On Observability in Timed Continuous Petri Net Systems

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Abstract

Fluidification is a classical relaxation technique for approximated performance evaluation of discrete systems. In this paper we deal with T-timed fully continuous Petri Nets working under infinite servers semantics, what leads to (deterministic) piecewise linear differential systems. Switches among dynamic linear systems are triggered by internal events through minimum operators on marking variables. The observability problem consists of estimating the (initial) marking from a partial measure. This paper is devoted to observability concepts and criteria in this particular class of systems, not to the observers design. The concept of structural observability, regarding to the possibility of estimating the marking of places for any speed of the transitions is introduced and studied for the subclass of Join-Free Petri Nets (JF). For non Join-Free Petri Nets, conditions to compute suitable estimates will be established.

1 Introduction

Petri nets represent a powerful formalism for the modelling of discrete concurrent systems [7, 12]. Stochastic T-timed Petri nets under infinite server semantics is a well-known performance evaluation discrete model [11, 1]. Under high traffic or heavy loads discrete event systems often suffer from the state explosion problem. One of the possibilities to tackle this problem is to relax the original discrete model.

Fluidification is a relaxation technique in which discrete elements of the system are taken as continuous. As in other discrete event systems, the continuous *relaxation* of Petri nets has been introduced in order to deal with the state explosion problem. Under infinite servers semantics, a timed continuous Petri net system can be seen as a deterministic piecewise linear system [13, 14], i.e., the evolution of the state of the system is ruled by a set of switching linear differential equation systems. Hence, there exists the chance of applying some results coming from Systems Theory to continuous Petri nets. The timed continuous Petri net system has the particularity that, at a given instant, the differential equations that rule its evolution depend uniquely on the state of the system (marking). Hence, the switch from one linear differential equation system to another one is activated by an *internal event*, i.e., by a certain change in the marking of the system.

Analysis and synthesis are two major issues of study regarding continuous Petri nets. Focusing on synthesis, a crucial topic of research is the design of control laws that drive the evolution of the system in a desired way. When considering discrete event systems, it has to be noticed that scheduling problems are in fact dynamic performance control problems.

In order to control a dynamic system, frequently it is necessary to know its current state. In order to gather this information, sensors can be placed on several locations of the plant being modeled. However, it may happen that some state variables of the system cannot be directly measured by sensors. It can also happen that the cost of the sensors required to measure every state variable is prohibitive. In a general dynamic system, under some conditions, some of the variables that cannot be directly measured can be estimated. This estimate constitutes the observation. The observabil-

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ity problem, i.e., the characterization of which state variables are observable and its observation, has been studied in detail in the framework of *linear* systems (see for example [8, 3]). For these systems, the observable subspace can be characterized algebraically. A system state estimation based on such algebraic equation can be *theoretically* obtained from the computation of the derivatives of the output signal.

Observability problems have also been studied in the discrete event systems setting (see for example [9, 4]).

The main goal of this paper is the study of observability in the framework of continuous Petri net systems [2, 13]. Our attention is first focused on the study of net systems without synchronizations, named Join Free (JF) systems. For this class of net systems a linear differential equation system describes its behavior, thus classical results on observability of linear systems apply here. For JF systems, an effort has been made to introduce and study the concept of *structural observability.* A JF system is said to be structurally observable if its marking can be estimated independently of the speeds of the transitions. Results on structural observability for JF systems that are presented in this paper can be extended to other linear systems. Afterwards, general systems including synchronizations will be considered. Some inherent features of continuous Petri net systems allow us to extract less restrictive observability conditions than the ones known for general piecewise linear systems [15].

The paper is structured as follows: In Section 2 continuous Petri nets are introduced. In Section 3 the observability problem for continuous Petri nets is stated in a similar way to the observability problem for linear systems. In Section 4 we concentrate on JF systems. For this type of systems, structural conditions of observability are obtained from the output of a fix point algorithm. Section 5 is devoted to the study of the problem for general Petri net systems. Section 6 sums up the main results of the paper.

2 Continuous Petri Net Systems

2.1 Untimed Continuous Petri Net Systems

The reader is assumed to be familiar with Petri nets (PNs) (see for example [7, 12]). The Petri net systems that will be considered are *continuous.* Continuous systems are obtained as a relaxation of *discrete* ones. Unlike 'usual' discrete systems, the amount in which a transition can be fired in a continuous Petri net system is not restricted to be a natural number. Firing of a transition a non-negative real amount of times may cause the marking of the system to become a vector of real numbers. A PN system is a pair $\langle \mathcal{N}, \mathbf{m_0} \rangle$, where $\mathcal{N}$ specifies the net structure, $\mathcal{N} = \langle P, T, \mathbf{Pre}, \mathbf{Post} \rangle$ and $\mathbf{m_0}$ is the initial marking. The sets of places and transitions are denoted by P and T respectively. Matrices $\mathbf{Post}$ and $\mathbf{Pre}$ are the arc weight arc matrices and $\mathbf{C} = \mathbf{Post} - \mathbf{Pre}$ is the token flow matrix. The set of input (output) places of a given set of transitions V is denoted as $^\bullet V$ $(V^\bullet)$. Respectively, the set of input (output) transitions of a given set of places W is denoted as $^\bullet W$ $(W^\bullet)$. As in discrete nets, continuous nets can be classified according to their structure. A net is Join Free (JF) iff every transition has only one input place (for every t, $|^\bullet t| = 1$).

In continuous Petri net systems a transition t is *enabled* at a marking $\mathbf{m}$ iff every input place of t is marked (every $p \in {}^\bullet t$, $\mathbf{m}[p] > 0$). As in discrete systems, the *enabling degree* at marking $\mathbf{m}$ of a transition measures the maximum amount in which the transition can be fired in a single occurrence, i.e., $\text{enab}(t, \mathbf{m}) = \min_{p \in {}^\bullet t}\{\mathbf{m}[p]/\mathbf{Pre}[p,t]\}$. The firing of t in an amount $\alpha \leq \text{enab}(t, \mathbf{m})$ produces a new marking $\mathbf{m'}$, and it is denoted as $\mathbf{m} \xrightarrow{\alpha t} \mathbf{m'}$. It holds $\mathbf{m'} = \mathbf{m} + \alpha \cdot \mathbf{C}[P, t]$, hence, as in discrete systems the state equation $\mathbf{m} = \mathbf{m_0} + \mathbf{C} \cdot \boldsymbol{\sigma}$ summarizes the way the marking evolves, where $\boldsymbol{\sigma}$ is the firing count vector.

