Proceedings of the 1996 IEEE/SICE/RSJ International Conference on Multisensor Fusion and Integration for Intelligent Systems

Detecting High Level Features for Mobile Robot Localization*

J. A. Castellanos† J. Neira† O. Strauss‡ J. D. Tardós†

†Departamento de Informática e Ingeniería de Sistemas, Universidad de Zaragoza
c/María de Luna 3, E-50015 Zaragoza, Spain
Email : {joseac,neira}@prometeo.cps.unizar.es, jdtardos@mcps.unizar.es

‡Laboratoire d'Informatique, de Robotique et de Micro-Electronique
rue Ada 161, 34392 Montpellier Cédex 5, France
Email: strauss@lirmm.fr

Abstract

Robust mobile robot localization requires the availability of highly reliable features obtained by the external sensors of the robot. Redundancy assures reliability and precision of the observed features. In this work we use two different sensors, namely, a laser rangefinder and a monocular vision system, whose complementary nature allows to robustly identify high level features, i.e. corners and semiplanes, in the environment of the robot. We present a general fusion mechanism, based on the Extended Information Filter, supported by a robust modelling of uncertain geometric information, to fuse information obtained by different sensors mounted on the robot. Localization of the robot is achieved by matching these observations with an a priori map of the environment. An a priori estimation of the robot location is not required. Experimental results are presented, showing the increase in reliability of the observed features after fusing information from both sensors.

1 Introduction

Robust sensing of the environment of a mobile robot is an important task both in localizing the robot and in building a complete and reliable map of such an environment. One of the fundamental ideas to achieve this robustness is the use of redundancy, that is, to combine environmental information obtained by several sensors. Such approach provides more reliable and accurate information about what the sensory system of the robot really observes. Credibility of the observed features is therefore enhanced. Dealing with redundancy requires both the availability of a robust modelling tool to represent uncertain geometric information and a multisensor fusion mechanism capable of handling information obtained by different sensors.

Some works have addressed the problem of combining information for both environment modelling and mobile robot localization. In [8] Song et al. combine sensory information from double ultrasonic sensors and a CCD camera for mobile robot self-localization. They use an EKF (Extended Kalman Filter) to fuse raw sensory data, that is low level features. Grandjean et al. [5] perform a segmentation of the raw laser information which is then merged with photometric information obtained by a stereovision system. They propose a fusion mechanism based on the EKF to calculate the robot location. Finally, Neira et al. [6] find the mobile robot localization by fusing two types of sensorial information obtained by a 3D laser rangefinder, namely, distance measurements and grey-level images. They use low level features, such as laser points, to obtain the localization of the robot. In their approach, robustness highly depends on the accuracy of the a priori model map of the environment and the initial estimation of the robot location.

By contrast, we address the problem of robust modelling of the environment of a mobile robot from a higher level perspective, considering the use of high level features, obtained by both a laser rangefinder and a CCD camera, thus, robustness is achieved at the level of observations. An overview of the process we perform might be sketched as follows. Segmentation of laser rangefinder readings (i.e. 2D points) provides a set of laser segments representing the structure of the environment of the mobile robot. Further processing of this set of segments produce two different types of high level geometric information, namely laser corners, which correspond to the intersection of consecutive segments, and laser semiplanes, which correspond to the free endpoints of laser segments (section 2). Simultaneously, information obtained by the

*This work was partially supported by CICYT, project TAP94-0390 and by a research grant FPI PN94-29099535.

vision subsystem is processed in order to find the vertical edges present in the image (section 3). Next, a fusion process is carried out to increase the credibility of the detected features and to reduce their location uncertainty (section 4). Finally, a matching between these reliable observations and an a priori map of the environment of the robot is performed to obtain the mobile robot localization (section 5). Two important contributions of this approach are: an initial estimation of the robot location is not required, and matching one of the high level features is enough to find the robot location (although more than one pairing is used to increase reliability). In section 6 we present experimental results showing raw sensory information and their corresponding high level features. There is a great credibility in the detected features because they have been observed by two different sensors. Finally, the estimated localization of the mobile robot is also presented for two different sample cases.

At each stage of the processing, a robust modelling tool, the SPmodel [9, 6], is used to efficiently consider the intrinsic properties of uncertain geometric information. Fusion of multisensor information is achieved by application of the Extended Information Filter (appendix A).

2 Processing of Laser Rangefinder Information

A tipical indoor structured environment is composed of walls, corners, doors, etc. which can be detected by a 2D laser rangefinder. Figure 1 shows a sample environment in which laser features have been drawn, namely, points, segments, corners and semiplanes. In this section we present the process to detect each of these features.

