STRUCTURED SOLUTION OF STOCHASTIC DSSP SYSTEMS

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Outline

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4. RS expression for St-DSSP

5. CTMC expression for St-DSSP

6. Advantages/disadvantages

Motivations

- St-DSSP are compositional by definition

- Approximate solution already exploits composition

- Exact numerical solution?

+ To work in a class in which properties can be proved without RG construction

Contributions

- Extend the asynchronous approach to a different class of net

- Put the construction in a general framework to allow an easy extension to other classes

DSSP definition

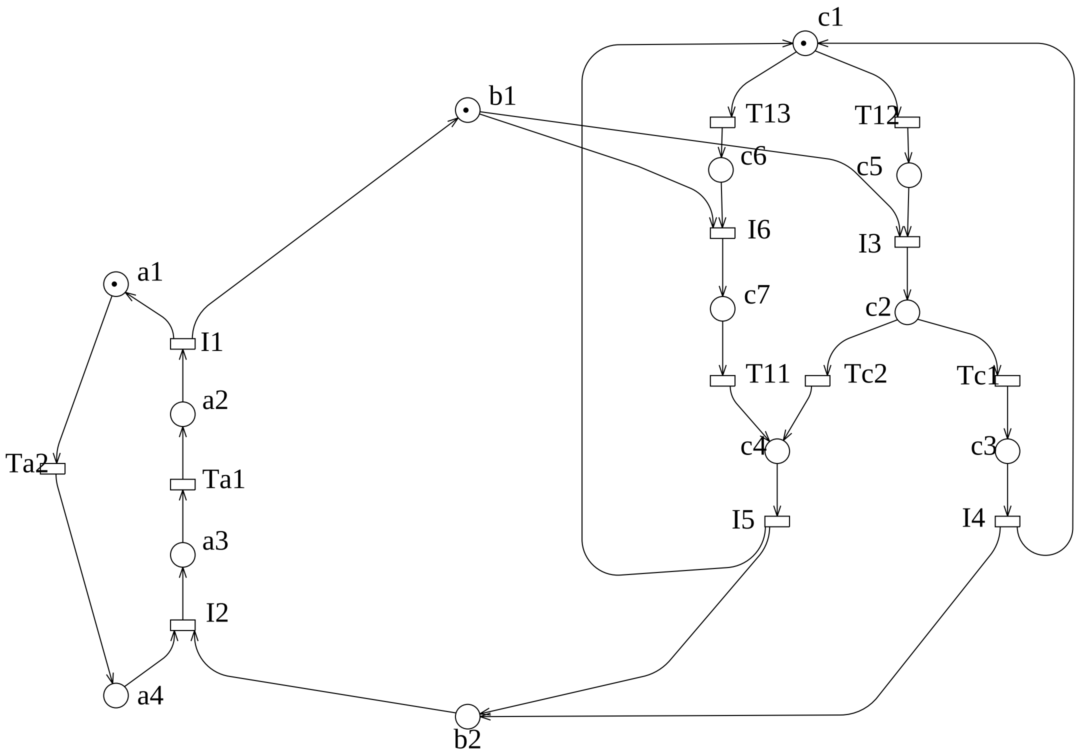
$\mathcal{S} = \{P_1 \cup \ldots \cup P_K \cup B, T_1 \cup \ldots \cup T_K, \mathbf{Pre}, \mathbf{Post}, \mathbf{m_0}\}$, is a DSSP (*Deterministically Synchronized Sequential Processes*) iff:

1. $P_i \cap P_j = \emptyset$, $T_i \cap T_j = \emptyset$, $P_i \cap B = \emptyset$,
2. $\langle \mathcal{SM}_i, \mathbf{m_0}_i \rangle$ is a **state machine**, strongly connected and 1–bounded
3. $\forall$ buffer $b \in B$:
   (a) Nor sink neither source
   $|{}^\bullet b| \geq 1$ and $|b^\bullet| \geq 1$,
   (b) Output private
   $\exists i \in \{1, \ldots, K\}$ such that $b^\bullet \subset T_i$,
   (c) Deterministically synchronized
   $\forall p \in P_1 \cup \ldots \cup P_K$: $t, t' \in p^\bullet \implies$
   $\mathbf{Pre}[b, t] = \mathbf{Pre}[b, t']$.

$\mathrm{TI} = {}^\bullet B \cup B^\bullet$ *interface transitions.*

$(T_1 \cup \ldots \cup T_K) \setminus \mathrm{TI}$ *internal transitions*

DSSP example



$$B = \{b1, b2\}$$

$$P_1 = \{a1, a2, a3, a4\}$$

$$P_2 = \{c1, \ldots, c7\}$$

$$TI = \{I1, \ldots, I6\}$$

St-DSSP definition

Stochastic DSSP (St-DSSP) $\{\mathcal{S}, w\}$

$$\{P, T, \mathbf{Pre}, \mathbf{Post}, \mathbf{m_0}, w\}$$

- $\mathcal{S}$ is a DSSP and
- $w : T \to \mathbb{R}^+$ is the rate of exponentially distributed firing time

Immediate transitions?