2.2 Timed Continuous Petri Net Systems

For the timing interpretation, a first order (or deterministic) approximation of the discrete case [10] will be used, assuming that the delays associated to the firing of the transitions can be approximated by their mean values. Each transition t has associated an internal firing speed $\boldsymbol{\lambda}[t] > 0$. The state equation has an explicit dependence on time $\mathbf{m}(\tau) = \mathbf{m_0} + \mathbf{C} \cdot \boldsymbol{\sigma}(\tau)$. Deriving with respect to time, $\dot{\mathbf{m}}(\tau) = \mathbf{C} \cdot \dot{\boldsymbol{\sigma}}(\tau)$ is obtained. Let us denote $\mathbf{f} = \dot{\boldsymbol{\sigma}}$, since it represents the *flow* through the transitions. In this paper it will be assumed that every transition has at least one input place. Infinite servers semantics will be considered. Under this semantics, the flow of a transition is given by the product of $\boldsymbol{\lambda}$ and its enabling degree, i.e., $\mathbf{f}[t] = \boldsymbol{\lambda}[t] \cdot \text{enab}(t, \mathbf{m}) = \boldsymbol{\lambda}[t] \cdot \min_{p \in {}^\bullet t}\{\mathbf{m}[p]/\mathbf{Pre}[p,t]\}$, what leads to a non-linear system.

In JF systems, transitions have only one input place, and so the computation of the enabling degrees does not require the *min* operator. Hence, the flow of the transitions can be expressed as $\mathbf{f} = \boldsymbol{\Psi} \cdot \mathbf{m}$ where $\boldsymbol{\Psi}[t,p] = \boldsymbol{\lambda}[t]/\mathbf{Pre}[p,t]$ if $p = {}^\bullet t$, $\boldsymbol{\Psi}[t,p] = 0$ otherwise. Consequently, the evolution of the marking can be described by an equation in the form $\dot{\mathbf{m}} = \mathbf{C} \cdot \mathbf{f} = \mathbf{A} \cdot \mathbf{m}$, where $\mathbf{A} = \mathbf{C} \cdot \boldsymbol{\Psi}$. Hence, a JF system can be interpreted as a linear system.

For a general system, matrix $\mathbf{A}$ is not constant but piecewise-constant. The value of $\mathbf{A}$ at a given instant is determined by the marking $\mathbf{m}$ at that instant. To compute $\mathbf{A}$, it is necessary to know the set of places that is actually enabling the transitions, i.e., the set of places that are giving the minimum in the expression for the enabling degree. Once this set is computed, it is easy to establish a linear relationship between the marking of the places in this set and the flow of the transitions: $\dot{\mathbf{m}} = \mathbf{A} \cdot \mathbf{m}$, with $\mathbf{A} = \mathbf{C} \cdot \mathbf{\Psi}$ where $\mathbf{\Psi}[t,p] = \boldsymbol{\lambda}[t]/\mathbf{Pre}[p,t]$ if $p \in {}^\bullet t$ and $\mathbf{m}[p]/\mathbf{Pre}[p,t] = \min_{q \in {}^\bullet t}\{\mathbf{m}[q]/\mathbf{Pre}[q,t]\}$, $\mathbf{\Psi}[t,p] = 0$ otherwise.

The marking of the places restricts the behaviour of their output transitions. For each marking $\mathbf{m}$, its PT-set can be defined as the set of all the pairs, (p,t), such that the marking of p is restricting the flow of transition t at marking $\mathbf{m}$.

Definition 1 *Given a net system, the* PT-set *at a marking* $\mathbf{m}$ *is*

$$\text{PT-set}(\mathbf{m}) = \{(p,t) \mid \mathbf{f}[t] = \boldsymbol{\lambda}[t] \cdot \mathbf{m}[p]/\mathbf{Pre}[p,t]\} \quad (1)$$

Obviously, for JF systems a unique PT-set exists, and $\dot{\mathbf{m}} = \mathbf{A} \cdot \mathbf{m}$. Otherwise, if the PT-set is known, the system evolves according to $\dot{\mathbf{m}} = \mathbf{A}_1 \cdot \mathbf{m}$ where $\mathbf{A}_1$ depends on PT-set($\mathbf{m}$) and the $\boldsymbol{\lambda}$ of the transitions. If at a given instant the PT-set changes, i.e., a transition is restricted by other input place, the system will be ruled by another linear system $\dot{\mathbf{m}} = \mathbf{A}_2 \cdot \mathbf{m}$. That is, every PT-set, k, has associated a square matrix $\mathbf{A}_k$ and a linear system $\Sigma_k : \dot{\mathbf{m}} = \mathbf{A}_k \cdot \mathbf{m}$. The set of PT-sets that will be active during the evolution of the system, i.e., *behavioral PT-sets*, depends on the net structure and the initial marking. If the initial marking is not known, the net structure defines the set of potential PT-sets, i.e., *structural PT-sets*, that might be active. Clearly, the set of structural PT-sets contains the set of behavioral PT-sets.

In this way, a continuous Petri net system can be seen as a piecewise linear system in which the switches among the linear systems are activated by internal events, i.e., the change from one PT-set to another does not need any external agent, just a certain change in the system marking. Due to the way in which the system evolution is defined, it can be assured that the marking of the system and its first derivative with respect to time are continuous.

In order to illustrate the evolution of a non JF system, let us consider the system in Figure 1 with initial marking $\mathbf{m_0} = (3\ 0\ 0)$ and transition speeds $\boldsymbol{\lambda} = (0.9\ 1\ 1)$. If $\mathbf{m}[p_1] \le \mathbf{m}[p_2]$, the flow of transition t_2 will be defined by the marking of p_1 (Σ_1) and the PT-set will be $\{(p_1,t_1),(p_1,t_2),(p_3,t_3)\}$. Similarly, if $\mathbf{m}[p_1] \ge \mathbf{m}[p_2]$ the flow of t_2 will be restricted by p_2 (Σ_2) and the PT-set will be $\{(p_1,t_1),(p_2,t_2),(p_3,t_3)\}$.

