

Display Adaptive 3D Content Remapping

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Abstract

Glasses-free automultiscopic displays are on the verge of becoming a standard technology in consumer products. These displays are capable of producing the illusion of 3D content without the need of any additional eyewear. However, due to limitations in angular resolution, they can only show a limited depth of field, which translates into blurred-out areas whenever an object extrudes beyond a certain depth. Moreover, the blurring is device-specific, due to the different constraints of each display. We introduce a novel display-adaptive light field retargeting method, to provide high-quality, blur-free viewing experiences of the same content on a variety of display types, ranging from hand-held devices to movie theaters. We pose the problem as an optimization, which aims at modifying the original light field so that the displayed content appears sharp while preserving the original perception of depth. In particular, we run the optimization on the central view and use warping to synthesize the rest of the light field. We validate our method using existing objective metrics for both image quality (blur) and perceived depth. The proposed framework can also be applied to retargeting disparities in stereoscopic image displays, supporting both dichotomous and non-dichotomous comfort zones.

Keywords: stereo, displays, automultiscopic, content retargeting.

1. Introduction

Within the last years, stereoscopic and automultiscopic displays have started to enter the consumer market from all angles. These displays can show three-dimensional objects that appear to be floating in front of or behind the physical screen, even without the use of additional eyewear. Capable of electronically switching between a full-resolution 2D and a lower-resolution 3D mode, parallax barrier technology [1] is dominant for hand-held and tablet-sized devices, while medium-sized displays most often employ arrays of microlenses [2]. Although most cinema screens today are stereoscopic and rely on additional eyewear, large-scale automultiscopic projection systems are an emerging technology [3]. Each technology has its own particular characteristics, including field of view, depth of field, contrast, resolution, and screen size. Counterintuitively, produced content is usually targeted toward a single display configuration, making labor-intensive, manual post-processing of the recorded or rendered data necessary.

Display-adaptive content retargeting is common practice for attributes such as image size, dynamic range (tone mapping), color gamut, and spatial resolution [4]. In order to counteract the accommodation-convergence mismatch of stereoscopic displays, stereoscopic disparity retargeting methods have recently been explored [5, 6, 7, 8, 9]. These techniques are successful in modifying the disparities of a stereo image pair so that visual discomfort of the observer is mitigated while preserving the three-dimensional appearance of the scene as much as possible. Inspired by these techniques, we tackle the problem of 3D content retargeting for glasses-free light field (i.e. automultiscopic) displays. These displays exhibit a device-specific

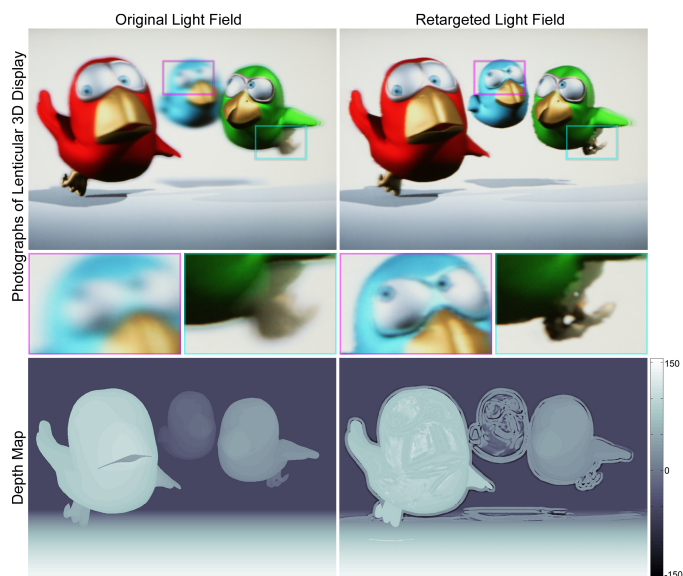


Figure 1: Our 3D content retargeting for a glasses-free lenticular display. Due to the limited depth of field of all light field displays, some objects in a 3D scene will appear blurred. Our remapping approach selectively fits the 3D content into the depth budget of the display, while preserving the perceived depth of the original scene. Top: actual photographs of the original and retargeted scenes, as seen on a Toshiba GL1 lenticular display. Notice the improvement in the blue bird or the legs of the green bird in the retargeted version. Middle: close-ups. Bottom: original and retargeted depths yielded by our method.

depth of field (DOF) that is governed by their limited angular resolution [10, 11]. Due to the fact that most light field displays only provide a low angular resolution, that is the number of viewing zones, the supported DOF is so shallow that virtual

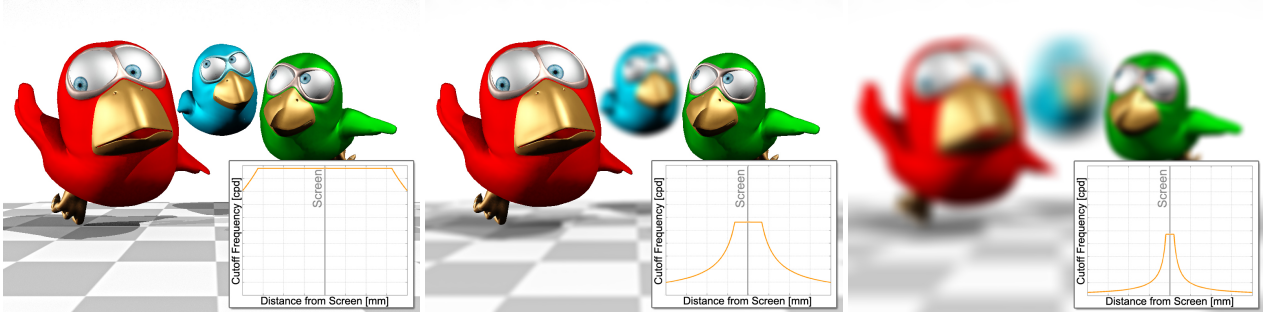


Figure 2: Simulated views of the *three-birds* scene for three different displays. From left to right: Holografika HoloVizio C80 movie screen, desktop and cell phone displays. The last two displays fail to reproduce it properly, due to their intrinsic depth-of-field limitations. The insets plot the depth vs. cut-off frequency charts for each display.

36 3D objects extruding from the physical display enclosure ap-
 37 pear blurred out (see Figs. 1, left, and 2 for a real photograph
 38 and a simulation showing the effect, respectively). We propose
 39 here a framework that remaps the disparities in a 3D scene to
 40 fit the DOF constraints of a target display by means of an opti-
 41 mization scheme that leverages perceptual models of the human
 42 visual system. Our optimization approach runs on the central
 43 view of an input light field and uses warping to synthesize the
 44 rest of the views.

45 **Contributions.** Our nonlinear optimization framework for
 46 3D content retargeting specifically provides the following con-
 47 tributions:
 48

- 49 • We propose a solution to handle the intrinsic trade-off
 50 between the spatial frequency that can be shown and the
 51 perceived depth of a given scene. This is a fundamental
 52 limitation of automultiscopic displays (see Section 3).
- 53 • We combine exact formulations of display-specific depth
 54 of field limitations with models of human perception, to
 55 find an optimized solution. In particular, we consider the
 56 frequency-dependent sensitivity to contrast of the human
 57 visual system, and the sensitivity to binocular disparity.
 58 Based on this combination, a first objective term mini-
 59 mizes the perceived luminance and contrast difference
 60 between the original and the displayed scene, effectively
 61 minimizing DOF blur, while a second term strives to pre-
 62 serve the perceived depth.
- 63 • We validate our results with existing state-of-the-art, ob-
 64 jective metrics for both image quality and perceived depth.
- 65 • We show how our framework can be easily extended to
 66 the particular case of *stereoscopic* disparity, thus demon-
 67 strating its versatility.
- 68 • For this extension, we account for a non-dichotomous
 69 zone of viewing comfort which constitutes a more ac-
 70 curate model of discomfort associated with the viewing
 71 experience.

72 As a result of our algorithm, the depth of a given 3D scene
 73 is modified to fit the DOF constraints imposed by the target

74 display, while preserving the perceived 3D appearance and the
 75 desired 2D image fidelity (Figure 1, right).

76 **Limitations.** We do not aim at providing an accurate model
 77 of the behavior of the human visual system; investigating all
 78 the complex interactions between its individual components re-
 79 mains an open problem as well, largely studied by both psy-
 80 chologists and physiologists. Instead, we rely on existing com-
 81 putational models of human perception and apply them to the
 82 specific application of 3D content retargeting. For this purpose,
 83 we currently consider sensitivities to luminance contrast and
 84 depth, but only approximate the complex interaction between
 85 these cues using a heuristic linear blending, which works well
 86 in our particular setting. Using the contrast sensitivity func-
 87 tion in our context (Section 4) is a convenient but conservative
 88 choice. Finally, depth perception from motion parallax exhibits
 89 strong similarities in terms of sensitivity with that of binocu-
 90 lar disparity, suggesting a close relationship between both [12];
 91 but existing studies on sensitivity to motion parallax are not as
 92 exhaustive as those on binocular disparity, and therefore a reli-
 93 able model cannot be derived yet. Moreover, some studies have
 94 shown that, while both cues are effective, stereopsis is more re-
 95 levant by an order of magnitude [13]. In any case, our approach
 96 is general enough so that as studies on these and other cues ad-
 97 vance and new, more sophisticated models of human perception
 98 become available, they could be incorporated to our framework.