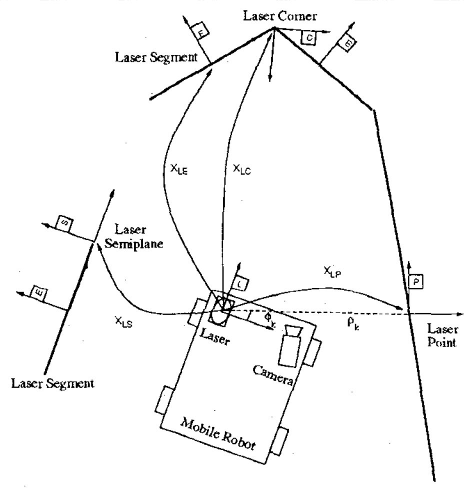

Figure 1: Relative location of features respect to a reference attached to the laser sensor.

2.1 Laser Rangefinder Data Segmentation

The most elementary geometric feature we deal with in this work is a *laser point*, obtained directly by the sensor as a pair of polar coordinates $P_k = (\rho_k,\ \phi_k)$, where ρ_k represents the distance between the scanned point and the sensor, and ϕ_k represents the azimuth angle of the location of the point respects to the reference system attached to the sensor. In the SPmodel, a reference P_k is attached to each laser point with the X-axis aligned with the laser beam (figure 1). Then, a laser point is represented by an uncertain location $\mathbf{L}_{LP_k} = (\hat{\mathbf{x}}_{LP_k}, \hat{\mathbf{p}}_{P_k}, C_{P_k}, B_{P_k})$, with respect to the laser rangefinder L, where $\hat{\mathbf{x}}_{LP_k}$ is the estimated location vector of the point with respect to the laser rangefinder; the real location of the point is obtained as:

$$\mathbf{x}_{LP_k} = \hat{\mathbf{x}}_{LP_k} \oplus \mathbf{d}_{P_k} = \hat{\mathbf{x}}_{LP_k} \oplus B_{P_k}^T \mathbf{p}_{P_k} \quad (1)$$

where $\oplus$ represents the composition of location vectors (the inverse is represented by $\ominus$); $\mathbf{p}_{P_k} \sim N(\hat{\mathbf{p}}_{P_k},\ C_{P_k})$ is the perturbation vector of the point, which takes into account sensor imprecision; and B_{P_k} is the self-binding matrix of the entity, which takes into account its symmetries. In the case of a point there is symmetry of rotation around the Z-axis, therefore angular uncertainty is not considered, thus:

$$\mathbf{d}_{P_k} = (d_x,\ d_y,\ 0)^T \quad (2)$$

and,

$$B_E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \ ; \ \mathbf{p}_{P_k} = (d_x,\ d_y)^T \quad (3)$$

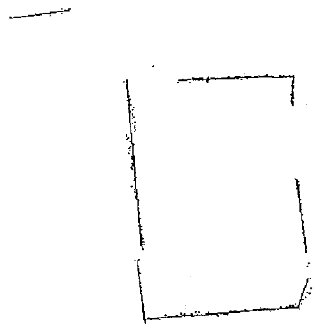

Figure 2: Laser Points obtained by the laser rangefinder and the segmentation result for a sample case.

Laser segments are obtained by application of the segmentation method presented in [3] where segmentation is achieved by dividing the laser data in regions formed by a unique polygonal line, which are subsequently divided into segments by an iterative method. We attach a reference E to each of the supporting edges (i.e. infinite lines) of the segments, placed in the middle point of the segment and with the X-axis aligned with the edge (figure 1). Thus, a laser segment is represented by an uncertain location $\mathbf{L}_{LE} = (\hat{\mathbf{x}}_{LE}, \hat{\mathbf{p}}_E, C_E, B_E)$ (obtained by application of the information filter to the set of points building the edge), where, taking into account the symmetry of traslation along the edge:

$$\mathbf{d}_E = (0,\ d_y,\ d_\phi)^T \tag{4}$$

and,

$$B_E = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \ ; \ \mathbf{p}_E = (d_y,\ d_\phi)^T \tag{5}$$

and its observed length $\mathbf{l}_E = \{\hat{l}_E, \sigma^2_{l_E}\}$, where the value of σ_{l_E} is obtained from the covariance of the extreme points of the segment. In figure 2 we show the segmentation of a sample scan obtained by the laser rangefinder.

Segmentation of laser rangefinder allows to easilly identify higher level features in the environment of the mobile robot, such as laser corners, found by intersection of two consecutive laser segments, and laser semiplanes, found at one of the extreme points of a laser segment.

2.2 Detecting Laser Corners

The most robust feature we consider in our work is a *laser corner*, which corresponds to the intersection of two consecutive laser segments. A laser corner is an uncertain geometric feature without symmetries, therefore its self-binding matrix is the identity matrix I_3. Lack of symmetries means that identification of a simple laser corner allows the robot to localize itself in an indoor structured environment.