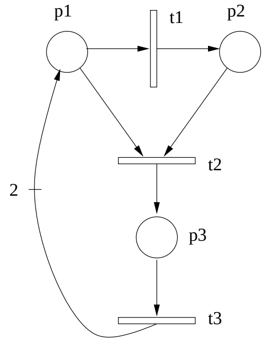

Figure 1. A non JF net system with two PT-sets.

$$\Sigma_1 : \ \dot{\mathbf{m}} = \begin{pmatrix} -1.9 & 0 & 2 \\ -0.1 & 0 & 0 \\ 1.0 & 0 & -1 \end{pmatrix} \cdot \mathbf{m}$$

$$\Sigma_2 : \ \dot{\mathbf{m}} = \begin{pmatrix} -0.9 & -1 & 2 \\ 0.9 & -1 & 0 \\ 0.0 & 1 & -1 \end{pmatrix} \cdot \mathbf{m}$$

At the time instant in which $\mathbf{m}[p_1] = \mathbf{m}[p_2]$, Σ_1 and Σ_2 behave in the same way and any of them can be taken. Figure 2 shows the evolution of the system along time. At the beginning the system evolves according to Σ_2. Then a switch occurs and the dynamics of the system is described by Σ_1. A second switch turns the system back to Σ_2, the system stabilizes and no more switches take place.

Notice that for a given marking, the set of places that are not in the PT-set do not play any role in the evolution of the system. Mathematically this is expressed by null columns in the system matrix $\mathbf{A}_j$ corresponding to the places that are not in the PT-set. Such places can be temporarily considered as a kind of *timed-implicit* places, since the system evolution does not depend on them. However, when a switch occurs, at least one place that was acting as timed-implicit becomes member of the new PT-set. For the net system in Figure 1 with $\mathbf{m_0} = (3\ 0\ 0)$, p_2 is timed-implicit only in the period when Σ_1 is describing the system dynamics.

3 Observability: Problem Statement

Let us consider first linear time invariant systems, for which observability has been thoroughly studied [6, 8, 3]. An unforced linear system (i.e., without

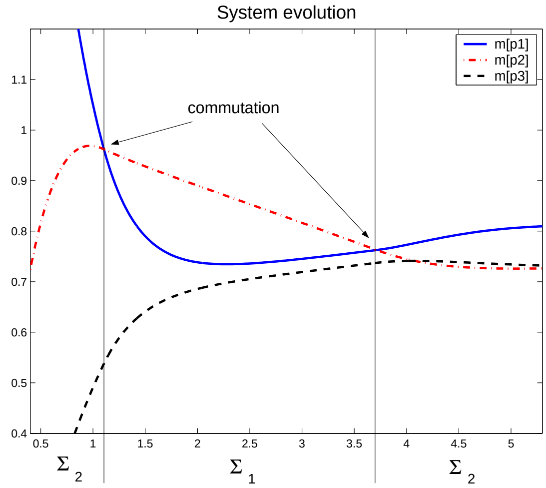

Figure 2. Marking evolution of the system in Figure 1 with $\mathbf{m_0} = (3\ 0\ 0)$.

inputs) is usually expressed by equations $\dot{\mathbf{x}} = \mathbf{A}\cdot\mathbf{x}$, $\mathbf{y} = \mathbf{S}\cdot\mathbf{x}$ where $\mathbf{x}$ is the state of the system and $\mathbf{y}$ is the output, i.e., the set of measured variables. The state space is denoted as $\mathbf{X}$. Knowing the matrices $\mathbf{A}$ and $\mathbf{S}$ and being able to watch the evolution of $\mathbf{y}$, a linear system is said to be observable iff it is possible to compute its initial state, $\mathbf{x}(t_0)$ (in fact, since the system is deterministic, knowing the state at the initial time is equivalent to knowing the state at any time).

In Systems Theory a well-known observability criterion exists that allows to decide whether a continuous (deterministic) time linear system is observable or not [8, 3]. Besides, several approaches exist to compute the initial state of continuous time linear system that is observable. Nevertheless, in order to simplify the presentation of the results and make them more intuitive, the evolution of the systems will be expressed in discrete time (continuous time will be used only when dealing with structural observability, in Section 4) .

Given a linear system of dimension n expressed in discrete time, $\mathbf{x}(k+1) = \mathbf{F}\cdot\mathbf{x}(k)$, $\mathbf{y}(k) = \mathbf{S}\cdot\mathbf{x}(k)$ the output of the system in the first $n-1$ periods is:

$$\begin{pmatrix} \mathbf{y}(0) \\ \mathbf{y}(1) \\ \mathbf{y}(2) \\ .. \\ \mathbf{y}(n-1) \end{pmatrix} = \begin{pmatrix} \mathbf{S} \\ \mathbf{S}\cdot\mathbf{F} \\ \mathbf{S}\cdot\mathbf{F}^2 \\ .. \\ \mathbf{S}\cdot\mathbf{F}^{n-1} \end{pmatrix} \cdot \mathbf{x}_0 = \vartheta \cdot \mathbf{x}_0 \qquad (2)$$

The matrix ϑ is called *observability matrix* [6, 8, 3]. The linear system is observable iff ϑ has full rank. For a non observable system it is possible to decompose the state space $\mathbf{X}$ into two subspaces: the *observable subspace*, $\mathbf{X}_o$, and the *non observable subspace*, $\mathbf{X}_{no}$. It can be verified that $\mathbf{X}_{no}$ is the kernel of ϑ, i.e., $\vartheta\cdot\mathbf{X}_{no} = \mathbf{0}$, because it does not have any influence on the vector of outputs.

