Constraint-Based Object Recognition in Multisensor Systems

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Abstract

In this paper we present a general procedure to compute and validate geometric constraints between uncertain geometric features. Uncertain geometric information is represented using the Symmetries and Perturbation model [8]. The proposed procedure allows to obtain the geometric relations between any pair of geometric elements in a systematic way, with an explicit consideration of the uncertainty due to the use of different sensors. This makes the representation model and constraint validation mechanism well suited for multisensor systems. We also show how to use information regarding geometric relations, not only in the validation of hypotheses during the recognition process, but also for the selection of observations that allow the determination of the object location.

1 Introduction

The object recognition process based on the matching between model features and sensor observations is of exponential complexity. One of the fundamental ideas to reduce this complexity is the use of geometric constraints: the validation that geometric relations between model features are satisfied in the observations we are trying to match. In multisensor systems, the validation of geometric constraints is complicated by the diverse nature of sensorial information and its uncertainty.

Considerable work has been done on the use geometric constraints for the validation of hypotheses in object recognition [3, 5, 6]. In [3], the most extensive, Grimson proposes a recognition process which relies on simple constraints to validate the consistency between a set of observations and an object model. These constraints are independent of the object location and thus can be calculated with no estimation of this location. Uncertainty is managed by considering error bounds on constraints.

This approach has three fundamental drawbacks:

- It is not well suited for multisensor systems, where it is not plausible to consider that all sensors will give observations with the same estimation errors.
- Bounds on errors are in general very conservative estimations (some work to tighten these bounds can be found in [1]).
- Location-independent constraints are less discriminant than location-dependent ones, and thus many inconsistent interpretations survive up to object location estimation.

In this paper we describe an alternative approach based on the early estimation of object location in order to allow the use of more discriminant location-dependent constraints. We account for uncertainty using the Symmetries and Perturbation Model (SPmodel) to represent the estimated location of observations [8]. The main advantage of this model is its *generality*: it is valid for any object, geometric feature or sensorial observation. The probabilistic representation of uncertainty adopted in the model allows to reflect the different capabilities of sensors, and thus, it is adequate for multisensor systems. Section 2 contains a brief description of the SPmodel. In section 3 we show how to systematically deduce the geometric relations between any set of features and express them using a uniform representation. This allows us to define a general mechanism for the validation of geometric constraints under uncertainty. In section 4 we show how the estimation of geometric constraints is central to the recognition process, not only for constraint validation, but also for the selection of observations that allow the determination of the object location. Finally, in section 5 we discuss the fundamental conclusions of this work.

2 Representation of Geometric Information

The most common approaches for the representation of geometric uncertainty use probabilistic models. In them, the location of an element is represented by a parameter vector, and the available knowledge about it is characterized by the mean and covariance of an associated probability distribution function (usually Gaussian). Based on this model, the fusion of sensorial information can be done using optimal estimation techniques, such as the extended Kalman filter.

The main drawback of these approaches is that they use a different set of parameters to represent the location of each type of geometric element. In [8] we have presented a general method for the representation of the location of any geometric entity and its uncertainty: the Symmetries and Perturbations model

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(SPmodel). It is used to establish a general integration mechanism that allows to obtain a ***suboptimal estimation of location*** for objects or features from a set of partial and uncertain sensorial observations. It is based on the theory of the iterated extended Kalman filter. Since this paper deals with the validation of geometric relations, we will not present the integration mechanism here. A more complete presentation of the model and the integration mechanism can be found in [8, 9].

2.1 The SPmodel

The SPmodel is a probabilistic model that associates a reference to every geometric element E. Its location is given by the transformation t_{WE} relative to a base reference W. To represent this transformation, we use a ***location vector*** $\mathbf{x}_{WE}$, composed of three cartesian coordinates and three Roll-Pitch-Yaw angles:

$$\mathbf{x}_{WE} = (x,\ y,\ z,\ \psi,\ \theta,\ \phi)^T$$

where:

$$t_{WE} = \text{Trans}(x,\ y,\ z)\cdot\text{Rot}(z,\ \phi)\cdot\text{Rot}(y,\ \theta)\cdot\text{Rot}(x,\ \psi)$$

The estimation of the location of an element is denoted by $\hat{\mathbf{x}}_{WE}$, and the estimation error is represented locally by a ***differential location vector*** $\mathbf{d}_E$ relative to the reference attached to the element. Thus, the true location of the element is :

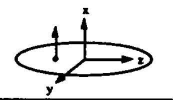
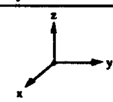
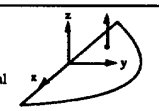
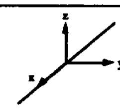
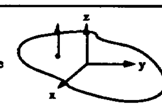
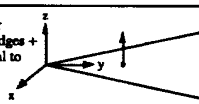
$$\mathbf{x}_{WE} = \hat{\mathbf{x}}_{WE} \oplus \mathbf{d}_E$$

where $\oplus$ represents the composition of location vectors (the inversion is represented with $\ominus$).

Our model also exploits the concept of ***symmetries*** of a geometric element, defined as the set S_E of transformations that preserve the element [8]. It has been shown that the symmetries of any geometric element are a subgroup of the group of transformation $(\mathcal{T}, \cdot)$. Figure 1 shows some fundamental subgroups of transformations and examples of the geometric elements they represent. For example, the symmetries of an infinite edge are the set of continuous translations (T_x) and rotations (R_x) along the edge. There is also a cyclic symmetry of 180 degrees around any axis perpendicular to the edge, corresponding to the two opposite edge orientations. In this paper we will concentrate on the symmetries of continuous motion of features, because they play a fundamental role in the determination of the geometric relations between features. Cyclic symmetries must be taken into account as alternate hypotheses when matching two features in the recognition process.

