	15	3	10	202.63	0.0051408	9.16	0.916
0	1	15	3	15	188.069	0.00552787	14.02	0.934667
0	1	15	3	20	188.386	0.00544692	18.32	0.916
0	1	15	3	25	192.505	0.00537049	23.08	0.9232
0	1	15	3	30	192.37	0.00527768	27.84	0.928
0	1	15	5	1	345.02	0.00306916	1	1
0	1	15	5	5	190.669	0.00568252	4.56	0.912
0	1	15	5	10	160.031	0.00647438	9.32	0.932
0	1	15	5	15	152.348	0.00686502	13.94	0.929333
0	1	15	5	20	153.241	0.00682901	18.72	0.936
0	1	15	5	25	148.545	0.0069084	22.76	0.9104
0	1	15	5	30	141.793	0.00727105	27.84	0.928
0	1	15	10	1	323.88	0.00326869	1	1
0	1	15	10	5	134.806	0.00779238	4.74	0.948
0	1	15	10	10	96.2382	0.0106207	9.2	0.92
0	1	15	10	15	88.1487	0.0115401	13.64	0.909333
0	1	15	10	20	81.2703	0.012409	18.88	0.944
0	1	15	10	25	80.9101	0.0125239	23.14	0.9256
0	1	15	10	30	79.297	0.0127211	27.82	0.927333
0	1	15	15	1	319.8	0.00325699	1	1
0	1	15	15	5	131.915	0.00787713	4.66	0.932
0	1	15	15	10	91.6442	0.0111266	9.42	0.942
0	1	15	15	15	81.1664	0.0125202	14.04	0.936
0	1	15	15	20	80.7324	0.0125151	18.32	0.916
0	1	15	15	25	78.9951	0.0128812	23.1	0.924

Aut	A	E	M	P	T_c	X	$\#token_{AVG}$	$\mathbb{P}$
0	**1**	**15**	**15**	**30**	**77.1699**	**0.0130509**	**27.9**	**0.93**
0	**5**	**1**	**3**	**1**	**325.12**	**0.00326292**	**1**	**1**
0	**5**	**1**	**3**	**5**	**223.219**	**0.00477554**	**4.56**	**0.912**
0	**5**	**1**	**3**	**10**	**199.62**	**0.00522011**	**9.1**	**0.91**
0	**5**	**1**	**3**	**15**	**194.822**	**0.00527688**	**13.86**	**0.924**
0	**5**	**1**	**3**	**20**	**193.321**	**0.00538013**	**18.64**	**0.932**
0	**5**	**1**	**3**	**25**	**187.466**	**0.00541701**	**23.22**	**0.9288**
0	**5**	**1**	**3**	**30**	**188.638**	**0.00534657**	**27.74**	**0.924667**
0	**5**	**1**	**5**	**1**	**330.5**	**0.00313168**	**1**	**1**
0	**5**	**1**	**5**	**5**	**197.458**	**0.00551927**	**4.68**	**0.936**
0	**5**	**1**	**5**	**10**	**164.665**	**0.0064652**	**9.32**	**0.932**
0	**5**	**1**	**5**	**15**	**144.973**	**0.00723329**	**13.88**	**0.925333**
0	**5**	**1**	**5**	**20**	**148.347**	**0.00689158**	**18.58**	**0.929**
0	**5**	**1**	**5**	**25**	**148.601**	**0.00692056**	**23.1**	**0.924**
0	**5**	**1**	**5**	**30**	**148.774**	**0.00690898**	**27.92**	**0.930667**
0	**5**	**1**	**10**	**1**	**332.58**	**0.00314664**	**1**	**1**
0	**5**	**1**	**10**	**5**	**148.335**	**0.00713938**	**4.68**	**0.936**
0	**5**	**1**	**10**	**10**	**108.895**	**0.00947519**	**9.3**	**0.93**
0	**5**	**1**	**10**	**15**	**97.1604**	**0.0104296**	**13.8**	**0.92**
0	**5**	**1**	**10**	**20**	**91.944**	**0.0109792**	**18.62**	**0.931**
0	**5**	**1**	**10**	**25**	**90.6667**	**0.0111172**	**23.4**	**0.936**
0	**5**	**1**	**10**	**30**	**89.3615**	**0.0112518**	**27.76**	**0.925333**
0	**5**	**1**	**15**	**1**	**319.28**	**0.00330242**	**1**	**1**
0	**5**	**1**	**15**	**5**	**129.616**	**0.00805581**	**4.76**	**0.952**
0	**5**	**1**	**15**	**10**	**100.461**	**0.010171**	**9.36**	**0.936**
0	**5**	**1**	**15**	**15**	**91.4909**	**0.0110434**	**13.88**	**0.925333**
0	**5**	**1**	**15**	**20**	**90.4444**	**0.0111821**	**18.58**	**0.929**
0	**5**	**1**	**15**	**25**	**89.2721**	**0.0112803**	**23.3**	**0.932**
0	**5**	**1**	**15**	**30**	**88.2593**	**0.0113853**	**27.78**	**0.926**
0	**5**	**2**	**3**	**1**	**355.84**	**0.00295585**	**1**	**1**
0	**5**	**2**	**3**	**5**	**204.111**	**0.00547241**	**4.56**	**0.912**
0	**5**	**2**	**3**	**10**	**202.771**	**0.00524065**	**9.18**	**0.918**
0	**5**	**2**	**3**	**15**	**189.932**	**0.00544802**	**13.92**	**0.928**
0	**5**	**2**	**3**	**20**	**188.924**	**0.00542123**	**18.36**	**0.918**
0	**5**	**2**	**3**	**25**	**179.077**	**0.00570649**	**23.56**	**0.9424**
0	**5**	**2**	**3**	**30**	**192.48**	**0.00528373**	**27.6**	**0.92**
0	**5**	**2**	**5**	**1**	**326.16**	**0.00321374**	**1**	**1**
0	**5**	**2**	**5**	**5**	**181.573**	**0.0061115**	**4.68**	**0.936**
0	**5**	**2**	**5**	**10**	**160.644**	**0.00677534**	**9.24**	**0.924**
0	**5**	**2**	**5**	**15**	**155.039**	**0.00685959**	**13.98**	**0.932**
0	**5**	**2**	**5**	**20**	**148.619**	**0.00695814**	**18.64**	**0.932**
0	**5**	**2**	**5**	**25**	**143.165**	**0.00722726**	**23.3**	**0.932**