The uncertain location of a laser corner is obtained from the edges which build up the corner (figure 1). We attach a reference C to each corner such that its X-axis is aligned with the bisector of the corner. Let $\mathbf{L}_{LC} = (\hat{\mathbf{x}}_{LC}, \hat{\mathbf{d}}_C, C_C, I_3)$ represent the estimated location of the corner C, respect to the sensor L, where:

$$\mathbf{p}_C = \mathbf{d}_C = (d_x,\ d_y,\ d_\phi)^T \tag{6}$$

Let E_k be a reference frame associated to an edge, whose location is represented by $\mathbf{L}_{LE_k} = (\hat{\mathbf{x}}_{LE_k}, \hat{\mathbf{p}}_{E_k}, C_{E_k}, B_{E_k})$. We apply the information filter to this problem by considering the perturbation vector of the corner $\mathbf{d}_C$ as the state to be estimated, and the perturbation vector of each observed edge $\mathbf{p}_{E_k}$ the measurements. The implicit non-linear function $\mathbf{f}_k(\mathbf{d}_C, \mathbf{p}_{E_k})$ expresses the fact that the laser corner belongs to each of the associated edges. Another important parameter in the representation of a laser corner is its observed angle $\phi_C = \{\hat{\phi}_C, \sigma^2_{\phi_C}\}$ formed by an estimation of the real angle, and its variance. Both values are obtained from the angle between the references attached to the laser segments building the laser corner, and their orientation covariances.

2.3 Detecting Laser Semiplanes

A *laser semiplane* is obtained from a free endpoint of a laser segment (figure 1). These endpoints might correspond to frames of open doors, convex corners, obstacles, etc. , which cannot be differentiated at this stage of the processing. Similarly to the case of a laser corner, a laser semiplane is also an uncertain geometric feature without symmetries with a self-binding matrix equal to the identity matrix I_3. Laser semiplanes are not as robust as laser corners because their estimated location is greatly influenced by the angular resolution of the laser rangefinder. Fusion with the monocular vision subsystem will increase their robustness.

We attach a reference S to each laser semiplane such that its X-axis is aligned with the associated laser segment. Let $\mathbf{L}_{LS} = (\hat{\mathbf{x}}_{LS}, \hat{\mathbf{d}}_S, C_S, I_3)$ represent the estimated location of the laser semiplane S, respect to the sensor L, where:

$$\mathbf{p}_S = \mathbf{d}_S = (d_x,\ d_y,\ d_\phi)^T \tag{7}$$

Let E be a reference frame associated to an edge, whose uncertain location is represented by $\mathbf{L}_{LE} = (\hat{\mathbf{x}}_{LE}, \hat{\mathbf{p}}_E, C_E, B_E)$ and whose observed length is given by $\mathbf{l}_E = \{\hat{l}_E, \sigma^2_{l_E}\}$. The relative transformation between the laser semiplane and the edge consists in a translation given by:

$$\mathbf{x}_{ES} = (\hat{l}_E/2, 0, 0)^T \tag{8}$$

Then the uncertain location of the laser semiplane will be given by the following composition:

$$\begin{aligned} \mathbf{x}_{LS} &= \mathbf{x}_{LE} \oplus \mathbf{x}_{ES} \\ &= \hat{\mathbf{x}}_{LE} \oplus B_E^T \hat{\mathbf{p}}_E \oplus \mathbf{x}_{ES} \\ &= \hat{\mathbf{x}}_{LE} \oplus \mathbf{x}_{ES} \oplus J_{SE} B_E^T \hat{\mathbf{p}}_E \end{aligned} \tag{9}$$

and:

$$\hat{\mathbf{x}}_{LS} = \hat{\mathbf{x}}_{LE} \oplus \mathbf{x}_{ES} \tag{10}$$

$$\hat{\mathbf{d}}_S = J_{SE} B_E^T \hat{\mathbf{p}}_E \tag{11}$$

$$C_S = J_{SE} B_E^T \ C_E \ B_E J_{SE}^T \ + C_R \tag{12}$$

where J_{SE} is the jacobian of the transformation $\mathbf{x}_{SE}$ [7]. A second summand C_R, has been added to the expression of C_V because, due to the symmetries of an edge, we obtain a covariance matrix which do not consider uncertainty along the X-axis of the reference attached to the vertical edge V. Thus, we need an uncertainty value for the X-axis, which is obtained from the angular resolution of the laser rangefinder.

3 Processing of Monocular Vision Sensor Information

In this section we obtain the angle of vertical edges in the environment of the robot with respect to an offthe-shelf CCD camera mounted on the robot. We are mainly interested in long vertical edges, corresponding to corners and doors of the robot's environment.

To detect vertical edges in the image, we use the Burns's segment extractor [2]. Burns's method to compute image segments is based on geometrical operations in a 3D space, where X and Y coordinates represent the image plane and Z represents the gray level of the corresponding pixel.

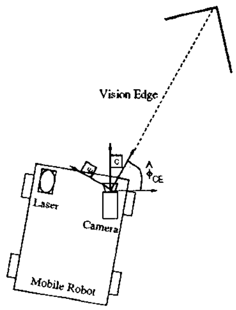

Figure 3: Representation of a vision edge.