$+$ Internal

Can be reduced in the class

$-$ Interface

Cannot be reduced in the class

DSSP properties

1. Live and bounded $\Longrightarrow$ home states.

2. For bounded and strongly connected DSSP:

   $$\text{Live} \Longleftrightarrow \text{deadlock-free}$$

3. Let $\mathbf{C}$ be the $n \times m$ incidence matrix.

   Structurally bounded $\Longleftrightarrow$

   $\exists \mathbf{y} \in \mathbb{N}^n : \mathbf{y} > \mathbf{0} \land \mathbf{y} \cdot \mathbf{C} \leq \mathbf{0}$.

4. Deadlock-free $\Longleftrightarrow$ "a" linear programming problem has no integer solution.

5. For live DSSP:

   bounded $\Longleftrightarrow$ it is structurally bounded

6. Structurally live and structurally bounded

   $\Longleftrightarrow$ consistent (i.e., $\exists \mathbf{x} > \mathbf{0}, \mathbf{C} \cdot \mathbf{x} = \mathbf{0}$)

   and conservative (i.e., $\exists \mathbf{y} > \mathbf{0}, \mathbf{y} \cdot \mathbf{C} = \mathbf{0}$)

   and rank($\mathbf{C}$) $= |\mathcal{E}| - 1$

Proving ergodicity

Step 1: check structural boundedness using statement *3*;

Step 2: check the characterization for structural liveness and structural boundedness using statement *6*;

Step 3: check deadlock-freeness using statement *4* (thus, by statement *2*, liveness).

Answer = YES if and only if St-DSSP is live and bounded (statement *5*) thus it has home state (statement *1*), therefore

the CTMC is finite and ergodic

and all the transitions have non-null throughput.

Structured solution methods

- **Goal:** avoid the explicit construction of RG and $\mathbf{Q}$ and its storing

- **How:** find (or use) a decomposition into K components
  - Express $\mathbf{Q}$ as tensor expression of $\mathbf{Q}_i$
  - Compute $\boldsymbol{\pi} \cdot \mathbf{Q}$ using the expression, without storing $\mathbf{Q}$

Synchronous approach
(without an abstract view)

Synchronous approach (without an abstract view)

- $\mathrm{RS} \subseteq \mathrm{PS} = \mathrm{RS}_1 \times \ldots \times \mathrm{RS}_K$

- $\mathbf{Q} = \mathbf{R} - \mathrm{rowsum}(\mathbf{R})$

- $\mathbf{R} \subseteq \bigoplus_{i=1}^{K} \mathbf{R}'_i \; + \sum_{t \in TS} w(t) \bigotimes_{i=1}^{K} \mathbf{K}_i(t)$

This is the approach of

- SAN (Plateau),
- SGSPN (Donatelli)
- Synchronized SWN (Haddad - Moreaux)

Asynchronous approach (with an abstract view)

Asynchronous approach (with an abstract view)

$$\mathrm{RS}(\mathcal{S}) = \biguplus_{\mathbf{z} \in \mathrm{RS}(\mathcal{BS})} \mathrm{RS}_{\mathbf{z}}(\mathcal{S}) \subseteq$$

$$\biguplus_{\mathbf{z} \in \mathrm{RS}(\mathcal{BS})} \{\mathbf{z}\} \times \mathrm{RS}_{\mathbf{z}}(\mathcal{LS}_1) \times \cdots \times \mathrm{RS}_{\mathbf{z}}(\mathcal{LS}_K)$$

$\mathbf{R}$ can be split in sub-blocks $\mathbf{R}(\mathbf{z}, \mathbf{z}')$

$$\mathbf{R}(\mathbf{z}, \mathbf{z}) = \bigoplus_{i=1}^{K} \mathbf{R}_i(\mathbf{z}, \mathbf{z})$$

$$\mathbf{R}(\mathbf{z}, \mathbf{z}') = \bigotimes_{i=1}^{K} \mathbf{R}_i(\mathbf{z}, \mathbf{z}')$$

This is the approach proposed for

- MG (Buchholz - Kemper),
- HCGSPN (Buchholz) and
- Asynchronous SWN (Haddad - Moreaux)