100 2. Related Work

101 Glasses-free 3D displays were invented more than a cen-
 102 tury ago, but even today, the two dominating technologies are
 103 parallax barriers [1] and integral imaging [2]. Nowadays, the
 104 palette of existing 3D display technologies, however, is much
 105 larger and includes holograms, volumetric displays, multilayer
 106 displays and directional backlighting among many others. State
 107 of the art reviews of conventional stereoscopic and automul-
 108 tiscopic displays [14] and computational displays [15] can be
 109 found in the literature. With the widespread use of stereoscopic
 110 image capture and displays, optimal acquisition parameters and
 111 capture systems [16, 17, 18, 19, 20], editing tools [21, 22],
 112 and spatial resolution retargeting algorithms for light fields [23]

113 have recently emerged. In this paper, we deal with the prob-
 114 lem of depth remapping of light field information to the specific
 115 constraints of each display.

116

117 Generally speaking, content remapping is a standard ap-
 118 proach to adapt spatial and temporal resolution, contrast, col-
 119 ors, and sizes of images to a display having limited capabilities
 120 in any of these dimensions [4]. For the particular case of dispar-
 121 ity remapping, Lang et al. [6] define a set of non-linear disparity
 122 remapping operators, and propose a new stereoscopic warping
 123 technique for the generation of the remapped stereo pairs. A
 124 metric to assess the magnitude of perceived changes in binocular
 125 disparity is introduced by Didyk et al. [8], who also investi-
 126 gate the use of the Cornsweet illusion to enhance perceived
 127 depth [24]. Recently, the original disparity metric has been fur-
 128 ther refined including the effect of luminance-contrast [9]. Kim
 129 and colleagues [7] develop a novel framework for flexible ma-
 130 nipulation of binocular parallax, where a new stereo pair is cre-
 131 ated from two non-linear cuts of the EPI volume corresponding
 132 to multi-perspective images [25]. Inspired by Lang and col-
 133 leagues [6], they explore linear and non-linear global remap-
 134 ping functions, and also non-linear disparity gradient compres-
 135 sion. Here we focus on a remapping function that incorporates
 136 the specific depth of field limitations of the target display [26].
 137 Section 8 provides direct comparisons with some of these ap-
 138 proaches.

139 3. Display-specific Depth of Field Limitations

140 Automultiscopic displays are successful in creating convinc-
 141 ing illusions of three-dimensional objects floating in front and
 142 behind physical display enclosures without the observer having
 143 to wear specialized glasses. Unfortunately, all such displays
 144 have a limited depth of field which, just as in wide-aperture
 145 photography, significantly blurs out-of-focus objects. The focal
 146 plane for 3D displays is directly on the physical device.
 147 Display-specific depth of field expressions have been derived
 148 for parallax barrier and lenslet-based systems [10], multilayer
 149 displays [11], and directional backlit displays [27]. In order to
 150 display an aliasing-free light field with any such device, four-
 151 dimensional spatio-angular pre-filters need to be applied before
 152 computing the display-specific patterns necessary to synthesize
 153 a light field, either by means of sampling or optimization. In
 154 practice, these filters model the depth-dependent blur of the in-
 155 dividual displays and are described by a depth of field blur ap-
 156 plied to the target light field. Intuitively, this approach fits the
 157 content into the DOF of the displays by blurring it as necessary.
 158 Figure 3 illustrates the supported depth of field of various auto-
 159 multiscopic displays for different display sizes.

160

161 Specifically, the depth of field of a display is modeled as the
 162 maximum spatial frequency f_ξ of a diffuse plane at a distance
 163 d_0 to the physical display enclosure. As shown by previous
 164 works [10, 11], the DOF of parallax barrier and lenslet-based
 165 displays is given by

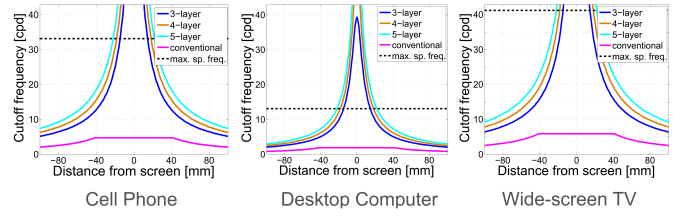


Figure 3: Depth of field for different display architectures and target displays. From left to right: cell phone ($p = 0.09mm$, $v_D = 0.35m$); desktop computer ($p = 0.33mm$, $v_D = 0.5m$); and widescreen TV ($p = 0.53mm$, $v_D = 2.5m$). For comparison purposes all depths of field are modeled for seven angular views.

$$|f_\xi| \leq \begin{cases} \frac{f_0}{N_a}, & \text{for } |d_0| + (h/2) \leq N_a h \\ \left(\frac{h}{(h/2)+|d_0|}\right)f_0, & \text{otherwise} \end{cases}, \quad (1)$$

166 where N_a is the number of angular views, d_0 is the distance
 167 to the front plane of the display (i.e. the parallax barrier or
 168 lenslet array plane), h represents the thickness of the display,
 169 $f_0 = 1/(2p)$, and p is the size of the view-dependent subpixels
 170 of the back layer of the display, making the maximum resolu-
 171 tion of the display at the front surface $f_\xi = f_0/N_a = 1/(2pN_a)$.
 172 For multilayered displays, the upper bound on the depth of field
 173 for a display of N layers was derived by Wetzstein et al. [11] to
 174 be

$$|f_\xi| \leq N f_0 \sqrt{\frac{(N+1)h^2}{(N+1)h^2 + 12(N-1)d_0^2}}. \quad (2)$$

175 Note that in this case d_0 represents the distance to the middle of
 176 the display, and p the pixel size of the layers.

177
 178 It can be seen how depth of field depends on display pa-
 179 rameters such as pixel size p , number of viewing zones N_a ,
 180 device thickness h , and number of layers N (for multilayer dis-
 181 plays), and thus varies significantly for different displays. It
 182 also depends on the viewing distance v_D when expressed in cy-
 183 cles per degree. The above expressions can then be employed
 184 to predict an image displayed on a particular architecture, in-
 185 cluding loss of contrast and blur. Figure 2 shows three simu-
 186 lated views of the *three-birds* scene for three different dis-
 187 plays: a Holografika HoloVizio C80 movie screen ($h = 100mm$,
 188 $p = 0.765mm$, $v_D = 6m$), a Toshiba automultiscopic monitor
 189 ($h = 20$, $p = 0.33$, $v_D = 1.5$) and a cell-phone-sized display
 190 ($h = 6$, $p = 0.09$, $v_D = 0.35$). The scene can be represented
 191 in the large movie screen without blurring artifacts (left); how-
 192 ever, when displayed on a desktop display (middle), some areas
 193 appear blurred due to the depth-of-field limitations described
 194 above (see the blue bird). When seen on a cell-phone display
 195 (right), where the limitations are more severe, the whole scene
 196 appears badly blurred. In the following, we show how these
 197 predictions are used to optimize the perceived appearance of
 198 a presented scene in terms of image sharpness and contrast,
 199 where the particular parameters of the targeted display are an
 200 input to our method.

201 4. Optimization Framework

202 In order to mitigate display-specific DOF blur artifacts, we
 203 propose to scale the original scene into the provided depth bud-
 204 get while preserving the perceived 3D appearance as best as
 205 possible. As detailed in Section 3, this is not trivial, since there
 206 is an intrinsic trade-off between the two goals. We formulate
 207 this as a multi objective optimization problem, with our objec-
 208 tive function made up of two terms. The first one minimizes
 209 the perceived luminance and contrast difference between the
 210 original and the displayed scene, for which display-specific ex-
 211 pressions of the displayable frequencies are combined with a
 212 perceptual model of contrast sensitivity. The second term pe-
 213 nalyzes loss in perceived depth, for which we leverage disparity
 214 sensitivity metrics. Intuitively, the disparity term prevents the
 215 algorithm from yielding the obvious solution where the whole
 216 scene is flattened onto the display screen; this would guarantee
 217 perfect focus at the cost of losing any sensation of depth. The
 218 input to our algorithm is the depth map and the luminance im-
 219 age of the central view of the original light field, which we term
 220 d_{orig} and L_{orig} , respectively. The output is a retargeted depth
 221 map d , which is subsequently used to synthesize the retargeted
 222 light field.

223 **Optimizing luminance and contrast:** We model the display-
 224 specific frequency limitations by introducing spatially-varying,
 225 depth-dependent convolution kernels $k(d)$. They are defined as
 Gaussian kernels whose standard deviation σ is such that fre-
 quencies above the cut-off frequency at a certain depth $f_{\xi}(d)$
 are reduced to less than 5% of its original magnitude. Al-
 though more accurate image formation models for defocus blur
 in scenes with occlusions can be found in the literature [28],
 their use is impractical in our optimization scenario, and we
 found the Gaussian spatially-varying kernels to give good re-
 sults in practice. Kernels are normalized so as not to modify
 the total energy during convolution. As such, the kernel for a
 pixel i is:

$$k(d) = \frac{\exp\left(-\frac{x_i^2 + y_i^2}{2(\sigma(d))^2}\right)}{\sum_j^K \left(\exp\left(-\frac{x_j^2 + y_j^2}{2(\sigma(d))^2}\right)\right)} \quad (3)$$

where K is its number of pixels. The standard deviation σ is
 computed as:

$$\sigma(d) = \frac{\sqrt{-2\log(0.05)}}{2\pi p f_{\xi}(d)} \quad (4)$$

224 with p being the pixel size in $mm/pixel$.

