An integrated methodology to solve optimization problems with Petri nets as disjunctive constraints.

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Methodology for decision taking based on alternatives aggregation Petri nets

Motivation

1. Research the application to the design problem of a DES of classical performance optimization techniques (which have been used extensively for the operation of a DES).

2. Develop a decision support methodology for the design of a DES with optimal operational behaviour as a complement to structural techniques.

3. Improve the efficiency of classical performance optimization techniques based on Petri net models.

Methodology for decision taking based on alternatives aggregation Petri nets

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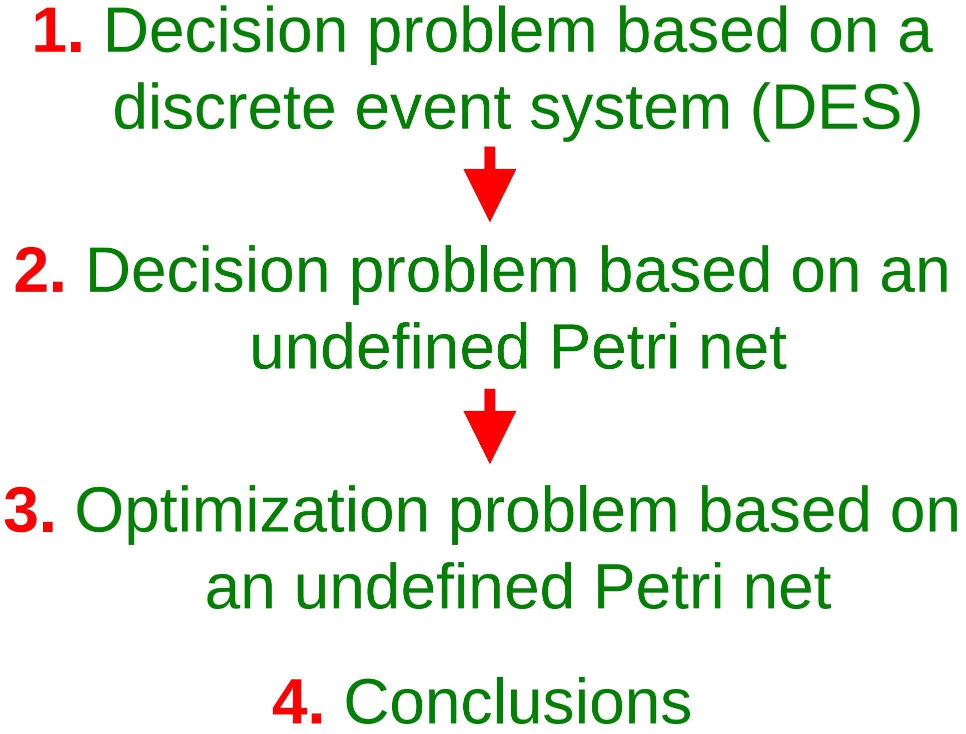

1. Decision problem based on a discrete event system (DES)

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2. Decision problem based on an undefined Petri net

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3. Optimization problem based on an undefined Petri net

4. Conclusions

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1. Decision problem based on a discrete event system

Outline

Undefined discrete event system

Undefined characteristics that allow to take a decision

- Of design (~structure)
- Of operation (~behaviour)

Decision problem

Question on a set of undefined characteristics.

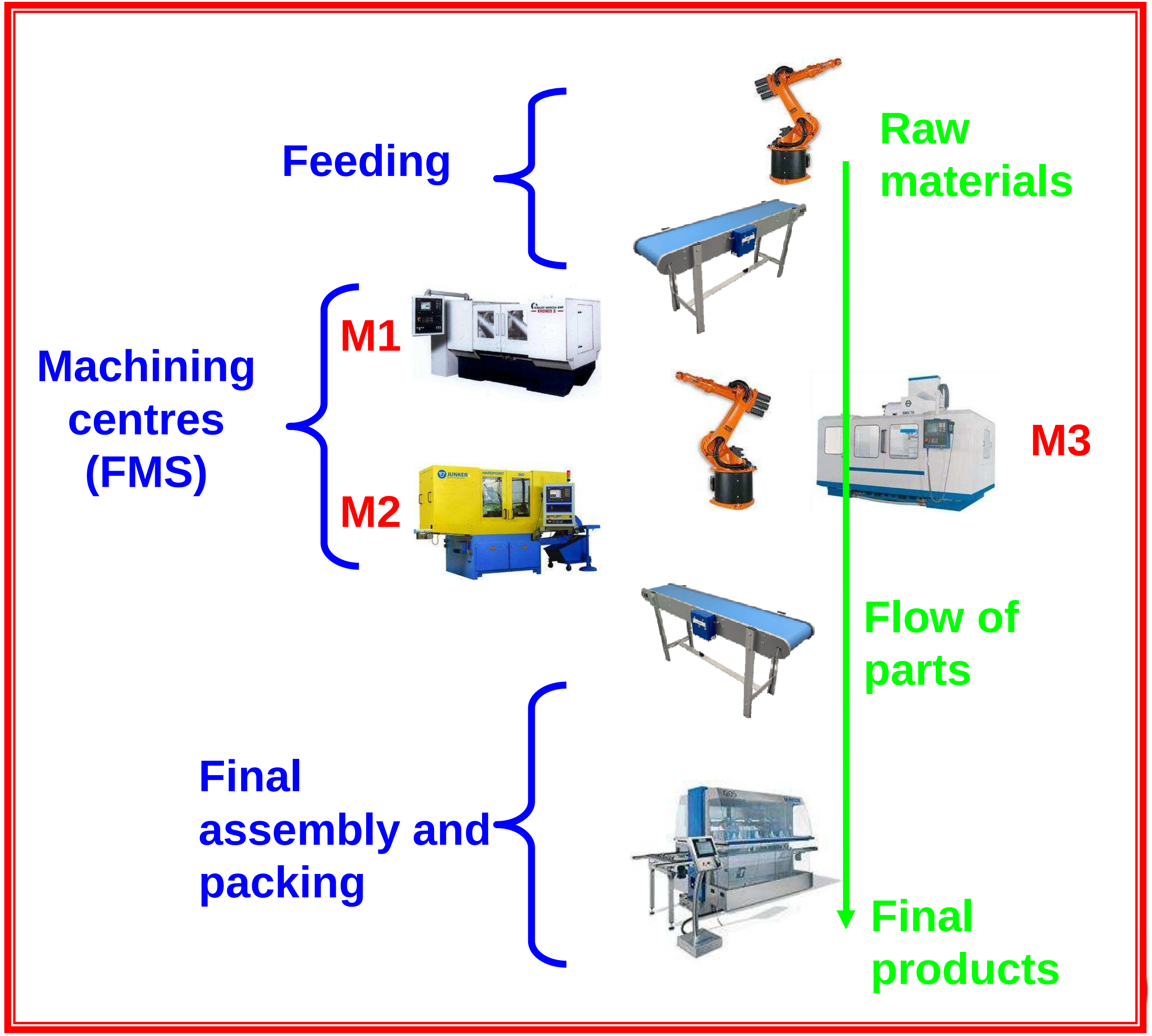
1. Decision problem based on a discrete event system

