Fusing Range and Intensity Images for Mobile Robot Localization

José Neira, Juan D. Tardós, Joachim Horn, and Günther Schmidt

***Abstract*—In this paper, we present the two-dimensional (2-D) version of the *symmetries and perturbation model* (SPmodel), a probabilistic representation model and an EKF integration mechanism for uncertain geometric information that is suitable for sensor fusion and integration in multisensor systems. We apply the SPmodel to the problem of location estimation in indoor mobile robotics, experimenting with the mobile robot MACROBE. We have chosen two types of complementary sensory information:**

1) **range images;**
2) **intensity images;**

obtained from a laser sensor. Results of these experiments show that fusing simple and computationally inexpensive sensory information can allow a mobile robot to precisely locate itself. They also demonstrate the generality of the proposed fusion and integration mechanism.

***Index Terms*—Computer vision, map-based localization, mobile robots, range finder, sensor fusion.**

I. Introduction

MOBILE robots cannot rely solely on dead-reckoning to determine their location because dead-reckoning errors are cumulative. For this reason, mobile robots must be equipped with *exteroceptive* sensors that obtain information from the environment to help the robot determine its location more accurately.

There has been considerable work in *map-based localization,* usually relying on one type of sensor or on sensing one type of feature from the environment. The most commonly used sensors are sonar [1], [2], laser or infrared rangefinders [3]–[5], monocular vision [6]–[10], and stereo vision [11]–[14]. In this work, we show that multisensor integration using simple and inexpensive sensor processing, allows a mobile robot to precisely locate itself, and at the same time makes the robotic system more robust.

With respect to *precision,* there are situations in which relying on one sensor or one type of feature may be insufficient. For example, a mobile robot equipped with a laser rangefinder, such as the MACROBE [5], is capable of obtaining information of the scene in front of the robot. This information can be used to determine the location of walls in its field of view. The detection of a wall allows the robot to reduce uncertainty in the direction normal to the wall. This proves to be quite effective in most cases. However, when the sensor only detects walls in front of it, uncertainty is only reduced in the direction of motion. In this situation, being able to obtain sensor information of different nature would help the robot to locate itself more precisely.

In relation with *robustness,* several processes in map-based localization can take advantage of multisensor integration.

1) *Feature Extraction*: Some sensor combinations, for example a laser rangefinder and a vision system, are *complementary.* The rangefinder cannot detect reliably a closed door, that can be easily extracted with vision. On the other hand, vision alone will require more complex processing to determine if a door is open or not; this is evident to the rangefinder. Cooperation of both sensors can be used to obtain better precision and robustness in the extraction of *high level features* such as corners or doors [15].
2) *Matching observations* with the map is known to be a difficult problem, especially with monocular vision systems [6], [10] because they obtain very incomplete information about the location of natural landmarks. However, this problem is much simpler using a laser rangefinder. The combination of both sensors increases the robustness of the system, avoiding the robot getting lost due to incorrect matchings.

Despite these potential advantages, there have been few works experimenting with mobile robot multisensor integration in real environments. The early work of Matthies and Elfes [16] integrates range information from a sonar sensor array and from a stereo vision system in an *occupancy grid,* using a Bayesian estimator to mark every cell as occupied, empty or unknown. However, such representation is not well suited to accommodate information from other sensors like monocular vision. A more general approach was used in the Hilare robot [17], integrating points obtained with a laser rangefinder and lines extracted with a stereo vision system, to obtain a set of three-dimensional (3-D) planes modeling the environment. For numeric fusion of both data and for calibrating the relative location between both sensors, an extended Kalman filter (EKF) was used. More recently, other works use a similar EKF technique to estimate the location of the robot from several

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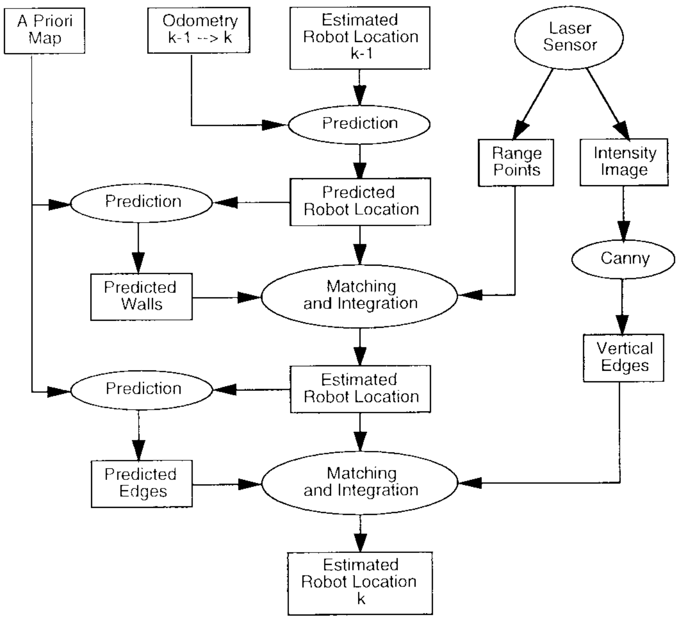


Fig. 1. Integration of range points and intensity edges for mobile robot localization.

exteroceptive sensors, such as infrared and ultrasonic [18], or laser range and vision [19].

