A SWN model for polling system

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1 Introduction

The model presented in this chapter represents an extension of *single polling system*, namely multiserver multiqueue polling system. While in the *single polling model* there is only one server which cyclically visits N queues where the costumers are waiting for service, in the *multiserver multiqueue (MSMQ) model*, there is more then one server visiting the queues.

The description of the behavior of this system requires the definition of several parameters and operating rules:

- **number of queues** $N \geq 1$;
- **number of servers** $1 \leq S \leq N$;
- **queues capacity** $Q \geq 1$;
- **arrival process**
- **service times**
- **walk times**
- **service discipline**

We consider two distinct models, one in which the servers visit the queues in a random order, the other in which the servers cyclically visit the queues in a predefined order.

Our example will have a *number of queues* $N = 4$, a *number of servers* $S = 2$ and a *storage capacity* $Q = 1$ so that one an only one customer at a time may be in each queue. The *arrival process* is a Poisson process with rate λ, the *service time* are exponentials distributed random variables with mean $\frac{1}{\mu}$ and the *walk times* exponentials distributed random variables with mean $\frac{1}{\omega}$. The *service discipline* for each queue will be FCFS.

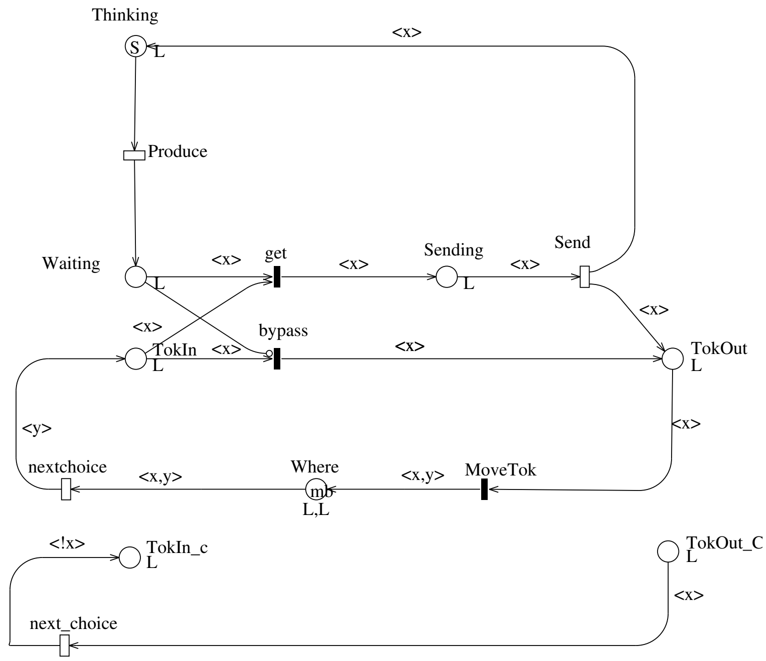

Figure 1: SWN model of a Multiserver random polling

2 The description of the model

Fig.1 shows a SWN model describing a Multiserver with four queues, in which only the identity of queues and not the servers identity is kept, in order to make easy the comprehension and the analysis. This choice renders very compact the SRG and easy the color domain's analysis for the places and the transitions. In fact table 1 shows that the model has only one color class representing the different queues.

Class id	**Description**	**Type**
L	different queues	unordered(random) ordered(cyclic)
Subclass id	**Description**	**Elements**
$L1$	subclass of L	l_1, l_2, l_3, l_4

Table 1: Characteristics of color class

The model can be divided in two sub-models: one representing the waiting queue in which the customers are scheduled, the other representing the server behavior.

2.1 The description of the sub-model of waiting queue

Transition *Produce* models the arrival of a customer at queue l_i. The customer, after entering queue l_i, waits in place *Waiting* for the arrival of token of color l_i in *ToKIn* (representing the arrival of server at queue l_i), so that the transition immediate *Get* can fire.

The firing of transition *Send* represents the end of the service and therefore the server proceeds and polls the next queue, while the token representing the customer can go to the place *Thinking* where, through the transition *Produce*, will again enter the waiting queue.

2.2 The description of the sub-model of servers

This sub-model represents the choice of the next queue to be visited and the movement towards that queue.The random polling version of the model is described first.

A server leaving queue l_i is modelled by token of color l_i in place *ToKOut*; the choice for the next queue is modelled by immediate transition *MoveToK*. The transition *nextchoice* models the walk time to reach the selected queue l_j from the just visited queue l_i.

For the cyclic polling system the sub-model of servers behavior does not need place *Where* and transition *MoveToken* because the choice of next queue to be polled is predefined by the ordered class. The color domain of transition *nextchoice* changes from $L \times L$(variables x and y) to L (variable x only), because we don't need to remember the last visited queue to calculate the walk times. It may be deduced directly knowing only the next queue.

place	**Domain(random)**	**Domain(cyclic)**
Waiting	L	L
Sendig	L	L
Thinking	L	L
ToKIn	L	L
ToKOut	L	L
Where	$L \times L$	(empty)

Table 2: Table of places

2.3 The description of the initial state

The initial state m_o corresponds to a situation in which N customers are in the place *Thinking* and S servers in the place *Where* for random model or S servers in the place *ToKIn* for cyclic model.

Transition	**Domain(random|cyclic)**	**Weight**	**Type**
Produce	$L \mid L$	λ	single-server per color
Get	$L \mid L$	1	immediate
Send	$L \mid L$	μ	single-server per color
bypass	$L \mid L$	1	immediate
MoveToK	$L \times L \mid$ (empty)	1	immediate
nextchoice	$L \times L \mid L$	ω	infinite-server per color

Table 3: Table of transitions

Example $N = 4$ **and** $S = 2$

Random model
Thinking(1<l10>1<l11>)Where(2<l11,l11>) |l10|=3 |l11|=1
Cyclic model
Thinking(1<l10>1<l11>)ToKIn(2<l11,l11>) |l10|=3 |l11|=1

3 Size of state of SRS and RS

Table 4 and Table5 show the number of symbolic and ordinary markings, for random and cyclic model, (tangible/vanishing) obtained for the model in Fig.1 with two servers and for a variable number of queues.

From an analysis of results, it's evident that SRS size is relatively small acceptable, also with ten queues; while the RG grows dramatically as the number of queues increases. For $N = 10$ the RS size is a orders of magnitude longer than the SRS: this is dine to the fact that the system is symmetric.

Queue	**Num. of markings SRG**	**Num. of marking RG**
2	31/31	57/62
3	91/69	474/384
4	161/107	2712/1744
5	231/145	12480/6640
6	301/221	49776/22560
7	371/221	179424/70805
8	441/259	599808/209436
9	511/297	$1.9 \cdot 10^6/0.6 \cdot 10^6$
10	581/335	$5.7 \cdot 10^6/1.6 \cdot 10^6$

Table 4: Table of the markings of random polling system

Queue	Num. of markings SRG	Num. of marking RG
2	6/4	12/8
3	12/8	36/24
4	24/16	96/64
5	48/32	240/160
6	96/64	576/684
7	192/128	1344/896
8	384/256	3072/2048
9	768/512	6912/4608
10	1536/1024	15360/10240

Table 5: Table of the markings of cyclic polling system

References

[1] M. Ajmone Marsan, S. Donatelli, G. Franceschinis and F. Neri. *Reduction in Generalized Stochastic Petri Nets and Stochastic Well-formed Net:an overview and an example of application.* In The State-of-the-art in Performace Modeling and Simulation: Network Theory, Tool and Tutorial, J. Walrand, K. Bagchi and G. Zobrist(editors). Gordon and Breach Publishers INC., 1997