Structured solution of St-DSSP

Determine an high level view that

- can coexist with the original net

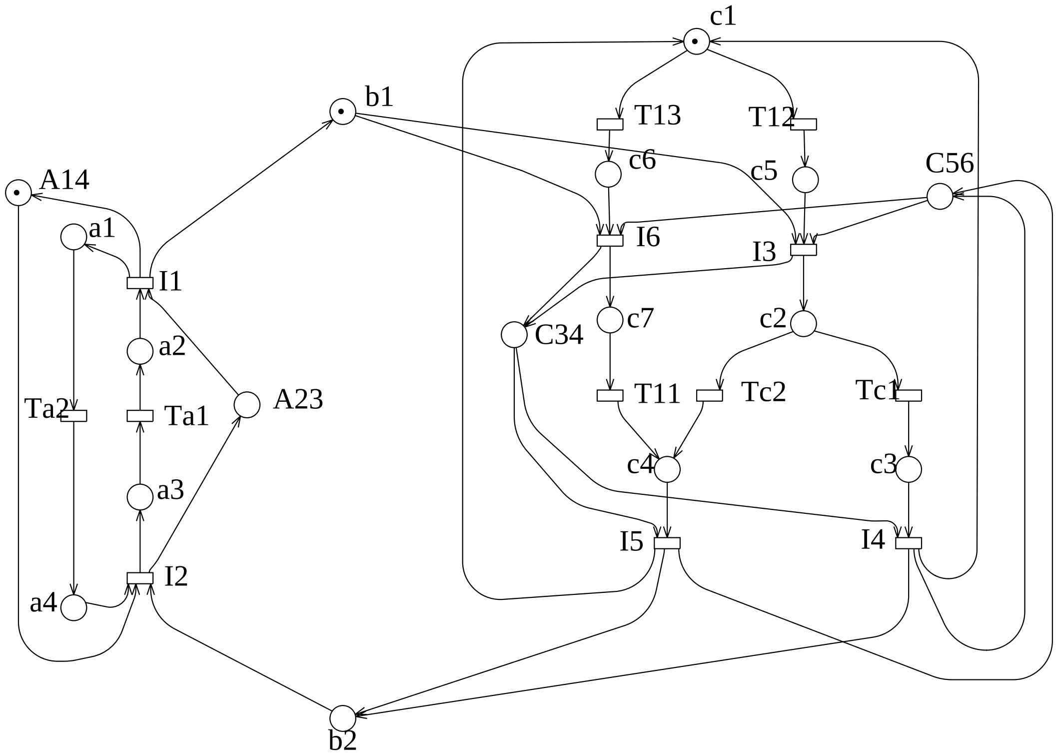
- it is easy to compute.

Show that it is possible to work with superset of state space and supermatrix of infinitesimal generator