In this work, we make use of the EKF in the experimental framework of the MACROBE project. Information provided by a laser sensor is used both to determine the location of walls in front of the robot, and by processing the intensity image, to determine the location of vertical edges that can correspond to corners, as well as door and window frames. At each step in the robot trajectory, a prediction of the robot location is obtained from previous computed locations through the integration of odometric sensors. Then the information from the range and intensity images is matched with the *a priori* map and, if accepted, is integrated to obtain a more precise estimation of the robot location (Fig. 1). Some preliminary results in this line were presented in [20]. In this paper, we present a more thorough experiment and we analyze in detail the contribution of multisensor fusion to the precision and robustness of the robot localization process.

In Section II, we describe our uncertainty representation model, the SPmodel [21], and in Section III we present the corresponding EKF based integration mechanism, the SPfilter. The process of extracting environmental information and fusing it to locate the MACROBE, as well as experimental results, are described in Section IV. Finally in Section V, the main conclusions derived from experimenting with the SPmodel and the MACROBE are drawn.

II. Uncertain Geometry: The SPmodel

For multisensor fusion to be possible, a model that allows to describe imprecise geometric information of diverse nature and fuse it appropriately is necessary. Most classical models of imprecision belong to one of two categories [22]: *set-based* models, where fusion of geometric information is accomplished by region intersection [14], [23]–[25], and *probabilistic* models, where fusion is carried out using optimal or suboptimal estimation methods, like the extended Kalman filter [26], [27].

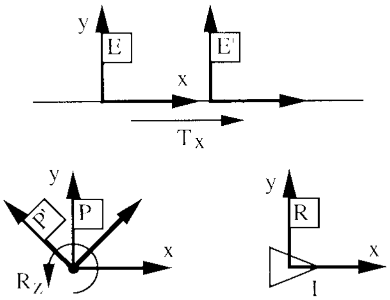


Fig. 2. Associated reference and symmetries of an edge, a point, and a robot in 2-D.

The main problem of set-based representations is the high complexity of propagating uncertainty and fusing information in multidimensional and nonlinear problems [22]. Some simplifications can be considered to reduce this complexity, but at the cost of loosing precision and obtaining pessimistic estimations [23], [24]. In contrast, probabilistic models offer mathematically simple tools that allow to obtain precise estimations. Additionally, the correlation between position and orientation errors, a critical issue in location problems, is simple to consider using probabilistic models.

In this work, we use a new probabilistic model to represent uncertain geometric information: the SPmodel [21]. The main advantage of our model against other probabilistic approaches is its generality: it offers a common representation for different types of geometric features observed by different sensors, using the concept of *symmetry*. In the following, the two-dimensional (2-D) version of the SPmodel is presented, and a description is given of how it deals with partiality and imprecision.

A. Partiality

In the SPmodel, a reference E is associated to the location of any type of geometric feature (Fig. 2). The location of this reference with respect to a base reference W is given by a transformation t_{WE} or equivalently, by a *location vector* $\mathbf{x}_{WE}$ composed by two Cartesian coordinates and an angle

$$\begin{aligned} t_{WE} &= \text{Trans}(x, y) \cdot \text{Rot}(z, \phi) \\ \mathbf{x}_{WE} &= (x, y, \phi)^T. \end{aligned}$$

The composition of two location vectors is denoted by $\oplus$, and the inversion of location vectors, as well as the composition with the inverse are denoted by $\ominus$.

Different geometric features have different d.o.f. associated to their location, that are related to its *symmetries of continuous motion*. The symmetries of a geometric entity E are defined as the set $\mathcal{S}_E$ of transformations that preserve the element. For example, the symmetries of an infinite edge are the set of continuous translations (T_x) along the edge, and the symmetries of a point are the set of continuous rotations (R_z) around it (Fig. 2). For a mobile robot, whose location is given by three d.o.f., the symmetries are the identity transformation. We represent the set of symmetries using a *row selection matrix* B_E, denominated *binding matrix of the feature.* The binding matrices for points, edges, and the robot are, respectively

$$B_P = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}; \quad B_E = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}; \quad B_R = I_3.$$

The binding matrix of a geometric entity allows us to express one of the fundamental geometric concepts, *coincidence.* Given two geometric entities of the same type, whose location is represented by references A and B, respectively, their locations coincide if

$$B_A \mathbf{x}_{AB} = 0 \tag{1}$$

where B_A denotes the binding matrix of both geometric entities. For example, in the case of two points, (1) expresses the fact that the two points coincide if the relative position of their associated references is zero, *regardless of what their relative orientation is.*

To express coincidence between different types of geometric elements, we use the *binding matrix of a pairing.* In the case of two geometric entities of different type, whose location is represented by A and B, respectively, one of the following equations expresses whether their locations coincide (up to symmetries):

$$B_{AB}\mathbf{x}_{AB} = 0 \quad \text{direct constraint} \tag{2}$$

$$B_{BA}\mathbf{x}_{BA} = 0 \quad \text{inverse constraint} \tag{3}$$

where B_{AB} or B_{BA} denote the binding matrix of the pairing. The use of the direct or inverse constraint depends on the type of geometric elements considered.