225 To take into account how frequency changes are perceived
 by a human observer, we rely on the fact that the visual sys-
 tem is more sensitive to near-threshold changes in contrast and
 less sensitive at high contrast levels [29]. We adopt a conserva-
 tive approach and employ sensitivities at near-threshold levels
 as defined by the contrast sensitivity function (CSF). We follow
 the expression for contrast sensitivities ω_{CSF} proposed by Man-
 tiuk et al. [30], which in turn builds on the model proposed by

Barten [31]:

$$\omega_{CSF}(l, f_i) = p_4 s_A(l) \frac{MTF(f_i)}{\sqrt{(1 + (p_1 f_i)^{p_2})(1 - e^{-(f_i/7)^2})^{-p_3}}}, \quad (5)$$

where l is the adapting luminance in $[cd/m^2]$, f_i represents the
 spatial frequency of the luminance signal in $[cpd]$ and p_i are the
 fitted parameters provided in Mantiuk's paper¹. MTF (modu-
 lation transfer function) and s_A represent the optical and the
 luminance-based components respectively, and are given by:

$$MTF(f_i) = \sum_{k=1..4} a_k e^{-b_k f_i} \quad (6)$$

$$s_A(l) = p_5 \left(\left(\frac{p_6}{l} \right)^{p_7} + 1 \right)^{-p_8} \quad (7)$$

226 where a_k and b_k can again be found in the original paper. Fig-
 227 ure 4 (left) shows contrast sensitivity functions for varying adap-
 228 tation luminances, as described by Equations 5-7. In our con-
 229 text we deal with complex images, as opposed to a uniform
 230 field; we thus use the steerable pyramid [32] $\rho_S(\cdot)$ to decom-
 231 pose a luminance image into a multi-scale frequency represen-
 232 tation. The steerable pyramid is chosen over other commonly
 233 used types of decomposition (e.g. Cortex Transform) since it
 234 is mostly free of ringing artifacts that can cause false masking
 235 signals [30].

Taking into account both the display-specific frequency lim-
 itations and the HVS response to contrast, we have the follow-
 ing final expression for the first term of our optimization:

$$\left\| \omega_{CSF} \left(\rho_S(L_{orig}) - \rho_S(\phi_b(L_{orig}, d)) \right) \right\|_2^2, \quad (8)$$

237 where ω_{CSF} , defined by Equation 5, are frequency-dependent
 238 weighting factors, and the operator $\phi_b(L, d) = k(d) * L$ models
 239 the display-specific, depth-dependent blur (see Section 3 and
 240 Figure 3). Note that we omit the dependency of ω_{CSF} on (l, f_i)
 241 for clarity. Figure 5 (left) shows representative weights ω_{CSF}
 242 for different spatial frequency luminance levels of the pyramid
 243 for a sample scene.

244 **Preserving perceived depth:** This term penalizes the per-
 ceived difference in depth between target and retargeted scene
 using disparity sensitivity metrics. As noted by different re-
 searchers, the effect of binocular disparity in the perception of
 depth works in a manner similar to the effect of contrast in the
 perception of luminance [8, 33, 34]. In particular, our ability to
 detect and discriminate depth from binocular disparity depends
 on the frequency and amplitude of the disparity signal. Human
 sensitivity to binocular disparity is given by the following equa-
 tion [8] (see also Figure 4, right):

$$\omega_{BD}(a, f) = (0.4223 + 0.007576a + 0.5593\log_{10}(f) + 0.03742a\log_{10}(f) + 0.0005623a^2 + 0.7114\log_{10}^2(f))^{-1} \quad (9)$$

¹sourceforge.net/apps/mediawiki/hdrvp/

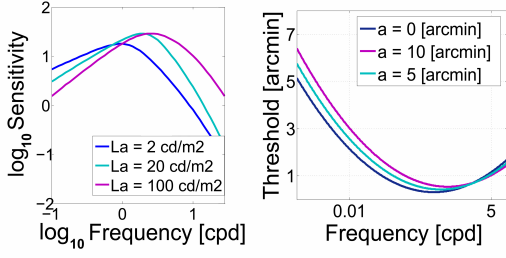


Figure 4: Thresholds and sensitivity values from which the weights for our optimization are drawn. Left: Contrast sensitivity functions. Right: Binocular disparity discrimination thresholds (thresholds are the inverse of sensitivities).

where frequency f is expressed in [cpd], a is the amplitude in [arcmin], and ω_{BD} is the sensitivity in [arcmin⁻¹]. In a similar way to ω_{CSF} in Equation 8, the weights ω_{BD} account for our sensitivity to disparity amplitude and frequency. Given this dependency on frequency, the need for a multi-scale decomposition of image disparities arises again, for which we use a Laplacian pyramid $\rho_L(\cdot)$ for efficiency reasons, following the proposal by Didyk et al. [8]. Figure 5 (right), shows representative weights ω_{BD} .

The error in perceived depth incorporating these sensitivities is then modeled with the following term:

$$\left\| \omega_{BD} \left(\rho_L \left(\phi_v \left(d_{orig} \right) \right) - \rho_L \left(\phi_v \left(d \right) \right) \right) \right\|_2^2. \quad (10)$$

Given the viewing distance v_D and interaxial distance e , the operator $\phi_v(\cdot)$ converts depth into vergence as follows:

$$\phi_v(d) = \text{acos} \left(\frac{\mathbf{v}_L \cdot \mathbf{v}_R}{\|\mathbf{v}_L\| \|\mathbf{v}_R\|} \right), \quad (11)$$

where vectors \mathbf{v}_L and \mathbf{v}_R are illustrated in Figure 6. The Laplacian decomposition transforms this vergence into frequency-dependent disparity levels.

Objective function: Our final objective function is a combination of Equations 8 and 10:

$$\arg \min_d \left(\mu_{DOF} \left\| \omega_{CSF} \left(\rho_S \left(L_{orig} \right) - \rho_S \left(\phi_b \left(L_{orig}, d \right) \right) \right) \right\|_2^2 + \mu_D \left\| \omega_{BD} \left(\rho_L \left(\phi_v \left(d_{orig} \right) \right) - \rho_L \left(\phi_v \left(d \right) \right) \right) \right\|_2^2 \right). \quad (12)$$

For multilayer displays, we empirically set the values of $\mu_{DOF} = 10$ and $\mu_D = 0.003$, while for conventional displays $\mu_D = 0.0003$ due to the different depth of field expressions.

5. Implementation Details

We employ a large-scale trust region method [35] to solve Equation 12. This requires finding the expressions for the analytic gradients of the objective function used to compute the

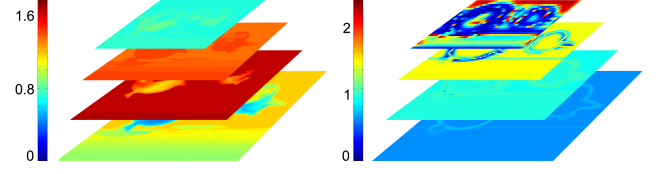


Figure 5: Left: Weights ω_{CSF} (contrast sensitivity values) for different luminance spatial frequency levels for a sample scene (birds). Right: Weights ω_{BD} (inverse of discrimination threshold values) for different disparity spatial frequency levels for the same scene.

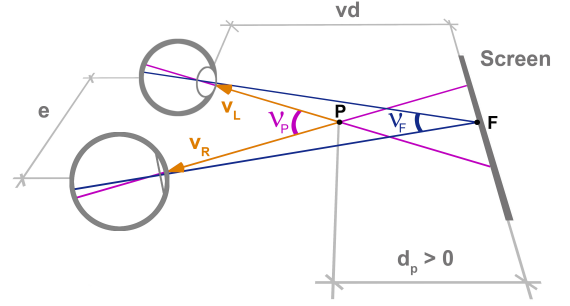


Figure 6: Computing vergence values. Vergence v_P of a point P depends on its position, the viewing distance v_D and the interaxial e . The corresponding disparity for P is $(v_P - v_F)$. v_d refers to the viewing distance and d_p is the depth of point P .

Jacobian, which can be found in Annex A. The objective term in Equation 8 models a single view of the light field, i.e. the central view, in a display-specific field of view (FOV). Within a moderate FOV, as provided by commercially-available displays, this is a reasonable approximation; we obtain the rest of the light field by warping. In the following, we describe this and other additional implementation details.

Sensitivity weights and target values: The weights used in the different terms, ω_{CSF} and ω_{BD} are pre-computed based on the values of the original depth and luminance, d_{orig} and L_{orig} . The transformation from d_{orig} to vergence, its pyramid decomposition and the decomposition of L_{orig} are also pre-computed.

Contrast sensitivity function: As reported by Mantiuk et al. [30], no suitable data exists to separate L- and M-cone sensitivity. Following their approach, we rely on the achromatic CSF using only luminance values.

Depth-of-field simulation: The depth-dependent image blur of automultiscopic displays is modeled as a spatially-varying convolution in each iteration of the optimization procedure. Due to limited computational resources, we approximate this expensive operation as a blend between multiple shift-invariant convolutions corresponding to a quantized depth map, making the process much more efficient. For all scenes shown in this paper, we use $n_c = 20$ quantized depth clusters.