Aut	A	E	M	P	T_c	X	$\#token_{AVG}$	$\mathbb{P}$
0	5	2	5	30	144.702	0.00709632	28.02	0.934
0	5	2	10	1	322.62	0.00325459	1	1
0	5	2	10	5	126.785	0.00852442	4.6	0.92
0	5	2	10	10	87.4729	0.0117967	9.38	0.938
0	5	2	10	15	72.4202	0.0144	13.96	0.930667
0	5	2	10	20	70.2	0.0146257	18.46	0.923
0	5	2	10	25	64.4118	0.0157957	23.42	0.9368
0	5	2	10	30	63.1318	0.0161391	27.9	0.93
0	5	2	15	1	334.44	0.00318014	1	1
0	5	2	15	5	115.121	0.00908087	4.66	0.932
0	5	2	15	10	72.882	0.0140551	9.08	0.908
0	5	2	15	15	60.6052	0.016946	13.84	0.922667
0	5	2	15	20	57.3548	0.0178895	18.38	0.919
0	5	2	15	25	52.4862	0.0193114	23.4	0.936
0	5	2	15	30	49.1456	0.0205836	27.94	0.931333
0	5	5	3	1	336.16	0.00308299	1	1
0	5	5	3	5	213.378	0.00505837	4.6	0.92
0	5	5	3	10	192.091	0.00552717	9.16	0.916
0	5	5	3	15	190.656	0.00544897	13.9	0.926667
0	5	5	3	20	182.48	0.00564685	18.2	0.91
0	5	5	3	25	185.225	0.0054887	23.32	0.9328
0	5	5	3	30	187.527	0.00542365	27.74	0.924667
0	5	5	5	1	336.1	0.00317037	1	1
0	5	5	5	5	184.609	0.00593478	4.56	0.912
0	5	5	5	10	148.717	0.00722649	9	0.9
0	5	5	5	15	143.284	0.00736479	14.18	0.945333
0	5	5	5	20	140.184	0.00737263	18.68	0.934
0	5	5	5	25	140.968	0.00726056	23.04	0.9216
0	5	5	5	30	151.813	0.00677927	27.76	0.925333
0	5	5	10	1	339.6	0.003125	1	1
0	5	5	10	5	115.861	0.00916145	4.7	0.94
0	5	5	10	10	86.5403	0.0124134	9.3	0.93
0	5	5	10	15	72.3296	0.0143892	13.78	0.918667
0	5	5	10	20	67.2357	0.0154686	18.36	0.918
0	5	5	10	25	64.6518	0.0159797	23.18	0.9272
0	5	5	10	30	61.5718	0.0165593	27.74	0.924667
0	5	5	15	1	348.02	0.00302837	1	1
0	5	5	15	5	117.955	0.00904623	4.66	0.932
0	5	5	15	10	70.8877	0.0149239	9.16	0.916
0	5	5	15	15	57.1535	0.0182799	13.96	0.930667
0	5	5	15	20	49.0506	0.0210762	18.62	0.931
0	5	5	15	25	47.0161	0.0218229	23.36	0.9344

Aut	A	E	M	P	T_c	X	$\#token_{AVG}$	$\mathbb{P}$
0	**5**	**5**	**15**	**30**	**44.6489**	**0.0227464**	**27.84**	**0.928**
0	**5**	**10**	**3**	**1**	**348.9**	**0.00296939**	**1**	**1**
0	**5**	**10**	**3**	**5**	**199.58**	**0.00540943**	**4.54**	**0.908**
0	**5**	**10**	**3**	**10**	**197.638**	**0.00528324**	**9.12**	**0.912**
0	**5**	**10**	**3**	**15**	**189.411**	**0.00543643**	**14.1**	**0.94**
0	**5**	**10**	**3**	**20**	**184.32**	**0.00551537**	**18.34**	**0.917**
0	**5**	**10**	**3**	**25**	**189.484**	**0.00540974**	**23.18**	**0.9272**
0	**5**	**10**	**3**	**30**	**193.999**	**0.0053096**	**27.68**	**0.922667**
0	**5**	**10**	**5**	**1**	**337.64**	**0.00311604**	**1**	**1**
0	**5**	**10**	**5**	**5**	**171.996**	**0.00623823**	**4.56**	**0.912**
0	**5**	**10**	**5**	**10**	**159.465**	**0.0067247**	**9.12**	**0.912**
0	**5**	**10**	**5**	**15**	**155.327**	**0.00668467**	**13.94**	**0.929333**
0	**5**	**10**	**5**	**20**	**151.478**	**0.00690434**	**18.38**	**0.919**
0	**5**	**10**	**5**	**25**	**145.689**	**0.00705257**	**23.14**	**0.9256**
0	**5**	**10**	**5**	**30**	**137.765**	**0.00743645**	**27.78**	**0.926**
0	**5**	**10**	**10**	**1**	**343.14**	**0.00305527**	**1**	**1**
0	**5**	**10**	**10**	**5**	**119.961**	**0.00885364**	**4.78**	**0.956**
0	**5**	**10**	**10**	**10**	**78.9167**	**0.0134807**	**9.34**	**0.934**
0	**5**	**10**	**10**	**15**	**76.6204**	**0.0134935**	**13.84**	**0.922667**
0	**5**	**10**	**10**	**20**	**65.9564**	**0.0155454**	**18.54**	**0.927**
0	**5**	**10**	**10**	**25**	**61.9893**	**0.0165768**	**23**	**0.92**
0	**5**	**10**	**10**	**30**	**60.7503**	**0.016916**	**27.7**	**0.923333**
0	**5**	**10**	**15**	**1**	**328.38**	**0.00321576**	**1**	**1**
0	**5**	**10**	**15**	**5**	**118.088**	**0.00911336**	**4.68**	**0.936**
0	**5**	**10**	**15**	**10**	**70.6022**	**0.0148716**	**9.12**	**0.912**
0	**5**	**10**	**15**	**15**	**57.0293**	**0.0183982**	**13.74**	**0.916**
0	**5**	**10**	**15**	**20**	**51.9896**	**0.0198374**	**18.62**	**0.931**
0	**5**	**10**	**15**	**25**	**49.0269**	**0.0208811**	**23.48**	**0.9392**
0	**5**	**10**	**15**	**30**	**47.3114**	**0.0218013**	**27.22**	**0.907333**
0	**5**	**15**	**3**	**1**	**341.6**	**0.00308123**	**1**	**1**
0	**5**	**15**	**3**	**5**	**185.661**	**0.00575258**	**4.76**	**0.952**
0	**5**	**15**	**3**	**10**	**162.992**	**0.00643212**	**9.44**	**0.944**
0	**5**	**15**	**3**	**15**	**159.918**	**0.00646927**	**14.1**	**0.94**
0	**5**	**15**	**3**	**20**	**140.993**	**0.00723662**	**18.26**	**0.913**
0	**5**	**15**	**3**	**25**	**149.576**	**0.00689211**	**23.24**	**0.9296**
0	**5**	**15**	**3**	**30**	**151.949**	**0.00681533**	**27.8**	**0.926667**
0	**5**	**15**	**5**	**1**	**343.48**	**0.00306221**	**1**	**1**
0	**5**	**15**	**5**	**5**	**175.477**	**0.00620676**	**4.62**	**0.924**
0	**5**	**15**	**5**	**10**	**154.182**	**0.00714331**	**9.52**	**0.952**
0	**5**	**15**	**5**	**15**	**146.545**	**0.00722437**	**13.62**	**0.908**
0	**5**	**15**	**5**	**20**	**147.298**	**0.00710225**	**18.5**	**0.925**
0	**5**	**15**	**5**	**25**	**143.02**	**0.00724595**	**23.34**	**0.9336**