A *vision edge* (figure 3) is represented, in 2D, by an attached reference with its origin on the optical center of the camera and its X-axis pointing to the middle point of the segment detected on the image and projected on the plane $z = 0$ of the reference attached to the camera. Let $\mathbf{L}_{CE} = (\hat{\mathbf{x}}_{CE}, \hat{\mathbf{p}}_E, C_E, B_E)$ represent the estimated location of the vision edge E, respect to the sensor C, where:

$$\hat{\mathbf{x}}_{CE} = (0,\ 0,\ \hat{\phi}_{CE})^T \qquad (13)$$

$$B_E = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \ ; \ \mathbf{p}_E = (0,\ d_\phi)^T \qquad (14)$$

There is an angular error, represented by σ_ϕ, associated to the uncertain location of each vision edge, which is related to the detection error (about 2 pixels) of the vertical edge in the image.

4 Fusing Laser and Monocular Vision Information

In this section we use a general mechanism to fuse information coming from a laser rangefinder and a monocular vision system. Basic features have been extracted in the previous sections of this work.

4.1 Matching Laser Rangefinder and Camera Features

Fusion of uncertain information requires all geometric features to be expressed in the same reference system. In our work we have selected the reference attached to the laser rangefinder L, as the base reference for expressing all the available features. By calibration we have obtained the relative transformation $\mathbf{x}_{LC} = (x, y, \phi)^T$, between the camera reference system C, and the laser reference system L, thus, a given feature E, which estimated location respects to the camera C is expressed by $\hat{\mathbf{x}}_{CE} = (0,\ 0,\ \hat{\phi}_{CE})^T$ will be expressed respect to the laser L, by the composition:

$$\hat{\mathbf{x}}_{LE} = \mathbf{x}_{LC} \oplus \hat{\mathbf{x}}_{CE} = (x,\ y,\ \phi + \hat{\phi}_{CE})^T \qquad (15)$$

Having the observed features expressed in the same reference system we need to match them, that is, we need to obtain pairings between laser corners and vision edges on one hand, and laser semiplanes and vision edges on the other. The criteria used to decide whether two given features can be matched is a hypothesis test based on the *Mahalanobis Distance* [1]. When a vision edge can be paired with more than one feature, either a laser corner or a laser semiplane, we apply an euclidean distance criterium, thus the closest feature to the reference attached to the camera will be matched with the vision edge.

4.2 Fusing Laser Corners and Vision Edges

The first set of pairings obtained by the matching process is formed by pairings of a laser corner (section 2.2) and a vision edge (section 3). Thus, by fusionating both observations we have features with greater credibility, that is, they are more likely to come from real features. The position, orientation and angle of the corner was obtained by the laser sensor, however, the integration of the vision edge allows the reduction of the uncertainty associated to the location of the corner relative to the robot.

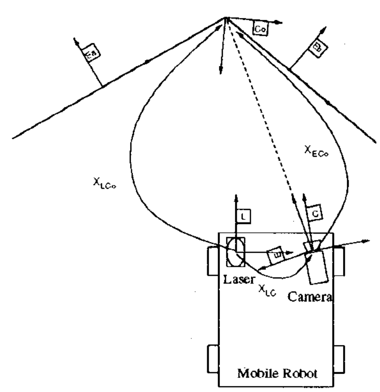

Figure 4: Fusion of a laser corner with a vision edge.

Fusion of both observations is achieved by application of the information filter presented in appendix A. The measurement $\mathbf{p}_E$, corresponding to the perturbation of the vision edge (figure 4), contributes to the estimation of the perturbation vector of the corner $\mathbf{d}_{LV}^{Co}$. In this case, the implicit measurement function $\mathbf{f}(\mathbf{d}_{LV}^{Co}, \mathbf{p}_E)$ expresses the fact that the corner belongs to the vision edge.

Let $\mathbf{L}_{LCo} = (\hat{\mathbf{x}}_{LCo}, \hat{\mathbf{d}}_{Co}, C_{Co}, I_3)$ the uncertain location of the corner estimated by the laser sensor, then the contribution of the vision edge to the perturbation vector of the corner will be given by:

$$\mathbf{d}_{LV}^{Co} = P_{LV}\, M_{LV} \tag{16}$$

$$P_{LV}^{-1} = P_L^{-1} + F_V \ ; \ M_{LV} = M_L - N_V \tag{17}$$

where:

$$P_L = C_{Co} \ ; \ M_L = P_L^{-1}\hat{\mathbf{d}}_{Co}$$

come from the integration of laser information in the estimation of the corner and F_V and N_V come from the use of the vision edge as redundant information in the estimation of the corner.

4.3 Fusing Laser Semiplanes and Vision Edges

A similar situation is found in the case of pairings formed by a laser semiplane (section 2.3) and a vision edge (section 3). The redundant information coming from the observation of the vision edge is used to increase the credibility on the detection of a real semiplane in the environment of the robot and to reduce its location uncertainty.