Example 1

Decision problem

Design of an industrial facility

Two alternative suppliers



Feeding

Raw materials

Machining centres (FMS)

M1

M2

M3

Flow of parts

Final assembly and packing

Final products

1. Decision problem based on a discrete event system

Example 1	Undefined characteristics

Of design

Feeding: Lenght of the conveyor.
Feeding: Number of robots.
FMS: Presence and capacity of the input buffer.
FMS: Distributions of the machining centres.
FMS: Number of robots.
FMS: Size of the output buffer.
System of assembly and packing (two alternative options).
Assembly and packing: Number of robots.

Of operation

Feeding: Number of pallets of raw materials in the wharehouse.
Feeding: Capacity of the pallets.
Manufacturing sequence of the products.

1. Decision problem based on a discrete event system

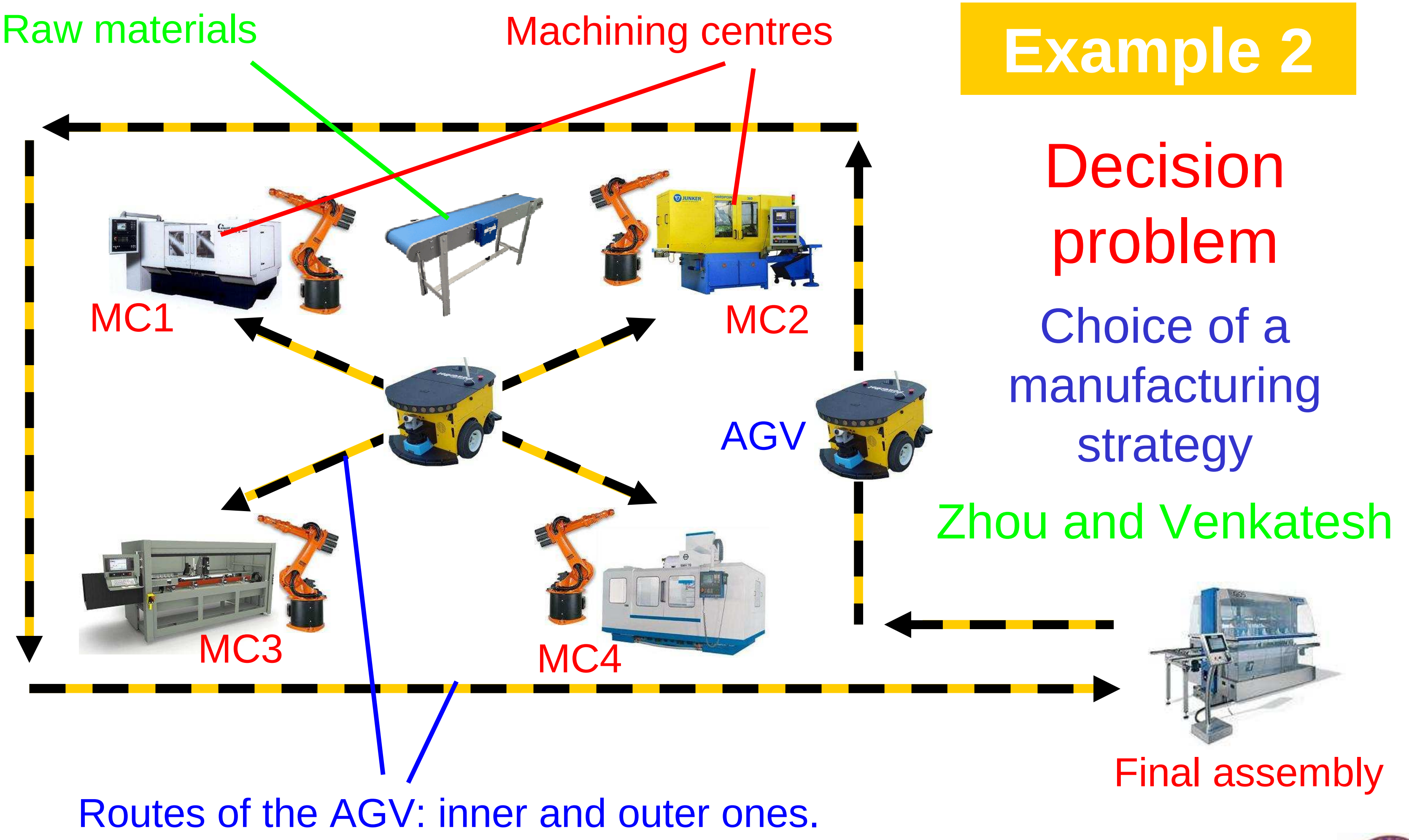
Example 1	Decision problem

Specify the undefined characteristics

to obtain the maximal financial performance from the facility.