Aut	A	E	M	P	T_c	X	$\#token_{AVG}$	$\mathbb{P}$
0	**5**	**15**	**5**	**30**	**144.062**	**0.00710733**	**27.88**	**0.929333**
0	**5**	**15**	**10**	**1**	**340.94**	**0.00303927**	**1**	**1**
0	**5**	**15**	**10**	**5**	**120.532**	**0.00869882**	**4.48**	**0.896**
0	**5**	**15**	**10**	**10**	**81.9538**	**0.0129387**	**9.06**	**0.906**
0	**5**	**15**	**10**	**15**	**70.2613**	**0.0148466**	**13.78**	**0.918667**
0	**5**	**15**	**10**	**20**	**67.4406**	**0.0152433**	**18.34**	**0.917**
0	**5**	**15**	**10**	**25**	**64.8276**	**0.015788**	**23.48**	**0.9392**
0	**5**	**15**	**10**	**30**	**60.4232**	**0.016754**	**27.76**	**0.925333**
0	**5**	**15**	**15**	**1**	**326.18**	**0.00320893**	**1**	**1**
0	**5**	**15**	**15**	**5**	**118.456**	**0.0088995**	**4.6**	**0.92**
0	**5**	**15**	**15**	**10**	**71.0182**	**0.0150447**	**9.28**	**0.928**
0	**5**	**15**	**15**	**15**	**56.2704**	**0.0186755**	**13.86**	**0.924**
0	**5**	**15**	**15**	**20**	**50.1574**	**0.0205874**	**18.52**	**0.926**
0	**5**	**15**	**15**	**25**	**46.4848**	**0.0220698**	**23.08**	**0.9232**
0	**5**	**15**	**15**	**30**	**45.9925**	**0.0221532**	**27.62**	**0.920667**

Table 3 - Results for 50 simulations

From the results we obtain that:

- the cycle time is decreasing if the number of available resources is adequate with respect to the number of patients, if the resources don't satisfy the requests the system exceeds the limit and the cycle time increases;
- the throughput is increasing if the available resources are able to manage the requests, in case the saturated system obviously this value decreases;
- the average token number on which the simulations are based is almost the one considered, thanks to the decision function the majority of patients complete the clinical course;
- the average token number on which the simulations are based is almost that considered, so it means, as expected, that thanks to the decision function, most patients complete the clinical pathway;
- the probability associated with patients is in the set range.

To obtain better and more reliable results the analysis is carried out considering a larger number of simulations.

Obviously, as you can see from the tables, the cycle time trend remains the same, but in some cases the time difference between a simulation and the next, as the resources change, has been highlighted most by increasing the number of simulations.

Furthermore, the results highlighted show the cases in which the resources are adequate and don't bring the system to saturation.

The comparison of the results obtained in CPN Tools and those obtained from previous works with TimeNET is the following:

A	E	M	P	T_c $\#Sim10$	T_c $\#Sim20$	T_c $\#Sim50$	T_c $TimeNET$
1	**1**	**3**	**1**	**309.7**	**347.8**	**359.32**	**447.853**
1	**1**	**3**	**5**	**227.44**	**240.155**	**234.238**	**230.987**
1	**1**	**3**	**10**	**185.859**	**199.3**	**204.128**	**218.2931**
1	**1**	**3**	**15**	**196.474**	**191.063**	**200.868**	**225.412**
1	**1**	**3**	**20**	**179.061**	**189.21**	**183.96**	**215.455**
1	**1**	**5**	**1**	**365.9**	**360.65**	**346.74**	**486.192**
1	**1**	**5**	**5**	**183.18**	**188.543**	**197.693**	**177.0514**
1	**1**	**5**	**10**	**158.093**	**168.907**	**169.043**	**152.256**
1	**1**	**5**	**15**	**163.457**	**160.9**	**160.802**	**149.023**
1	**1**	**5**	**20**	**144.195**	**149.926**	**151.098**	**155.553**
1	**1**	**10**	**1**	**337.7**	**349.6**	**342.46**	**485.193**
1	**1**	**10**	**5**	**138.995**	**143.35**	**150.736**	**142.835**
1	**1**	**10**	**10**	**110.123**	**113.129**	**117.5**	**110.868**
1	**1**	**10**	**15**	**107.224**	**103.819**	**102.42**	**102.475**
1	**1**	**10**	**20**	**101.366**	**97.9202**	**98.3491**	**101.396**

Table 4 - Comparison results TimeNET - CPN Tools

As you can see from the results in the table, as mentioned previously, the values aren't exactly the same as we expected, given that the model we are considering is the real and not approximate model, but the cycle time trend when the resources vary turns out to be the same.

Therefore, from the analysis carried out it is possible to state that the transformation carried out is consistent with the case in question.

Another analysis of the system is performed in which it is considered that patients associated with the clinical pathway are all present in the Autotransfusion program.

The results obtained are as follows:

Aut	*A*	*E*	*M*	*P*	T_c – $\#Sim = 10$	T_c – $\#Sim = 20$	T_c – $\#Sim = 50$
1	**1**	**1**	**3**	**1**	**339.6**	**329.9**	**324.46**
5	**1**	**1**	**3**	**5**	**198.475**	**207.725**	**219.038**
10	**1**	**1**	**3**	**10**	**194.091**	**198.521**	**198.007**
15	**1**	**1**	**3**	**15**	**194.655**	**195.521**	**192.43**
20	**1**	**1**	**3**	**20**	**186.29**	**190.736**	**190.43**
25	**1**	**1**	**3**	**25**	**178.56**	**185.448**	**192.868**
30	**1**	**1**	**3**	**30**	**198.887**	**193.694**	**188.798**
1	**1**	**1**	**5**	**1**	**367.4**	**351.2**	**331.98**
5	**1**	**1**	**5**	**5**	**194.899**	**185.405**	**179.48**
10	**1**	**1**	**5**	**10**	**177.378**	**163.962**	**163.876**
15	**1**	**1**	**5**	**15**	**157.098**	**153.283**	**158.005**
20	**1**	**1**	**5**	**20**	**154.913**	**156.275**	**158.009**
25	**1**	**1**	**5**	**25**	**151.358**	**148.016**	**151.401**
30	**1**	**1**	**5**	**30**	**147.836**	**146.765**	**143.564**
1	**1**	**1**	**10**	**1**	**353.5**	**317.45**	**304.04**
5	**1**	**1**	**10**	**5**	**137.405**	**139.736**	**137.376**
10	**1**	**1**	**10**	**10**	**103.843**	**106.884**	**106.898**
15	**1**	**1**	**10**	**15**	**88.2228**	**86.0184**	**92.9292**
20	**1**	**1**	**10**	**20**	**87.451**	**90.1744**	**87.934**
25	**1**	**1**	**10**	**25**	**86.0195**	**87.1534**	**85.2547**
30	**1**	**1**	**10**	**30**	**85.4438**	**84.5465**	**84.4269**
1	**1**	**1**	**15**	**1**	**341.7**	**322.7**	**323.82**
5	**1**	**1**	**15**	**5**	**120.195**	**141.387**	**139.915**
10	**1**	**1**	**15**	**10**	**101.386**	**103.249**	**97.8008**
15	**1**	**1**	**15**	**15**	**88.612**	**90.0043**	**91.2164**
20	**1**	**1**	**15**	**20**	**82.762**	**83.5993**	**84.6794**
25	**1**	**1**	**15**	**25**	**76.5667**	**79.8039**	**81.9264**
30	**1**	**1**	**15**	**30**	**82.8015**	**82.7268**	**82.1564**
1	**1**	**2**	**3**	**1**	**320**	**322.5**	**330.28**
5	**1**	**2**	**3**	**5**	**204.865**	**185.32**	**176.902**
10	**1**	**2**	**3**	**10**	**141.762**	**157.471**	**162.678**
15	**1**	**2**	**3**	**15**	**138.639**	**151.211**	**151.442**