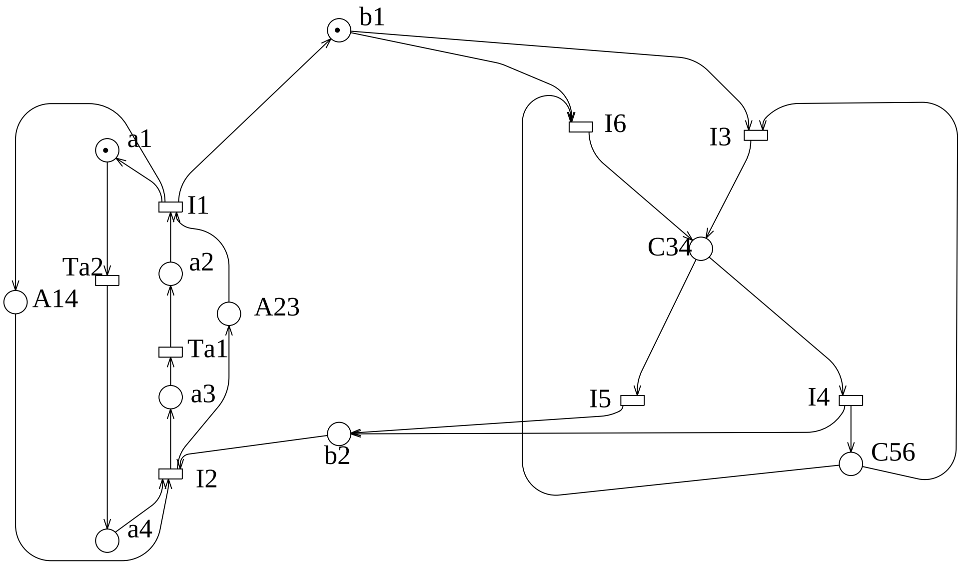
Example

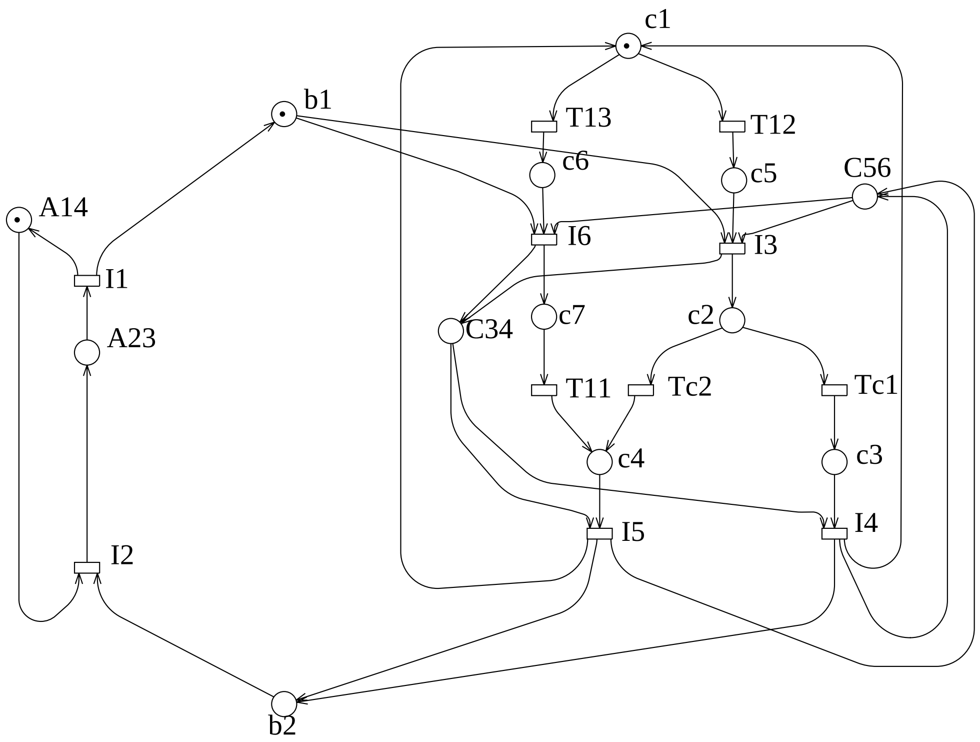
Definition

Auxiliary systems

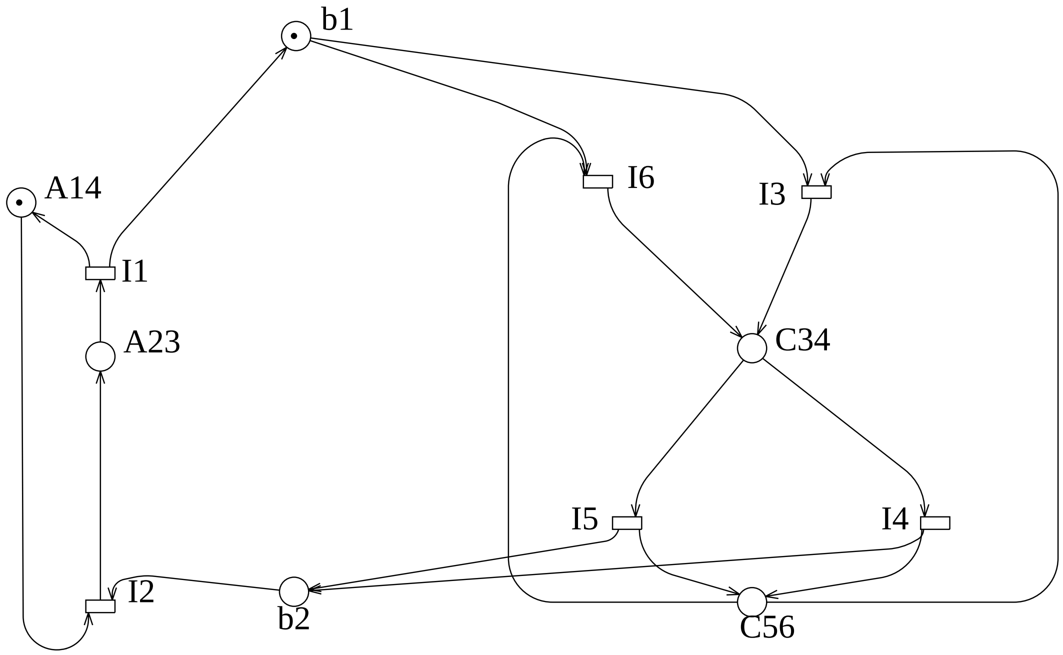


Low level


b1
a1
I6
I3
I1
C34
a2
Ta2
A14
A23
Ta1
a3
I5
I4
C56
b2
I2
a4


c1
b1
T13
T12
c6
c5
C56
A14
I6
I3
I1
c7
c2
C34
A23
T11
Tc2
Tc1
c4
c3
I4
I5
I2
b2

Basic skeleton



Definitions

Equivalence relation R

R is defined on $P \setminus B$ by: $\langle p_1^i, p_2^i \rangle \in \mathrm{R}$ for $p_1^i, p_2^i \in P_i$ iff there exists a non-directed path np in $\mathcal{SM}_i$ from p_1^i to p_2^i such that $\mathrm{np} \cap \mathrm{TI} = \emptyset$ (i.e., containing only internal transitions). Let $[p_1^i]$ be the corresponding equivalence classes.

$$[p_1^i] \quad \Longleftrightarrow \quad h_j^i$$

Definitions

Extended System $\mathcal{ES}$

i) $P_{\mathcal{ES}} = P \cup H_1 \cup \ldots \cup H_K$,
with $H_i = \{h_1^i, \ldots, h_{r(i)}^i\}$

ii) $T_{\mathcal{ES}} = T$;

iii) $\mathbf{Pre}_{\mathcal{ES}}|_{P \times T} = \mathbf{Pre}$; $\mathbf{Post}_{\mathcal{ES}}|_{P \times T} = \mathbf{Post}$;

iv) for $t \in \mathrm{TI} \cap T_i$:

$$\mathbf{Pre}[h_j^i, t] = \sum_{p \in [p_j^i]} \mathbf{Pre}[p, t];$$

$$\mathbf{Post}[h_j^i, t] = \sum_{p \in [p_j^i]} \mathbf{Post}[p, t],$$

v) $\mathbf{m_0}^{\mathcal{ES}}[p] = \mathbf{m_0}[p]$, for all $p \in P$;

vi) $\mathbf{m_0}^{\mathcal{ES}}[h_j^i] = \sum_{p \in [p_j^i]} \mathbf{m_0}[p]$.

Definitions

Low Level Systems $\mathcal{LS}_i$

$\mathcal{LS}_i$ $(i = 1, \ldots, K)$ of $\mathcal{S}$ is obtained from $\mathcal{ES}$ deleting all the nodes in $\bigcup\limits_{j \neq i} (P_j \cup (T_j \setminus \mathrm{TI}))$ and their adjacent arcs.