Warping: View warping is orthogonal to the proposed re-targeting approach; we implement here the method described by Didyk et al. [36], although other methods could be em-

301 ployed instead ([7, 37, 38]). To reduce warping artifacts due
 302 to large depth gradients at the limits of the field of view for
 303 each light field, we median-filter the depth and constrain depth
 304 values around the edges.

305 6. Retargeting for Stereoscopic Displays

306 One of the advantages of our framework is its versatility,
 307 which allows to adapt it for display-specific disparity remap-
 308 ping of stereo pairs. We simply drop the depth of field term
 309 from Equation 12, and incorporate a new term that models the
 310 comfort zone. This is an area around the screen within which
 311 the 3D content does not create fatigue or discomfort in the
 312 viewer in stereoscopic displays, and is usually considered as a
 313 dichotomous subset of the fusional area. Although any comfort-
 314 zone model could be directly plugged into our framework, we
 315 incorporate the more accurate, non-dichotomous model sug-
 316 gested by Shibata et al. [39]. This model provides a more ac-
 317 curate description of its underlying psychological and physio-
 318 logical effects. Additionally, this zone of comfort depends on
 319 the viewing distance v_D , resulting on different expressions for
 320 different displays, as shown in Figure 7. Please refer to Annex
 321 B for details on how to incorporate the simpler, but less precise,
 322 dichotomous model.

323 Our objective function thus becomes:

$$\left\| \omega_{BD} \left(\rho_L(\phi_v(D_{orig})) - \rho_L(\phi_v(d)) \right) \right\|_2^2 + \mu_{CZ} \|\varphi(d)\|_2^2, \quad (13)$$

where $\varphi(\cdot)$ is a function mapping depth values to visual dis-
 comfort:

$$\varphi(d) = \begin{cases} 1 - \frac{s_{far}}{v_D - d} - T_{far} & \text{for } d < 0 \\ 1 - \frac{s_{near}}{v_D - d} - T_{near} & \text{for } d \geq 0 \end{cases} \quad (14)$$

324 where v_D is the distance from the viewer to the central plane of
 325 the screen and s_{far} , s_{near} , T_{far} , and T_{near} are values obtained in
 326 a user study carried out with 24 subjects.

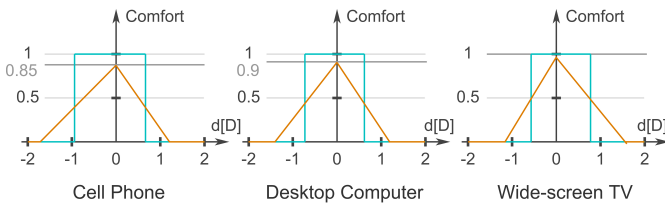


Figure 7: Dichotomous (blue) and non-dichotomous (orange) zones of comfort for different devices. From left to right: cell phone ($v_D = 0.35m$), desktop computer ($v_D = 0.5m$) and wide-screen TV ($v_D = 2.5m$).

327 7. Results

328 We have implemented the proposed algorithm for differ-
 329 ent types of automultiscopic displays including a commercial
 330 Toshiba GL1 lenticular-based display providing horizontal-only

331 parallax with nine discrete viewing zones, and custom multi-
 332 layer displays. The Toshiba panel has a native resolution of
 333 3840×2400 pixels with a specially engineered subpixel struc-
 334 ture that results in a resolution of 1280×800 pixels for each of
 335 the nine views. Note that even a highly-engineered device such
 336 as this suffers from a narrow depth of field due to the limited
 337 angular sampling. We consider a viewing distance of 1.5 m for
 338 the Toshiba display and 0.5 m for the multilayer prototypes.

339 Figures 1 and 8 show results of our algorithm for the Toshiba
 340 display. The target scenes have been originally rendered as light
 341 fields with a resolution of 9×9 , with a field of view of 10° .
 342 Since the Toshiba display only supports horizontal parallax, we
 343 only use the nine horizontal views for these examples. Note
 344 how depth is compressed to fit the display’s constraints in those
 345 areas with visible loss of contrast due to blur (blue bird or far
 346 away pins, for instance), while enhancing details to preserve the
 347 perceived depth; areas with no visible blur are left untouched
 348 (eyes of the green bird, for instance). This results into sharper
 349 retargeted scenes that can be shown within the limitations of the
 350 display. The remapping for the teaser image took two hours for
 351 a resolution of 1024×768 , using our unoptimized Matlab code.

352 We have also fabricated a prototype multilayer display (Fig-
 353 ure 9). This display is composed of five inkjet-printed trans-
 354 parency patterns spaced by clear acrylic sheets. The size of
 355 each layer is 60×45 mm, while each spacer has a thickness
 356 of $1/8''$. The transparencies are conventional films for office
 357 use and the printer is an Epson Stylus Photo 2200. This mul-
 358 tilayer display supports 7×7 views within a field of view of
 359 7° for both horizontal and vertical parallax. The patterns are
 360 generated with the computed tomography solver provided by
 361 Wetzstein et al. [11]. Notice the significant sharpening of the
 362 blue bird and, to a lesser extent, of the red bird. It should be
 363 noted that these are lab prototypes: scattering, inter-reflections
 364 between the acrylic sheets, and imperfect color reproduction
 365 with the desktop inkjet printer influence the overall quality of
 366 the physical results. In Figure 10, we show sharper, simulated
 367 results for the *dice* scene for a similar multilayer display.

370 We show additional results using more complex data sets,
 371 with varying degrees of depth and texture, and different object
 372 shapes and surface material properties. In particular, we use
 373 the Heidelberg light field archive², which includes ground-truth
 374 depth information. The scenes are optimized for a three-layer
 375 multilayer display, similar to the one shown in Figure 9. They
 376 have been optimized for a viewing distance of 0.5 m and have
 377 resolutions ranging from 768×768 to 1024×720 . The weights
 378 used in the optimization are again $\mu_{DOF} = 10$ and $\mu_D = 0.003$.
 379 Figure 11 shows the results for the *papillon*, *buddha2* and *statue*
 380 data sets. Our algorithm recovers most of the high frequency
 381 content of the original scenes, lost by the physical limitations
 382 of the display. The anaglyph representations allow to compare
 383 the perceived depth of the original and the retargeted scenes

²http://hci.iwr.uni-heidelberg.de/HCI/Research/LightField/lf_archive.php

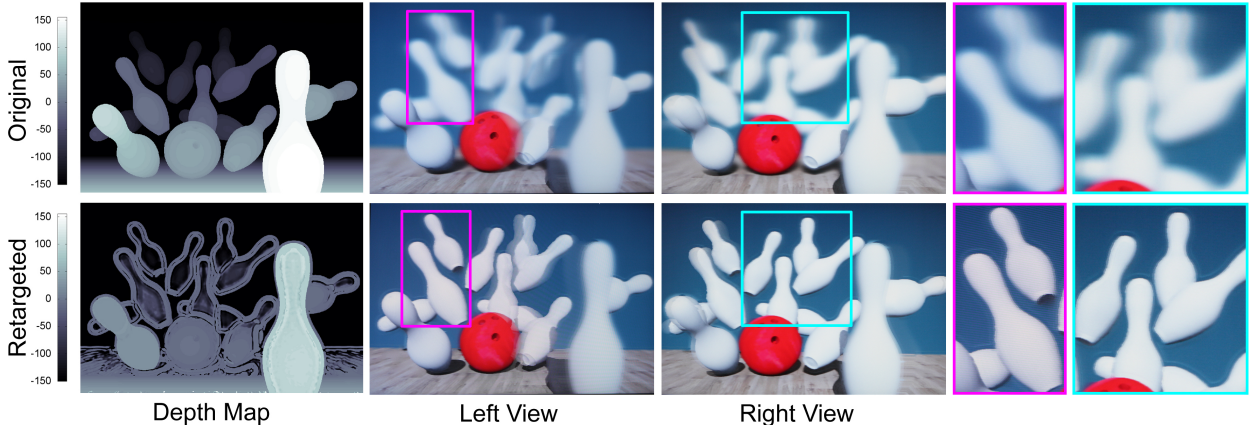


Figure 8: Additional results for commercial lenticular display (actual photographs). Top row: depth map, perspective from left, and perspective from right for original scene. Bottom row: depth map and similar perspectives for the retargeted scene. The slight double-view of some of the pins in the left view is due to interview cross-talk in the Toshiba display.

385 (please refer to the supplementary material for larger versions
 386 to ensure proper visualization). Figure 12 shows additional
 387 views of the *buddha2* and *statue* light fields.