Let us now consider timed continuous Petri net systems. As it has been seen, the evolution of a Petri net system is ruled by a set of switching linear systems, each one associated to a PT-set, where the state vector is the marking of the net, $\mathbf{m}$. Every linear system $\Sigma_i : \dot{\mathbf{m}} = \mathbf{A}_i\cdot\mathbf{m}$ associated to a PT-set of the Petri net can be discretized in time. The associated discrete time system can be written as $\Sigma_i^d : \mathbf{m}(k+1) = \mathbf{F}_i\cdot\mathbf{m}(k)$, with $\mathbf{F}_i = e^{\mathbf{A}_i\cdot\delta}$ where δ is the time period. The output of the net system is given by $\mathbf{y} = \mathbf{S}\cdot\mathbf{m}$. Here it will be assumed that each place is either *measured* or *unmeasured*. It will be said that a place p_i is measured iff there exists a row j in $\mathbf{S}$ such that $\mathbf{S}(j,i) \neq 0$ and $\mathbf{S}(j,k) = 0$ for every $k \neq i$.

Let us define the concept of observability for a continuous Petri net system:

Definition 2 *Let $\mathcal{N}$ be a continuous Petri net system, $\boldsymbol{\lambda}$ the internal speeds of the transitions, and $\mathcal{D}$ the set of measured places.*

- *A place $p \in P$ is* observable *from $\mathcal{D}$ iff it is possible to compute its initial marking $\mathbf{m_0}[p] = \mathbf{m}(\tau_0)[p]$ by measuring the marking evolution of the places in $\mathcal{D}$.*
- *$\mathcal{N}$ is* observable *from $\mathcal{D}$ iff every place $p \in P$ is* observable.

Applying [8, 3] an observability criteria is immediately deduced:

Property 3 *Given a Petri net system and $\Sigma_i^d : \mathbf{m}(k+1) = \mathbf{F}_i\cdot\mathbf{m}(k)$ the linear system associated to PT-set i. The PT-set i is observable iff its associated observability matrix ϑ_i has full rank.*

Clearly, when the net system is ruled by an observable PT-set the marking of all the places can be computed through Equation 2.

For a general PT-set, the places not in the PT-set can be considered as *timed-implicit* and do not play any role in the dynamics of the system (see Subsection 2.2). Consequently, no information about the marking of the timed-implicit places can be inferred from the marking of the places in the PT-set. Therefore, if a PT-set is wanted to be observable, the only way to compute the marking of the timed-implicit places is to take them directly in the output matrix $\mathbf{S}$.

Notice that every $\mathbf{F}_i$ is an exponential matrix and therefore it can be inverted. Hence continuous Petri net systems can be simulated backwards if the actual marking is known. Observe that the PT-set changes can be detected also from the backwards simulation. This implies that if the marking of the system at a given instant is known then the marking of the system at any previous instant can be computed.

4 Observability in Join Free Systems

As already pointed out, in continuous JF systems the evolution of the marking can be modelled as $\dot{\mathbf{m}} = \mathbf{A} \cdot \mathbf{m}$ (or $\mathbf{m}(k+1) = \mathbf{F} \cdot \mathbf{m}(k)$ if time is discrete or discretized). As in Property 3, the existing observability criterion for linear systems can be directly applied. In this section, an effort is made to extract an observability criterion that is independent of the internal speeds of the transitions, i.e., vector $\boldsymbol{\lambda}$.

4.1 Structural Observability

Definition 4 *Let $\mathcal{N}$ be a continuous Petri net and $\mathcal{D}$ the set of measured places of the system:*

- *Place p is* structurally observable *from $\mathcal{D}$ iff it is observable from $\mathcal{D}$ for any $\boldsymbol{\lambda} > 0$.*
- *$\mathcal{N}$ is* structurally observable *from $\mathcal{D}$ iff every place p is* structurally observable.

In other words, structural observability looks for observability for any $\boldsymbol{\lambda}$, like structural boundedness looks for boundedness for any $\mathbf{m_0}$. As an example, let us suppose that the only measured place of the system in Figure 3 is p_3 and that the vector $\boldsymbol{\lambda}$ is known. The variation, i.e., the derivative, of the marking of a place is given by the difference between its input and output flows. For p_3, we have $\dot{\mathbf{m}}[p_3] = \mathbf{f}_2 - \mathbf{f}_3$ where: $\mathbf{f}_2 = \boldsymbol{\lambda}[t_2] \cdot \mathbf{m}[p_2]$ and $\mathbf{f}_3 = \boldsymbol{\lambda}[t_3] \cdot \mathbf{m}[p_3]$, and so $\mathbf{m}[p_2] = (\dot{\mathbf{m}}[p_3] + \boldsymbol{\lambda}[t_3] \cdot \mathbf{m}[p_3])/\boldsymbol{\lambda}[t_2]$. Therefore, from the evolution of $\mathbf{m}[p_3]$, $\mathbf{m}[p_2]$ can be computed. Furthermore, it holds $\dot{\mathbf{m}}[p_2] = \mathbf{f}_1 - \mathbf{f}_2$ and $\mathbf{f}_1 = \boldsymbol{\lambda}[t_2] \cdot \mathbf{m}[p_1]$. Thus, being $\mathbf{m}[p_2]$ computable, $\mathbf{m}[p_1]$ can also be computed. This procedure can be carried out whatever the value of $\boldsymbol{\lambda}$ is, i.e. this net is structurally observable.

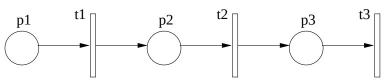

Figure 3. A JF net system whose marking can be computed from the observation of p_3.

This result can be easily generalized:

Proposition 5 *Let $\mathcal{N}$ be a continuous JF Petri net and $\mathcal{D}$ the set of measured places. Let p be a place such that a path from p to $\mathcal{D}$ exists in which all the places have only one input transition (i.e., it is attribution free). Then, p is structurally observable.*

Proof: In a JF system, the output flow of a place p is proportional to its marking, $\mathbf{f}_{out}[p] = \sum_{t \in p^\bullet} \mathbf{Pre}[p,t] \cdot \boldsymbol{\lambda}[t] \cdot \text{enab}(t, \mathbf{m}) = \sum_{t \in p^\bullet} \boldsymbol{\lambda}[t] \cdot \mathbf{m}[p]$. From $\dot{\mathbf{m}}[p] = \mathbf{f}_{in}[p] - \mathbf{f}_{out}[p]$, if $\mathbf{m}[p]$, and $\dot{\mathbf{m}}[p]$ are known, the total input flow of p can be obtained. If place p has only one input transition it is easy to obtain the marking of the input place of that transition, $\mathbf{f}_{in}[p]/\mathbf{Post}[p, {}^\bullet p] = \boldsymbol{\lambda}[{}^\bullet p] \cdot \mathbf{m}[{}^\bullet({}^\bullet p)]/\mathbf{Pre}[{}^\bullet({}^\bullet p), {}^\bullet p]$. ■

However, for the system in Figure 4, if p_3 is measured, it is not possible to observe p_1 or p_2 if $\boldsymbol{\lambda}[t_1] = \boldsymbol{\lambda}[t_2]$ (it can be seen that the observability matrix does not have full rank unless this condition holds). Intuitively, if the flow of both transitions comes with "the same speed" it cannot be decided how much comes from each source. Hence, this net is not structurally observable. However, it is observable when $\boldsymbol{\lambda}[t_1] \neq \boldsymbol{\lambda}[t_2]$.