Basic Skeleton $\mathcal{BS}$

$\mathcal{BS}$ is obtained from $\mathcal{ES}$ deleting all the nodes in $\bigcup\limits_{j} (P_j \cup (T_j \setminus \mathrm{TI}))$ and their adjacent arcs.

Properties

$\mathcal{S}$ is a DSSP

$\mathcal{LS}_i$ its low level systems ($i = 1, \ldots, K$),

$\mathcal{BS}$ its basic skeleton,

$\mathrm{L}(\mathcal{S})$ the language of $\mathcal{S}$

1. $\mathrm{RG}(\mathcal{ES}) \cong \mathrm{RG}(\mathcal{S})$

2. $\mathrm{L}(\mathcal{S})|_{T_i \cup \mathrm{TI}} \subseteq \mathrm{L}(\mathcal{LS}_i)$, for $i = 1, \ldots, K$.

3. $\mathrm{L}(\mathcal{S})|_{\mathrm{TI}} \subseteq \mathrm{L}(\mathcal{BS})$.

Reachability set construction

$$\mathrm{RS}_{\mathbf{z}}(\mathcal{ES}) \;=\; \{\mathbf{m} \in \mathrm{RS}(\mathcal{ES}) \;:\; \mathbf{m}|_{H_1 \cup \ldots \cup H_K \cup B} = \mathbf{z}\}$$

$$\mathrm{RS}_{\mathbf{z}}(\mathcal{S}) \;=\; \{\mathbf{m} \in \mathrm{RS}(\mathcal{S}) \;\text{ such that }\; \exists \mathbf{m}' \in \mathrm{RS}_{\mathbf{z}}(\mathcal{ES}) : \mathbf{m}'|_{P_1 \cup \ldots \cup P_K \cup B} = \mathbf{m}\}$$

$$\mathrm{RS}_{\mathbf{z}}(\mathcal{LS}_i) \;=\; \{\mathbf{m}_i \in \mathrm{RS}(\mathcal{LS}_i) \;:\; \mathbf{m}_i|_{H_1 \cup \ldots \cup H_K \cup B} = \mathbf{z}\}$$

$$\mathrm{PS}_{\mathbf{z}}(\mathcal{S}) = \{\mathbf{z}|_B\} \times \mathrm{RS}_{\mathbf{z}}(\mathcal{LS}_1)|_{P_1} \times \cdots \times \mathrm{RS}_{\mathbf{z}}(\mathcal{LS}_K)|_{P_K}$$

$$\mathrm{PS}(\mathcal{S}) = \biguplus_{\mathbf{z} \in \mathrm{RS}(\mathcal{BS})} \mathrm{PS}_{\mathbf{z}}(\mathcal{S})$$

$$\mathrm{RS}(\mathcal{S}) \subseteq \mathrm{PS}(\mathcal{S}) = \biguplus_{\mathbf{z} \in \mathrm{RS}(\mathcal{BS})} \mathrm{PS}_{\mathbf{z}}(\mathcal{S})$$

$$\mathrm{RS}_{\mathbf{z}}(\mathcal{S}) \subseteq \mathrm{PS}_{\mathbf{z}}(\mathcal{S})$$

Example of RS construction

RS of $\mathcal{S}$	
$\mathbf{v}_1$	a1, b1, c1
$\mathbf{v}_2$	a1, b1, c6
$\mathbf{v}_3$	a1, b1, c5
$\mathbf{v}_4$	a4, b1, c1
$\mathbf{v}_5$	a1, c7
$\mathbf{v}_6$	a4, b1, c6
$\mathbf{v}_7$	a4, b1, c5
$\mathbf{v}_8$	a1, c2
$\mathbf{v}_9$	a1, c4
$\mathbf{v}_{10}$	a4, c7
$\mathbf{v}_{11}$	a4, c2
$\mathbf{v}_{12}$	a1, c3
$\mathbf{v}_{13}$	a4, c4

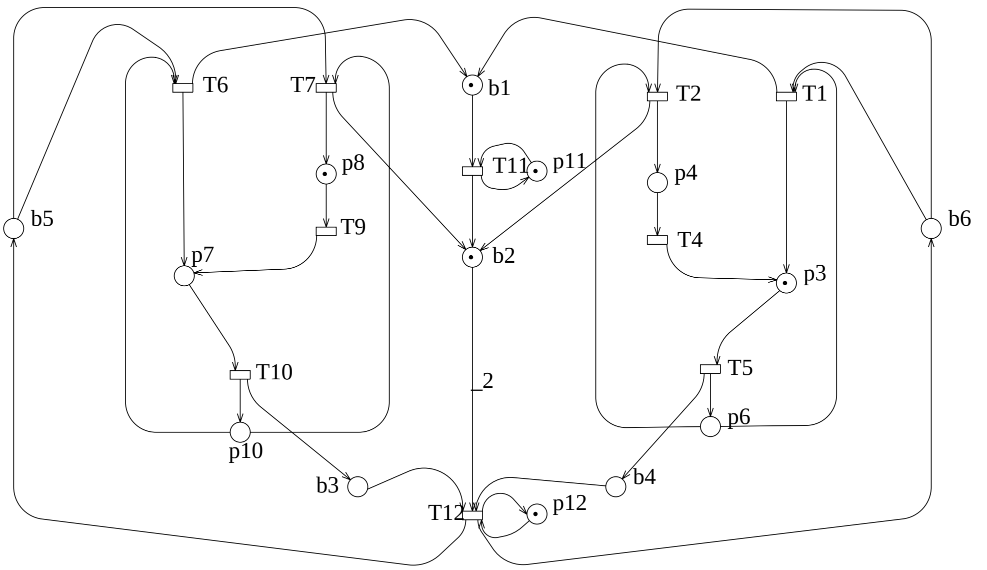
RS of $\mathcal{S}$	
$\mathbf{v}_{14}$	a1, c1, b2
$\mathbf{v}_{15}$	a4, c3
$\mathbf{v}_{16}$	a4, c1, b2
$\mathbf{v}_{17}$	a1, b2, c6
$\mathbf{v}_{18}$	a1, b2, c5
$\mathbf{v}_{19}$	a4, b2, c6
$\mathbf{v}_{20}$	a4, b2, c5
$\mathbf{v}_{21}$	a3, c1
$\mathbf{v}_{22}$	a3, c6
$\mathbf{v}_{23}$	a3, c5
$\mathbf{v}_{24}$	a2, c1
$\mathbf{v}_{25}$	a2, c6
$\mathbf{v}_{26}$	a2, c5