Aut	A	E	M	P	T_c	X	$\#token_{AVG}$
20	**1**	**2**	**3**	**20**	**132.282**	**139.696**	**152.817**
25	**1**	**2**	**3**	**25**	**130.679**	**149.787**	**142.709**
30	**1**	**2**	**3**	**30**	**156.113**	**147.462**	**150.042**
1	**1**	**2**	**5**	**1**	**348.3**	**322.5**	**330.28**
5	**1**	**2**	**5**	**5**	**195.195**	**185.35**	**176.902**
10	**1**	**2**	**5**	**10**	**169.919**	**157.471**	**162.678**
15	**1**	**2**	**5**	**15**	**140.597**	**147.462**	**151.442**
20	**1**	**2**	**5**	**20**	**140.21**	**139.696**	**152.817**
25	**1**	**2**	**5**	**25**	**143.093**	**149.787**	**142.709**
30	**1**	**2**	**5**	**30**	**158.662**	**151.211**	**150.042**

Table 5 - Simulation results in which all patients are in Autotransfusion

Also in this case, as can be seen from the results in the table, the trend of the cycle time decreases as the resources change. We can verify that in some cases of functioning the saturated system because the number of available resources isn't sufficient to dispose of the requests, i.e. the number of common resources is not sufficient to dispose of the number of patients.

In the case where $A = 1, E = 1, M = 10$ and varying the number of patients the system is controllable under these conditions, but as we can see from the following simulations increasing the number of doctors doesn't get any benefit because the system starts to be again not controllable (saturated).

Consequently, that in this case it is necessary to increase the other resources.

5.3 Evaluation of clinical scenario performance

Cycle time is evaluated considering a clinical scenario defined by two clinical pathways (CADERA and RODILLA).

The SWN model obtained by HEAT-Designer has been simulated in CPN Tools and the network obtained is as follows:

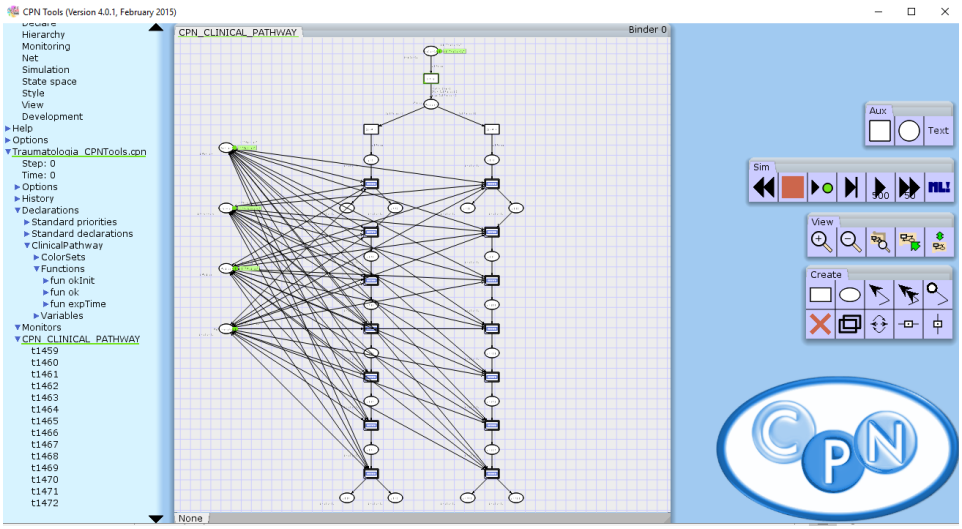

Figure 24 - SWN model of a clinical scenario consisting of two clinical paths in CPN Tools

In the case contemplated are considered $\#\,Sim = 10$, $\#\,Sim = 20$ simulations.

In particular, 100 simulations of the network were carried out in CPN Tools and some simulations were considered for the analysis.

Patients present in the clinical scenario were also considered in the Autotransfusion program.

The input parameters corresponding to the first 4 columns are:

- Aut - Autotransfusion
- A - Auxiliaries
- E - Nurses
- M - Doctors
- P - Patients

While in output we have the cycle time T_c.

In order to perform the analysis, the decision function was used again so that the patients (tokens) were divided according to the probability value assigned to the clinical course.

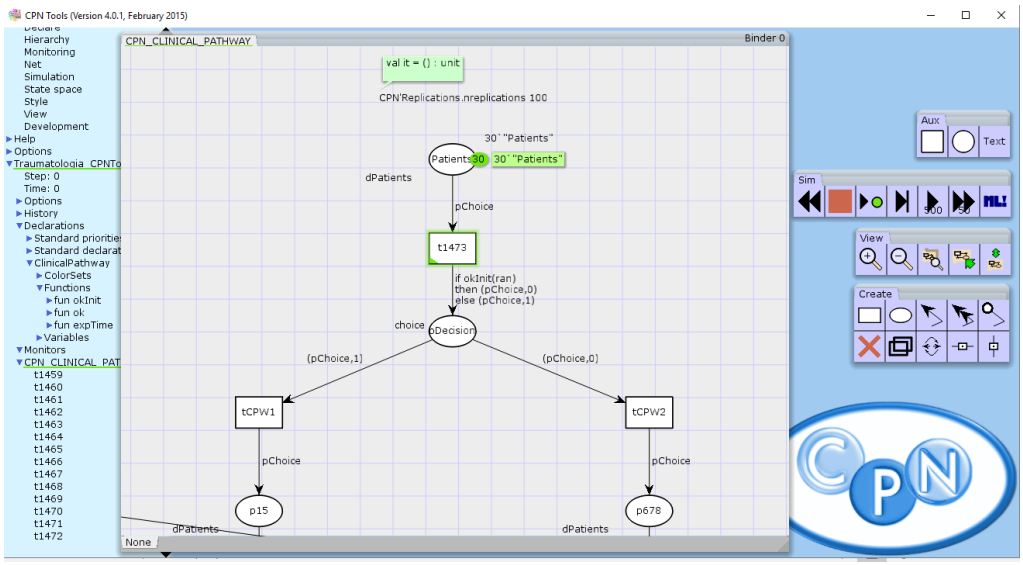

Figure 25 - CPN Tools decision function

Three different simulations were performed in which the following values are considered:

$$CADERA\ 50\% - RODILLA\ 50\%$$

$$CADERA\ 30\% - RODILLA\ 70\%$$

$$CADERA\ 70\% - RODILLA\ 30\%$$

The results obtained from the tests are shown below.