388

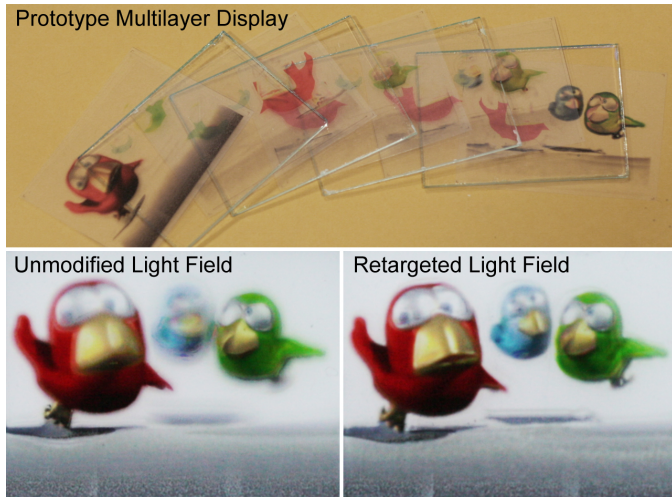


Figure 9: 3D content retargeting for multilayer light field displays (actual photographs). Even five attenuating layers (top) can only provide a limited depth of field for a displayed scene (bottom left). Our retargeting algorithm maps the multiview content into the provided depth budget (bottom right).

389 As shown in this section, our algorithm works well within a
 390 wide range of displays and data sets of different complexities.
 391 However, in areas of very high frequency content, the warp-
 392 ing step may accumulate errors which end up being visible in
 393 the extreme views of the light fields. Figure 13 shows this:
 394 the *horses* data set contains a background made up of a texture
 395 containing printed text. Although the details are successfully
 396 recovered by our algorithm, the warping step cannot deal with
 397 the extremely high frequency of the text, and the words appear
 398 broken and illegible.

399

400 Finally, Figure 14 shows the result of applying our adapted
 401 model to the particular case of stereo retargeting, as described

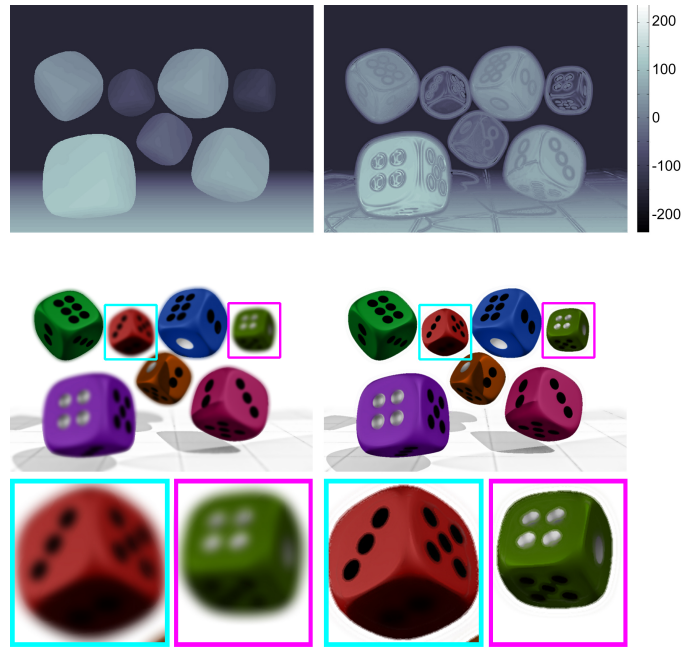


Figure 10: Results of simulations for a multilayer display (five layers). Top row: initial and retargeted depth. Middle row: initial and retargeted luminance. Bottom row: close-ups.

402 in Section 6.

403 8. Comparison to Other Methods

404 Our method is the first to specifically deal with the par-
 405 ticular limitations of automultiscopic displays (depth vs. blur
 406 trade-off), and thus it is difficult to directly compare with others.
 407 However, we can make use of two recently published *objective*
 408 computational metrics, to measure distortions both in the ob-
 409 served 2D image fidelity, and in the perception of depth. This
 410 also provides an objective background to compare against exist-
 411 ing approaches for stereoscopic disparity retargeting, for which

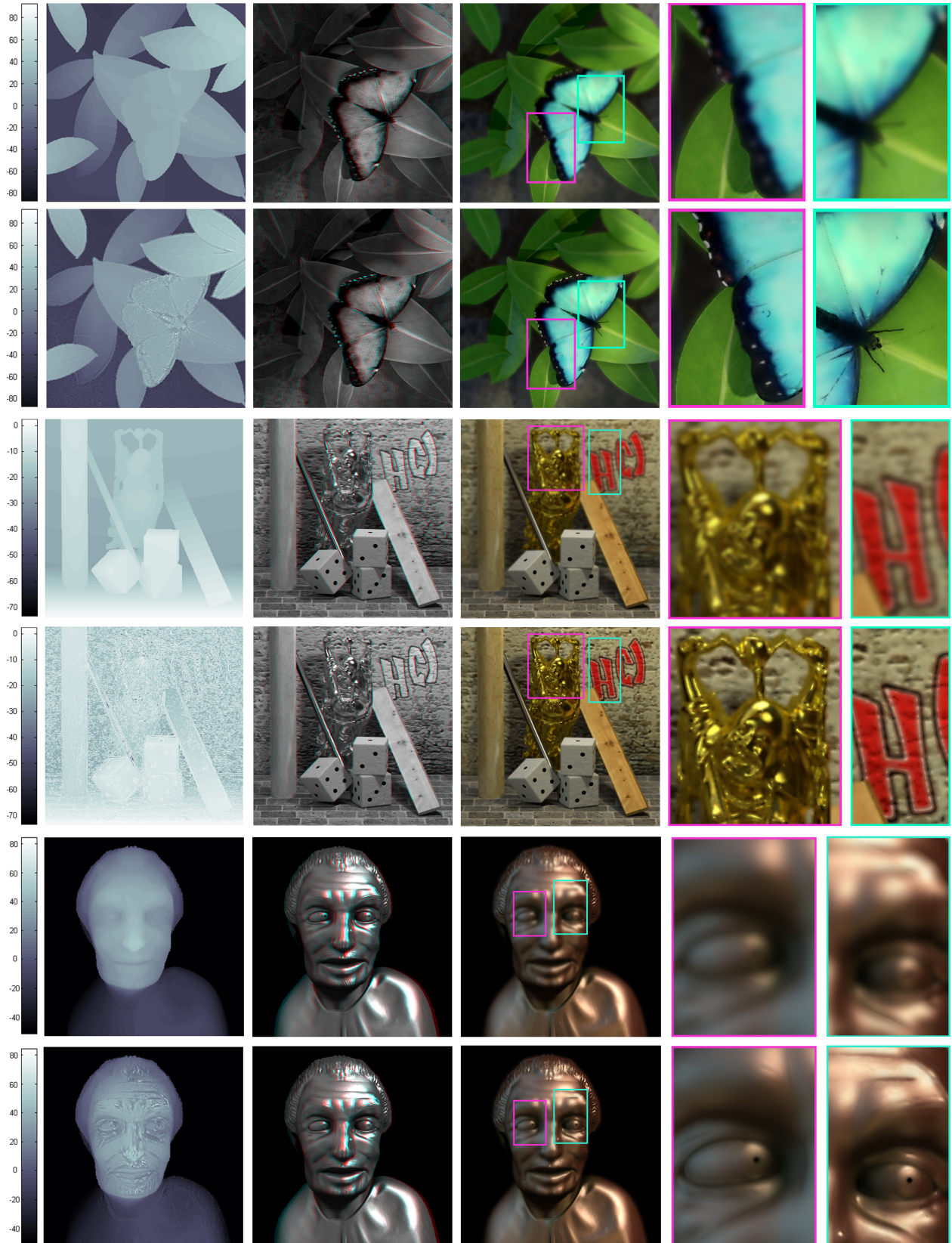


Figure 11: Results for the *papillon* (top), *buddha2* (middle) and *statue* (bottom) data sets from the Heidelberg light field archive. For each data set, the top row shows the original scene, while the bottom row shows our retargeted result. From left to right: depth map, anaglyph representation, central view image, and selected zoomed-in regions. Notice how our method recovers most of the high frequency details of the scenes, while preserving the sensation of depth (larger versions of the anaglyphs appear in the supplementary material). Note: please wear anaglyph glasses with cyan filter on left and red filter on right eye; for an optimal viewing experience please resize the anaglyph to about 10 cm wide in screen space and view it at a distance of 0.5 m.

412 alternative methods do exist.

413



Figure 12: Additional non-central views of the retargeted *buddha2* and *statue* light fields, with corresponding close-ups.

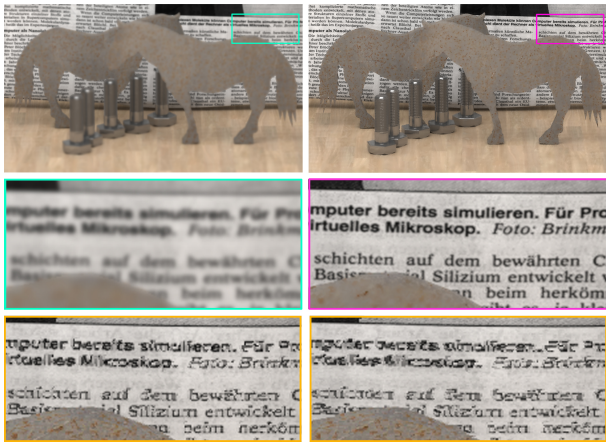


Figure 13: Results for the *horses* data set from the Heidelberg light field archive. Very high frequencies that have been initially cut off by the display (green box) are successfully recovered by our algorithm (pink). However, subsequent warping can introduce visible artifacts in those cases, which progressively increase as we depart from the central view of the light field. This progression is shown in the bottom row (yellow boxes).