B. Imprecision

In the SPmodel, the estimate of the location of a given entity E is denoted by $\hat{\mathbf{x}}_{WE}$, and the error associated to this estimate is expressed using a *differential location vector* $\mathbf{d}_E$, relative to the reference associated to the element, so that the true location of E is given by

$$\mathbf{x}_{WE} = \hat{\mathbf{x}}_{WE} \oplus \mathbf{d}_E. \tag{4}$$

Since the d.o.f. of $\mathbf{d}_E$ corresponding to the symmetries of continuous motion contain no location information, we assign zero to their corresponding values. We call *perturbation vector* a vector $\mathbf{p}_E$ formed by the non-null elements of $\mathbf{d}_E$. These two vectors are related by the binding matrix B_E

$$\mathbf{d}_E = B_E^T \mathbf{p}_E; \quad \mathbf{p}_E = B_E \mathbf{d}_E. \tag{5}$$

The information associated to the estimated location of a geometric element E is represented by a quadruple

$$\mathbf{L}_{WE} = (\hat{\mathbf{x}}_{WE}, \hat{\mathbf{p}}_E, C_E, B_E)$$

where

$$\begin{aligned} \mathbf{x}_{WE} &= \hat{\mathbf{x}}_{WE} \oplus B_E^T \mathbf{p}_E \\ \hat{\mathbf{p}}_E &= E[\mathbf{p}_E] \\ C_E &= \text{Cov}(\mathbf{p}_E). \end{aligned}$$

Note that the error associated with a location is expressed relative to the feature reference E and not to the base reference W. In this way the value of the covariance is not magnified by the distance of the feature to the base reference. This guarantees that covariance values have a clear interpretation. The use of the binding matrix also avoids the representation being overparameterized.

C. Matching Elements under Uncertainty

Let $\mathbf{L}_{WA}$ and $\mathbf{L}_{WB}$ represent the uncertain location of two geometric entities. Assuming that the coincidence between A and B is described by the binding matrix B_{AB}, the location of A and B coincide if $B_{AB}\hat{\mathbf{x}}_{AB}$ can be considered zero. Under uncertainty, the discrepancy between the locations of A and B can be measured using the Mahalanobis distance [28]

$$D^2 = (B_{AB}\hat{\mathbf{x}}_{AB})^T [B_{AB}\text{Cov}(\mathbf{x}_{AB})B_{AB}^T]^{-1} B_{AB}\hat{\mathbf{x}}_{AB}.$$

Under the Gaussianity hypothesis, distance D^2 follows a χ^2 distribution with $m = \text{rank}(B_{AB})$ degrees of freedom. For a given significance level α, $\mathbf{x}_{WA}$, and $\mathbf{x}_{WB}$ can be considered coincident if

$$D^2 \le \chi^2_{m,\,\alpha}. \tag{6}$$

III. Multisensor Fusion: The SPfilter

In this paper, we present the use of a specialized version of the EKF for the SPmodel, the SPfilter, to estimate the location of a mobile robot from a set of partial and imprecise observations of features in the robot's environment.

The extended information filter is formulated as follows: let $\mathbf{x}$ be the state vector whose value is to be estimated, and let there be n independent and possibly partial observations $\mathbf{y}_k$ of $\mathbf{x}$, where $k \in \{1, \cdots, n\}$, affected by white Gaussian noise

$$\hat{\mathbf{y}}_k = \mathbf{y}_k + \mathbf{u}_k; \quad \mathbf{u}_k \sim N(0, S_k).$$

Let each observation $\mathbf{y}_k$ be related to $\mathbf{x}$ by an implicit nonlinear function of the form $\mathbf{f}_k(\mathbf{x}, \mathbf{y}_k) = 0$. We use a first order approximation of $\mathbf{f}_k$

$$\mathbf{f}_k(\mathbf{x}, \mathbf{y}_k) \simeq \mathbf{h}_k + H_k(\mathbf{x} - \hat{\mathbf{x}}) + G_k(\mathbf{y}_k - \hat{\mathbf{y}}_k)$$

where

$$\begin{aligned} \mathbf{h}_k &= \mathbf{f}_k(\hat{\mathbf{x}}, \hat{\mathbf{y}}_k) \\ H_k &= \left.\frac{\partial \mathbf{f}_k}{\partial \mathbf{x}}\right|_{(\hat{\mathbf{x}}, \hat{\mathbf{y}}_k)} \\ G_k &= \left.\frac{\partial \mathbf{f}_k}{\partial \mathbf{y}}\right|_{(\hat{\mathbf{x}}, \hat{\mathbf{y}}_k)}. \end{aligned} \tag{7}$$

The estimate $\hat{\mathbf{x}}_n$ of the state vector and its covariance P_n after integrating the n measurements are

$$\begin{aligned} \hat{\mathbf{x}}_n &= P_n M_n \\ P_n^{-1} &= \sum_{k=1}^{n} H_k^T (G_k S_k G_k^T)^{-1} H_k \\ M_n &= -\sum_{k=1}^{n} H_k^T (G_k S_k G_k^T)^{-1} \mathbf{h}_k. \end{aligned} \tag{8}$$

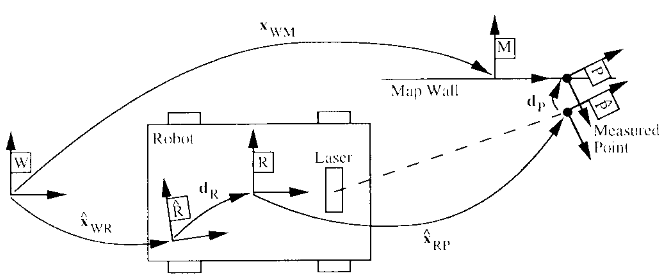


Fig. 3. References involved in the integration of a laser point to the estimation of the robot location.