$$CASE - CADERA\ 50\%\ and\ 50\%\ RODILLA$$

$$\#Sim = 10$$

$$CADERA\ RESULTS$$

Aut	A	E	M	P	T_c	X	$\#token_{AVG}$	$\mathbb{P}$
1	1	1	3	1	324.7	0.00328527	1	1
5	1	1	3	5	321.643	0.00331928	3.2	0.64
10	1	1	3	10	310.789	0.00352985	6	0.6
15	1	1	3	15	280.997	0.00377109	8.1	0.54
20	1	1	3	20	343.951	0.00297842	12.2	0.488
25	1	1	3	25	365.73	0.00281798	15.4	0.513333
30	1	1	3	30	375.845	0.00276309	8.9	0.445
1	1	1	5	1	368.1	0.00284846	1	1
5	1	1	5	5	319.565	0.00333558	6.6	0.44
10	1	1	5	10	308.865	0.00340031	8.6	0.43
15	1	1	5	15	307.675	0.00352872	2.9	0.58
20	1	1	5	20	300.992	0.00352849	4.5	0.45
25	1	1	5	25	287.899	0.00358933	11.4	0.456
30	1	1	5	30	261.255	0.00386333	14.7	0.49
1	1	1	10	1	306.9	0.0033586	1	1
5	1	1	10	5	262.895	0.00422325	4.6	0.46
10	1	1	10	10	236.95	0.00472437	2.9	0.58
15	1	1	10	15	212.753	0.00541739	8	0.533333
20	1	1	10	20	207.715	0.00490628	13.6	0.453333
25	1	1	10	25	196.366	0.00532277	12.2	0.488
30	1	1	10	30	193.997	0.00527078	9.3	0.465
1	1	1	15	1	296.963	0.003918	1	1
5	1	1	15	5	279.3	0.00402787	2.4	0.48
10	1	1	15	10	249.26	0.00445207	4.8	0.48
15	1	1	15	15	232.562	0.00439258	6.8	0.453333
20	1	1	15	20	194.03	0.00532273	9.8	0.49
25	1	1	15	25	185.046	0.00560564	12.7	0.508
30	1	1	15	30	209.127	0.00496807	13.3	0.443333

Table 6 - Clinical Result: Cadera (50% - 10 simulations)

$$CASE - CADERA\ 50\%\ and\ 50\%\ RODILLA$$

$$Sim = 10$$

$$RODILLA\ RESULTS$$

Aut	A	E	M	P	T_c	X	$\#token_{AVG}$	$\mathbb{P}$
1	1	1	3	1	397	0.00259918	1	1
5	1	1	3	5	391.1	0.00262694	3.1	0.62
10	1	1	3	10	378.203	0.00273142	5.3	0.53
15	1	1	3	15	339.675	0.00301815	7.8	0.52
20	1	1	3	20	363.498	0.00281208	10.1	0.505
25	1	1	3	25	368.903	0.00274386	11.6	0.464
30	1	1	3	30	378.572	0.00277757	15.1	0.503333
1	1	1	5	1	376.6	0.00269844	1	1
5	1	1	5	5	316.658	0.00326959	2.7	0.54
10	1	1	5	10	313.859	0.00329384	5.4	0.54
15	1	1	5	15	310.426	0.00343874	7	0.466667
20	1	1	5	20	294.249	0.00348192	9.5	0.475
25	1	1	5	25	288.918	0.00358786	11.9	0.476
30	1	1	5	30	283.416	0.00361956	13.2	0.44
1	1	1	10	1	277.658	0.00395887	1	1
5	1	1	10	5	375.1	0.00273526	2.7	0.54
10	1	1	10	10	251.962	0.00413558	4.7	0.47
15	1	1	10	15	241.129	0.00445566	7	0.466667
20	1	1	10	20	233.51	0.00435704	10.4	0.416
25	1	1	10	25	220.303	0.00466202	9	0.45
30	1	1	10	30	219.9	0.00464577	14.1	0.47
1	1	1	15	1	427.1	0.00234946	1	1
5	1	1	15	5	301.508	0.0034966	2.5	0.5
10	1	1	15	10	252.633	0.00417363	5.2	0.52
15	1	1	15	15	227.875	0.00452567	8	0.533333
20	1	1	15	20	224.045	0.00463609	10.8	0.432
25	1	1	15	25	213.75	0.0047806	14.1	0.47
30	1	1	15	30	237.853	0.00432563	8.6	0.43

Table 7 - Clinical Result: Rodilla (50% - 10 simulations)

$$CASE - CADERA\ 50\%\ and\ 50\%\ RODILLA$$

$$\#Sim = 20$$

$$CADERA\ RESULTS$$

Aut	A	E	M	P	T_c	X	$\#token_{AVG}$	$\mathbb{P}$
1	1	1	3	1	330.214	0.00324617	1	1
5	1	1	3	5	329.9	0.00319602	3.1	0.62
10	1	1	3	10	322.457	0.00331134	5.65	0.565
15	1	1	3	15	316.768	0.00335503	7.6	0.506667
20	1	1	3	20	346.356	0.00299169	9.05	0.4525
25	1	1	3	25	348.64	0.00296493	12.5	0.5
30	1	1	3	30	392.87	0.00264632	15.55	0.518333
1	1	1	5	1	343.5	0.00306382	1	1
5	1	1	5	5	302.858	0.00357225	2.9	0.58
10	1	1	5	10	311.147	0.00338114	4.5	0.45
15	1	1	5	15	301.104	0.00354321	7	0.466667
20	1	1	5	20	299.403	0.00355382	8.4	0.42
25	1	1	5	25	265.126	0.00390916	11.95	0.478
30	1	1	5	30	260.056	0.00395523	14.35	0.478333
1	1	1	10	1	305.9	0.0034198	1	1
5	1	1	10	5	251.158	0.00433486	2.5	0.5
10	1	1	10	10	235.09	0.00466841	4.9	0.49
15	1	1	10	15	234.832	0.00486062	7.2	0.48
20	1	1	10	20	212.675	0.00500761	9.05	0.4525
25	1	1	10	25	210.363	0.00485751	11.5	0.46
30	1	1	10	30	198.302	0.00516592	13.6	0.453333
1	1	1	15	1	304	0.0036107	1	1
5	1	1	15	5	268.726	0.00437903	2.65	0.53
10	1	1	15	10	217.316	0.00497689	5.3	0.53
15	1	1	15	15	221.867	0.00468629	7.1	0.473333
20	1	1	15	20	198.814	0.0053236	9.5	0.475
25	1	1	15	25	193.831	0.00545274	12.3	0.492
30	1	1	15	30	200.999	0.00514562	13.8	0.46