RS of $\mathcal{BS}$	
$\mathbf{z}_1$	A14, C56, b1
$\mathbf{z}_2$	A14, C34
$\mathbf{z}_3$	A14, C56, b2
$\mathbf{z}_4$	A23, C56

RS of $\mathcal{LS}_1$		
$\mathbf{x}_1$	a1, b1, C56, A14	$\mathbf{z}_1$
$\mathbf{x}_2$	a4, b1, C56, A14	$\mathbf{z}_1$
$\mathbf{x}_3$	a1, C34, A14	$\mathbf{z}_2$
$\mathbf{x}_4$	a4, C34, A14	$\mathbf{z}_2$
$\mathbf{x}_5$	a1, b2, C56, A14	$\mathbf{z}_3$
$\mathbf{x}_6$	a4, b2, C56, A14	$\mathbf{z}_3$
$\mathbf{x}_7$	a3, C56, A23	$\mathbf{z}_4$
$\mathbf{x}_8$	a2, C56, A23	$\mathbf{z}_4$

RS of $\mathcal{LS}_2$		
$\mathbf{y}_1$	A14, b1, c1, C56	$\mathbf{z}_1$
$\mathbf{y}_2$	A14, b1, c6, C56	$\mathbf{z}_1$
$\mathbf{y}_3$	A14, b1, c5, C56	$\mathbf{z}_1$
$\mathbf{y}_4$	A14, c7, C34	$\mathbf{z}_2$
$\mathbf{y}_5$	A14, c2, C34	$\mathbf{z}_2$
$\mathbf{y}_6$	A14, c4, C34	$\mathbf{z}_2$
$\mathbf{y}_7$	A14, c3, C34	$\mathbf{z}_2$
$\mathbf{y}_8$	A14, b2, c1, C56	$\mathbf{z}_3$
$\mathbf{y}_9$	A14, b2, c6, C56	$\mathbf{z}_3$
$\mathbf{y}_{10}$	A14, b2, c5, C56	$\mathbf{z}_3$
$\mathbf{y}_{11}$	A23, c1, C56	$\mathbf{z}_4$
$\mathbf{y}_{12}$	A23, c6, C56	$\mathbf{z}_4$
$\mathbf{y}_{13}$	A23, c5, C56	$\mathbf{z}_4$

Counter-example for state space



z= [b1, b2, P34, P78] is in RS($\mathcal{BS}$)

[p8, b1, b2, P34] and [p7, b1, b2, P34] are in $\text{RS}_{\mathbf{z}}(\mathcal{LS}_1)$

[p3, b1, b2, P78] and [p4, b1, b2, P78] is in $\text{RS}_{\mathbf{z}}(\mathcal{LS}_2)$

The cross product gives a $\text{PS}_{\mathbf{z}}$ equal to

```
[ p3, p8, b1, b2 ]      [ p3, p7, b1, b2 ]
[ p4, p8, b1, b2 ]      [ p4, p7, b1, b2 ]
```

but state [p4, p8, b1, b2] does not belong to the RS of the DSSP

CTMC generation

Basic idea: split the behaviour in two

1. transitions that *change* the high level view
2. transitions that *do not change* the high level view

CTMC generation

$$\mathbf{Q} = \mathbf{R} - \text{rowsum}(\mathbf{R})$$

For the $\mathcal{LS}_i$ components:

$$\mathbf{Q}_i = \mathbf{R}_i - rowsum(\mathbf{R}_i)$$

Technique:

(1) Consider $\mathbf{Q}$ and $\mathbf{R}$ in blocks $(\mathbf{z}, \mathbf{z'})$, of size $|\mathrm{RS}_{\mathbf{z}}(\mathcal{S})| \cdot |\mathrm{RS}_{\mathbf{z'}}(\mathcal{S})|$

(2) Consider $\mathbf{Q}_i$ and $\mathbf{R}_i$ in blocks $(\mathbf{z}, \mathbf{z'})$ of size $|\mathrm{RS}_{\mathbf{z}}(\mathcal{LS}_i)| \cdot |\mathrm{RS}_{\mathbf{z'}}(\mathcal{LS}_i)|$

(3) Describe each block of $\mathbf{Q}$ and $\mathbf{R}$ as tensor expression of the blocks of $\mathbf{Q}_i$ and $\mathbf{R}_i$.