To account for the continuous motion symmetries, we assign in $\mathbf{d}_E$ a null value to the degrees of freedom corresponding to them, because they do not represent an effective location error. We call ***perturbation vector*** the vector $\mathbf{p}_E$ formed by the non null elements of $\mathbf{d}_E$. Both vectors can be related by a row selection matrix B_E that we call ***self-binding matrix*** of the geometric element:

$$\mathbf{d}_E = B_E^T \mathbf{p}_E \quad ; \quad \mathbf{p}_E = B_E \mathbf{d}_E$$

Subgroup Of Transformations	Example	Other Examples
Rx	Circle 	Conical Surface Edge with extremes
Rxyz	Vertex 	Spherical surface
Tx	Semidihedral (edge + normal to plane) 	Dihedral (edge + two normals)
Tx Rx	Infinite Edge 	Infinite Cylindrical Surface
Txy Rz	Infinite Plane 	2D Edge (Vision)
1	Corner (two edges + normal to plane) 	Coplanar Edges (non parallel)

$$T_x = \{\text{Trans}(a, 0, 0) \mid a \in \mathbb{R}\}$$
$$T_{xyz} = \{\text{Trans}(a, b, c) \mid a, b, c \in \mathbb{R}\}$$
$$R_x = \{\text{Rot}(x, \psi) \mid \psi \in (-\pi,\ \pi]\}$$
$$R_{xyz} = \{\text{Rot}(\mathbf{u}, \theta) \mid \mathbf{u} \in \mathbb{R}^3, \theta \in (-\pi, \pi]\}$$

Figure 1: Representation of some geometric elements

For example, in the case of an edge, we have:

$$\mathbf{d}_E = (0,\ dy,\ dz,\ 0,\ d\theta,\ d\phi)^T$$
$$\mathbf{p}_E = (dy,\ dz,\ d\theta,\ d\phi)^T$$
$$B_E = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Based on these ideas, the SPmodel represents the information about the location of a geometric element by a triplet $(\hat{\mathbf{x}}_{WE}, \hat{\mathbf{p}}_E, C_E)$ where:

$$\mathbf{x}_{WE} = \hat{\mathbf{x}}_{WE} \oplus B_E^T \mathbf{p}_E$$
$$\hat{\mathbf{p}}_E = E[\mathbf{p}_E] \quad ; \quad C_E = Cov(\mathbf{p}_E)$$

Transformation $\hat{\mathbf{x}}_{WE}$ is an estimation taken as base for perturbations, $\hat{\mathbf{p}}_E$ the estimated value of the perturbation vector, and C_E its covariance. When $\hat{\mathbf{p}}_E = 0$, we say that the estimation is *centered*.

The main advantage of this model is its generality: it is valid for any object, geometric feature or sensorial observation. Moreover, the representation of uncertainty using a perturbation vector does not de-

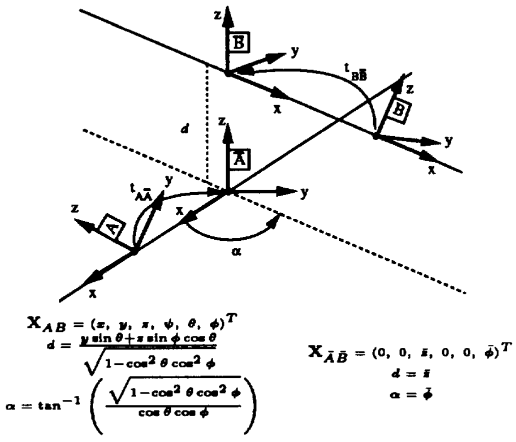


Figure 2: Reference alignment for two edges

pend on the base reference used, has a clear interpretation, and is not overparameterized. Problems related to singularities found in other representations are also avoided.

3 Geometric Relations and Constraint Validation

Geometric relations between features are a set of parameters that derive from the geometry of each feature (dimensions), and from their relative location (distances and angles). They constitute one of the most important sources of information to establish the validity of an interpretation of a set of observations with respect to an object model. In systems that rely on one sensor and only consider a specific type of feature (stereo vision and edges, for example), these parameters are easily derived. However, in multisensor systems, where different sensors or combinations of sensors can give geometric information of diverse type, such as dihedrals, corners or circles, we have the problem of deriving them for each pair of geometric elements.

We are interested in the geometric relations that determine the relative location of the involved geometric elements, because they can be represented in a uniform way. Also, we only consider parameters that are invariant under partial occlusion. The next subsections will be devoted to the estimation of such geometric relations between uncertain observations of features, and their validation against the nominal relations computed in the model.

3.1 Geometric Relations

In general, geometric relations are non-linear functions of the relative location of the involved geometric features. As an example consider a pair of model edges whose location is represented by references A and B respectively (fig. 2). Intuitively, we can see that their relative location is defined by two parameters: their perpendicular distance and the angle between them. As it can be seen in the figure, the distance d and the angle α are non-linear functions of $\mathbf{x}_{AB}$. Thus, to compute the uncertainty of parameters d and α due to the uncertainty of $\mathbf{x}_{AB}$, we would need to use a first-order approximation of d and α. An alternative is to find two *aligned* references $\bar{A}$ and $\bar{B}$ which equivalently describe the location of the edges, such that d and α are linear functions of $\mathbf{x}_{\bar{A}\bar{B}}$. Note that the aligning transformations $t_{A\bar{A}}$ and $t_{B\bar{B}}$ belong to the symmetries of their respective geometric elements, that we denote by $\mathcal{S}_A$ and $\mathcal{S}_B$.