Table 8 – Clinical Scenario Result: Cadera (50% - 20 simulations)

$$CASE - CADERA\ 50\%\ and\ 50\%\ RODILLA$$

$$\#Sim = 20$$

$$RODILLA\ RESULTS$$

Aut	A	E	M	P	T_c	X	$\#token_{AVG}$	$\mathbb{P}$
1	1	1	3	1	378.75	0.00273216	1	1
5	1	1	3	5	360.808	0.00287621	3.35	0.67
10	1	1	3	10	377.423	0.00272673	5.25	0.525
15	1	1	3	15	341.3	0.00301537	7.8	0.52
20	1	1	3	20	371.981	0.00273635	10.4	0.52
25	1	1	3	25	381.412	0.0027311	11.3	0.452
30	1	1	3	30	371.766	0.00273016	13.95	0.465
1	1	1	5	1	371.25	0.00277444	1	1
5	1	1	5	5	342.464	0.00311304	2.7	0.54
10	1	1	5	10	288.811	0.00359935	5.3	0.53
15	1	1	5	15	315.103	0.00337236	6.8	0.453333
20	1	1	5	20	287.019	0.00360365	10.1	0.505
25	1	1	5	25	293.491	0.00352038	11.55	0.462
30	1	1	5	30	301.747	0.00343303	13.55	0.451667
1	1	1	10	1	381.55	0.00268854	1	1
5	1	1	10	5	264.896	0.00409554	2.85	0.57
10	1	1	10	10	267.681	0.00399864	4.4	0.44
15	1	1	10	15	232.149	0.00454958	7.4	0.493333
20	1	1	10	20	232.306	0.00445182	9.2	0.46
25	1	1	10	25	223.913	0.00455993	11.25	0.45
30	1	1	10	30	218.067	0.00466945	14.55	0.485
1	1	1	15	1	398.2	0.00257319	1	1
5	1	1	15	5	311.713	0.00342258	2.4	0.48
10	1	1	15	10	257.256	0.00408607	4.6	0.46
15	1	1	15	15	211.982	0.00479895	7.5	0.5
20	1	1	15	20	222.757	0.00459211	9	0.45
25	1	1	15	25	222.534	0.00459465	11.25	0.45
30	1	1	15	30	228.523	0.00452372	13.65	0.455

Table 9 – Clinical Scenario Result: Rodilla (50% - 20 simulations)

$$CASO - CADERA\ 70\%\ and\ 30\%\ RODILLA$$

$$\#Sim = 10$$

$$CADERA\ RESULTS$$

Aut	A	E	M	P	T_c	X	$\#token_{AVG}$	$\mathbb{P}$
1	1	1	3	1	338	0.00310032	1	1
5	1	1	3	5	331.283	0.00318458	3	0.6
10	1	1	3	10	290.07	0.00377785	6.8	0.68
15	1	1	3	15	258.059	0.00394592	10.3	0.686667
20	1	1	3	20	266.477	0.00382631	13.2	0.66
25	1	1	3	25	276.25	0.00384267	16.7	0.668
30	1	1	3	30	280.012	0.00370585	18.6	0.62
1	1	1	5	1	307.6	0.00336011	1	1
5	1	1	5	5	255.6	0.00410868	3.2	0.64
10	1	1	5	10	252.91	0.00407617	6.3	0.63
15	1	1	5	15	256.937	0.00423321	9.6	0.64
20	1	1	5	20	209.872	0.00493191	13.3	0.665
25	1	1	5	25	216.799	0.00479245	16.5	0.66
30	1	1	5	30	228.068	0.00446546	19.6	0.653333
1	1	1	10	1	326.1	0.00321324	1	1
5	1	1	10	5	196.753	0.00599681	3.5	0.7
10	1	1	10	10	160.3	0.00673587	6.6	0.66
15	1	1	10	15	131.123	0.00790814	10.3	0.686667
20	1	1	10	20	131.075	0.00784146	14.6	0.73
25	1	1	10	25	129.76	0.00785903	17.3	0.692
30	1	1	10	30	128.962	0.00803054	20	0.666667
1	1	1	15	1	307.4	0.00349706	1	1
5	1	1	15	5	163.68	0.00640803	4	0.8
10	1	1	15	10	163.543	0.00679474	6.4	0.64
15	1	1	15	15	159.135	0.0069105	8.7	0.58
20	1	1	15	20	132.697	0.00769809	13.6	0.68
25	1	1	15	25	143.954	0.00719707	15.3	0.612
30	1	1	15	30	154.66	0.00660712	17.7	0.59

Table 10 – Clinical Scenario Result: Cadera (70% - 10 simulations)

$$CASO - CADERA\ 70\%\ and\ 30\%\ RODILLA$$

$$\#Sim = 10$$

RODILLA RESULTS

Aut	*A*	*E*	*M*	*P*	T_c	*X*	$\#token_{AVG}$	$\mathbb{P}$
1	1	1	3	1	427.3	0.00237841	1	1
5	1	1	3	5	347.518	0.00306118	2.9	0.58
10	1	1	3	10	407.3	0.00247701	4	0.4
15	1	1	3	15	414.939	0.00242385	5.2	0.346667
20	1	1	3	20	403.233	0.00250134	6.8	0.34
25	1	1	3	25	400.196	0.00252715	7.8	0.312
30	1	1	3	30	416.763	0.00242226	9.6	0.32
1	1	1	5	1	409.1	0.00247649	1	1
5	1	1	5	5	336.208	0.00314342	2.4	0.48
10	1	1	5	10	352.792	0.00288282	3.4	0.34
15	1	1	5	15	392.258	0.00259465	4.9	0.326667
20	1	1	5	20	344.493	0.00300022	5.3	0.265
25	1	1	5	25	372.312	0.00275188	6.9	0.276
30	1	1	5	30	372.26	0.00280436	7.8	0.26
1	1	1	10	1	391.9	0.0025832	1	1
5	1	1	10	5	338.565	0.00311718	2.2	0.44
10	1	1	10	10	323.383	0.003212	3.1	0.31
15	1	1	10	15	319.818	0.00326746	4.2	0.28
20	1	1	10	20	314.606	0.00322383	5.2	0.26
25	1	1	10	25	306.704	0.00339418	5.9	0.236
30	1	1	10	30	280.548	0.00367338	8.3	0.276667
1	1	1	15	1	427.3	0.00237841	1	1
5	1	1	15	5	327.167	0.00313407	2	0.4
10	1	1	15	10	293.772	0.00356741	3.8	0.38
15	1	1	15	15	264.373	0.00388492	5.7	0.38
20	1	1	15	20	258.601	0.00391378	5.5	0.275
25	1	1	15	25	298.073	0.0034492	6.9	0.276
30	1	1	15	30	304.659	0.00345948	9.3	0.31

Table 11 – Clinical Scenario Result: Rodilla (30% - 10 simulations)