1. Decision problem based on a discrete event system

Example 2



Routes of the AGV: inner and outer ones.

Decision problem

Choice of a manufacturing strategy

Zhou and Venkatesh

1. Decision problem based on a discrete event system

Undefined characteristics **Example 2**

Of design

***Transport:* Number of AGV to acquire and set up.**

Of operation

***Manufacturing:* Manufacturing strategy (push, pull, hybrid).**
***Manufacturing:* Size of the manufacturing lot.**
***Transport:* Size of the transport lot.**
***Transport:* Number of routes for the AGV.**

1. Decision problem based on a discrete event system

Decision problem	Example 2

Specify the manufacturing strategy

to obtain the maximal financial performance from the facility.

2. Decision problem based on an undefined Petri net

Formalization

Algorithmic solution of the problem

It is necessary to formalize the data of the problem:

- Expression in an unambiguous format.
- Transformation of the system in a formal model.
- Formalism of the Petri nets (for DES).

Possibility to use the model to perform operations of:

- Simulation and optimization.

2. Decision problem based on an undefined Petri net

Paradigm of the Petri nets

Matrix-based definition of Petri net (unmarked):

4-tuple N = <*P*, *T*, **Pre**=W^-, **Post**=W^+>.

P is the set of places (*state variables*).

T is the set of transitions (*transform the values of the state variables*).

C = **Post** - **Pre** are the incidence matrices.(*arc weights between nodes*).

Definition of Petri net system (marked):

Couple < N, m_0>.

m_0 is the initial marking (*initial state vector*).

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2. Decision problem based on an undefined Petri net

Formalism of the Petri nets

Evolution:

Evolution rule (firing of transitions).

Characteristic or fundamental equation (spurious solutions).

$\boldsymbol{m} = \boldsymbol{m_0} + \mathbf{C} \cdot \sigma$, where

m is the final marking (state vector of the system)

σ is the firing count (or Parikh) vector.

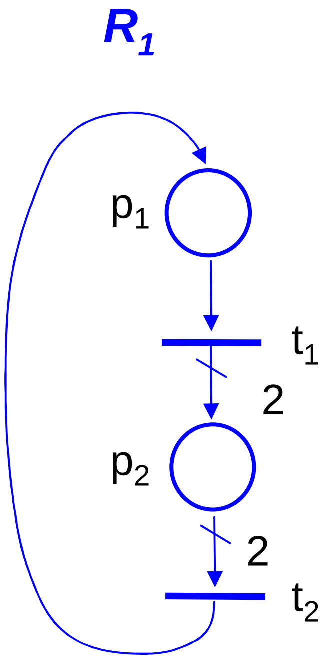
State space: set of reachable markings.

2. Decision problem based on an undefined Petri net

Formalism of the Petri nets

Double representation

Graphical



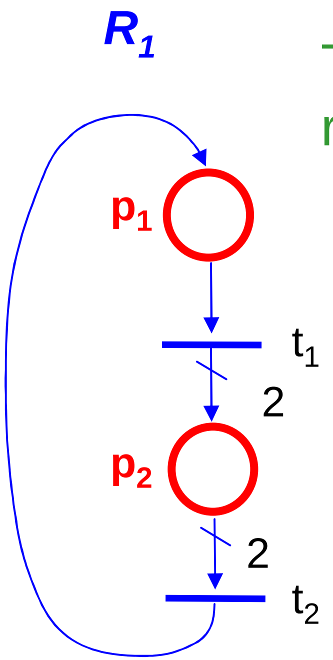
Matrix

$$W(R_1) = \begin{array}{cc} t_1 & t_2 & \\ \left(\begin{array}{cc} -1 & 1 \\ 2 & -2 \end{array}\right) & & \begin{array}{c} p_1 \\ p_2 \end{array} \end{array}$$

2. Decision problem based on an undefined Petri net

Formalism of the Petri nets

Places (*state variables*)



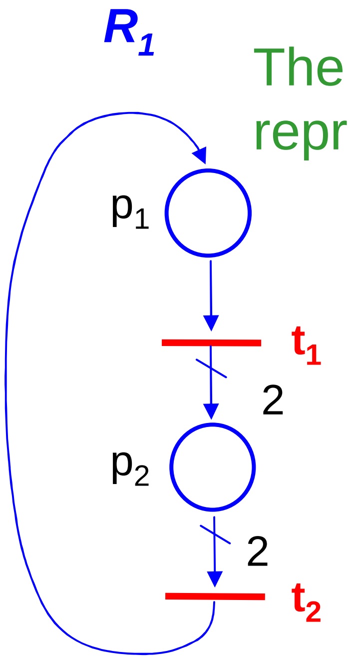
The ith-row of the incidence matrix (*pure PN*) represents the arcs to/from the ith-place.

$$W(R_1) = \begin{array}{ccc} t_1 & t_2 & \\ -1 & 1 & p_1 \\ 2 & -2 & p_2 \end{array}$$

2. Decision problem based on an undefined Petri net

Formalism of the Petri nets

Transitions (*transformers of state variables*)



The ith-column of the incidence matrix (*pure PN*) represents the arcs to / from the ith-transition.

$$\begin{array}{cc} & \begin{array}{cc} t_1 & t_2 \end{array} \\ W(R_1) = & \left(\begin{array}{cc} \boxed{\begin{matrix}-1\\2\end{matrix}} & \boxed{\begin{matrix}1\\-2\end{matrix}} \end{array}\right) \begin{array}{c} p_1 \\ p_2 \end{array} \end{array}$$

2. Decision problem based on an undefined Petri net

Outline

Undefined Petri net

Paradigm for modelling DES with parallel and concurrent evolution.