Let t_{WA} and t_{WB} express the location of two geometric elements A and B with respect to a base reference W, and let t_{AB} be the transformation that expresses the relative location between both elements. The sets of transformations that equivalently describe the locations of A and B are:

$$L_A = t_{WA} \cdot \mathcal{S}_A \quad ; \quad L_B = t_{WB} \cdot \mathcal{S}_B$$

The set of all possible transformations between the references associated to the two geometric elements is given by:

$$\begin{aligned} L_{AB} &= L_A^{-1} \cdot L_B \\ &= \mathcal{S}_A \cdot t_{AB} \cdot \mathcal{S}_B \end{aligned}$$

Each transformation that belongs to the set L_{AB} equivalently describes the relative location of the features. The most simple transformation belonging to L_{AB} (the one that contains the minimum possible number of translations and rotations) has one translation or rotation corresponding to each of the parameters defining the geometric relations between the elements. This means that there exist references $\bar{A}$ and $\bar{B}$ where the geometric relations between the two elements are linear functions of $\mathbf{x}_{\bar{A}\bar{B}}$. In the above example $\mathbf{x}_{\bar{A}\bar{B}}$ contains one translation and one rotation corresponding with the distance and angle between the edges. This transformation is then, the simplest transformation that satisfies:

$$L_{AB} = \mathcal{S}_A \cdot t_{\bar{A}\bar{B}} \cdot \mathcal{S}_B$$

For all the combinations of features considered, $t_{\bar{A}\bar{B}}$ can be uniquely determined (table 2). Let us study an example.

Geometric Relations between Semidihedrals. Let A and B represent the location of two semidihedrals and let $\mathbf{x}_{AB} = (x,\ y,\ z,\ \psi,\ \theta,\ \phi)^T$. The symmetries of semidihedrals are T_x (figure 1), and so $\mathcal{S}_A = \mathcal{S}_B = T_x$. Thus:

$$\begin{aligned} \mathbf{x}_{A\bar{A}} &= (x_a,\ 0,\ 0,\ 0,\ 0,\ 0)^T \\ \mathbf{x}_{B\bar{B}} &= (x_b,\ 0,\ 0,\ 0,\ 0,\ 0)^T \end{aligned}$$

Case for $\mathbf{x}_{AB}$	$\mathbf{x}_{A\bar{A}}$	$\mathbf{x}_{B\bar{B}}$	$\mathbf{x}_{\bar{A}\bar{B}}$
$\theta = \phi = 0$	$(x + x_b,\ 0,\ 0,\ 0,\ 0,\ 0)^T$	$(x_b,\ 0,\ 0,\ 0,\ 0,\ 0)^T$	$(0,\ y,\ z,\ \psi,\ 0,\ 0)^T$
$\theta = 0$	$(\frac{x \sin\phi - y \cos\phi}{\sin\phi},\ 0,\ 0,\ 0,\ 0,\ 0)^T$	$(-\frac{y}{\sin\phi},\ 0,\ 0,\ 0,\ 0,\ 0)^T$	$(0,\ 0,\ z,\ \psi,\ 0,\ \phi)^T$
otherwise	$(\frac{x \sin\theta \cos\phi + z \cos\theta}{\sin\theta},\ 0,\ 0,\ 0,\ 0,\ 0)^T$	$(\frac{z}{\sin\theta},\ 0,\ 0,\ 0,\ 0;\ 0)^T$	$(0,\ \frac{y \sin\theta + z \sin\phi \cos\theta}{\sin\theta},\ 0,\ \psi,\ \theta,\ \phi)^T$

Table 1: Solutions for two semidihedrals

Our purpose is to find the values of x_a and x_b such that $\mathbf{x}_{\bar{A}\bar{B}} = (\bar{x},\ \bar{y},\ \bar{z},\ \bar{\psi},\ \bar{\theta},\ \bar{\phi})^T$ contains the minimum number of translations and rotations. This is given by:

$$\begin{aligned}
\mathbf{x}_{\bar{A}\bar{B}} &= \ominus \mathbf{x}_{A\bar{A}} \oplus \mathbf{x}_{AB} \oplus \mathbf{x}_{B\bar{B}} \\
&= (x - x_a + x_b \cos\theta \cos\phi,\ y + x_b \cos\theta \sin\phi, \\
&\quad z - x_b \sin\theta,\ \psi,\ \theta,\ \phi)^T
\end{aligned}$$

The solutions in this case are resumed in table 1. Note that in the general case the geometric relations between the dihedrals are one distance and three angles. In this case the solution is unique. In the case where $\theta = 0$, the set of parameters is reduced to a distance and two angles, but this solution is also unique. On the other hand, when $\theta = \phi = 0$, which corresponds to the case when the edges of the dihedrals are parallel, x_b remains a free variable, and thus we cannot uniquely determine the values of $\mathbf{x}_{A\bar{A}}$ and $\mathbf{x}_{B\bar{B}}$. Later we will see that this information is useful in determining whether a set of observations can completely determine the location of the object.