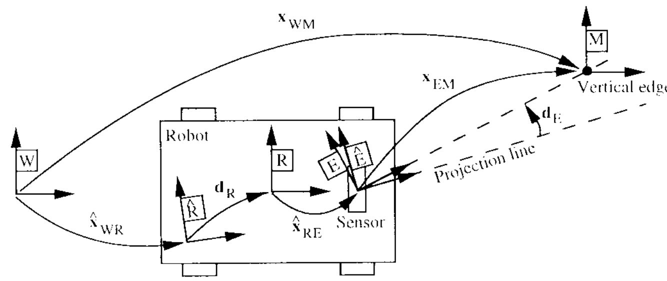


Fig. 4. References involved in the integration of a vertical edge to the estimation of the robot location.

This scheme is applicable to a wide range of estimation problems. In this work we apply it to two different estimation problems in mobile robotics:

1) estimation of the location of the robot from laser points measured on a wall;
2) estimation of the location of the robot from edges extracted from an intensity image.

A. Estimating the Location of a Mobile Robot from Range Points

Let $\mathbf{L}_{WR} = (\hat{\mathbf{x}}_{WR}, \hat{\mathbf{p}}_R, C_R, I_3)$ be the estimated location of a mobile robot. Let $\mathbf{x}_{WM}$ represent the location of a vertical wall (a 2-D edge) according to the map. Let $\mathbf{L}_{RP} = (\hat{\mathbf{x}}_{RP}, \hat{\mathbf{p}}_P, C_P, B_P)$ be the estimated location of a point on the wall, observed by a laser range system (Fig. 3). In this case, we have

$$B_M = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}; \quad B_P = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$
$$B_{MP} = [0 \quad 1 \quad 0].$$

To improve the estimation of the robot location we impose the condition that the observed point P must belong to the model wall M. This fact, which should be verified during the matching process using (6), can be stated by means of the direct constraint (2)

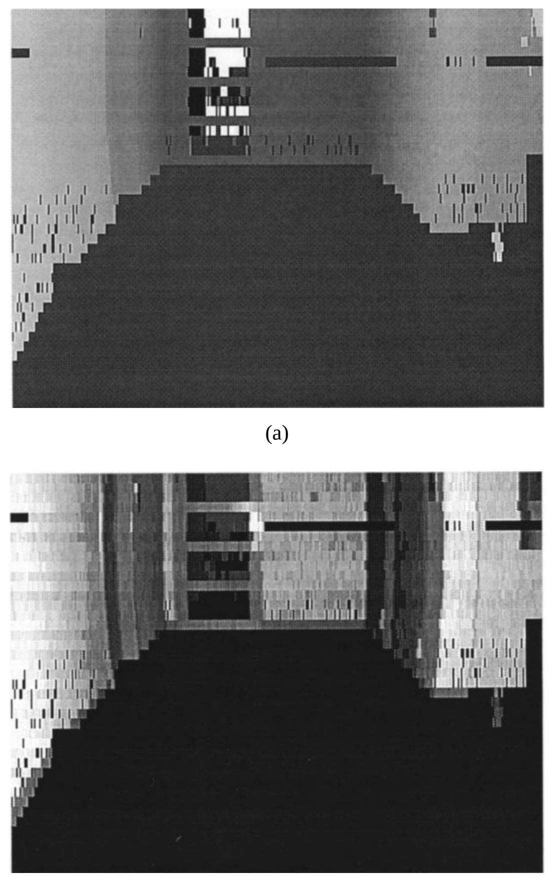
$$\begin{aligned} \mathbf{f}_k(\mathbf{x}, \mathbf{y}_k) &= B_{MP}\mathbf{x}_{MP} \\ &= B_{MP}(\ominus \mathbf{x}_{WM} \oplus \hat{\mathbf{x}}_{WR} \oplus \mathbf{d}_R \oplus \hat{\mathbf{x}}_{RP} \oplus \mathbf{d}_P) \\ &= 0. \end{aligned}$$

In this case, the state to be estimated is represented by the perturbation vector of R ($\mathbf{x} = \mathbf{d}_R$), and the measurement by the perturbation vector of P ($\mathbf{y}_k = \mathbf{p}_P$). Considering $\hat{\mathbf{d}}_R = 0$ and $\hat{\mathbf{p}}_P = 0$, and applying (5) and (7), we have (see appendix)

$$\begin{aligned} \mathbf{h}_k &= B_{MP}\hat{\mathbf{x}}_{MP} \\ H_k &= B_{MP}J_{1\ominus}\{0, \hat{\mathbf{x}}_{MP}\}J_{MR} \\ G_k &= B_{MP}J_{2\ominus}\{\hat{\mathbf{x}}_{MP}, 0\}B_P^T. \end{aligned} \tag{9}$$

B. Estimating the Location of a Mobile Robot from Vertical Edges

Let $\mathbf{L}_{WR}$ be the estimated location of a mobile robot. Let $\mathbf{x}_{WM}$ represent the location of a vertical edge (a 2-D point) according to the map. Using only one image, a vertical edge gives us information about the direction in which the



(a)

(b)

Fig. 5. Images of (a) range and (b) intensity at location 15 of the robot trajectory.

corresponding scene feature is located relative to the camera. Let $\mathbf{L}_{RE}$ be the estimated location of the projection line of the vertical edge, relative to the robot (Fig. 4). In this case we have

$$B_M = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}; \quad B_E = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$
$$B_{EM} = [0 \quad 1 \quad 0].$$

Assuming that the projection line E corresponds to model point M (Fig. 4), again a fact that should be verified during the matching process, we use it to improve the estimation of