On the contrary, from the knowledge of the marking of place p it is not possible to compute the marking of the place(s) $(p^\bullet)^\bullet$: Let $p' \in (p^\bullet)^\bullet$, and assume to simplify that $p^\bullet = {}^\bullet p' = t$. To compute the marking of p' it would be necessary to solve $\dot{\mathbf{m}}[p'] = \mathbf{f}_{out}[p] \cdot \mathbf{Post}[p', t]/\mathbf{Pre}[p, t] - \mathbf{f}_{out}[p']$. If the initial marking of p' is not known that equation cannot be solved whatever the value of $\mathbf{Pre}, \mathbf{Post}$ and $\boldsymbol{\lambda}$ are. This means that from the measured places at most the marking of the supplying places can be inferred.

Moreover, it can be proved that the set of places whose marking can be computed does not depend on the "output" of the measured places.

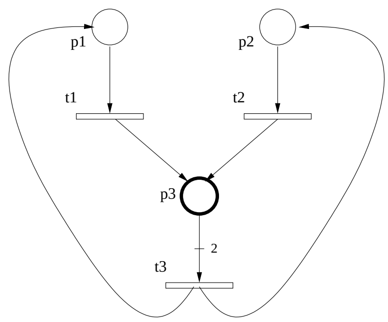

Figure 4. A JF net system whose marking cannot be computed from the observation of p_3 if $\boldsymbol{\lambda}[t_1] = \boldsymbol{\lambda}[t_2]$, because t_1 and t_2 make an attribution to p_3.

Proposition 6 *Let $\mathcal{N}$ be a continuous JF Petri net and $\mathcal{D}$ the set of measured places. The observable subspace (and so the set of structurally observable places) neither depends on the output arc weights of the measured places, $\mathbf{Pre}[p, T]$ $\forall p \in \mathcal{D}$, nor on the firing* *speeds of their output transitions,* $\boldsymbol{\lambda}[t]\ \forall\ t \in \mathcal{D}^\bullet$*, nor on the output arc weights of their output transitions,* $\mathbf{Post}[p, T]\ \forall\ p \in (\mathcal{D}^\bullet)^\bullet$.

Applying Proposition 6 to the system in Figure 5(a) with p_4 as the only measured place, the observable subspace of the system does not depend on the values of $r, s, t, q, \boldsymbol{\lambda}[t_4]$. Even after removing the input/output arcs of transition t_4 as in Figure 5(b) (this is equivalent to $\boldsymbol{\lambda}[t_4] = 0$), the obtained system is identical to the original one in terms of observability.

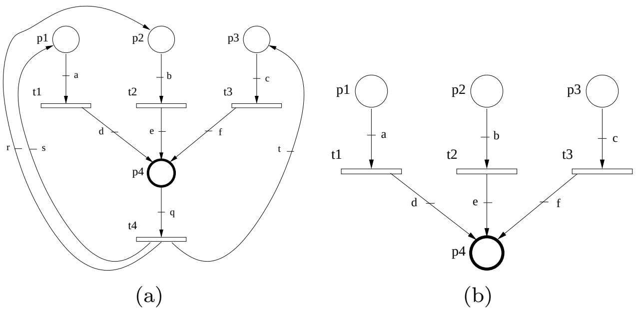

Figure 5. Two net systems with identical observable subspaces if the only measured place is p_4.

According to Proposition 6 the output transitions of the measured places can be removed (by setting their $\boldsymbol{\lambda}$ to zero) without affecting the observable subspace of the system. Therefore:

Corollary 7 *Let* $\mathcal{N}$ *be a continuous Petri net and* $\mathcal{D}$ *the set of measured places. If a place* p *is structurally observable then a forward path from* p *to* $\mathcal{D}$ *exists.*

As an example, let us consider that the only measured place in Figure 3 is place p_2. Using the derivative of the marking of p_2 the marking of p_1 can be computed. However, there exists no forward path going from p_3 to p_2 and therefore, according to Corollary 7, p_3 is not structurally observable. Intuitively, the marking of p_3 cannot be deduced from its input flow.

4.2 Computation Algorithm

A similar approach to the one taken to observe p_1 and p_2 in the system in Figure 3 can be used to observe p_1 and p_2 in the system in Figure 6, where the measured places are p_3, p_4 and p_5. Let us consider a matrix $\mathbf{Post}^u \in \mathbb{R}^{|P|\times|T|}$ containing only the output arc weights of the transitions whose flow is, "in principle", unknown, i.e., the marking of their input places is not known. More formally, for the iterative algorithm that will be proposed, $\mathbf{Post}^u[i,j] = 0$ if the marking of the place $^\bullet t_j$ is measured or has been computed in previous iterations, and $\mathbf{Post}^u[i,j] = \mathbf{Post}[i,j]$ otherwise.

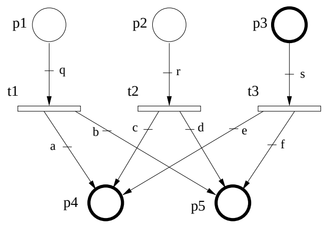

Figure 6. A JF system whose marking is computable from the evolution of p_3, p_4 and p_5.