The solutions for all the combinations of symmetries are obtained in the same way and are summarized in table 2. Thus, given two geometric features A and B, whose subgroups of symmetries are $\mathcal{S}_A$ and $\mathcal{S}_B$, the location vector corresponding to the geometric relations between the two elements is computed by first calculating the values of $\mathbf{x}_{A\bar{A}}$ and $\mathbf{x}_{B\bar{B}}$ using the results shown in the table and then computing $\mathbf{x}_{\bar{A}\bar{B}}$. In the next subsection we will use this result to estimate the geometric relations between uncertain geometric features and validate them.

3.2 Geometric Relations under Uncertainty

Let us consider now the estimation of geometric relations between uncertain observations of features. Given two geometric features with associated references A and B, whose uncertain locations are expressed by $(\hat{\mathbf{x}}_{WA}, \hat{\mathbf{p}}_A, C_A)$ and $(\hat{\mathbf{x}}_{WB}, \hat{\mathbf{p}}_B, C_B)$ respectively, the geometric relations between them can be estimated from the relative transformation $\mathbf{x}_{\bar{A}\bar{B}}$, using the results obtained in subsection 3.1. The procedure follows these steps:

First, calculate the location vectors $\mathbf{x}_{A\bar{A}}$ and $\mathbf{x}_{B\bar{B}}$ using table 2. The uncertain location of $\bar{A}$, $(\hat{\mathbf{x}}_{W\bar{A}}, \hat{\mathbf{p}}_{\bar{A}}, C_{\bar{A}})$ can be calculated as follows:

$$\begin{aligned}
\mathbf{x}_{W\bar{A}} &= \hat{\mathbf{x}}_{WA} \oplus B_A^T \mathbf{p}_A \oplus \mathbf{x}_{A\bar{A}} \\
&= (\hat{\mathbf{x}}_{WA} \oplus \mathbf{x}_{A\bar{A}}) \oplus J_{\bar{A}A} B_A^T \mathbf{p}_A \\
&= \hat{\mathbf{x}}_{W\bar{A}} \oplus B_{\bar{A}}^T \mathbf{p}_{\bar{A}} \\
\mathbf{p}_{\bar{A}} &= B_{\bar{A}} J_{\bar{A}A} B_A^T\, \mathbf{p}_A \\
C_{\bar{A}} &= B_{\bar{A}} J_{\bar{A}A} B_A^T\ C_A\ B_A J_{\bar{A}A}^T B_{\bar{A}}^T
\end{aligned}$$

where $J_{\bar{A}A}$ is the Jacobian [4] of the transformation represented by $\mathbf{x}_{A\bar{A}}$. The uncertain location of $\bar{B}$, $(\hat{\mathbf{x}}_{W\bar{B}}, \hat{\mathbf{p}}_{\bar{B}}, C_{\bar{B}})$, is calculated in the same way. Second, the location vector $\mathbf{x}_{\bar{A}\bar{B}}$ and its covariance are calculated as follows:

$$\begin{aligned}
\mathbf{x}_{\bar{A}\bar{B}} &= \ominus \mathbf{x}_{W\bar{A}} \oplus \mathbf{x}_{W\bar{B}} \\
&= \ominus(\mathbf{x}_{W\bar{A}} \oplus B_{\bar{A}}^T \mathbf{p}_{\bar{A}}) \oplus (\mathbf{x}_{W\bar{B}} \oplus B_{\bar{B}}^T \mathbf{p}_{\bar{B}}) \\
&= \ominus \mathbf{d}_{\bar{A}} \oplus \hat{\mathbf{x}}_{\bar{A}\bar{B}} \oplus \mathbf{d}_{\bar{B}} \\
Cov(\mathbf{x}_{\bar{A}\bar{B}}) &= J_{1\oplus}\{0, \hat{\mathbf{x}}_{\bar{A}\bar{B}}\} B_{\bar{A}}^T C_{\bar{A}} B_{\bar{A}} J_{1\oplus}^T\{0, \hat{\mathbf{x}}_{\bar{A}\bar{B}}\} \\
&+ J_{2\oplus}\{\hat{\mathbf{x}}_{\bar{A}\bar{B}}, 0\} B_{\bar{B}}^T C_{\bar{B}} B_{\bar{B}} J_{2\oplus}^T\{\hat{\mathbf{x}}_{\bar{A}\bar{B}}, 0\}
\end{aligned}$$

where $J_{1\oplus}$ and $J_{2\oplus}$ are the Jacobians of the composition of location vectors [7]:

$$\begin{aligned}
J_{1\oplus}\{\mathbf{x}_1, \mathbf{x}_2\} &= \left. \frac{\partial(\mathbf{y} \oplus \mathbf{z})}{\partial \mathbf{y}} \right|_{\mathbf{y}=\mathbf{x}_1,\ \mathbf{z}=\mathbf{x}_2} \\
J_{2\oplus}\{\mathbf{x}_1, \mathbf{x}_2\} &= \left. \frac{\partial(\mathbf{y} \oplus \mathbf{z})}{\partial \mathbf{z}} \right|_{\mathbf{y}=\mathbf{x}_1,\ \mathbf{z}=\mathbf{x}_2}
\end{aligned}$$

The location vector $\mathbf{x}_{\bar{A}\bar{B}}$ and its covariance constitute an estimation of the geometric relations between A and B, to which we can apply a general constraint validation mechanism as follows: some components of this location vector correspond to the geometric relations $\mathbf{r}$ between the features and the rest are zero, so we use a row selection matrix S to extract them in the following way:

$$\mathbf{r} = S\, \mathbf{x}_{\bar{A}\bar{B}} \ ;\ \hat{\mathbf{r}} = S\, \hat{\mathbf{x}}_{\bar{A}\bar{B}}$$
$$Cov(\mathbf{r}) = S\, Cov(\mathbf{x}_{\bar{A}\bar{B}})\ S^T$$