CTMC generation

Blocks $\mathbf{R}(\mathbf{z}, \mathbf{z})$ have non null entries that are due *only* to non interface transitions

$$\mathbf{G}(\mathbf{z}, \mathbf{z}) = \bigoplus_{i=1}^{K} \mathbf{R}_i(\mathbf{z}, \mathbf{z})$$

Blocks $\mathbf{R}(\mathbf{z}, \mathbf{z}')$ with $\mathbf{z} \neq \mathbf{z}'$ have non null entries that are due *only* to the firing of transitions in TI.

$$\mathbf{K}_i(t)(\mathbf{z}, \mathbf{z}')[\mathbf{m}, \mathbf{m}'] = \begin{cases} 1 & \text{if } \mathbf{m} \xrightarrow{t} \mathbf{m}' \\ 0 & \text{otherwise} \end{cases}$$

$$\mathbf{G}(\mathbf{z}, \mathbf{z}') = \sum_{t \in \mathrm{TI}_{\mathbf{z},\mathbf{z}'}} w(t) \bigotimes_{i=1}^{K} \mathbf{K}_i(t)(\mathbf{z}, \mathbf{z}')$$

CTMC definition

1. Transition rates among reachable states are correctly computed
   $\forall \mathbf{z}$ and $\mathbf{z}' \in \mathrm{RS}(\mathcal{BS})$:
   $$\mathbf{R}(\mathbf{z}, \mathbf{z}') \text{ is a submatrix of } \mathbf{G}(\mathbf{z}, \mathbf{z}')$$

2. Unreachable states are never assigned a non-null probability
   $\forall \mathbf{m} \in \mathrm{RS}(\mathcal{S})$ and $\forall \mathbf{m}' \in \mathrm{PS}(\mathcal{S}) \setminus \mathrm{RS}(\mathcal{S})$:
   $$\mathbf{G}[\mathbf{m}, \mathbf{m}'] = 0$$

CTMC example

$$\mathbf{R} = \left(\begin{array}{c|c|c|c} \mathbf{R}(\mathbf{z}_1,\mathbf{z}_1) & \mathbf{R}(\mathbf{z}_1,\mathbf{z}_2) & \mathbf{R}(\mathbf{z}_1,\mathbf{z}_3) & \mathbf{R}(\mathbf{z}_1,\mathbf{z}_4) \\ \hline \mathbf{R}(\mathbf{z}_2,\mathbf{z}_1) & \mathbf{R}(\mathbf{z}_2,\mathbf{z}_2) & \mathbf{R}(\mathbf{z}_2,\mathbf{z}_3) & \mathbf{R}(\mathbf{z}_2,\mathbf{z}_4) \\ \hline \mathbf{R}(\mathbf{z}_3,\mathbf{z}_1) & \mathbf{R}(\mathbf{z}_3,\mathbf{z}_2) & \mathbf{R}(\mathbf{z}_3,\mathbf{z}_3) & \mathbf{R}(\mathbf{z}_3,\mathbf{z}_4) \\ \hline \mathbf{R}(\mathbf{z}_4,\mathbf{z}_1) & \mathbf{R}(\mathbf{z}_4,\mathbf{z}_2) & \mathbf{R}(\mathbf{z}_4,\mathbf{z}_3) & \mathbf{R}(\mathbf{z}_4,\mathbf{z}_4) \end{array}\right)$$

$$\begin{aligned} \mathbf{G}(\mathbf{z}_1,\mathbf{z}_1) &= \mathbf{R}_1(\mathbf{z}_1,\mathbf{z}_1) \oplus \mathbf{R}_2(\mathbf{z}_1,\mathbf{z}_1) \\ \mathbf{G}(\mathbf{z}_1,\mathbf{z}_2) &= w(I_3)(\mathbf{K}_1(I_3)(\mathbf{z}_1,\mathbf{z}_2) \otimes \mathbf{K}_2(I_3)(\mathbf{z}_1,\mathbf{z}_2)) + \\ &\quad w(I_6)(\mathbf{K}_1(I_6)(\mathbf{z}_1,\mathbf{z}_2) \otimes \mathbf{K}_2(I_6)(\mathbf{z}_1,\mathbf{z}_2)) \end{aligned}$$

CTMC example

$\mathbf{R}_1(\mathbf{z}_1, \mathbf{z}_1)$

	$\mathbf{x}_1$	$\mathbf{x}_2$
$\mathbf{x}_1$		$w(Ta_2)$
$\mathbf{x}_2$		

$\mathbf{R}_2(\mathbf{z}_1, \mathbf{z}_1)$

	$\mathbf{y}_1$	$\mathbf{y}_2$	$\mathbf{y}_3$
$\mathbf{y}_1$		$w(T_{13})$	$w(T_{12})$
$\mathbf{y}_2$			
$\mathbf{y}_3$			