$$CASO - CADERA\ 70\%\ and\ 30\%\ RODILLA$$

$$\#Sim = 20$$

$$CADERA\ RESULTS$$

Aut	A	E	M	P	T_c	X	$\#token_{AVG}$	$\mathbb{P}$
1	1	1	3	1	340.25	0.00305317	1	1
5	1	1	3	5	306.143	0.00353718	3.5	0.7
10	1	1	3	10	273.183	0.00403861	7.1	0.71
15	1	1	3	15	272.552	0.00378501	9.7	0.646667
20	1	1	3	20	276.085	0.00371612	12.55	0.6275
25	1	1	3	25	274.444	0.00371604	16.75	0.67
30	1	1	3	30	294.257	0.00355202	19	0.633333
1	1	1	5	1	310.1	0.00338956	1	1
5	1	1	5	5	277.912	0.00389241	2.9	0.58
10	1	1	5	10	236.624	0.00444351	6.95	0.695
15	1	1	5	15	236.361	0.00456016	15.55	0.622
20	1	1	5	20	229.444	0.00464091	9.75	0.65
25	1	1	5	25	225.764	0.00465717	12.55	0.6275
30	1	1	5	30	217.269	0.00471891	18.95	0.631667
1	1	1	10	1	325.6	0.00319313	1	1
5	1	1	10	5	213.081	0.00537062	3.35	0.67
10	1	1	10	10	166.299	0.00644683	6.25	0.625
15	1	1	10	15	151.77	0.00695042	9.95	0.663333
20	1	1	10	20	144.782	0.00726642	13	0.65
25	1	1	10	25	144.022	0.00713684	17.25	0.69
30	1	1	10	30	137.513	0.00751666	19.1	0.636667
1	1	1	15	1	306.45	0.00351117	1	1
5	1	1	15	5	186.797	0.00581384	3.65	0.73
10	1	1	15	10	181.465	0.0060359	5.95	0.595
15	1	1	15	15	152.21	0.00709673	9.15	0.61
20	1	1	15	20	141.187	0.00734244	13.2	0.66
25	1	1	15	25	141.205	0.00731592	15.85	0.634
30	1	1	15	30	154.148	0.00675629	18.1	0.603333

Table 12 – Clinical Scenario Result: Cadera (70% - 20 simulations)

$$CASO - CADERA\ 70\%\ and\ 30\%\ RODILLA$$

$$\#Sim = 20$$

$$RODILLA\ RESULTS$$

Aut	A	E	M	P	T_c	X	$\#token_{AVG}$	$\mathbb{P}$
1	1	1	3	1	411.735	0.00245317	4.1	0.41
5	1	1	3	5	416.41	0.00241554	5.1	0.34
10	1	1	3	10	408.457	0.00248297	7	0.35
15	1	1	3	15	397.231	0.00255698	7.85	0.314
20	1	1	3	20	390.058	0.00263902	9.6	0.32
25	1	1	3	25	334.204	0.00313654	2.35	0.47
30	1	1	3	30	359.54	0.00284701	3.4	0.34
1	1	1	5	1	368.848	0.0027819	4.85	0.323333
5	1	1	5	5	364.478	0.00283645	5.85	0.2925
10	1	1	5	10	357.507	0.00288321	7.6	0.304
15	1	1	5	15	347.449	0.00301756	8.95	0.298333
20	1	1	5	20	347.483	0.00298632	2	0.4
25	1	1	5	25	326.13	0.00325646	3.25	0.325
30	1	1	5	30	313.898	0.00332194	4.35	0.29
1	1	1	10	1	317.845	0.00325628	6.05	0.3025
5	1	1	10	5	291.284	0.00352136	6.05	0.242
10	1	1	10	10	276.829	0.00369653	9.05	0.301667
15	1	1	10	15	293.655	0.00353517	1.95	0.39
20	1	1	10	20	302.964	0.00350394	3.8	0.38
25	1	1	10	25	307.149	0.00336577	4.9	0.326667
30	1	1	10	30	345.15	0.00299475	5.5	0.275
1	1	1	15	1	296.424	0.00351329	7.1	0.284
5	1	1	15	5	266.341	0.00380579	9.2	0.306667
10	1	1	15	10	397.231	0.00255698	4.1	0.41
15	1	1	15	15	390.058	0.00263902	5.1	0.34
20	1	1	15	20	408.457	0.00248297	7	0.35
25	1	1	15	25	411.735	0.00245317	7.85	0.314
30	1	1	15	30	416.41	0.00241554	9.6	0.32

Table 13 – Clinical Scenario Result: Cadera (30% - 20 simulations)

CASE* 30% *CADERA and* 70% *RODILLA

$\#Sim = 10$

CADERA RESULTS

Aut	*A*	*E*	*M*	*P*	T_c	*X*	$\#token_{AVG}$	$\mathbb{P}$
1	1	1	3	1	313.5	0.00338089	1	1
5	1	1	3	5	364.717	0.00279888	2.4	0.48
10	1	1	3	10	374.03	0.00270687	3.9	0.39
15	1	1	3	15	390.671	0.0025889	5	0.333333
20	1	1	3	20	391.329	0.00260438	5.8	0.29
25	1	1	3	25	409.478	0.00247617	7.3	0.292
30	1	1	3	30	419.115	0.00245136	8.5	0.283333
1	1	1	5	1	310.3	0.00347607	1	1
5	1	1	5	5	356.261	0.00294795	2.2	0.44
10	1	1	5	10	341.443	0.00305918	4.3	0.43
15	1	1	5	15	327.772	0.00318802	4.3	0.286667
20	1	1	5	20	296.515	0.00371597	5	0.25
25	1	1	5	25	295.2	0.00353187	7.7	0.308
30	1	1	5	30	291.171	0.00362062	9.1	0.303333
1	1	1	10	1	298.75	0.00380886	1.8	1.8
5	1	1	10	5	296.053	0.00355273	2.1	0.42
10	1	1	10	10	290.205	0.00377835	3.5	0.35
15	1	1	10	15	286.092	0.00364755	4.4	0.293333
20	1	1	10	20	280.6	0.00400055	6.6	0.33
25	1	1	10	25	270.773	0.00393425	6.6	0.264
30	1	1	10	30	229.573	0.00450001	8	0.266667
1	1	1	15	1	320.167	0.00338284	1	1
5	1	1	15	5	314.738	0.0034162	1.7	0.34
10	1	1	15	10	313.343	0.0034034	3.3	0.33
15	1	1	15	15	312.3	0.00342882	4.2	0.28
20	1	1	15	20	265.271	0.00398705	5.8	0.29
25	1	1	15	25	224.563	0.00458621	6.5	0.26
30	1	1	15	30	310.105	0.00345621	9.5	0.316667

Table 14 – Clinical Scenario Result: Cadera (30% - 10 simulations)

CASO* 30% *CADERA and* 70% *RODILLA

$\#Sim = 10$

RODILLA RESULTS

Aut	*A*	*E*	*M*	*P*	T_c	*X*	$\#token_{AVG}$	$\mathbb{P}$
1	1	1	3	1	377.6	0.00276323	1	1
5	1	1	3	5	374.37	0.00275145	3.6	0.72
10	1	1	3	10	307.812	0.00338953	7.3	0.73
15	1	1	3	15	299.844	0.00341712	10.2	0.68
20	1	1	3	20	311.103	0.0033121	12.4	0.62
25	1	1	3	25	301.839	0.00342194	13.1	0.524
30	1	1	3	30	295.772	0.00349856	18.7	0.623333
1	1	1	5	1	403.4	0.00250835	1	1
5	1	1	5	5	311.033	0.00358543	2.8	0.56
10	1	1	5	10	277.85	0.00380941	6.4	0.64
15	1	1	5	15	264.091	0.00396715	10.4	0.693333
20	1	1	5	20	261.176	0.00404326	13.3	0.665
25	1	1	5	25	239.022	0.00428649	15.6	0.624
30	1	1	5	30	241.488	0.00422125	18.6	0.62
1	1	1	10	1	223.085	0.00474672	4.1	4.1
5	1	1	10	5	220.495	0.00479709	3.2	0.64
10	1	1	10	10	208.99	0.00500555	6.4	0.64
15	1	1	10	15	185.083	0.00550228	9.7	0.646667
20	1	1	10	20	190.109	0.0053236	11.8	0.59
25	1	1	10	25	181.923	0.00553818	15.2	0.608
30	1	1	10	30	177.338	0.00570406	20.2	0.673333
1	1	1	15	1	346.2	0.00297356	1	1
5	1	1	15	5	244.773	0.00448056	3.7	0.74
10	1	1	15	10	195.826	0.00530446	6.6	0.66
15	1	1	15	15	165.931	0.00616218	10.7	0.713333
20	1	1	15	20	168.656	0.0059484	12.9	0.645
25	1	1	15	25	171.039	0.00586668	16.8	0.672
30	1	1	15	30	174.686	0.00578558	19.4	0.646667