Given a vector $\mathbf{r}_m$, which contains the value of the geometric relations in the model, we can measure the discrepancy between $\mathbf{r}_m$ and $\mathbf{r}$ using the Mahalanobis distance:

$$D^2 = (\hat{\mathbf{r}} - \mathbf{r}_m)^T Cov(\mathbf{r})^{-1} (\hat{\mathbf{r}} - \mathbf{r}_m)$$

Under the gaussianity hypothesis, D^2 follows a chi-square distribution χ_n^2, with $n = dim(\mathbf{r})$ degrees of freedom. For a given significance level α, $\mathbf{r}$ can be considered compatible with $\mathbf{r}_m$ if:

$$D^2 \leq D_\alpha^2$$

where D_α^2 is a threshold value, obtained from the χ_n^2

$\mathcal{S}_1$	$\mathcal{S}_2$	Case of t_{AB}	General Form of $t_{\bar{A}\bar{B}}$
R_x	R_x	$y=\theta=\phi=0$ $\theta=\phi=0$ *otherwise*	$t_{\bar{x}} \cdot t_{\bar{z}}$ $t_{\bar{x}} \cdot t_{\bar{y}}$ $t_{\bar{x}} \cdot t_{\bar{y}} \cdot t_{\bar{z}} \cdot r_{\bar{\phi}}$
R_x	R_{xyz}	$y=z=0$ *otherwise*	$t_{\bar{x}}$, $t_{\bar{x}} \cdot t_{\bar{z}}$
T_x	T_x	$\theta=\phi=0$ $\theta=0$ *otherwise*	$t_{\bar{y}} \cdot t_{\bar{z}} \cdot r_{\bar{\psi}}$ $t_{\bar{z}} \cdot r_{\bar{\phi}} \cdot r_{\bar{\psi}}$ $t_{\bar{y}} \cdot r_{\bar{\phi}} \cdot r_{\bar{\theta}} \cdot r_{\bar{\psi}}$
T_x	R_x		$t_{\bar{y}} \cdot t_{\bar{z}} \cdot r_{\bar{\phi}} \cdot r_{\bar{\theta}}$
T_x	$T_x\, R_x$	$\theta=\phi=0$ $\theta=0$ *otherwise*	$t_{\bar{y}} \cdot t_{\bar{z}}$ $t_{\bar{z}} \cdot r_{\bar{\phi}}$ $t_{\bar{y}} \cdot r_{\bar{\phi}} \cdot r_{\bar{\theta}}$
T_x	R_{xyz}		$t_{\bar{y}} \cdot t_{\bar{z}}$
$T_x\, R_x$	R_x	$\theta=\phi=\pi/2$ *otherwise*	$t_{\bar{y}} \cdot t_{\bar{z}} \cdot r_{\bar{\theta}}(\bar{\theta}=\pi/2)$ $t_{\bar{y}} \cdot t_{\bar{z}} \cdot r_{\bar{\phi}}$
$T_x\, R_x$	$T_x\, R_x$	$\theta=\phi=0$ *otherwise*	$t_{\bar{z}}$ $t_{\bar{z}} \cdot r_{\bar{\phi}}$
$T_x\, R_x$	R_{xyz}		$t_{\bar{z}}$
$T_{xy}\, R_z$	R_x		$t_{\bar{z}} \cdot r_{\bar{\theta}}$
$T_{xy}\, R_z$	R_{xyz}		$t_{\bar{z}}$
$T_{xy}\, R_z$	T_x	$\theta=0$ *otherwise*	$t_{\bar{z}} \cdot r_{\bar{\psi}}$ $r_{\bar{\theta}} \cdot r_{\bar{\psi}}$
$T_{xy}\, R_z$	$T_x\, R_x$	$\theta=0$ *otherwise*	$t_{\bar{z}}$ $r_{\bar{\theta}}$
$T_{xy}\, R_z$	$T_{xy}\, R_z$	$\theta=\psi=0$ *otherwise*	$t_{\bar{z}}$ $r_{\bar{\theta}}$
R_{xyz}	R_{xyz}		$t_{\bar{z}}$

$$t_{\bar{x}} = \text{Trans}(\bar{x},0,0);\quad r_{\bar{\psi}} = \text{Rot}(x,\bar{\psi})$$
$$t_{\bar{y}} = \text{Trans}(0,\bar{y},0);\quad r_{\bar{\theta}} = \text{Rot}(y,\bar{\theta})$$
$$t_{\bar{z}} = \text{Trans}(0,0,\bar{z});\quad r_{\bar{\phi}} = \text{Rot}(z,\bar{\phi})$$

Table 2: Geometric relations for all combinations of symmetries of geometric elements

distribution, such that the probability of rejecting a good matching is α.

4 Constraint-based Recognition

There are two fundamental approaches to the recognition of objects by the validation of geometric constraints between observations and model features:

- An interpretation tree is generated with all possible interpretations of the observations which satisfy unary and binary constraints. For the surviving hypotheses, object location is calculated and global consistency verified. This approach is called *recognizing before locating*. The most extensive work based on this approach can be found in [3].
- Object location hypotheses are generated with the minimum number of observations needed to estimate object location. A hypothesis is then verified by a process in which the location of new features is predicted, and its presence is verified among the available data or by a sensor [2]. This approach is called *recognizing while locating*.