$\mathbf{R}(\mathbf{z}_1, \mathbf{z}_1) =$

	$\mathbf{x}_1, \mathbf{y}_1$	$\mathbf{x}_1, \mathbf{y}_2$	$\mathbf{x}_1, \mathbf{y}_3$	$\mathbf{x}_2, \mathbf{y}_1$	$\mathbf{x}_2, \mathbf{y}_2$	$\mathbf{x}_2, \mathbf{y}_3$
$\mathbf{x}_1, \mathbf{y}_1$		$w(T_{13})$	$w(T_{12})$	$w(Ta_2)$		
$\mathbf{x}_1, \mathbf{y}_2$					$w(Ta_2)$	
$\mathbf{x}_1, \mathbf{y}_3$						$w(Ta_2)$
$\mathbf{x}_2, \mathbf{y}_1$					$w(T_{13})$	$w(T_{12})$
$\mathbf{x}_2, \mathbf{y}_2$						
$\mathbf{x}_2, \mathbf{y}_3$						

CTMC example

$$
\mathbf{R}(\mathbf{z}_1,\mathbf{z}_2) = \left(\begin{array}{c|cccccccc}
 & \mathbf{x}_3,\mathbf{y}_3 & \mathbf{x}_3,\mathbf{y}_4 & \mathbf{x}_3,\mathbf{y}_5 & \mathbf{x}_3,\mathbf{y}_6 & \mathbf{x}_4,\mathbf{y}_3 & \mathbf{x}_4,\mathbf{y}_4 & \mathbf{x}_4,\mathbf{y}_5 & \mathbf{x}_4,\mathbf{y}_6 \\
\hline
\mathbf{x}_1,\mathbf{y}_1 & & & & & & & & \\
\mathbf{x}_1,\mathbf{y}_2 & w(I_6) & & & & & & & \\
\mathbf{x}_1,\mathbf{y}_3 & & w(I_3) & & & & & & \\
\mathbf{x}_2,\mathbf{y}_1 & & & & & & & & \\
\mathbf{x}_2,\mathbf{y}_2 & & & & & w(I_6) & & & \\
\mathbf{x}_2,\mathbf{y}_3 & & & & & & w(I_3) & &
\end{array}\right)
$$

$$
\mathbf{K}_1(I_3)(\mathbf{z}_1,\mathbf{z}_2) = \left(\begin{array}{c|cc}
 & \mathbf{x}_3 & \mathbf{x}_4 \\
\hline
\mathbf{x}_1 & 1 & \\
\mathbf{x}_2 & & 1
\end{array}\right)
\quad
\mathbf{K}_2(I_3)(\mathbf{z}_1,\mathbf{z}_2) = \left(\begin{array}{c|cccc}
 & \mathbf{y}_4 & \mathbf{y}_5 & \mathbf{y}_6 & \mathbf{y}_7 \\
\hline
\mathbf{y}_1 & & & & \\
\mathbf{y}_2 & & & & \\
\mathbf{y}_3 & & 1 & &
\end{array}\right)
$$

$$
\mathbf{K}_1(I_6)(\mathbf{z}_1,\mathbf{z}_2) = \left(\begin{array}{c|cc}
 & \mathbf{x}_3 & \mathbf{x}_4 \\
\hline
\mathbf{x}_1 & 1 & \\
\mathbf{x}_2 & & 1
\end{array}\right)
\quad
\mathbf{K}_2(I_6)(\mathbf{z}_1,\mathbf{z}_2) = \left(\begin{array}{c|cccc}
 & \mathbf{y}_4 & \mathbf{y}_5 & \mathbf{y}_6 & \mathbf{y}_7 \\
\hline
\mathbf{y}_1 & & & & \\
\mathbf{y}_2 & 1 & & & \\
\mathbf{y}_2 & & & &
\end{array}\right)
$$

Computational costs

To solve an SPN

- build the RG,
- compute the associated CTMC
- solve the characteristic equation $\boldsymbol{\pi} \cdot \mathbf{Q} = \mathbf{0}$.

To solve a DSSP:

- build the $K + 1$ auxiliary models,
- compute the RG_i of each auxiliary model,
- compute the $\mathbf{R}_i(\mathbf{z}, \mathbf{z}')$ and $\mathbf{K}_i(t)(\mathbf{z}, \mathbf{z}')$ matrices
- solve the characteristic equation $\boldsymbol{\pi} \cdot \mathbf{G} = \mathbf{0}$

The advantages/disadvant‘ges depend on the relative size of the re‘chability graphs of $\mathcal{S}$, $\mathcal{BS}$, and $\mathcal{LS}_i$.

Storage costs

The storage cost of the classical solution method is proportional to $|\mathrm{RS}(\mathcal{S})|$ and to the number of arcs in the $\mathrm{RG}(\mathcal{S})$.

The storage cost for DSSP is proportional to $|\mathrm{PS}(\mathcal{S})|$, and to the sum of the number of arcs in the K reachability graphs $\mathrm{RG}_i(\mathcal{LS}_i)$.

The difference between the number of arcs in $\mathrm{RG}(\mathcal{S})$ and the sum of the number of arcs in the K $\mathrm{RG}_i(\mathcal{LS}_i)$ is what makes the method applicable in cases in which a direct solution is not possible, due to the lack of memory to store $\mathbf{Q}$.