Table 15 – Clinical Scenario Result: Cadera (70% - 10 simulations)

CASO* 30% *CADERA and* 70% *RODILLA

$\#Sim = 20$

CADERA RESULTS

Aut	*A*	*E*	*M*	*P*	T_c	*X*	$\#token_{AVG}$	$\mathbb{P}$
1	1	1	3	1	391.64	0.00261615	1	1
5	1	1	3	5	363.612	0.00280354	2.4	0.48
10	1	1	3	10	367.189	0.00286554	4.1	0.41
15	1	1	3	15	303.25	0.00347131	4.85	0.323333
20	1	1	3	20	376.794	0.00271905	5.9	0.295
25	1	1	3	25	394.328	0.00259743	6.75	0.27
30	1	1	3	30	405.855	0.00255613	9.4	0.313333
1	1	1	5	1	356.311	0.00292744	2.15	0.43
5	1	1	5	5	332.822	0.00314466	3.85	0.385
10	1	1	5	10	332.088	0.00311344	4.4	0.293333
15	1	1	5	15	327.717	0.00329868	5.1	0.255
20	1	1	5	20	323.75	0.00335888	7.4	0.296
25	1	1	5	25	311.345	0.00351608	8.9	0.296667
30	1	1	5	30	300.708	0.00366849	1.95	0.95
1	1	1	10	1	322.5	0.00332723	1	1
5	1	1	10	5	288.642	0.00376745	2.15	0.43
10	1	1	10	10	292.858	0.00357662	3.25	0.325
15	1	1	10	15	287.026	0.0037869	4.85	0.323333
20	1	1	10	20	283.511	0.00380901	6.85	0.274
25	1	1	10	25	272.071	0.00402722	6.1	0.305
30	1	1	10	30	271.791	0.00388172	8.55	0.285
1	1	1	15	1	363.612	0.00280354	1.85	0.37
5	1	1	15	5	294.194	0.00374085	3.45	0.345
10	1	1	15	10	293.68	0.00360468	4.5	0.3
15	1	1	15	15	281.979	0.00373798	5.75	0.2875
20	1	1	15	20	248.349	0.00424598	7.1	0.284
25	1	1	15	25	287.95	0.00362664	9.05	0.301667
30	1	1	15	30	319.083	0.00339929	2.4	0.48

Table 16 – Clinical Scenario Result: Cadera (30% - 20 simulations)

CASO* 30% *CADERA and* 70% *RODILLA

$\#Sim = 20$

RODILLA RESULTS

Aut	*A*	*E*	*M*	*P*	T_c	*X*	$\#token_{AVG}$	$\mathbb{P}$
1	1	1	3	1	368.8	0.00287822	1	1
5	1	1	3	5	354.386	0.00298052	3.55	0.71
10	1	1	3	10	324.12	0.0032182	6.65	0.665
15	1	1	3	15	324.794	0.00323681	9.6	0.64
20	1	1	3	20	297.653	0.0034904	13.15	0.6575
25	1	1	3	25	284.216	0.00360199	13.8	0.552
30	1	1	3	30	290.253	0.00359355	19.6	0.653333
1	1	1	5	1	401.45	0.00254352	1	1
5	1	1	5	5	301.04	0.00361011	3.2	0.64
10	1	1	5	10	284.331	0.00365175	5.9	0.59
15	1	1	5	15	252.432	0.00409109	10.15	0.676667
20	1	1	5	20	242.314	0.00428132	13.45	0.6725
25	1	1	5	25	235.532	0.00436033	15.95	0.638
30	1	1	5	30	237.193	0.00434108	18.8	0.626667
1	1	1	10	1	244.744	0.00451147	3.6	3.6
5	1	1	10	5	220.854	0.0048096	3.5	0.7
10	1	1	10	10	202.989	0.00513774	6.45	0.645
15	1	1	10	15	181.238	0.00560869	10.1	0.673333
20	1	1	10	20	182.623	0.00556936	12.3	0.615
25	1	1	10	25	179.322	0.00568687	15.7	0.628
30	1	1	10	30	178.792	0.00566407	19.1	0.636667
1	1	1	15	1	355.9	0.00290372	1	1
5	1	1	15	5	259.678	0.00416234	3.4	0.68
10	1	1	15	10	191.671	0.00553877	6.85	0.685
15	1	1	15	15	180.746	0.00577561	9.95	0.663333
20	1	1	15	20	170.771	0.00589674	12.85	0.6425
25	1	1	15	25	167.244	0.0060069	16.35	0.654
30	1	1	15	30	177.782	0.00574691	19.45	0.648333

Table 17 – Clinical Scenario Result: Cadera (70% - 20 simulations)

Also in this case from the analysis carried out with respect to the clinical scenario it is possible to notice the behavior of the system, in fact the cycle time is decreasing if the number of available resources is adequate with respect to the number of patients, if the resources do not satisfy the the saturated system is required and the cycle time increases; the throughput is increasing if the available resources manage the requests, in case the saturated system obviously this value decreases.

With regard to the average token token number and the patient-dependent probability values that terminate the clinical pathways this time, the probabilistic value associated with the clinical route must also be taken into account.

About the cycle time it is possible to notice that in the case in which the clinical paths are unlikely distributed the times are comparable with each other, while in the case in which a greater probability value is associated to a clinical path the cycle time will be lower because the available resources will be better exploited compared to the number of patients.

Conclusions

This thesis presents a methodology for hospital management based on modeling and analysis of clinical pathways.

A UML-based graphic model with few primitives is proposed that allows physicians to model paths in a simple way and to which are added HSS specifications that allow to add information to the considered path.

To support system simulations, the UML model is transformed into a mathematical model, using SWN models, through well-defined rules.

The methodology was used to develop the HEAT software platform that has been installed in the hospital and to which changes have been made in order to be able to perform a better performance analysis by inserting the automatic transformation mechanism that allowed the SWN network to be obtained with CPN Tools.

The transformation made was reliable and led to results that realistically reflect what happens in the real system.

Using the CPN Tools software it has been possible to obtain satisfactory results in terms of cycle time of the process.

In particular, the analysis allowed to highlight the behavior of the system with respect to the time taken by the patients to complete the clinical pathways and allowed to define which are the combinations of resources that allow patients to follow the clinical path in the shortest time possible.

Once the analysis and the synthesis are finished, the results can be shown to the doctors in a form that can be understood by them, for example, by means of a monitor.

This last step was not considered and could be one of the future tasks.

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