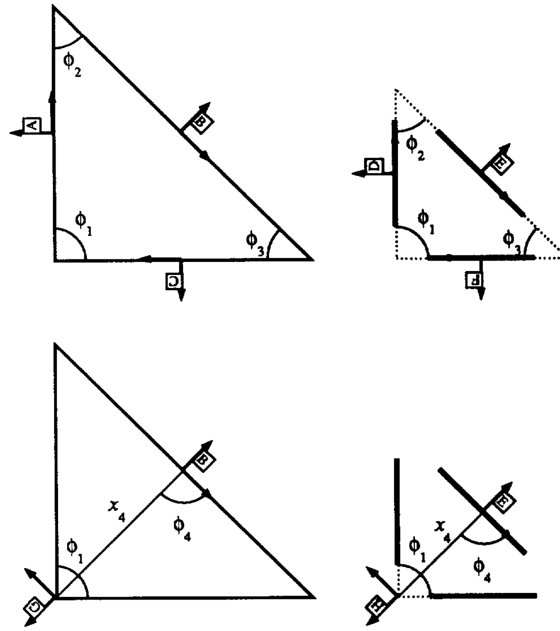


Figure 3: Constraint Tightness of Compound Features

The *recognizing before locating* approach is based on the idea of using very simple geometric relations that are fast to validate, and then estimating once the location of the object to validate global consistency. However, in comparison to the *recognizing while locating* approach it has several disadvantages that we analyze next.

The number of constraints to verify is larger. Given a hypothesis with n observation-model pairings, the inclusion of another pairing implies the validation of it with each of the n existing pairings in the hypothesis. This means that $n(n-1)/2$ constraint validations have been performed for this hypothesis. On the other hand, in the *recognizing while locating* approach the number of constraints that would have been validated for the hypothesis is $n-1$, because having located the object, the only constraint that must be validated for a new pairing is that the observation and predicted model feature locations coincide. Let us study as an example the case of three edges (Fig. 3). We have three observed edges $\{D,E,F\}$ and we must verify if they can be paired with edges $\{A, B, C\}$ of a model. Using the first approach we would need to verify constraints for each pair of observations $\{(D, E), (E, F), (D, F)\}$. On the other hand, with the validation of one constraint, D and F can be used to determine the location of the object, represented by H, and then we can predict the location of B and validate it against the location of E.

Binary constraints are not tight. A set of pairings may satisfy all binary constraints, and yet the interpretation may not be globally consistent. This implies that many inconsistent hypotheses survive until the object location is estimated. Studying the same example of figure 3 we can see that the pairings $\{(A, D), (B, E), (C, F)\}$ satisfy all constraints even though the interpretation is not globally consistent. The second approach has the advantage that it allows to validate the tightest possible geometric constraint, which is feature location. In the example we would see that when the location of B in the scene is predicted, it does not coincide with the location of E, and thus the interpretation would be discarded.

Goal-directed perception is not possible. The first approach has a limited use due to the fact that the provided information must be sufficient to recognize the object. If all the necessary information is not available, there is no way of predicting where more information should be acquired, and it is not possible to direct any sensor to obtain it.

4.1 Recognizing while Locating

For the above reasons, we use the *recognizing while locating* approach in which unary and binary constraints are used to validate the minimum number of observations that allow to generate object-location hypotheses. Their further verification is based on a prediction-verification scheme, in which a feature is predicted and its location in the scene is searched for in the data or obtained by a sensor. This approach confronts us with two fundamental problems:

- How can we choose an initial set of observations such that they completely determine the location of the object? According to the observations, where is the object located? How good is the estimation of the location?
- Once the object is located, how can we decide the order in which features should be predicted and verified so as to either accept or reject the hypothesis as soon as possible?

We will answer the first question next. The second one, closely related to sensor models and goal directed perception strategies, will be the subject of future work.

Selection of an initial set of Observations As we have said, this selection is fundamentally based on the possibility of the set to completely determine the location of the object. Each feature determines some degrees of freedom in the location of the object, depending on the symmetries of the feature. To completely determine the location of the object, the set of features must ***not*** have common symmetries of continuous motion (i.e. the intersection of the symmetries of the features must be equal to the identity transformation $\mathbb{1}$). The determination of the set of common symmetries depends on the values of the geometric relations between the features. This work is related to that of [10], in which the common symmetries of a set of geometric elements is used to determine assembly plans satisfying the constraints imposed by each geometric element. Here we will obtain the intersections of symmetries in an appropriate way to take into account the uncertainty related to the location of the geometric elements, and to obtain an estimation of the object location.

Let $\{F_1, F_2, \ldots, F_n\}$ represent the location of a set of n geometric elements with symmetries $\{S_{F_1}, S_{F_2}, \ldots, S_{F_n}\}$, respectively. The symmetries of each feature are expressed with respect to the reference associated to the feature. In order to calculate the intersection, we must express each set of symmetries with respect to a common reference F. Thus, the common symmetries of the set of features with respect to F can be calculated as:

$$ {}^{F}S_F = \bigcap_{i=1}^{n} t_{FF_i} \cdot S_{F_i} \cdot t_{FF_i}^{-1} \qquad (1) $$

Let us for the moment concentrate on the case when $n = 2$. In section 3 we have seen that given two features whose location is represented by F_1 and F_2, we can find references $\bar{F}_1$ and $\bar{F}_2$ such that $t_{\bar{F}_1\bar{F}_2}$ contains the minimum number of translations and rotations. Taking advantage of this, we will choose to calculate the set of common symmetries with respect to $\bar{F}_1$. In case the resulting set of symmetries is equal to $\mathbb{1}$, $\bar{F}_1$ can be used to calculate the location of the object, as we will see later. Thus, according to equation (1) we have:

$$ {}^{\bar{F}_1}S_F = S_{\bar{F}_1} \cap \left(t_{\bar{F}_1\bar{F}_2} \cdot S_{\bar{F}_2} \cdot t_{\bar{F}_2\bar{F}_1}\right) $$

The solution of this equation for all combinations of symmetries are shown in table 3. Let us study an example.