Here, the first three rows (that correspond to places p_1, p_2 and p_3) of $\mathbf{Post}^u$ are zeros and the forth and fifth rows (that correspond to places p_4 and p_5) are $(a\ c\ 0)$ and $(b\ d\ 0)$, respectively. The marking evolution of places p_4 and p_5 is known (because they are measured) and here it is equal to their input flow. Subtracting the flow coming from p_3, $\mathbf{f}_{i4}^{p_3}$ and $\mathbf{f}_{i5}^{p_3}$, we will obtain the flow coming from the unknown places p_1 and p_2:

$$\begin{pmatrix} \dot{\mathbf{m}}[p_4] - \mathbf{f}_{i4}^{p_3} \\ \dot{\mathbf{m}}[p_5] - \mathbf{f}_{i5}^{p_3} \end{pmatrix} = \begin{pmatrix} a & c \\ b & d \end{pmatrix} \cdot \begin{pmatrix} \boldsymbol{\lambda}[t_1] \cdot \dfrac{\mathbf{m}[p_1]}{q} \\ \boldsymbol{\lambda}[t_2] \cdot \dfrac{\mathbf{m}[p_2]}{r} \end{pmatrix}$$

Hence, if the matrix $(a\ c; b\ d)$ has full rank it will be possible to compute the markings of p_1 and p_2 independently of the $\boldsymbol{\lambda}$ of the transitions.

The procedure developed for the above examples can be generalized leading to a fix point algorithm. The goal of the algorithm is, given a set of measured places, $\mathcal{D}$, deduce which places of the net system can be observed for whatever value of $\boldsymbol{\lambda}$. The basis of such iterative algorithm is to look for sets of places whose marking has been computed in previous iterations and such that the matrix composed of their input arcs weights has full rank. If such a set exists, then it is possible to compute the marking of the supplying places.

Given a set of places $\mathcal{H}$, $\mathbf{Post}_{\mathcal{H}}^u$ denotes a matrix composed by the rows of $\mathbf{Post}^u$ corresponding to the places in $\mathcal{H}$, and whose null columns have been removed. The input data of the algorithm are the net structure, $\mathcal{N}$, and the set of measured places, $\mathcal{D}$. The output of the algorithm is the set of places, $\mathcal{Q}$, that are observable for every $\boldsymbol{\lambda}$. At a given iteration, $\mathcal{Q}$ stores the set of places whose markings are known to be computed till that instant.

Algorithm 8

Input $(\mathcal{N}, \mathcal{D})$

Output $\mathcal{Q}$ % places that can be observed $\forall\, \boldsymbol{\lambda} > 0$
$\mathcal{Q} := \mathcal{D}$
Compute $\mathbf{Post}^u$
While $\exists\, \mathcal{H} \subseteq \mathcal{Q}$, such that $^\bullet(^\bullet\mathcal{H}) \not\subset \mathcal{Q}$ and $\mathbf{Post}^u_{\mathcal{H}}$ has full rank **do**
$\mathcal{Q} := \mathcal{Q} \cup {}^\bullet(^\bullet H)$
Compute $\mathbf{Post}^u$ according to $\mathcal{Q}$
End_While

The following statements establish sufficient conditions for structural observability:

Proposition 9 *Let $\mathcal{N}$ be a JF net, $\mathcal{D}$ the set of measured places and $\mathcal{Q}$ the output of the Algorithm 8 applied on $(\mathcal{N}, \mathcal{D})$:*

- *Every place $p \in \mathcal{Q}$ is structurally observable.*
- *If $\mathcal{Q} = P$ (the set of places of $\mathcal{N}$) then the net is structurally observable.*

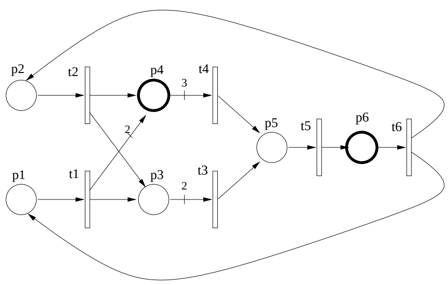

Figure 7. A JF net system that is structurally observable.

If the measured places of the system in Figure 7 are p_4 and p_6, the first iteration of Algorithm 8 on this system includes p_5 in the set of observable places. The second iteration includes p_3 and the third and last iteration includes p_1 and p_2. Therefore, it can be concluded that the whole net is structurally observable.

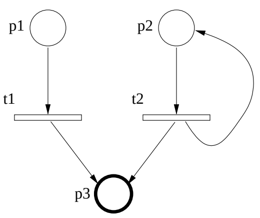

Figure 8. A JF system for which Algorithm 8 does not conclude that it is structurally observable.

Unfortunately, the conditions given in Proposition 9 are only sufficient for structural observability. The execution of Algorithm 8 on the system in Figure 8, whose only measured place is p_3, yields $\mathcal{Q} = p_3$, i.e., Proposition 9 cannot decide whether the system is structurally observable or not. Nevertheless, the observability matrix, ϑ, for that system is $\vartheta = (0\ 0\ 1; \boldsymbol{\lambda}[t_1]\ \boldsymbol{\lambda}[t_2]\ 0; -\boldsymbol{\lambda}[t_1]^2\ 0\ 0)$ which has full rank for every $\boldsymbol{\lambda} > 0$. Therefore the system is structurally observable.

Although a formal proof is missing, after testing a wide variety of examples, it seems reasonable to think that the condition on Proposition 9 is necessary and sufficient for those JF systems that are conservative.

5 Observability in General Net Systems

As mentioned in Section 2, continuous Petri net systems under infinite servers semantics can be seen as piecewise linear systems. The goal of this Section is the study of the conditions under which the initial PT-set as well as the initial marking can be unequivocally determined.

A way to face this problem consists of computing an estimate for every structural PT-set of the net. For simplicity, let the estimates be obtained by means of Equation 2 defined for $n-1$ consecutive periods. Theoretically and assuming that there is no noise, time discretization (δ) can be done as small as desired. It can be assumed that, for a small enough δ, no switch between PT-sets takes place in the first $n-1$ periods. The computed estimates can be used to filter those PT-sets that for sure are not ruling the evolution of the system. If only one PT-set remains, then the system evolves according to it.

The sorts of non suitable estimates that allow to filter a PT-set are:

- *Infeasible* estimates: No solution of Equation 2
- *Incoherent* estimates: The PT-set of the estimate is not the one for which it was computed
- *Suspicious* estimates: The estimate belongs simultaneously to several PT-sets.