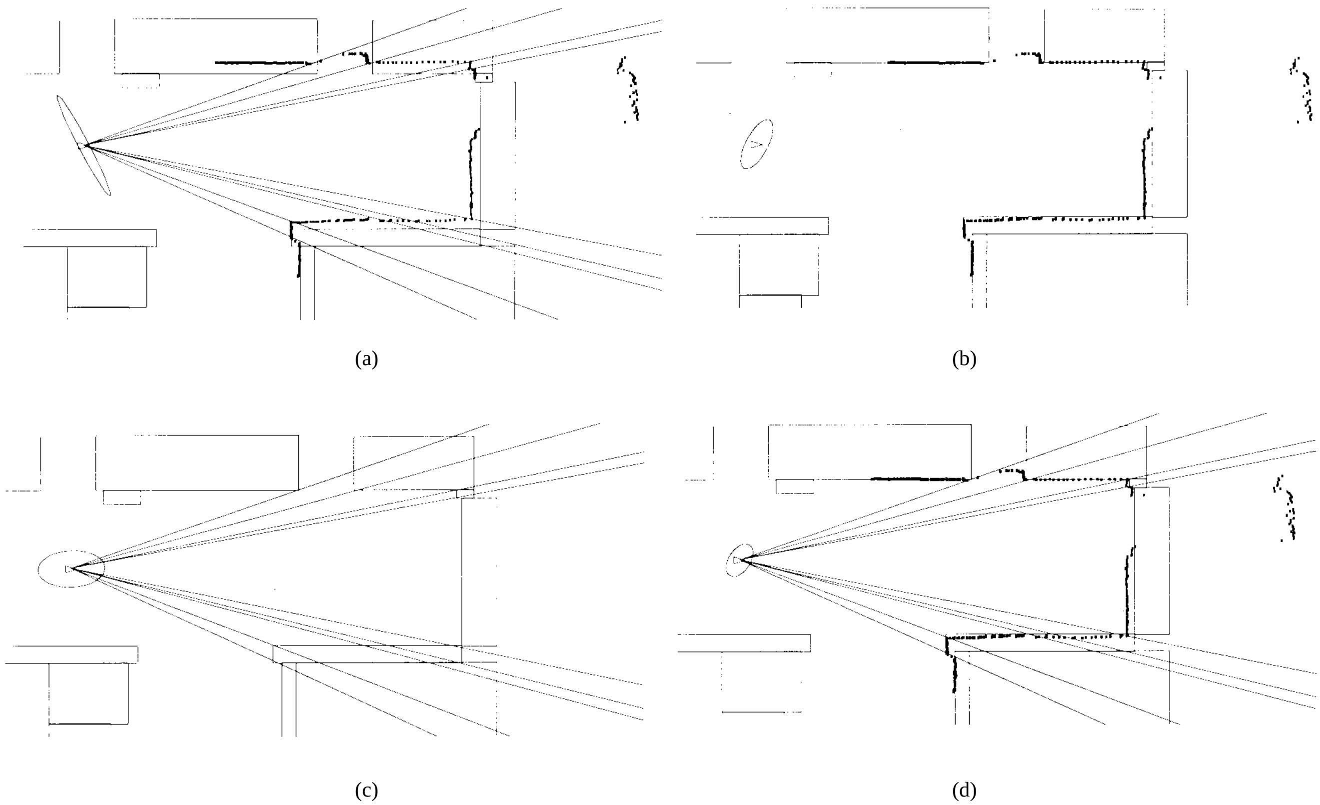


Fig. 6. (a) Range points and vertical edges observed from location 15 of the trajectory according to (d) odometry. Estimated robot location after integration of (b) range points, (c) intensity edges, and (d) both. Uncertainty after integration is magnified $30\times$.

the robot location by means of the inverse constraint (3)

$$\begin{aligned}\mathbf{f}_k(\mathbf{x}, \mathbf{y}_k) &= B_{EM}\mathbf{x}_{EM}\\ &= B_{EM}(\ominus\mathbf{d}_E \ominus \hat{\mathbf{x}}_{RE} \ominus \mathbf{d}_R \ominus \hat{\mathbf{x}}_{WR} \oplus \mathbf{x}_{WM})\\ &= 0.\end{aligned}$$

In this case, $\mathbf{x} = \mathbf{d}_R$ and $\mathbf{y}_k = \mathbf{p}_E$. If we consider $\hat{\mathbf{d}}_R = 0$ and $\hat{\mathbf{p}}_E = 0$, applying (5) and (7), we have

$$\begin{aligned}\mathbf{h}_k &= B_{EM}\hat{\mathbf{x}}_{EM}\\ H_k &= B_{EM}J_{2\ominus}\{\hat{\mathbf{x}}_{EM}, 0\}B_M^T\\ G_k &= -B_{EM}J_{1\ominus}\{0, \hat{\mathbf{x}}_{EM}\}B_E^T.\end{aligned} \tag{10}$$

IV. Experimental Results

In our experimental setup, the MACROBE robot is programmed to follow a path along a corridor. At 24 different locations of the robot trajectory the laser sensor acquires a 41 $\times$ 321 array of environmental points. The laser sensor provides not only the distance to each point of the array but also gives intensity information of each sensed point (Fig. 5). Fig. 6(a) shows the information we extract from range (points on the walls) and intensity images (vertical edges). Some of the edges obtained correspond to corners or doors; the others correspond to features visible to the sensor but not described in the map, such as door and window frames.