In this case, the information filter (figure 5) is applied to the fusion of both observations by considering the perturbation vector of the semiplane $\mathbf{d}_{LV}^{S}$, as the state to be re-estimated, and the perturbation vector of the observed vision edge $\mathbf{p}_E$, as the new measurement. The implicit measurement function $\mathbf{f}(\mathbf{d}_{LV}^{S}, \mathbf{p}_E)$ expresses the fact that the semiplane belongs to the vision edge.

The updated estimation of the perturbation vector of the semiplane is given by:

$$\mathbf{d}_{LV}^{S} = P_{LV}\, M_{LV} \tag{18}$$

$$P_{LV}^{-1} = P_L^{-1} + F_V \ ; \ M_{LV} = M_L - N_V \tag{19}$$

where P_L and M_L come from the integration of laser information in the estimation of the semiplane, similarly as we presented for a laser corner, and F_V and N_V come from the use of the vision edge as redundant information in the estimation of the semiplane.

5 Mobile Robot Localization

Knowlegde of the mobile robot localization is one of the fundamental tasks to be performed to succeed in navigating the robot. We state the mobile robot localization problem as a matching between high level features (i.e.

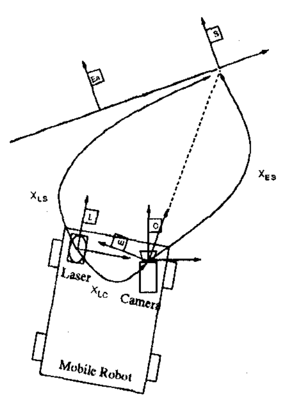

Figure 5: Fusion of a laser semiplane with a vision edge.

corners and semiplanes) obtained by the external sensors mounted on the robot and model features stored in a database, representing the structure of the environment. Note that an initial estimation of the robot location is not required for the following processing.

5.1 Mobile Robot Localization Formulation

Mathematically, the mobile robot localization problem is stated using the notation proposed by the SPmodel. Thus, an uncertain location $\mathbf{L}_{WR} = (\hat{\mathbf{x}}_{WR}, \hat{\mathbf{d}}_R, C_R, I_3)$ is assigned to the robot, with respect to a global reference W attached to the model map. A model feature m is represented by a reference frame M, whose location respect to the global reference is represented by the transformation $\mathbf{x}_{WM}$. An observation e (i.e. either a corner or a semiplane) is represented by a reference frame E with an associated uncertain location $\mathbf{L}_{RE} = (\hat{\mathbf{x}}_{RE}, \hat{\mathbf{d}}_E, C_E, I_3)$.

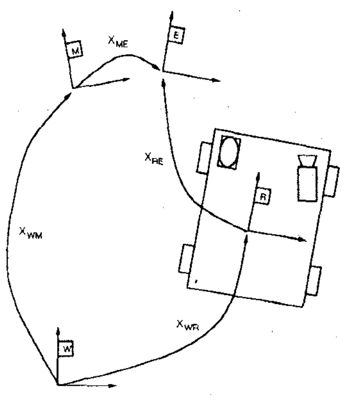

Figure 6: References involved in the problem of mobile robot localization.

Each of the pairings between a model feature and an observation, contributes to the estimation of the mobile robot localization. Using the information filter, we considered the perturbation vector of the robot $\mathbf{d}_R$ as the state to be estimated, and the perturbation vector of the observed feature $\mathbf{d}_E$, as the measurement. The implicit non-linear function $\mathbf{f}_k(\mathbf{d}_R, \mathbf{d}_E)$ corresponds to the pairing between the observation and the model feature and is obtained from figure 6 considering the relationship between location vectors.

It is important to note that due to the lack of symmetries of both a corner and a semiplane, the identification of a simple feature allows the system to completely localize the robot. A potential problem is that of a mismatch between an observation and a model feature, therefore, we prefer to use more than one pairing to find the robot localization. In the following section we present the mechanisms used to establish the set of pairings between observations and model features.

5.2 Searching for Observation-Model Pairings

We have identified three possible types of pairings between observations and model features (figure 7). Considering that our observations are corners and semiplanes we have:

1. Pairing between an observed corner and a model corner. Both the laser range finder and the CCD camera have detected a corner in the environment of the robot. This situation occurs in either the observation of a concave corner or the complete observation of a convex corner.

2. Pairing between an observed semiplane and a model semiplane. Both sensors have detected a semiplane in the environment of the robot. It might correspond to the observation of the frame of an open door.

3. Pairing between an observed semiplane and a model corner. In this case the laser rangefinder has observed one of the semiplanes of a convex corner, while the other semiplane remains hidden. The CCD camera is able to detect the vertical edge provided that there is enough contrast in the image.

The mobile robot localization problem implies carrying out two tasks: determining the observation-model pairings (i.e. identification), and computing its location in the model map. Identification is a search problem, while computing the robot location is an estimation problem. In [4] Castellanos et al. show that simultaneous identification and localization greatly reduces the complexity of the matching process, therefore, in this work we use such an scheme.