414 **Metrics:** We need to measure *both* observed 2D image
 415 quality *and* resulting degradations in perceived depth. For image
 416 quality, numerous metrics exist. We rely on the HDR-VDP
 417 2 calibration reports provided by Mantiuk and colleagues [30]
 418 in their website³, where the authors compare quality predic-
 419 tions from six different metrics and two image databases: LIVE
 420 [40] and TID2008 [41]. According to the prediction errors, re-
 421 ported as Spearman’s correlation coefficient, multi-scale SSIM
 422 (MS-SSIM, [42]) performs best across both databases for the
 423 blurred image distortions observed in our application. The map-
 424 ping function we use, $\log(1 - \text{MS-SSIM})$, yields the highest cor-
 425 relation for Gaussian blur distortions.

426

427 Fewer metrics exist to evaluate distortions in depth. We use
 428 the metric recently proposed by Didyk and colleagues to esti-
 429 mate the magnitude of the perceived disparity change between
 430 two stereo images [8]. The metric outputs a heat map of the dif-
 431 ferences between the original and the retargeted disparity maps

³http://hdrvdp.sourceforge.net/reports/2.1/quality_live/ http://hdrvdp.sourceforge.net/reports/2.1/quality_tid2008/

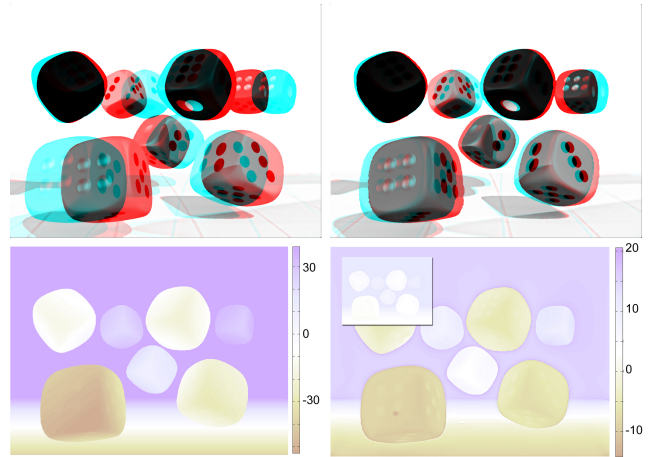


Figure 14: Retargeting for stereo content. *Left column:* Anaglyph and corresponding pixel disparity map of the original scene. For a common (around 0.5m) viewing distance on a desktop display, left and right images cannot be fused. *Right column:* Anaglyph and corresponding pixel disparity map of the retargeted scene. Images can now be fused without discomfort, and perception of depth is still present despite the aggressive depth compression. Note that the scales of the disparity maps are different for visualization purposes; the small inset shows the retargeted disparity map for the same scale as the original. Note: please wear anaglyph glasses with cyan filter on left and red filter on right eye; for an optimal viewing experience please resize the anaglyph to about 10 cm wide in screen space and view it at a distance of 0.5 m.

432 in Just Noticeable Difference (JND) units.

433

434 **Alternative Methods:** There is a large space of linear and
 435 non-linear global remapping operators, as well as of local ap-
 436 proaches. Also, these operators can be made more sophisti-
 437 cated, for instance by incorporating information from saliency
 438 maps, or adding the temporal domain [6]. To provide some
 439 context to the results of the objective metrics, we compare our
 440 method with a representative subset of alternatives, including
 441 global operators, local operators, and a recent operator based
 442 on a perceptual model for disparity. In particular, we compare
 443 against six other results using different approaches for stereo
 444 retargeting: a linear scaling of pixel disparity (*linear*), a linear
 445 scaling followed by the addition of bounded Cornsweet pro-
 446 files at depth discontinuities (*Cornsweet* [24])⁴, a logarithmic
 447 remapping (*log*, see e.g. [6]), and the recently proposed
 448 remapping of disparity in a perceptually linear space (*perc. lin-*
 449 *ear* [8]). For the last two, we present two results using different
 450 parameters. This selection of methods covers a wide range from
 451 very simple to more sophisticated.

452

The linear scaling is straightforward to implement. For the bounded Cornsweet profiles method, where profiles are carefully controlled so that they do not exceed the given disparity bounds and create disturbing artifacts, we choose $n = 5$ levels as suggested by the authors. For the logarithmic remapping, we

⁴In our tests, this consistently yielded better results than a naive application of unbounded Cornsweet profiles, as originally reported by Didyk and colleagues [24]

use the following expression, inspired by Lang et al. [6]:

$$\delta_o = K \cdot \log(1 + s \cdot \delta_i), \quad (15)$$

where δ_i and δ_o are the input and output pixel disparities, s is a parameter that controls the scaling and K is chosen so that the output pixel disparities fit inside the allowed range. We include results for $s = 0.5$ and $s = 5$. Finally, for the perceptually linear method, disparity values are mapped via transducers into a perceptually linear space, and then linearly scaled by a factor k . The choice of k implies a trade-off between the improvement in contrast enhancement and how faithful to the original disparities we want to remain. We choose $k = 0.75$ and $k = 0.95$ as good representative values for both options respectively.

Comparisons: Some of the methods we compare against (*linear*, *Cornsweet* and *log*) require to explicitly define a minimum spatial cut-off frequency, which will in turn fix a certain target depth range. We run comparisons on different data sets and for a varied range of cut-off frequencies: For the *birds* scene, where the viewing distance is $v_D = 1.5$ m, we test two cut-off frequencies: $f_{cpmm} = 0.12$ cycles per mm ($f_{cpd} = 3.14$ cycles per degree), and $f_{cpmm} = 0.19$ ($f_{cpd} = 5.03$), the latter of which corresponds to remapping to the depth range which offers the maximum spatial resolution of the display (see DOF plots in Figure 16b). For the *statue*, *papillon* and *buddha2* scenes, optimized for a multilayer display with $v_D = 0.5$ m, we set the frequencies to $f_{cpmm} = 0.4, 0.5$ and 1.1 , respectively (corresponding $f_{cpd} = 3.49, 4.36$ and 9.60). The frequencies are chosen so that they yield a fair compromise between image quality and perceived depth, given the trade-off between these magnitudes; they vary across scenes due to the different spatial frequencies of the image content in the different data sets.

Figure 15 shows a comparison to the results obtained with the other methods both in terms of image quality and of perceived depth for three different scenes from the Heidelberg data set (*papillon*, *buddha2*, and *statue*). Heat maps depict the error in perceived depth (in JNDs) given by Didyk et al.’s metric. Visual inspection shows that our method consistently leads to less error in perceived depth (white areas mean error below the 1 JND threshold). Close-ups correspond to zoomed-in regions from the resulting images obtained with each of the methods, where the amount of DOF blur can be observed (please refer to the supplementary material for the complete images). Our method systematically yields sharper images, even if it also preserves depth perception better. Only in one case, in the *statue* scene, perceptually linear remapping yields sharper results, but at the cost of a significantly higher error in depth perception, as the corresponding heat maps show.

To better assess the quality of the deblurring of the retargeted images, Figure 16a shows the MS-SSIM metric for the different methods averaged over the scenes tested, together with the associated standard error (we plot the absolute value of $\log(1 - \text{MS-SSIM})$). We have added the result of the original image, without any retargeting method applied (N for *none* in

the chart). Our method yields the best perceived image quality (highest MS-SSIM value), and as shown in Figure 15, the lowest error in depth perception as well. This can be intuitively explained by the fact that our proposed multi-objective optimization (Eq. 12) explicitly optimizes *both* luminance and depth, whereas existing algorithms are either heuristic or take into account only one of the two aspects.

To further explore this image quality vs. depth perception trade-off, we have run the comparisons for the *birds* scene for two different cut-off spatial frequencies. Figure 16b shows comparisons of all tested algorithms for the *birds* scene retargeted for a lenslet-based display. For two of the methods, ours and the perceptually linear remapping (with $k = 0.75$ and $k = 0.95$), defining this minimum spatial frequency is not necessary. Error in depth for these is shown in the top row. For the other four methods (*linear*, *Cornsweet*, *log s = 0.5*, *log s = 5*), the cut-off frequency needs to be explicitly defined: we set it to two different values of $f_{cpmm} = 0.12$ and $f_{cpmm} = 0.19$, which correspond to an intermediate value and to remapping the content to the maximum spatial frequency of the display, respectively. The resulting error in depth is shown in the middle and bottom rows of Figure 16b. Error in perceived depth clearly increases as the cut-off frequency is increased. The bar graph at the top left of Figure 16b shows image quality results for $f_{cpmm} = 0.12$. Note that for $f_{cpmm} = 0.19$, the methods *linear*, *Cornsweet* and *log* yield perfectly sharp images (since we explicitly chose that frequency to remap to the maximum resolution of the display), but at the cost of large errors in perceived depth.

9. Conclusions and Future Work

Automultiscopic displays are an emerging technology with form factors ranging from hand-held devices to movie theater screens. Commercially successful implementations, however, face major technological challenges, including limited depth of field, resolution, and contrast. We argue that compelling multi-view content will soon be widely available and tackle a crucial part of the multiview production pipeline: display-adaptive 3D content retargeting. Our computational depth retargeting algorithm extends the capabilities of existing glasses-free 3D displays, and deals with a part of the content production pipeline that will become commonplace in the future.