Example: Common Symmetries of a circular arc and a planar surface: Let F_1 and F_2 represent the location of a planar surface and a circular arc, respectively (fig. 4). In this case we have:

$$ S_{\bar{F}_1} = T_{xy} \cdot R_z \;\; ; \;\; S_{\bar{F}_2} = R_x $$
$$ t_{\bar{F}_1\bar{F}_2} = \text{Trans}(0,0,\bar{z}) \cdot \text{Rot}(y,\bar{\theta}) $$

The set of common symmetries of the features with respect to $\bar{F}_1$ is:

$$ \begin{aligned} {}^{\bar{F}_1}S_F \;=\; & (T_{xy} \cdot R_z) \\ & \cap \left(\text{Trans}(0,0,\bar{z}) \cdot \text{Rot}(y,\bar{\theta}) \cdot R_x \right. \\ & \left. \cdot \text{Rot}(y,-\bar{\theta}) \cdot \text{Trans}(0,0,-\bar{z})\right) \qquad (2) \end{aligned} $$

The only transformation that satisfies this condition is the identity $\mathbb{1}$, except when the circle and the plane are parallel (fig. 4 (b)). That is, when $\bar{\theta} = \pm\pi/2$. In this case, equation 2 becomes:

S_{F_1}	S_{F_2}	Case of $t_{\bar{A}\bar{B}}$	$S_{F_1} \cap S_{F_2}$
R_z	R_z	$\bar{y} = \bar{z} = \bar{\phi} = 0$ *otherwise*	R_z $\mathbb{1}$
R_z	R_{xyz}	$\bar{z} = 0$ *otherwise*	R_z $\mathbb{1}$
T_z	T_z	$\bar{\theta} = \bar{\phi} = 0$ *otherwise*	T_z $\mathbb{1}$
T_z	R_z		$\mathbb{1}$
T_z	$T_z\, R_z$	$\bar{\theta} = \bar{\phi} = 0$ *otherwise*	T_z $\mathbb{1}$
T_z	R_{xyz}		$\mathbb{1}$
$T_z\, R_z$	R_z	$\bar{y} = \bar{z} = \bar{\phi} = 0$	R_z
$T_z\, R_z$	$T_z\, R_z$	$\bar{y} = \bar{z} = \bar{\phi} = 0$ $\bar{\phi} = 0$ *otherwise*	$T_z\, R_z$ T_z $\mathbb{1}$
$T_z\, R_z$	R_{xyz}	$\bar{z} = 0$ *otherwise*	R_z $\mathbb{1}$
$T_{xy}\, R_z$	R_z	$\bar{\theta} = \pi/2$ *otherwise*	R_z $\mathbb{1}$
$T_{xy}\, R_z$	R_{xyz}		R_z
$T_{xy}\, R_z$	T_z	$\bar{\theta} = 0$ *otherwise*	T_z $\mathbb{1}$
$T_{xy}\, R_z$	$T_z\, R_z$	$\bar{\theta} = 0$ *otherwise*	T_z $\mathbb{1}$
$T_{xy}\, R_z$	$T_{xy}\, R_z$	$\bar{\phi} = 0$ *otherwise*	T_y $T_{xy}\, R_z$
R_{xyz}	R_{xyz}	$\bar{z} = 0$ *Otherwise*	R_{xyz} R_z

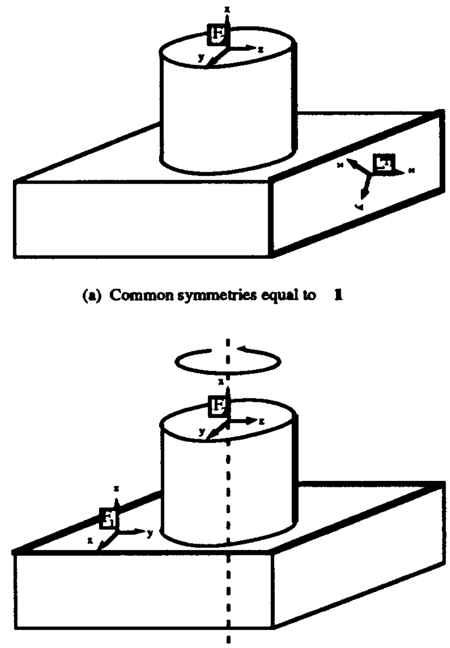
Table 3: Intersection of the symmetries of two geometric elements, expressed in $\bar{F}_1$.

$$
\begin{aligned}
{}^{\bar{F}_1}S_F &= (T_{xy} \cdot R_z) \\
&\quad \cap (\mathrm{Trans}(0,0,\bar{z}) \cdot \mathrm{Rot}(y,\pi/2) \cdot R_x \\
&\quad \cdot \mathrm{Rot}(y,-\pi/2) \cdot \mathrm{Trans}(0,0,-\bar{z})) \\
&= (T_{xy} \cdot R_z) \cap (\mathrm{Trans}(0,0,\bar{z}) \cdot R_z \\
&\quad \cdot \mathrm{Trans}(0,0,-\bar{z})) \\
&= (T_{xy} \cdot R_z) \cap R_z \\
&= R_z
\end{aligned}
$$

Thus, a planar surface and a circular arc can be used to establish the location of the object when $\bar{\theta} \neq \pm\pi/2$. The solutions of all the combinations of subgroups of symmetries can be derived in a similar way, and they are shown in table 3.