5.1 Infeasible and Suspicious Estimates

Let us show through an example how infeasible and suspicious estimates can be used to filter PT-sets.

Consider a system composed of a single synchronization with two input places p_1 and p_2 (see Figure 9). The net has two structural PT-sets, either $W_1 = \{(p_1, t_1)\}$ or $W_2 = \{(p_2, t_1)\}$. If the time period is one time unit, the evolution of the system according to PT-set W_1 is

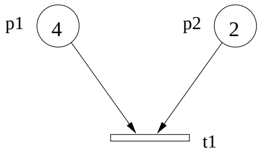

Figure 9. A simple synchronization with two input places.

ruled by the matrix $\mathbf{F}_1 = (e^{-1} \;\; 0; e^{-1} - 1 \;\; 1)$. The system matrix for PT-set W_2 is $\mathbf{F}_2 = (1 \;\; e^{-1} - 1; 0 \;\; e^{-1})$. Considering that the initial marking is $\mathbf{m_0} = (4; 2)$, the initial PT-set for the system is W_2, and after one time unit the marking will be $\mathbf{m}(\tau = 1) = (2 \cdot e^{-1} + 2; 2 \cdot e^{-1})$.

As external agents of the system we will consider 4 cases (see Figure 10): The output of the system, i.e., measured place, can be either p_1 or p_2; the PT-set that is assumed to be ruling the net system can be either W_1 or W_2.

Estimate PT−set \ System output	p1 S=(1 0)	p2 S=(0 1)
W2	case 1 Right estimation of m1 and m2	case 3 Right estimation of m2
W1	case 2 Infeasible estimation	case 4 Suspicious estimation (m1 = m2)

Figure 10. The four possible cases for an estimate for the system in Figure 9.

In the first two cases $\mathbf{m}[p_1]$ is the output of the system ($\mathbf{y} = (4; 2 \cdot e^{-1} + 2)$):

Case 1: W_2 is assumed to be the PT-set. For this case $\vartheta = (1 \;\; 0; 1 \;\; e^{-1} - 1)$ whose rank is 2. Using Equation 2 the initial marking $\mathbf{m_0} = (4; 2)$ is obtained.

Case 2: W_1 is assumed to be the PT-set. The observability matrix is $\vartheta = (1 \;\; 0; e^{-1} \;\; 0)$. Equation 2 has no solution.

In this way, by means of an "infeasible" estimate (case 2), it has been detected that the PT-set of the system is W_2.

If p_2 is measured, ($\mathbf{y} = (2; 2 \cdot e^{-1})$), the two cases are:

Case 3: W_2 is assumed to be the PT-set. $\vartheta = (0 \;\; 1; 0 \;\; e^{-1})$, which is not a full rank matrix. The observable subspace is in this case $\mathbf{m}[p_2]$, i.e., only the marking of p_2 is known. The application of Equation 2 yields $\mathbf{m_0}[p_2] = 2$.

Case 4: W_1 is assumed to be the PT-set. The observability matrix is $\vartheta = (0 \;\; 1; e^{-1} - 1 \;\; 1)$ which has full rank. The solution for Equation 2 is $\mathbf{m_0} = (2 \;\; 2)$, different from the real initial marking of the system.

The at first glance surprising result yielded in Case 4 will be obtained for any initial marking of p_1 greater than or equal to 2. The reason for this phenomenon is that the output of the system, $\mathbf{m}[p_2]$, is evolving according to the flow of the transition t_1 that depends only on $\mathbf{m}[p_2]$. The estimator "thinks" that the flow of the transition is ruled by p_1 (W_1 is assumed), so the only way in which p_2 can evolve according to the output $\mathbf{y}$ is assigning the same initial marking to both places.

An initial marking $\mathbf{m_0} = (2 \;\; 2)$ would mean that at the beginning the system is in both PT-sets, W_1 and W_2. If we assume that the initial marking was really (2 2) both PT-sets (case 3 and case 4) would correctly estimate here the marking of p_2. The PT-set W_1 can also estimate the marking of p_1, but it is not possible to know whether this estimate is correct since the same estimate would have been obtained for any $\mathbf{m_0}[p_1] \geq 2$. Therefore, it seems safer to stick to the PT-set W_2, even if it might mean losing some information. Those estimates that belong to several PT-sets will be considered "suspicious estimates". Notice that the only non desirable effect that may happen after filtering suspicious estimates is that some information (in case 4 the estimate of p_1) is lost if the system was really in several PT-sets. However, for sure the estimate that is not filtered (case 3) is correct.

5.2 Incoherent Estimates

Another rule to filter a PT-set is that the estimates should be *coherent* with the PT-set for which they are computed. In other words, it does not make sense to consider an estimate that assigns a greater marking to p_1 than to p_2, if the PT-set for which it is computed happens when $\mathbf{m}[p_1] \leq \mathbf{m}[p_2]$.

By means of Equation 2, it is possible to compute the set of estimates for a given PT-set. If the matrix ϑ has full rank, then only one estimate is possible for the PT-set. Otherwise, a set of possible estimates appears. In order to avoid those estimates that are not coherent with the PT-set, a set of inequalities may be added to Equation 2.

As an example, suppose that we are interested in computing an estimate for the system in Figure 1. That net has two structural PT-sets: we will say that the system is in PT-set W_1 if $\mathbf{m}[p_1] \leq \mathbf{m}[p_2]$ and the system is in PT-set W_2 if $\mathbf{m}[p_1] \geq \mathbf{m}[p_2]$. In the case that $\mathbf{m}[p_1] = \mathbf{m}[p_2]$, the system is considered to be in both PT-sets simultaneously. Two estimates will be computed for this system, one per PT-set. The estimate corresponding to PT-set W_1, $\hat{\mathbf{m}}_0^{(1)}$, (W_2, $\hat{\mathbf{m}}_0^{(2)}$) has to be solution of Equation 2 with ϑ computed for the linear system associated to the PT-set and $\hat{\mathbf{m}}_0^{(1)}$ ($\hat{\mathbf{m}}_0^{(2)}$) has to fulfill $\hat{\mathbf{m}}_0^{(1)}[p_1] < \hat{\mathbf{m}}_0^{(1)}[p_2]$ ($\hat{\mathbf{m}}_0^{(2)}[p_1] > \hat{\mathbf{m}}_0^{(2)}[p_2]$). The use of strict inequalities allows to filter also suspicious estimates like the one shown in Subsection 5.1. If there were no solution with strict inequalities, equalities would have to be added taking care of the suspicious cases.