An *a priori* map of the environment, composed of 2-D segments corresponding to walls and 2-D points corresponding to corners, is used to search for correspondences for the sensed features. Sensed features for which a pairing with a map feature is established are fused with the purpose of reducing the error introduced by the robot motion.

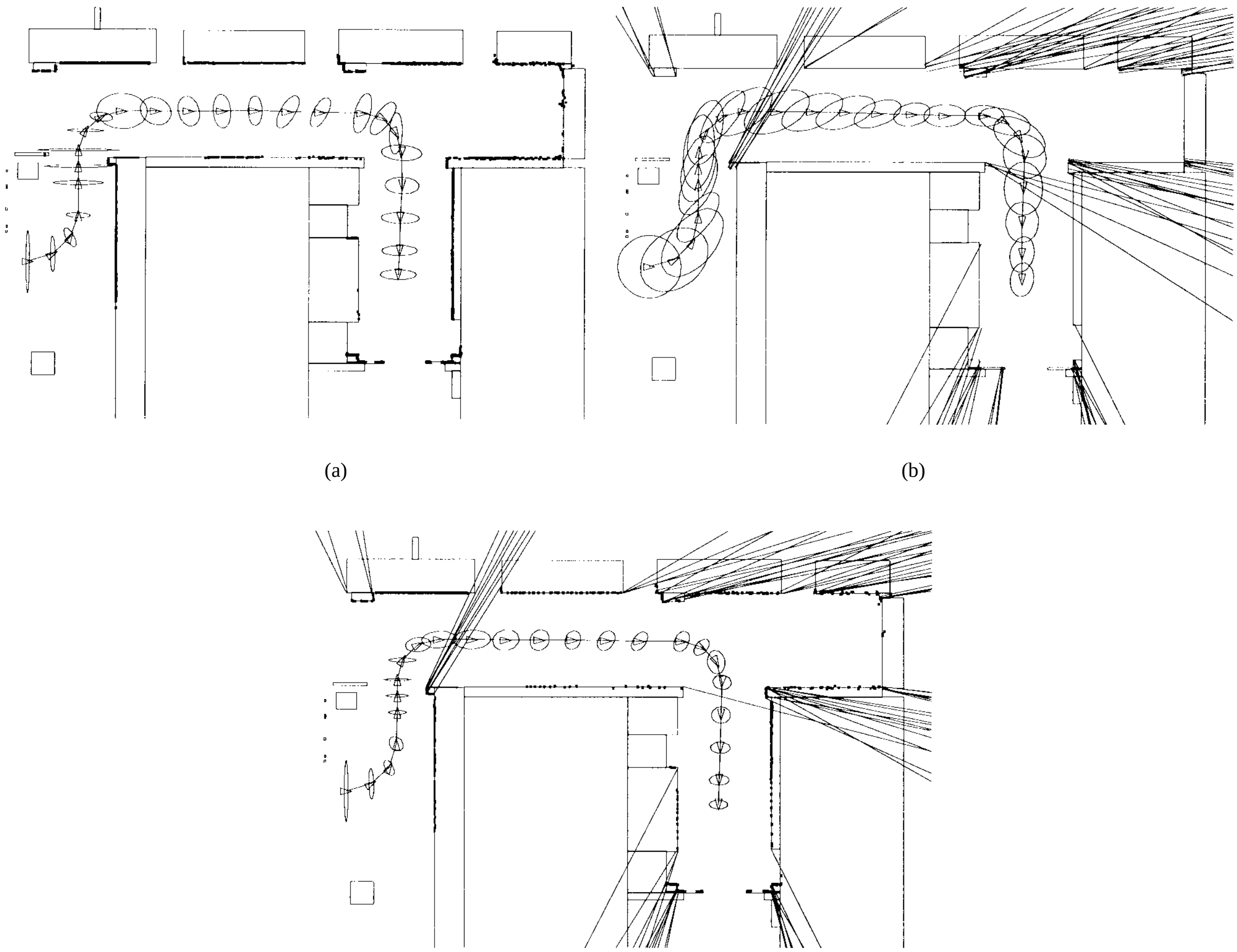
We have conducted three experiments: localizing the robot fusing only range points, fusing only the vertical edges corresponding to corners, and fusing both.

A. Fusing Range Points

In our first experiment we only consider the use of the range image of the laser sensor. Fig. 6(a) shows one scan line of range points, according to odometry. At each point of the trajectory, the predicted location of potentially visible walls is calculated, and a suitable pairing for each range point is obtained by applying the χ^2 compatibility test (6) with each of the predicted walls. The point is paired to the wall that satisfies the test, and whose extension includes the point location. If there is more than one, the wall with the smallest Mahalanobis distance is selected. Fig. 6(b) shows the result of the fusion of the paired points, using (8) and (9): the estimated robot location is much more precise and the error in the location of the sensed points has been significantly reduced.

We repeat this correspondence search and fusion process at each location along the trajectory. Fig. 7(a) shows the resulting trajectory when processing the laser points, as well as the resulting estimated location of the paired range points.

If we take a closer look to some parts of the trajectory, we can see potential problems that arise in most indoor trajectories. When crossing the first door, the laser sensor can only perceive the wall in front of it, thus the observed points can only contribute information in the normal direction to the wall.



(a) (b)

(c)

Fig. 7. Estimated robot trajectory after integration of (a) range points, (b) intensity edges, and (c) both. Only matched points and edges are shown. Uncertainty is magnified 30×.