The calculation of the set of common symmetries of more than two features can be easily done in an iterative way: a pair of features is selected and the intersection of their symmetries is calculated. If it is not equal to $\mathbb{1}$, the pair is grouped into a compound feature and another feature is selected to calculate its common symmetries with the compound feature. At the end of the process, a reference that expresses the location of the set of features has been obtained, and the set of common symmetries of the features expressed in that reference is $\mathbb{1}$.

Once we have established in which cases the common symmetries of a set of observations is $\mathbb{1}$, we must investigate how sensible this solution is to the relative location of the observations and their location uncertainty. We have seen that the set of common symmetries of a pair of observations depends on the parameters of the relative location between both. Continuing with our example above, the value of the variable $\bar{\theta}$ determines the final set of common symmetries. Thus, we prefer using pairs of observations where the value of this component of the location vector is far from to $\pi/2$ and its uncertainty is small.



(a) Common symmetries equal to $\mathbb{1}$

(b) Common symmetries equal to R_z

Figure 4: Symmetries for a circle and a planar surface

Generation of object-location hypotheses. In summary, given a set of observations, the hypothesis generation process follows these steps:

1. Select a pair of observations $O = \{O_1, O_2\}$, whose uncertain location in the scene are represented by $(\hat{\mathbf{x}}_{WO_1}, \hat{\mathbf{p}}_{O_1}, C_{O_1})$ and $(\hat{\mathbf{x}}_{WO_2}, \hat{\mathbf{p}}_{O_2}, C_{O_2})$ with respect to reference W. This selection is based on criteria such as size, relative scarceness and low location uncertainty.
2. Using the relative location vector $\mathbf{x}_{O_1O_2}$, estimate the geometric relations between O_1 and O_2, as ex-

plained in subsection 3.2. This gives us $\mathbf{x}_{O_1\bar{O}_1}$, $\mathbf{x}_{O_2\bar{O}_2}$, and $\mathbf{x}_{\bar{O}_1\bar{O}_2}$.

3. Decide whether O_1 and O_2 can be used to determine the location of the object, by verifying that the values of $\mathbf{x}_{\bar{O}_1\bar{O}_2}$ are far from the singular cases of the intersection for their symmetries. If not, the observations are discarded and another pair is selected.
4. Search in the model a pair of features $F = \{F_1, F_2\}$, which belong to the same object, and satisfy unary constraints with O_1 and O_2. The feature location is represented by $\mathbf{x}_{MF_1}$ and $\mathbf{x}_{MF_2}$ with respect to the location of the object, represented by M.
5. Using the relative location vector $\mathbf{x}_{F_1F_2}$, calculate (if not available) the geometric relations between F_1 and F_2, as explained in subsection 3.1. This gives us $\mathbf{x}_{F_1\bar{F}_1}$, $\mathbf{x}_{F_2\bar{F}_2}$, and $\mathbf{x}_{\bar{F}_1\bar{F}_2}$.
6. Validate that $\mathbf{x}_{\bar{O}_1\bar{O}_2}$ is compatible with $\mathbf{x}_{\bar{F}_1\bar{F}_2}$ using the chi-square test. If not, the hypothesis is abandoned and another pair of model features $\{F_1, F_2\}$ is selected.
7. Choose $\bar{O}_1$ and $\bar{F}_1$ to represent the location of the set of observations and features respectively (that is, $O = \bar{O}_1$ and $F = \bar{F}_1$).
8. Since the pairings of observations to model features implies $O = F$, the location of the model in the scene can be calculated as:

$$\mathbf{x}_{WM} = \mathbf{x}_{WO} \ominus \mathbf{x}_{MF}$$

   Given that the set of common symmetries of O is equal to $\mathbb{1}$, the location of the object in the scene is uniquely determined.
9. From the object location $\mathbf{x}_{WM}$ and the relative feature locations $\mathbf{x}_{MF_1}$ and $\mathbf{x}_{MF_2}$, the location of the features in the scene is predicted, and it is verified that the observations O_1 and O_2 actually are located within the area occupied by F_1 and F_2. This verification step applies the denominated *extension constraint* [9].

5 Conclusions

In this paper we have presented a method to estimate the geometric relations between a pair of uncertain observations of geometric features, using the Symmetries and Perturbations model. We have shown how to derive the geometric relations that depend on the relative location and symmetries of the involved geometric elements, and how they can be expressed in the form of an uncertain location vector. This allows to define a general constraint validation mechanism, which is based of statistical tests on the components of the location vector. This representation model and constraint validation mechanism allows the use of diverse geometric information in the recognition process, which makes it suitable for multisensor systems.

We have shown the advantages of having an early estimation of object location in speeding up the recognition process. Essentially it allows to validate a smaller set of more discriminant geometric constraints. We have also shown how the estimation of the geometric relations is also useful in determining whether a set of the observations completely determine the location of the object, and how this location can be determined.

Once the location of the object has been obtained, a hypothesis verification step must take place, in which the locations of features in the scene is predicted and their presence is verified among the available data or by a sensor. The choice of the feature to predict at each moment and the sensor that should be used, as well as its optimal location, will be the subject of future work.

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