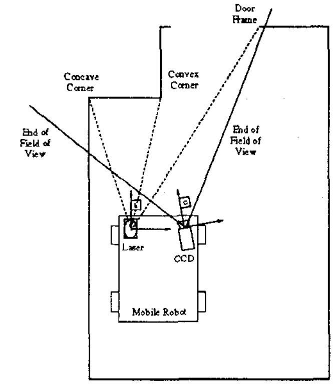

Figure 7: Different types of observations and their corresponding model features.

6 Experimental Results

In this section we present the experimental results obtained by application of the previously described techniques to two different localizations of the mobile robot in structured indoor environments. Each of the examples shows the following results (figures 8 and 9):

- Gray-level image obtained by the CCD camera. Long vertical segments have been extracted by application of the Burns algorithm and have been drawn on the image, as white arrows. We are just interested in long vertical segments (longer than 150 pixels in the image) because they are more likely to belong to corners and door frames in the environment of the robot.

- Representation of the estimated uncertainty, obtained by the laser rangefinder, of the high level features matched with the vision edges. We represent uncertainty by an ellipse obtained from the probability distribution (up to 95% limit) of the considered random variable, in this case, the perturbation vector of the geometric entity. Note that, in the results presented, uncertainty has been magnified by an scaling factor.

- Uncertainty reduction experimented by high level features after fusing information obtained by the vision sensor. Results show an important reduction in the location estimation of semiplanes due to the higher angular resolution of the CCD camera compared to the resolution of the laser rangefinder. However, in the case of corners this reduction is not so important due to the high accuracy obtained with the laser rangefinder.

- Mobile robot localization obtained by the system. We represent laser points superimposed to a hand-drawn map of the room in which the robot has been located. Note that even though the map in not precise, the estimated localization of the robot is greatly accurate. Existing objects in the environment do not prevent the robot from finding its localization.

7 Conclusions and Further Work

We have addressed the problem of mobile robot localization from a multisensor fusion point of view. From the raw laser rangefinder data we have obtained high level features, namely, laser corners and laser semiplanes. A second sensor, a CCD camera, has been used to increase the credibility of observations and to improve the location estimation of these features, due to its high angular resolution. Fusion has been performed by application of a general integration mechanism based on the extended information filter. We have observed an important reduction in the location uncertainty of semiplanes, although in the case of corners it has not been so important due to the high accuracy obtained with the laser rangefinder. Mobile robot localization has been achieved by application of a matching scheme between the fused features obtained by the system and an a priori map of the environment, stored in a database. An a priori estimation of the robot location was not required, although in the case of more complex indoor environments, with a great number of features, an estimation from the dead-reckoning system of the robot will be helpful in reducing the complexity of the matching process.

The method presented in this paper to detect reliable high level features is not limited to the problem of mobile robot localization but it might be extended to the problem of dynamic map building, obtaining a complete map of the environment of the robot formed by those high level features. This application is in one of our future research directions.

A Extended Information Filter

Let $\mathbf{x}$ be a state vector whose value is to be estimated, and let there be n observations $\mathbf{y}_k$ of $\mathbf{x}$, where $k \in \{1, \ldots, n\}$, affected by white Gaussian noise:

$$\hat{\mathbf{y}}_k = \mathbf{y}_k + \mathbf{u}_k \ ; \ \mathbf{u}_k \sim N(0, S_k)$$

Let each observation $\mathbf{y}_k$ be related to $\mathbf{x}$ by an implicit non-linear function of the form $\mathbf{f}_k(\mathbf{x}, \mathbf{y}_k) = 0$. Since $\mathbf{f}_k$ is nonlinear, we use a first order approximation:

$$\mathbf{f}_k(\mathbf{x}, \mathbf{y}_k) \simeq \mathbf{h}_k + H_k(\mathbf{x} - \hat{\mathbf{x}}) + G_k(\mathbf{y}_k - \hat{\mathbf{y}}_k)$$

where:

$$\mathbf{h}_k = \mathbf{f}_k(\hat{\mathbf{x}}, \hat{\mathbf{y}}_k) \ ; \ H_k = \left.\frac{\partial \mathbf{f}_k}{\partial \mathbf{x}}\right|_{(\hat{\mathbf{x}}, \hat{\mathbf{y}}_k)} \ ; \ G_k = \left.\frac{\partial \mathbf{f}_k}{\partial \mathbf{y}}\right|_{(\hat{\mathbf{x}}, \hat{\mathbf{y}}_k)}$$

The estimation $\hat{\mathbf{x}}_n$ of the state vector and its covariance P_n after integrating the n measurements are:

$$\hat{\mathbf{x}}_n = P_n M_n \ ; \ P_n^{-1} = \sum_{k=1}^{n} F_k \ ; \ M_n = -\sum_{k=1}^{n} N_k$$

where:

$$F_k = H_k^T (G_k S_k G_k^T)^{-1} H_k \quad (20)$$

$$N_k = H_k^T (G_k S_k G_k^T)^{-1} h_k \quad (21)$$

Matrix P_n^{-1}, the inverse of the covariance matrix is denominated *information matrix of the estimation*, while matrix F_k is denominated *information matrix of the observation.*

Acknowledgements

The work presented in this paper has been developed during a research stay of Mr. Castellanos at LIRMM, France. For the experiments we used the exteroceptives sensors mounted on the mobile robot VEGA. I would like to thank to Mr. P. Dauchez and Ms. M. J. Aldon for accepting me in the Robotics Department of the LIRMM.