Thus, uncertainty grows unbounded in the direction normal to the direction of motion. This situation arises commonly in large rooms, or work areas, and it limits the robustness of the localization process based solely on range points.

B. Fusing Vertical Edges

Fig. 6(a) shows the observed vertical edges at location 15 of the trajectory, according to odometry. The corresponding range points show the fact that some of the vertical edges correspond to corners, and for that reason may be paired correctly, while the rest correspond to visible wall features not described in the map, that cannot be paired.

The correspondence process for vertical edges is more complex than in the case of laser points, due to the fact that, since the projection ray is infinite, it may have many candidate corners for pairing, even after visibility constraints are applied. For this reason, the search for correspondences is an iterative process, in which at each iteration only projection rays that have one candidate map corner as pairing are integrated. This has the double effect of correcting the estimated position of the robot, as well as reducing its uncertainty, so that the probability of finding correct pairings in the next iteration is higher.

The result of this process is shown in Fig. 6(c). The reduction of uncertainty is less apparent than in the case of laser points. This is because less information is fused in this case. Applying this process to the whole robot trajectory using (8) and (10), we obtain the result shown in Fig. 7(b). In fusing vertical edges for the estimation of the robot location, we obtain a less precise result in the direction of motion. However, uncertainty is bounded in all directions.

C. Fusing Range and Intensity Images

Fusing range points seems to have an opposite effect on the estimation of the robot location, to that of fusing intensity edges. Range points make the estimation of the robot location more precise in the direction of motion, while intensity edges make it more precise in the direction *normal* to the direction of motion [Fig. 6(d)]. We have used a two-step approach in which range points are processed first because the matching process is simpler and less error-prone. In a second step, the intensity edges are fused to improve the estimated robot

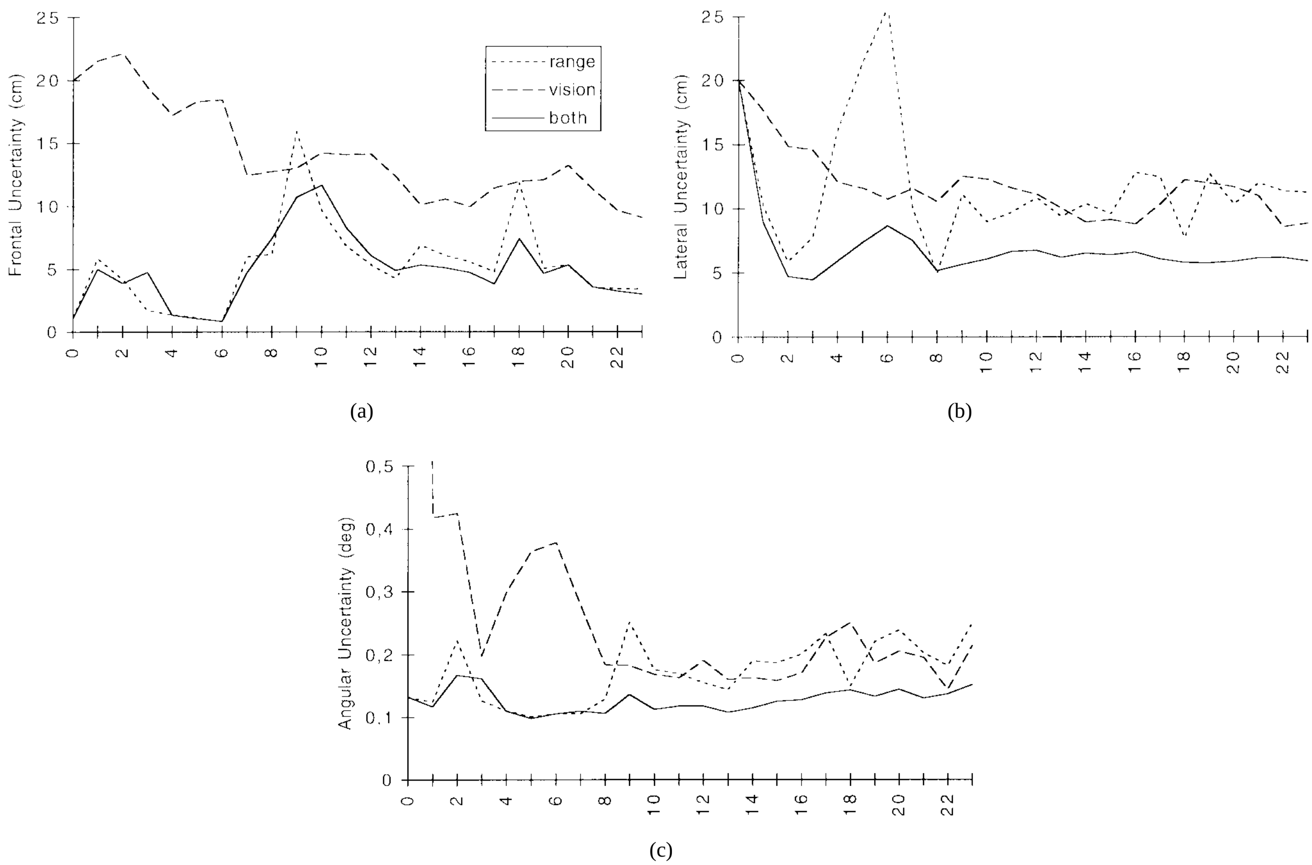


Fig. 8. Evolution of uncertainty along the trajectory (95% confidence level).

location obtained by range, allowing to have a more precise estimation in the direction normal to the direction of motion.