Undefined parameters that model undefined characteristics of the discrete event system.

1. Structural

2. Marking

3. Of transition firing (priorities in conflicts)

4. Interpretation

2. Decision problem based on an undefined Petri net

Outline

Definition of undefined parameter of a Petri net:

β is said to be a parameter of a PN $\boldsymbol{R}$ if it represents a numerical value in the PN model.

α_k is said to be an undefined parameter of a PN $\boldsymbol{R}$ iff

$\exists\ S_{val\alpha k}(\boldsymbol{R})=\{\ v_i \mid 1 \leq i \leq q\}$, known, where $\mathrm{card}(S_{val\alpha k}(\boldsymbol{R}))=q>1$

The assignment $\alpha_\mathrm{k} = v_j$ is made as a result of a decision.

2. Decision problem based on an undefined Petri net

Outline

Definition of an undefined Petri net system:

7-tuple R^c = <*P*, *T*, **Pre**, **Post**, m_0, S_α, $S_{val\alpha}$> that verifies:

- card(S_α)>0 (There exists at least one undefined parameter in R^c).
- $\forall\ \alpha_i \in S_\alpha$, card($S_{val\alpha i}$)>1 (It is possible to take a decision on the value of every undefined parameter), which implies that card($S_{val\alpha i}$)>1 .

Where: S_α is the set of undefined parameters in R^c.

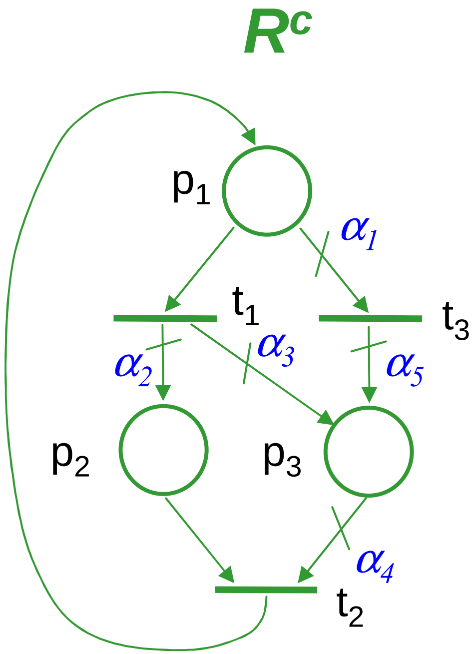
$S_{val\alpha}$ is the set of feasible combinations of values for the undefined parameters of R^c.

$S_{val\alpha i}$ is the set of feasible values for the undefined parameter $\alpha_i \in S_\alpha$.

2. Decision problem based on an undefined Petri net

Undefined Petri net

Parameters of an undefined Petri net



Undefined structural parameters

$$S_\alpha(\boldsymbol{R^c}) = S_{str\alpha}(\boldsymbol{R^c}) = (\ \alpha_1\ ,\ \alpha_2\ ,\ \alpha_3\ ,\ \alpha_4\ ,\ \alpha_5\ )$$

Feasible combination of values for the undefined structural parameters

$$S_{valstr\alpha}(\boldsymbol{R^c}) = \{\ (0,2,0,0,0)\ ,\ (0,1,1,1,0)\ ,\ (1,2,0,1,2)\ \}$$

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2. Decision problem based on an undefined Petri net

Undefined Petri net

Representations of an undefined Petri net

Objectives:

- To find different equivalent representations of the undefined Petri net.
- To use those different representations to solve a decision problem in an efficient way, profiting from the characteristics of the representations.
- To understand the relation between different representations of the undefined PN.

2. Decision problem based on an undefined Petri net

Properties of an undefined Petri net

Definition of partition

Given a set $S_{valstr\alpha}(R^c)$ with the feasible combinations of values for the undefined structural parameters of R^c.

It can be defined a partition of $S_{valstr\alpha}(R^c)$ as the set $\{ S_{valstr\alpha}(R_1)\ ,\ S_{valstr\alpha}(R_2)\ , \ldots ,\ S_{valstr\alpha}(R_{nr})\ \}$ such that:

i. $S_{valstr\alpha}(R^c) = S_{valstr\alpha}(R_1) \cup S_{valstr\alpha}(R_2) \cup \ldots \cup S_{valstr\alpha}(R_{nr})$

ii. $S_{valstr\alpha}(R_i) \neq \emptyset\ ,\ 1 \leq i \leq n_r$

iii. $S_{valstr\alpha}(R_i) \cap S_{valstr\alpha}(R_j) = \emptyset\ ,\ 1 \leq i,j \leq n_r$

2. Decision problem based on an undefined Petri net

Properties of an undefined Petri net

Example of partitions

The set of 3 elements $S_{valstr\alpha}(R^c)$ = {*a*, *b*, *c*} can generate 5 different partitions:

{*a*}, {*b*}, {*c*}

{*a*}, {*b*, *c*}

{*b*}, {*a*, *c*}

{*c*}, {*a*, *b*}

{*a*, *b*, *c*}

2. Decision problem based on an undefined Petri net

Properties of an undefined Petri net

Alternative Petri net systems

Given:

- An undefined Petri net system $\boldsymbol{R}^c$ = <P, T, Pre, Post, m_0, S_α, $S_{val\alpha}$>, where card($S_{str\alpha}$)>0 (hence card($S_{valstr\alpha}$)>1).
- A partition of $S_{valstr\alpha}(R^c)$, $\{ S_{valstr\alpha}(R_1)$, $S_{valstr\alpha}(R_2)$,…, $S_{valstr\alpha}(R_{nr})\}$

It is possible to define a set of alternative PN $S_R = \{ R_1, \ldots, R_{nr}\}$, where

R_i is given by <P, T, Pre, Post, m_0, $S_\alpha(R_i)$, $S_{val\alpha}(R_i)$>

$$S_{str\alpha}(R_i) \subseteq S_\alpha(R_i) \subseteq S_\alpha$$

$$S_{valstr\alpha}(R_i) \subseteq S_{val\alpha}(R_i) \subset S_{val\alpha}$$

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2. Decision problem based on an undefined Petri net

Properties of an undefined Petri net

Alternative Petri nets

They are generated from a partition of the set of feasible values for the undefined structural parameters of the undefined PN.

They are exclusive models or sets of models for a DES modelled by an undefined Petri net.

They show a pairwise exclusive evolution.

They have structural differences (incidence matrices).

2. Decision problem based on an undefined Petri net

Properties of an undefined Petri net

Types of alternative Petri nets

Simple:

- They lack any undefined structural parameter: card($S_{valstr\alpha}(R_i)$)=1.
- They represent a single model for the DES

Compound:

- They contain undefined structural parameters: card($S_{valstr\alpha}(R_j)$)>1
- Set of exclusive models for a DES

2. Decision problem based on an undefined Petri net

Properties of an undefined Petri net

Equivalent representations

(Equivalence = same behaviour / graph of reachable markings)

- Compound Petri net: $S_{valstr\alpha}(R^c)$ = {*a*, *b*, *c*}
- Set of simple alternative Petri nets: {*a*}, {*b*}, {*c*}
- Set of compound alternative Petri nets.
- Mixed set of compound alternative and simple alternative Petri nets: {*a*}, {*b*, *c*} or {*b*}, {*a*, *c*} or {*b*}, {*a*, *c*}.

2. Decision problem based on an undefined Petri net

Example 3 **Compound Petri net**

Partition {*a*, *b*, *c*}

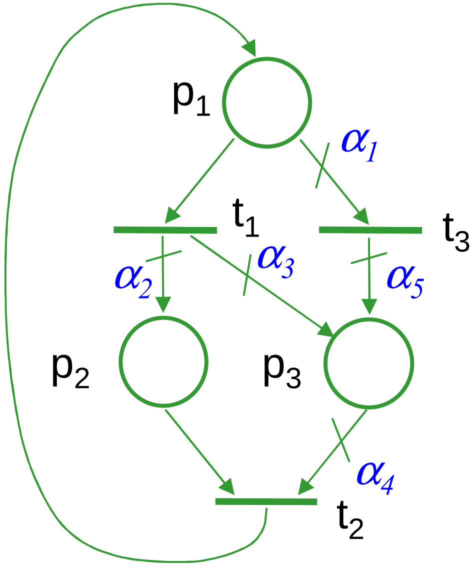
Feasible combinations of values for the undefined structural parameters (α_1 , α_2 , α_3 , α_4 , α_5)

$$S_{valstr\alpha}(\boldsymbol{R}^c) = \{ (0,2,0,0,0) , (0,1,1,1,0) , (1,2,0,1,2) \}$$

2. Decision problem based on an undefined Petri net

Example 3	**Compound Petri net**

R^c



Undefined structural parameters

$$(\alpha_1 , \alpha_2 , \alpha_3 , \alpha_4 , \alpha_5)$$

Feasible combinations of values for the undefined structural parameters

$$S_{valstr\alpha}(\boldsymbol{R^c}) = \{ (0,2,0,0,0) , (0,1,1,1,0) , (1,2,0,1,2) \}$$

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2. Decision problem based on an undefined Petri net

Example 3 **Simple alternative Petri nets**

Partition {*a*}, {*b*}, {*c*}

Feasible combination of values for the undefined structural parameters (α_1 , α_2 , α_3 , α_4 , α_5)

$$S^{o}_{valstr\alpha}(\boldsymbol{R_1}) = \{\ (0,2,0,0,0)\ \}$$

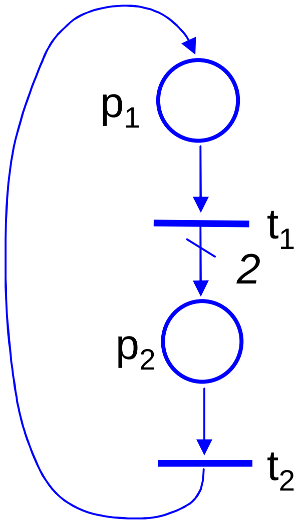
$$S^{o}_{valstr\alpha}(\boldsymbol{R_2}) = \{\ (0,1,1,1,0)\ \}$$

$$S^{o}_{valstr\alpha}(\boldsymbol{R_3}) = \{\ (1,2,0,1,2)\ \}$$

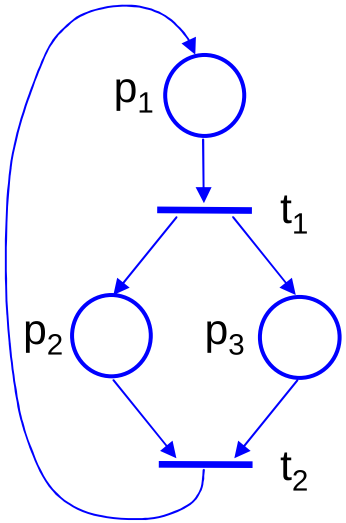
2. Decision problem based on an undefined Petri net