As shown in the paper, there is an inherent trade-off in automultiscopic displays between depth budget and displayed spatial frequencies (blur): depth *has to* be altered if spatial frequencies in luminance are to be recovered. This is not a limitation of our algorithm, but of the targeted hardware (Figure 3). Our algorithm aims at finding the best possible trade-off, so that the inevitable depth distortions introduced to improve image quality have a minimal perceptual impact. Therefore, the amount of blur (the cut-off frequency) in the retargeted scene depends on the actual visibility of the blur in a particular area, according to the CSF. Should the user need to further control the amount of defocus deblurring, it could be added to the optimization in the form of constraints over the depth values according to the

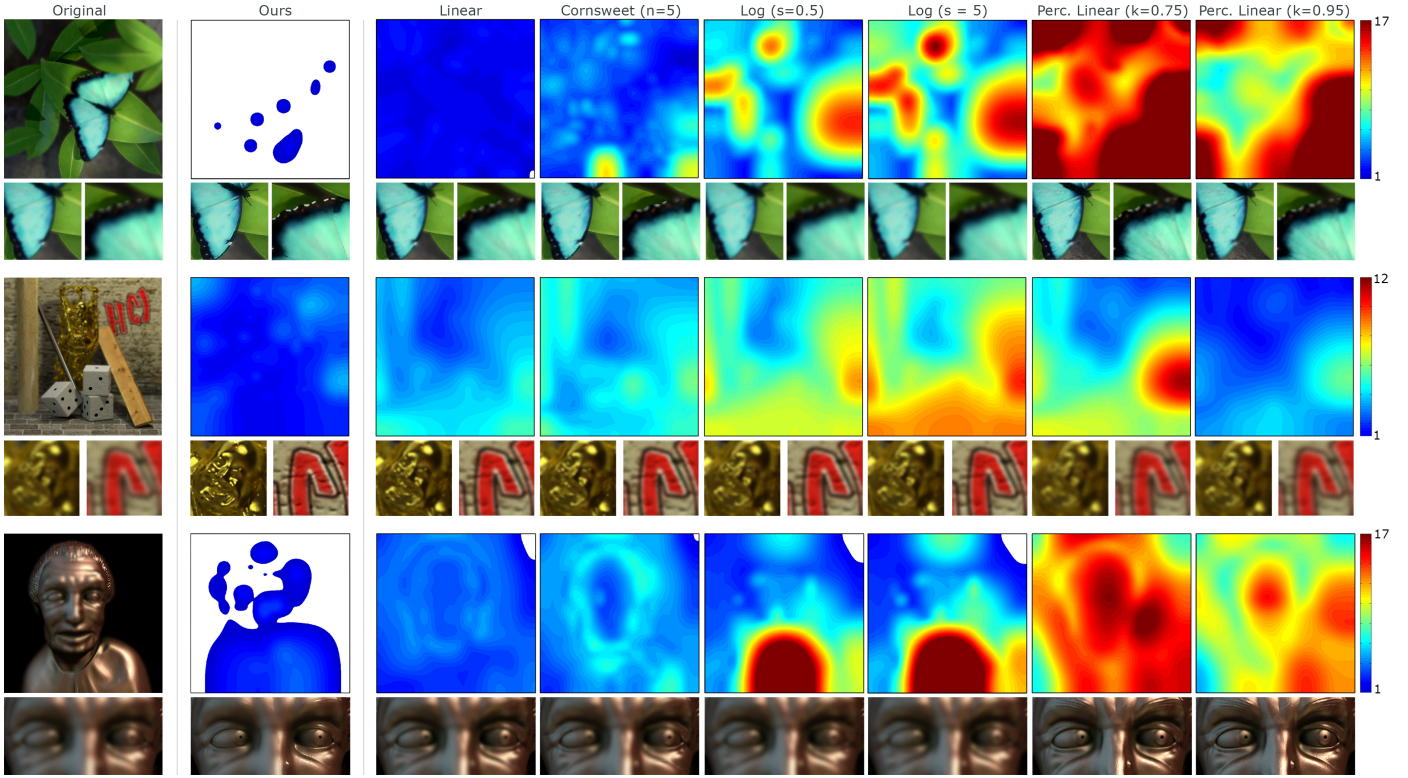


Figure 15: Comparison against other methods for three different scenes from the Heidelberg light field archive. From top to bottom: *papillon* ($f_{cpmm} = 0.4$, $f_{cpd} = 3.49$), *buddha2* ($f_{cpmm} = 1.1$, $f_{cpd} = 9.60$), and *statue* ($f_{cpmm} = 0.5$, $f_{cpd} = 4.36$). Errors in depth are shown as heat maps (lower is better) according to the metric by Didyk and colleagues [8]; white areas correspond to differences below one JND. Viewing distance is 0.5 m.

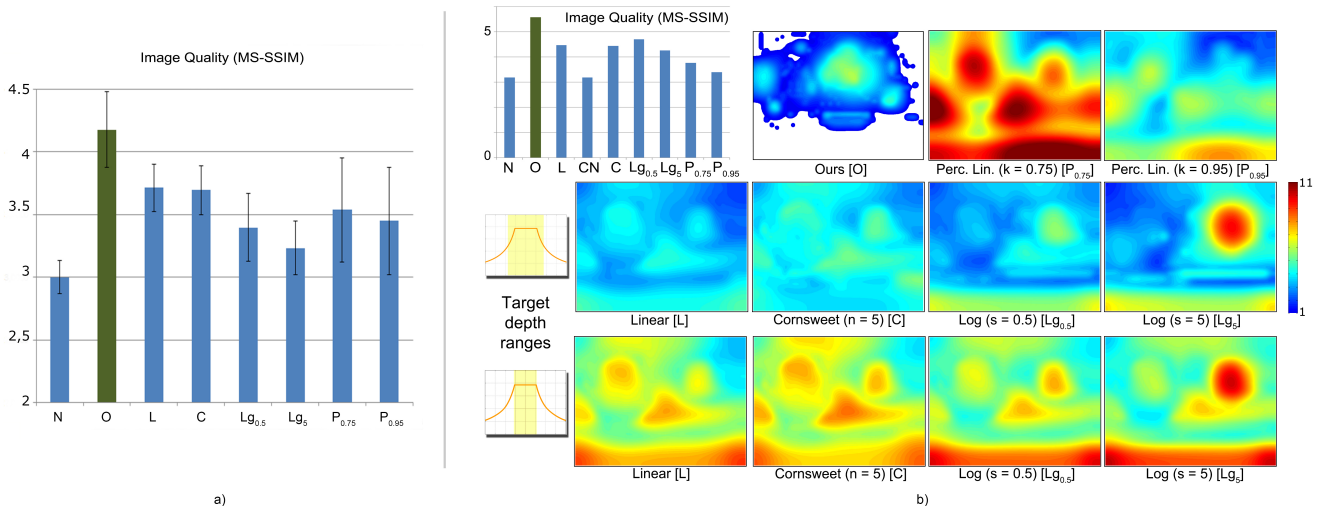


Figure 16: (a) Comparison of average luminance quality (lack of blur) according to the MS-SSIM metric for all the data sets used in this comparisons (higher is better). (b) Comparison against other methods for the *birds* scene, for two different cut-off frequencies. Top row, from left to right: resulting image quality as predicted by MS-SSIM for $f_{cpmm} = 0.12$, and error in depth for the two methods that do not require providing a target depth range. Middle row: error in depth for the three methods requiring a target depth range, for a cut-off frequency $f_{cpmm} = 0.12$ ($f_{cpd} = 3.14$). The smaller image represents the depth vs. cut-off frequency function of the display, with the target depth range highlighted in yellow. Bottom row: same as middle row for a cut-off frequency $f_{cpmm} = 0.19$ ($f_{cpd} = 5.03$), corresponding to the maximum spatial frequency allowed by the display (flat region of the DOF function). Errors in depth are shown as heat maps (lower is better) according to Didyk et al's metric [8]; white areas correspond to differences below one JND. Note the intrinsic trade-off between image quality and depth perception for the methods requiring a specific target depth range: when remapping to the maximum spatial frequency of the display, error in perceived depth significantly increases. Viewing distance is 1.5 m.

561 corresponding DOF function.

562

563 We have demonstrated significant improvements in sharp-
564 ness and contrast of displayed images without compromising

the perceived three-dimensional appearance of the scene, as our results and validation with objective metrics show. For the special case of disparity retargeting in stereoscopic image pairs, our method is the first to handle display-specific non-dichotomous zones of comfort: these model the underlying physical and physiological aspects of perception better than binary zones used in previous work. In the supplementary video, we also show an animated sequence for retargeted content. It is shown as an anaglyph, so it can be seen in 3D on a regular display. Although the frames of this video clip have been processed separately, our algorithm provides temporally stable retargeting results.

A complete model of depth perception remains an open problem. One of the main challenges is the large number of cues that our brain uses when processing visual information, along with their complex interactions [43, 44]. A possible avenue of future work would be to extend the proposed optimization framework by including perceptual terms modeling human sensitivity to accommodation, temporal changes in displayed images, sensitivity of depth perception due to motion parallax or the interplay between different perceptual cues. However, this is not trivial and will require significant advances in related fields. Another interesting avenue of future work would be to extend our optimization framework to deal with all the views in the light field, thus exploiting angular resolution.