Repeating this process along all the trajectory [Fig. 7(c)] we can obtain the desired result, an estimation of the robot location that is precise and bounded in all directions.

D. Discussion

Fig. 8 shows the resulting uncertainty bounds for the robot location. We can see that in general, the fusion of range points gives more precision than vision in the direction of motion. This is true as long as there is something in front of the robot that range can detect. With regard to the lateral direction, range and vision seem to render similar results in precision. However, there are some situations, such as when crossing the door (steps 4–6), in which vision obtains a more precise lateral estimation, which is critical for such a task.

The combination of range and vision results in a more precise estimation of the robot location than if either is used separately. However, there are a couple of situations where the precision of range seems to be better than in the multisensor case:

1) *At Step 3, Before Crossing the Door:* In this step, range gives smaller uncertainty bounds in the direction of motion, but the real accuracy obtained is less because some incorrect pairings have been accepted due to map inconsistencies at the left side of the door.
2) *At Steps 10–13, Along the Corridor:* The situation here is similar. The *a priori* map does not describe the end of the corridor accurately, so incorrect pairings are included in the estimation.

This means that range alone is not robust against objects inaccurately described or not described in the map. In both cases, the combination of range and vision allows the system to discard spurious pairings and thus the coherence with the map increases. Also, the resulting estimation is more adjusted to the real precision that the system is capable of attaining. Furthermore, the improvement in robustness allows this procedure to be used in the presence of obstacles, both static and mobile, that the self location system will ignore. Successful navigation avoiding obstacles requires the incorporation of tightly coupled perception and motion strategies [29].

V. Conclusion

Previous work in the context of the MACROBE project was concentrated on using range information for the purpose of robot self-localization. In this work we have extended the self-localization process to include both range (wall points) and intensity images (vertical edges corresponding to corners) obtained by the laser sensor. These two complementary types of sensor information are easily fused with the SPmodel. Experimental results show that multisensor integration is a practical approach to robust and precise mobile robot localization: the whole process shown in Fig. 1 is performed in less than 1 s per step, obtaining a precision of around 10 cm in position and 0.15° in orientation.

Our experiments have also highlighted the limitations inherent to the use of *a priori* maps for mobile robot localization. *A priori* maps may contain incorrect information, or may not be sufficiently detailed [see Fig. 6(d)]. These limitations are not an argument against the use of maps. Rather, we wish to emphasize that it is costly and difficult to have a really good *a priori* map of the environment. Since sensors can determine better the characteristics of the scene, we can have them *build* the map [30]. However, map building in general indoor environments, with static and mobile obstacles, is a more difficult problem. We believe it also constitutes a problem where the use of the SPmodel is useful and appropriate [31].

APPENDIX
DIFFERENTIAL LOCATION VECTORS AND JACOBIANS

If $\mathbf{d}_A$ and $\mathbf{d}_B$ are differential location vectors then

$$\mathbf{d}_A \oplus \mathbf{d}_B \simeq \mathbf{d}_A + \mathbf{d}_B$$
$$\ominus \mathbf{d}_A \simeq -\mathbf{d}_A.$$

The expressions to transform differential location vectors between references are

$$\mathbf{d}_A \oplus \mathbf{x}_{AB} = \mathbf{x}_{AB} \oplus \mathbf{d}_B$$
$$\mathbf{d}_B = J_{AB}^{-1}\mathbf{d}_A = J_{BA}\mathbf{d}_A$$
$$\mathrm{Cov}(\mathbf{d}_B) = J_{BA}\,\mathrm{Cov}(\mathbf{d}_A)J_{BA}^T$$

where J_{AB} is the Jacobian [32] corresponding to transformation $\mathbf{x}_{AB} = (x, y, \phi)^T$

$$J_{AB} = \begin{bmatrix} \cos\phi & -\sin\phi & y \\ \sin\phi & \cos\phi & -x \\ 0 & 0 & 1 \end{bmatrix}.$$

The Jacobians of the composition of two location vectors are [33]

$$\begin{aligned} J_{1\ominus}\{\mathbf{x}_1, \mathbf{x}_2\} &= \left.\frac{\partial(\mathbf{y}\oplus\mathbf{z})}{\partial\mathbf{y}}\right|_{\mathbf{y}=\mathbf{x}_1,\,\mathbf{z}=\mathbf{x}_2} \\ &= \begin{bmatrix} 1 & 0 & -x_2\sin\phi_1 - y_2\cos\phi_1 \\ 0 & 1 & x_2\cos\phi_1 - y_2\sin\phi_1 \\ 0 & 0 & 1 \end{bmatrix} \\ J_{2\ominus}\{\mathbf{x}_1, \mathbf{x}_2\} &= \left.\frac{\partial(\mathbf{y}\oplus\mathbf{z})}{\partial\mathbf{z}}\right|_{\mathbf{y}=\mathbf{x}_1,\,\mathbf{z}=\mathbf{x}_2} \\ &= \begin{bmatrix} \cos\phi_1 & -\sin\phi_1 & 0 \\ \sin\phi_1 & \cos\phi_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}. \end{aligned}$$

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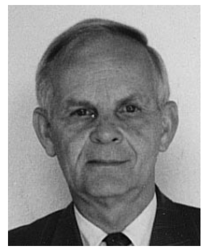

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