Example 3	Simple alternative Petri nets

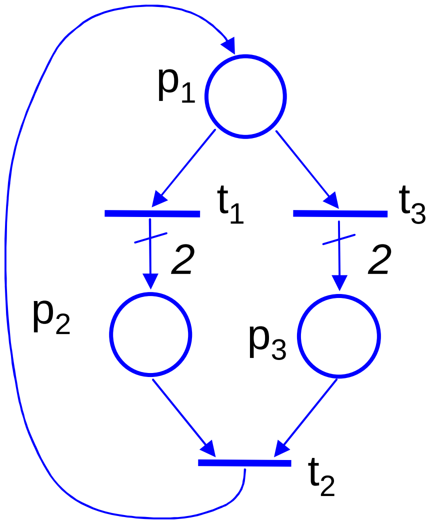
R_1



R_2



R_3



$S^o_{valstr\alpha}(R_1) = \{ (0,2,0,0,0) \}$ $S^o_{valstr\alpha}(R_2) = \{ (0,1,1,1,0) \}$ $S^o_{valstr\alpha}(R_3) = \{ (1,2,0,1,2) \}$

2. Decision problem based on an undefined Petri net

Properties of an undefined Petri net

Simple alternative Petri nets

Usual input in a decision problem based on a DES:

- Different alternative systems for the design of the DES.
- Those alternative systems usually can be modelled by means of PN models with differences in their incidence matrices.
- They can be represented by:
  - A set of alternative PN.
  - A compound PN with undefined structural parameters.
  - A mixed set of simple and compound alternative PN.

2. Decision problem based on an undefined Petri net

Example 3 | **Mixed set of simple alternative PN**

Partition $\{a, b\}, \{c\}$

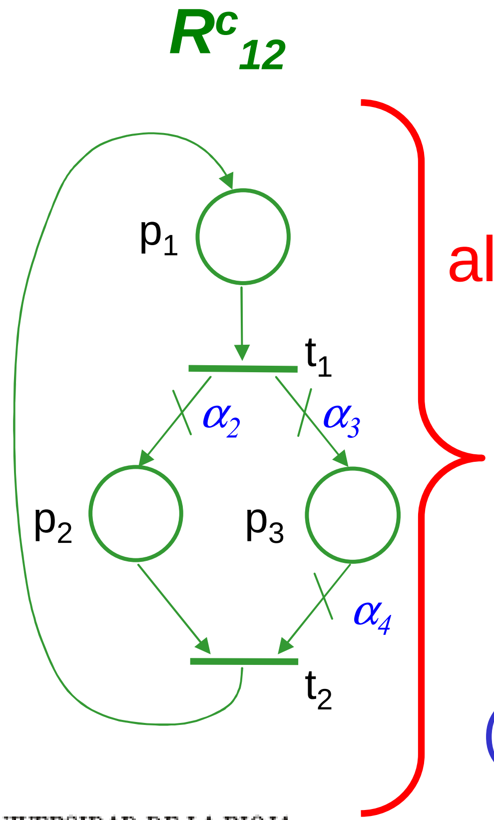
Feasible combination of values for the undefined structural parameters (α_1 , α_2 , α_3 , α_4 , α_5)

$$S^{o}_{valstr\alpha}(\boldsymbol{R}^{\boldsymbol{c}}_{\boldsymbol{12}}) = \{ (0,2,0,0,0), (0,1,1,1,0) \}$$

$$S^{o}_{valstr\alpha}(\boldsymbol{R}_{\boldsymbol{3}}) = \{ (1,2,0,1,2) \}$$

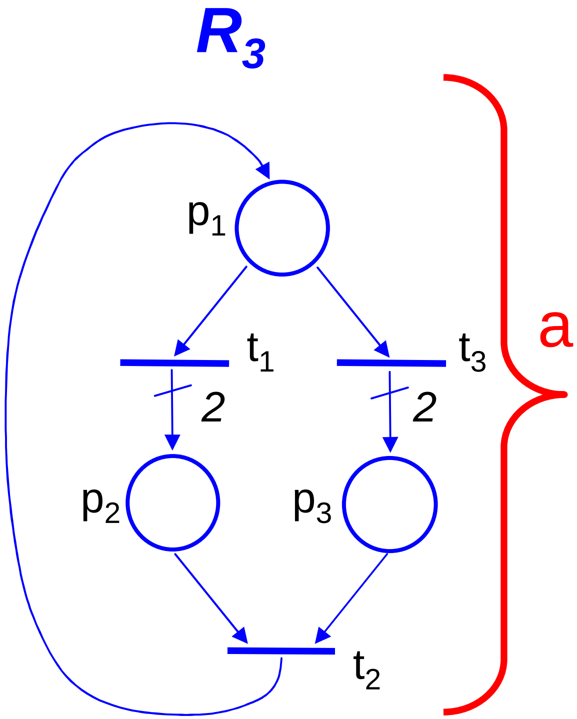
2. Decision problem based on an undefined Petri net

Example 3 **Mixed set of simple alternative PN**



Compound alternative PN

Undefined structural parameters (α_2 , α_3, α_4)



Simple alternative PN

2. Decision problem based on an undefined Petri net

Example 3 **2nd. Set of alternative Petri nets**

Partition {*a*, *c*}, {*b*}

Feasible combinations of values for the undefined structural parameters (α_1 , α_2 , α_3 , α_4 , α_5)