References

[1] T. Bar-Shalom and T.E. Fortmann. *Tracking and Data Association.* Academic Press In., 1988.

[2] J.B. Burns, A.R. Hanson, and E.M. Riseman. Extracting straight lines. *IEEE Trans. on Pattern Analysis and Machine Intelligence*, 8(4):425–455, 1986.

[3] J.A. Castellanos and J.D. Tardós. Laser-based segmentation and localization for a mobile robot. In *Proc. of the Second World Automation Congress*, Montpellier, France, May 1996.

[4] J.A. Castellanos, J.D. Tardós, and J. Neira. Constraint-based mobile robot localization. In *Proc. of the 1996 Int. Workshop on Advanced Robotics and Intelligent Machines*, Salford, Great Britain, April 1996. Paper No. 14.

[5] P. Grandjean and A.R. Robert de Saint Vincent. 3D modeling of indoor scenes by fusion of noisy range and stereo data. In *Proc. 1989 IEEE Int. Conf. Robotics and Automation*, pages 681–687, Scottsdale, Arizona, 1989.

[6] J. Neira, J. Horn, J.D. Tardós, and G. Schmidt. Multisensor mobile robot localization. In *Proc. 1996 IEEE Int. Conf. on Robotics and Automation*, pages 673–679, Minneapolis, USA, 1996.

[7] R.P. Paul. *Robot Manipulators: Mathematics, Programming, and Control.* MIT Press, Cambridge, Mass., 1981.

[8] K.T. Song and W.H. Tang. Environment recognition for a mobile robot using double ultrasonic sensors and a ccd camera. In *Proc. 1994 Int. Conf. on Multisensor Fusion and Integration for Intelligent Systems*, pages 715–722, Las Vegas, USA, 1994.

[9] J.D. Tardós. Representing partial and uncertain sensorial information using the theory of symmetries. In *Proc. 1992 IEEE Int. Conf. on Robotics and Automation*, pages 1799–1804, Nice, France, 1992.