We hope that our work will provide a foundation for the emerging multiview content production pipeline and inspire others to explore the close relationship between light field acquisition, processing, and display limitations in novel yet unforeseen ways. We believe bringing the human visual system into the design pipeline [45, 46] is a great avenue of future work to overcome current hardware limitations in all areas of the imaging pipeline, from capture to display.

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719 Appendix A. Objective Function and Analytical Derivatives

720 In this section we go through the mathematical expressions
721 of the two terms of the objective function in detail. We also
722 include their derivatives, necessary for computing the analytical
723 Jacobian used in the optimization process.

724 Appendix A.1. Term 1: Optimizing Luminance and Contrast

This term, as shown in Equation (8) of the main text, has the following form:

$$T_1 = \omega_{CSF} (\rho_S (L_{orig}) - \rho_S (\phi_b (L_{orig}, d))) \quad (A.1)$$

725 Note that this expression yields a vector of length N_{pyr} (N_{pyr}
726 being the number of pixels in the pyramid $\rho_S (L_{orig})$ or
727 $\rho_S (\phi_b (L_{orig}, d))$), which is a vector of differences with respect
728 to the target luminance L_{orig} , weighted by contrast sensitivity
729 values. This vector of errors thus contains the residuals that
730 `lsqnonlin` optimizes for the depth of field term. The weight-
731 ing factor μ_{DOF} is left out of this derivation for the sake of sim-
732 plicity, since it is just a product by a constant both in the objec-
733 tive function term and in its derivatives. This is valid also for
734 the second term of the objective function.

735 Since the multi-scale decomposition is a linear operation, we can write:

$$T_1 = \omega_{CSF} (M_S \cdot L_{orig} - M_S \cdot \phi_b (L_{orig}, d)) \quad (A.2)$$

where M_S is a matrix of size $N_{pyr} \times N_{im}$, N_{im} being the number of pixels in the luminance image L_{orig} . Substituting the blurring function $\phi_b (\cdot, \cdot)$ by its actual expression

$$\frac{\partial T_{1,i}}{\partial d} = \omega_{CSF,i} \left(-M_{S,i} \cdot (L_{orig} * \frac{\partial k(d)}{\partial d}) \right), \quad (A.3)$$

where $M_{S,i}$ is the i -th row of M_S . The derivative of the kernels $k(d)$ is:

$$\frac{\partial k(d)}{\partial d} = \frac{\left(\exp\left(-\frac{x_i^2+y_i^2}{2(\sigma(d))^2}\right) \right) \left(\frac{(x_i^2+y_i^2)4\sigma(d)\frac{\partial\sigma(d)}{\partial d}}{(2(\sigma(d))^2)^2} \right) \sum_j^K \left[\exp\left(-\frac{x_j^2+y_j^2}{2(\sigma(d))^2}\right) \right]}{\left(\sum_j^K \left[\exp\left(-\frac{x_j^2+y_j^2}{2(\sigma(d))^2}\right) \right] \right)^2} \quad (A.4)$$

$$\frac{\sum_j^K \left[\left(\exp\left(-\frac{x_j^2+y_j^2}{2(\sigma(d))^2}\right) \right) \left(\frac{(x_j^2+y_j^2)4\sigma(d)\frac{\partial\sigma(d)}{\partial d}}{(2(\sigma(d))^2)^2} \right) \right] \left(\exp\left(-\frac{x_i^2+y_i^2}{2(\sigma(d))^2}\right) \right)}{\left(\sum_j^K \left[\exp\left(-\frac{x_j^2+y_j^2}{2(\sigma(d))^2}\right) \right] \right)^2}.$$

The derivative of the standard deviation σ is straightforward, knowing $\partial(f_\xi(d))/\partial d$. As described in the main text, the expression for $f_\xi(d)$ depends on the type of automultiscopic display. For a conventional display [10]:

$$f_\xi(d) = \begin{cases} \frac{f_0}{N_a}, & \text{for } |d| + (h/2) \leq N_a h \\ \left(\frac{f_0}{(h/2)+|d|} \right) f_0, & \text{otherwise} \end{cases}, \quad (A.5)$$

where N_a is the number of angular views, h represents the thickness of the display and $f_0 = 1/(2p)$ is the spatial cut-off frequency of a mask layer with a pixel of size p . For multilayered displays, the upper bound on the depth of field for a display of N layers is [11]:

$$f_\xi(d) = N f_0 \sqrt{\frac{(N+1)h^2}{(N+1)h^2 + 12(N-1)d^2}}. \quad (A.6)$$

The derivatives are as follows:

$$\frac{\partial f_\xi(d)}{\partial d} = \begin{cases} 0, & \text{for } |d| + (h/2) \leq N_a h \\ \left(\frac{-hd/|d|}{((h/2)+|d|)^2} \right) f_0, & \text{otherwise} \end{cases} \quad (A.7)$$

for a conventional display and

$$\frac{\partial f_\xi(d)}{\partial d} = N f_0 \frac{12\sqrt{N+1}(N-1)hd}{((N+1)h^2 + 12(N-1)d^2)^{3/2}}. \quad (A.8)$$

736 for a multilayered display.

737 Appendix A.2. Term 2: Preserving Perceived Depth

This term, introduced in Equation 10 of the main text, is modeled as follows:

$$T_2 = \omega_{BD} (\rho_L (\phi_v (D_{orig})) - \rho_L (\phi_v (d))) \quad (A.9)$$

Again, since the multi-scale decomposition is a linear operation, we write:

$$T_2 = \omega_{BD} (M_L \cdot \phi_v (D_{orig}) - M_L \cdot \phi_v (d)) \quad (A.10)$$

where M_L is a matrix of size $N_{dpyr} \times N_d$, N_d being the number of pixels in the depth map D_{orig} . Taking the derivative with respect to d yields the following expression for each element $T_{2,i}$ of the residuals vector for this term:

$$\frac{\partial T_{2,i}}{\partial d} = \omega_{BD,i} \left(-M_{L,i} \cdot \frac{\partial \phi_v (d)}{\partial d} \right), \quad (A.11)$$

where $M_{L,i}$ is the i -th row of M_L . As explained in the main text, $\phi_v(d)$ converts depth d_P of a point P into vergence v_P . This, given the viewing distance v_D and the interaxial distance e , is done using function $\phi_v(\cdot)$:

$$\phi_v(d) = \text{acos}\left(\frac{\mathbf{v}_L \cdot \mathbf{v}_R}{\|\mathbf{v}_L\| \|\mathbf{v}_R\|}\right), \quad (\text{A.12})$$

where vectors \mathbf{v}_L and \mathbf{v}_R have their origins in P and end in the eyes (please also see Figure 6 in the main text). Placing the coordinate origin in the center of the screen (z-axis normal to the screen, x-axis in the horizontal direction) we can rewrite the previous equation for a point $P = (x_i, y_i, d_i)$ as:

$$v_d = \phi_v(d) = \text{acos}\left(\frac{\kappa}{\sqrt{\eta} \sqrt{\zeta}}\right), \quad (\text{A.13})$$

738 where:

$$739 \kappa = (x_L - x_i)(x_R - x_i) + (v_D - d_i)^2,$$

740

$$741 \eta = (x_L - x_i)^2 + (v_D - d_i)^2,$$

742

$$743 \zeta = (x_R - x_i)^2 + (v_D - d_i)^2.$$

744

Finally, differentiating Equation A.13 with respect to depth:

$$\frac{\partial \phi_v(d)}{\partial d} = -\left(1 - \left(\frac{\kappa}{\sqrt{\eta} \sqrt{\zeta}}\right)^2\right)^{-1/2} \cdot \left(\frac{-2(v_D - d_i) \sqrt{\eta} \sqrt{\zeta} - \kappa \Psi(d_i)}{\eta \zeta}\right)$$

745 where $\Psi(d_i)$ is as follows:

$$\Psi(d_i) = -d_i(v_D - d_i)\eta^{-1/2}\zeta^{1/2} - d_i(v_D - d_i)\zeta^{-1/2}\eta^{1/2}$$

746 Appendix B. A Dichotomous Zone of Comfort

As explained in the paper, Equation B.1 describes our objective function for the simplified case of stereo remapping:

$$\left\| \omega_{BD}(\rho_L(\phi_v(D_{orig})) - \rho_L(\phi_v(d))) \right\|_2^2 + \mu_{CZ} \|\varphi(d)\|_2^2, \quad (\text{B.1})$$

where $\varphi(\cdot)$ is a function mapping depth values to visual discomfort. To incorporate a dichotomous model (such as those shown in cyan in Figure 7 for different devices and viewing distances v_D), instead of the non-dichotomous model described in the paper (shown in orange in the same figure), we can define a binary indicator function, such as

$$\varphi_{dc}(d) = \begin{cases} 0 & \text{for } d_{comfort}^{min} \leq d \leq d_{comfort}^{max} \\ \infty & \text{otherwise} \end{cases} \quad (\text{B.2})$$

For a practical, numerically-robust implementation, a smooth function that approximates Equation B.2 is preferable, ensuring C^1 continuity. Our choice for such a function is the Butterworth function which is commonly used as a low-pass filter in signal processing:

$$\varphi_{bf}(d) = 1 - \sqrt{\frac{1}{1 + (\gamma d)^{2s}}} \quad (\text{B.3})$$

747 where γ controls the position of the cut-off locations and s the
748 slope of such cut-offs.