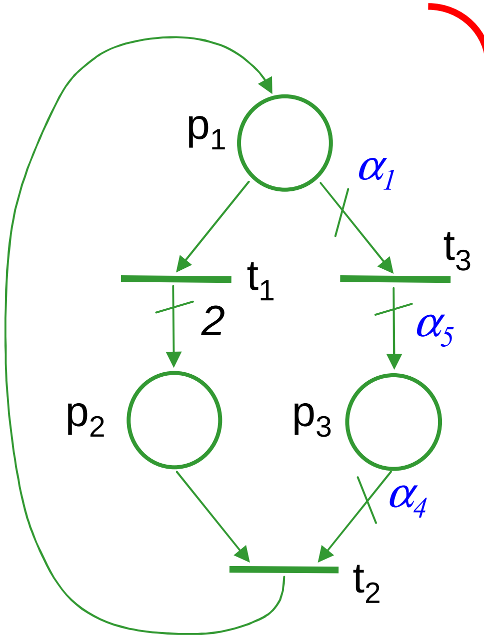
$$S^{o}_{valstr\alpha}(\boldsymbol{R}^{c}_{13}) = \{ (0,2,0,0,0), (1,2,0,1,2) \}$$

$$S^{o}_{valstr\alpha}(\boldsymbol{R}_{2}) = \{ (0,1,1,1,0) \}$$

2. Decision problem based on an undefined Petri net

Example 3 | **2nd. Set of alternative Petri nets**

R^{c}_{13}

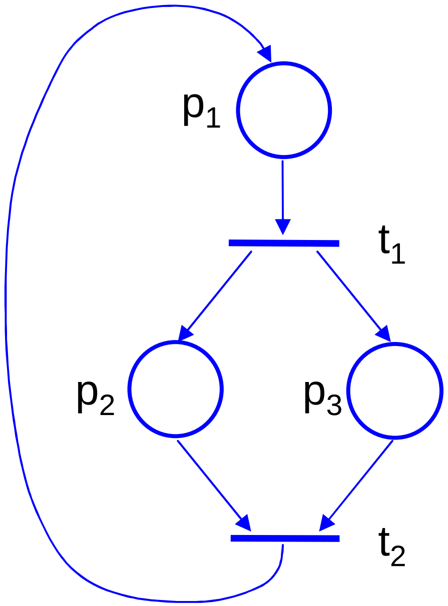


Compound alternative PN

Undefined structural parameters

(α_1 , α_4, α_5)

R_2



Simple alternative PN

2. Decision problem based on an undefined Petri net

Example 3 **3rd. Set of alternative Petri nets**

Partition {*a*}, {*b*, *c*}

Feasible combination of values for the undefined structural parameters (α_1 , α_2 , α_3 , α_4 , α_5)

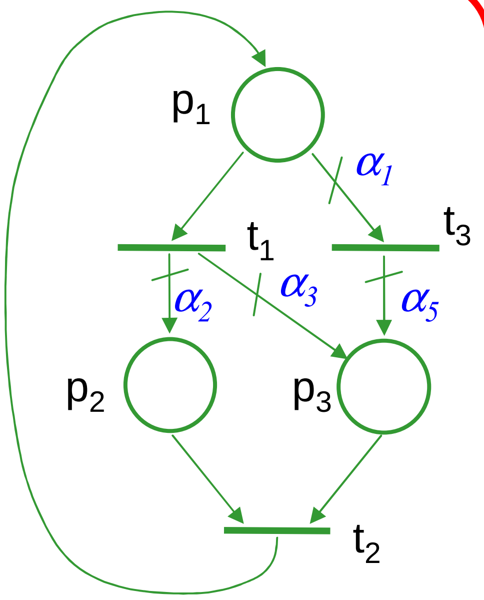
$$S^{o}_{valstr\alpha}(\boldsymbol{R_1}) = \{ (0,2,0,0,0) \}$$

$$S^{o}_{valstr\alpha}(\boldsymbol{R^{c}}_{23}) = \{ (0,1,1,1,0), (1,2,0,1,2) \}$$

2. Decision problem based on an undefined Petri net

Example 3 | **3rd. Set of alternative Petri nets**

R^c_{23}

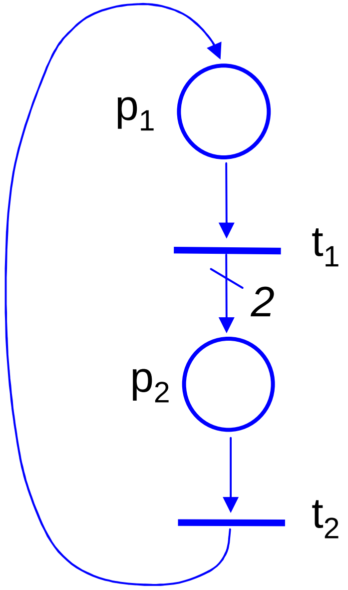


Compound alternative PN

Undefined structural parameters

(α_1 , α_2, α_3 , α_5)

R_1



Simple alternative PN

2. Decision problem based on an undefined Petri net

Properties of an undefined Petri net

Equivalence classes

The set $S_{valstr\alpha}(R_i)$, with the feasible combinations of values for the undefined structural parameters of R_i, define an equivalence class.

Equivalence relation: R_i and R_j are equivalent if the behaviour is the same (the graph of reachable states are the same or isomorphous).