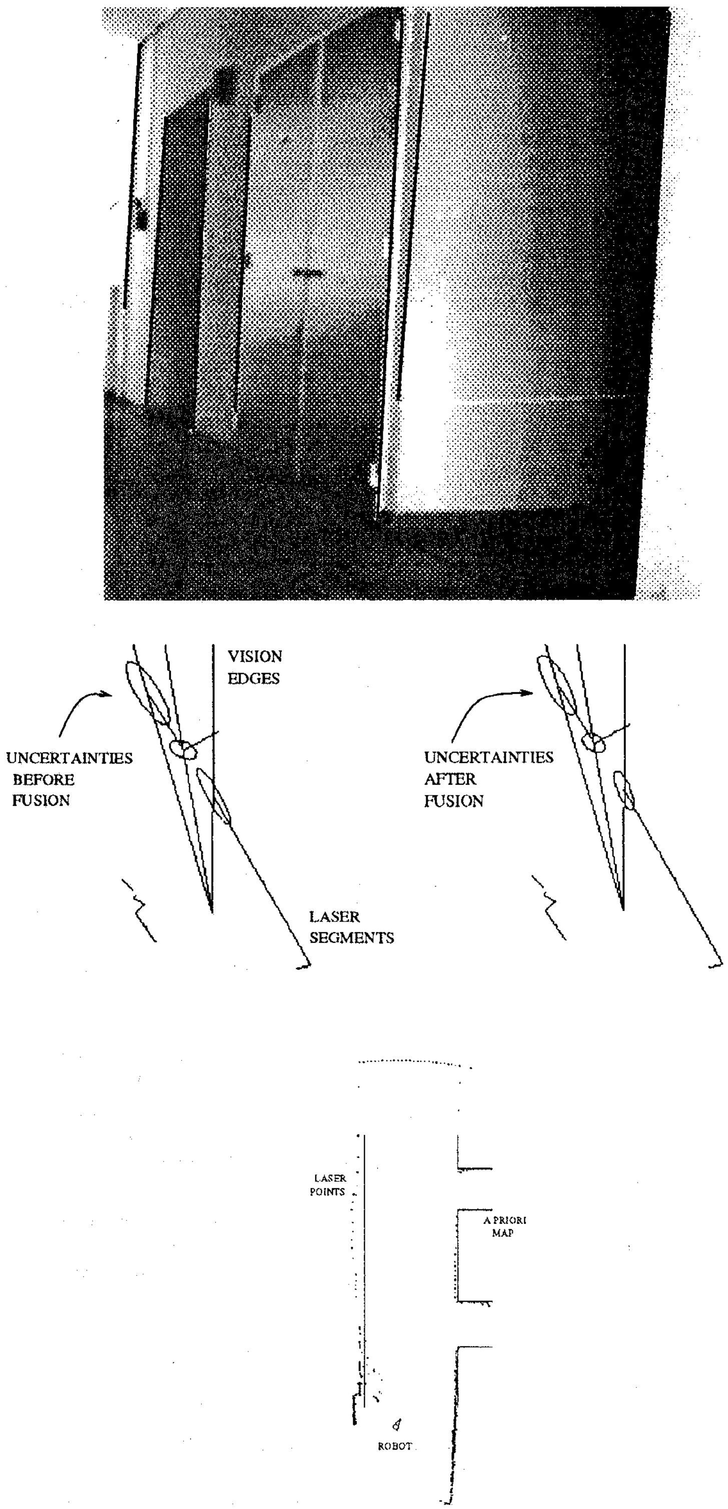

Figure 8: In the corridor. Extracted Vertical Edges (Top), Uncertainty of Features before Fusion (Medium-Left), Uncertainty of Features after Fusion (Medium-Right), Mobile Robot Localization (Bottom).

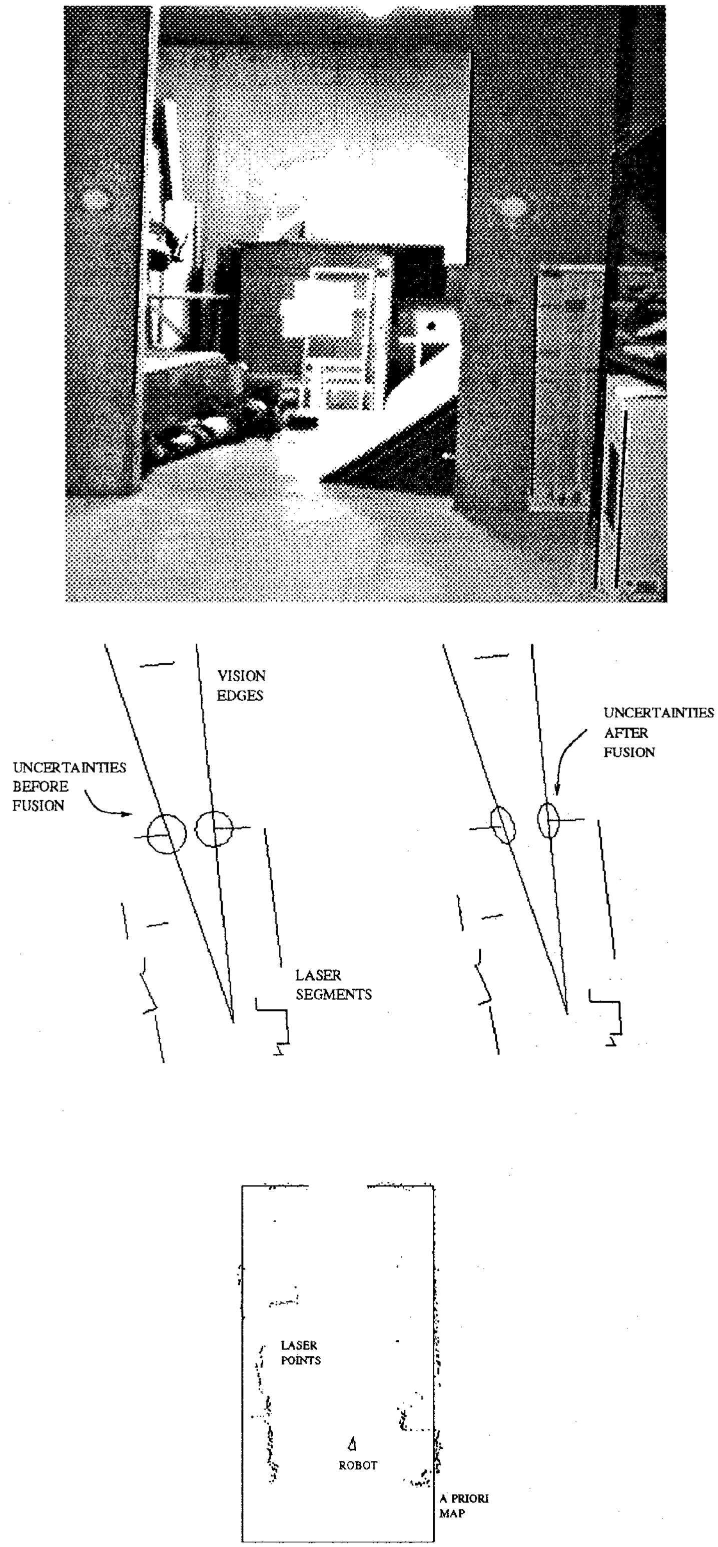

Figure 9: In the Printer Room. Extracted Vertical Edges (Top), Uncertainty of Features before Fusion (Medium-Left), Uncertainty of Features after Fusion (Medium-Right), Mobile Robot Localization (Bottom).