5.3 Deciding on Observability

Let us observe the output of the system in Figure 1 during three time periods in order to have enough output information to use Equation 2. Let us assume that no change of PT-set has taken place during these three time periods. After the observation, two equation systems, E_1 and E_2, can be defined to compute an estimate for the initial marking. The system E_1 (resp. E_2) contains Equation 2 with ϑ_1 (resp. ϑ_2) and the set of inequalities that defines the PT-set W_1, i.e., $\hat{\mathbf{m}}_0^{(1)}[p_1] < \hat{\mathbf{m}}_0^{(1)}[p_2]$ (resp. W_2, $\hat{\mathbf{m}}_0^{(2)}[p_1] > \hat{\mathbf{m}}_0^{(2)}[p_2]$). If only one of those equation systems has solution, that equation system corresponds to the initial PT-set of the net system. If both equation systems, E_1 and E_2 have solution, it is not possible to decide the initial PT-set of the system. The case in which none of the equation systems has solution happens when the initial marking of the system is in both PT-sets at the same time, i.e., $\mathbf{m}_0[p_1] = \mathbf{m}_0[p_2]$. In this case, the inequalities in E_1 and E_2 must be substituted by $\hat{\mathbf{m}}_0^{(1)}[p_1] = \hat{\mathbf{m}}_0^{(1)}[p_2]$. Any solution of E_1 or E_2 is a suitable estimate for the initial marking.

For a general Petri net system with k structural PT-sets, a set of equation systems, $E_1 \ldots E_k$, can be defined. Each E_i contains Equation 2 with ϑ_i and the set of inequalities that defines the PT-set.

Proposition 10 *Let $\langle \mathcal{N}, \boldsymbol{\lambda}, \mathbf{m_0} \rangle$ be a continuous system, whose initial marking, $\mathbf{m_0}$, is unknown but belongs to only one PT-set. Let $\mathbf{S}$ be the output matrix of the system and E_i the set of equations associated to the i-th PT-set.*

Then the PT-set of $\mathbf{m_0}$ can be determined before a switch to another PT-set happens iff only one system E_i, $1 \leq i \leq k$ has solution.

Proof:
($\Rightarrow$) Let us assume that more than one system, E_i and E_j, have solution. This would imply that at least two estimates for the initial marking exist, one for E_i and one for E_j. Taking those estimates as initial marking, the constraints for the PT-sets that E_i and E_j represent are verified, and the system output evolves according to the dynamics of E_i and E_j. This means, that both estimates are feasible, and so the initial PT-set of the system cannot be determined.
($\Leftarrow$) If only one system E_i has solution, it means that from the initial marking the output of the system can only evolve according to the equations in E_i. ■

In relation to general piecewise linear systems, determining the PT-set of a continuous Petri net is equivalent to determining the linear system that at a certain moment rules the evolution of the piecewise linear system. However, the condition that establishes whether it is possible to determine the linear system is much harder in general piecewise linear systems: in contrast to the condition in Proposition 10, it is required that the *joint observability matrix* of every couple of linear systems has full rank [15, 16]. This difference is due to the fact that in Petri nets the PT-set depends only on the marking (and the net structure).

For a system $\langle \mathcal{N}, \boldsymbol{\lambda}, \mathbf{m_0} \rangle$, the verification of the condition in Proposition 10 implies that the initial PT-set can be determined. However, it does not imply that the initial marking can be obtained. This happens when there exists only one system E_i that has solution, but the solution is not unique, i.e., there exists a non observable subspace. Assume that $\langle \mathcal{N}, \boldsymbol{\lambda}, \mathbf{m_0} \rangle$ switches to a PT-set that corresponds to an observable linear system. By using Equation 2 during the evolution of the system in this new PT-set, the complete marking can be computed. Once the complete marking of the system is known, the system can be "simulated" backwards, see Section 3. Since timing is deterministic, a backwards simulation till the initial time yields the initial marking, i.e., the system is observable.

Proposition 11 *Let $\langle \mathcal{N}, \boldsymbol{\lambda}, \mathbf{m_0} \rangle$ be a continuous system and $\mathbf{S}$ the output matrix. If the evolution of the marking of $\langle \mathcal{N}, \boldsymbol{\lambda}, \mathbf{m_0} \rangle$ passes through an observable PT-set then the system is observable.*

In other words, Proposition 11 implies that having a period in the evolution of the system during which the PT-set allows to compute the complete current marking is enough to determine the initial marking.

6 Conclusions

The performance model of continuous Petri nets working under infinite servers semantics has been considered. Structural observability has been introduced and studied for continuous Petri net systems without synchronizations. The main advantage of considering structural observability is that it depends only on $\mathcal{N}$, thus it can be parametrized by the internal speeds of transitions, $\boldsymbol{\lambda}$. That is, a structurally observable system can be observed for any $\boldsymbol{\lambda}$. A fix point algorithm to compute the set of places that are structurally observable has been presented. It is based on the net graph of the system and some elementary algebraic properties.

A general (with synchronizations) timed continuous Petri net can be analyzed as a specific kind of piecewise linear system in which switches are triggered by internal events. By using some concepts of linear systems theory, a marking estimate can be computed for each structural PT-set. This may lead to a large number of estimates. However, several cases have been shown in which the estimate for a given PT-set cannot be a suitable marking estimate: the estimate is either *infeasible* or *incoherent* or *suspicious*. Such non suitable estimates must be "filtered". The PT-set ruling the evolution of the system can be identified iff only one estimate is not filtered. Given that a (deterministic) continuous Petri net system can be simulated backwards, it is enough that the system passes through a observable horizon in order to be able to estimate the initial marking. Therefore, the conditions for observability in continuous Petri nets that have been obtained are much less restrictive than those for general piecewise linear systems [15].

Based on the results here introduced, the design of suitable observers for continuous Petri nets can be faced. In [5] a multi-linear observer is proposed. The switches among the linear observers running in parallel are triggered in such a way that the residual is minimized.

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