2. Decision problem based on an undefined Petri net

Properties of an undefined Petri net

Transformations that preserve the equivalence

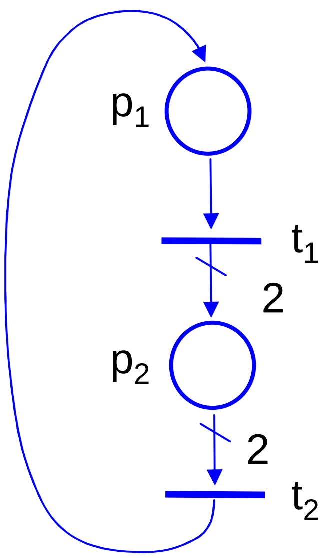
Operations that allow to transform R_i in R_j , both belonging to the same equivalence class:

1. Addition / removal of a row (*column*) of zeros in the incidence matrix of R_i. A row (*column*) of zeros corresponds to a non-connected place (*transition*).
2. To swap the position of two rows (*column*) in the incidence matrix.

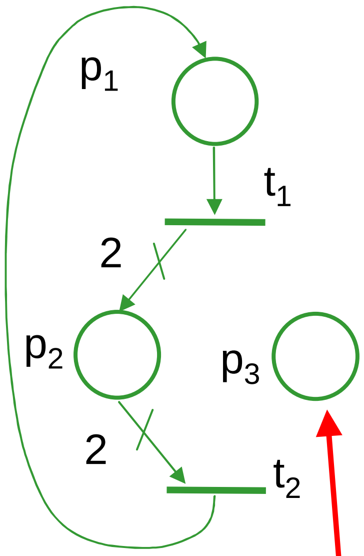
2. Decision problem based on an undefined Petri net

Addition / removal of a row of zeros

R_1



R'_1



$$W(R_1) = \begin{array}{ccc} \mathbf{t}_1 & \mathbf{t}_2 & \\ -1 & 1 & \mathbf{p}_1 \\ 2 & -2 & \mathbf{p}_2 \end{array}$$

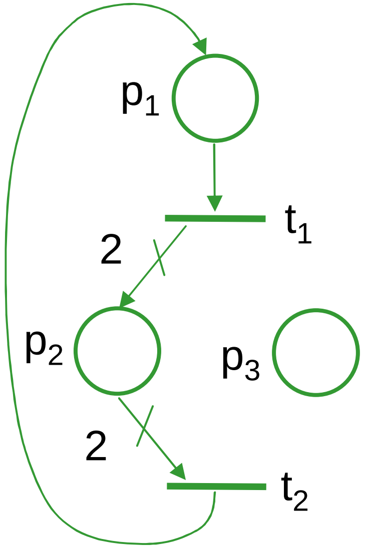
$$W(R'_1) = \begin{array}{ccc} \mathbf{t}_1 & \mathbf{t}_2 & \\ -1 & 1 & \mathbf{p}_1 \\ 2 & -2 & \mathbf{p}_2 \\ 0 & 0 & \mathbf{p}_3 \end{array}$$

Non-connected place and row of zeros

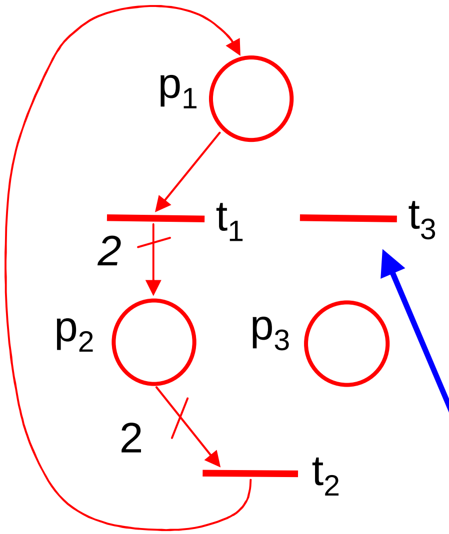
2. Decision problem based on an undefined Petri net

Addition / removal of a column of zeros

R'₁



R''₁



$$W(R'_1) = \begin{array}{c} \\ \\ \\ \end{array}\begin{array}{cc} t_1 & t_2 \\ \end{array}$$

$$W(R'_1) = \begin{array}{c|cc|c} & t_1 & t_2 & \\ & -1 & 1 & p_1 \\ & 2 & -2 & p_2 \\ & 0 & 0 & p_3 \end{array}$$

$$W(R''_1) = \begin{array}{c|ccc|c} & t_1 & t_2 & t_3 & \\ & -1 & 1 & \boxed{0} & p_1 \\ & 2 & -1 & \boxed{0} & p_2 \\ & 0 & 0 & \boxed{0} & p_3 \end{array}$$

Non-connected transition and column of zeros

2. Decision problem based on an undefined Petri net

Properties of an undefined Petri net

Significative elements of an equivalent class

Canonical alternative Petri net.

- It lacks non-connected places (*transitions*).
- It is not unique (possible rows/columns swapping).

Set of matching alternative Petri nets.

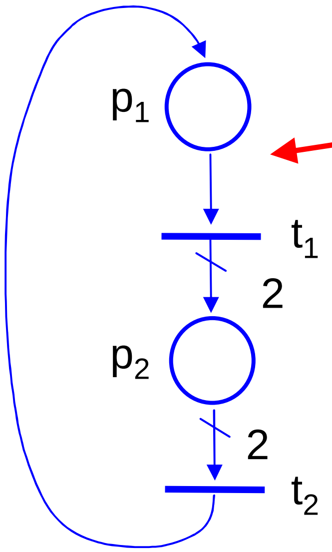
- Set with a representative of every equivalence class.
- They have the same set of parameters.

2. Decision problem based on an undefined Petri net

Properties of an undefined Petri net

Example of canonical Petri net

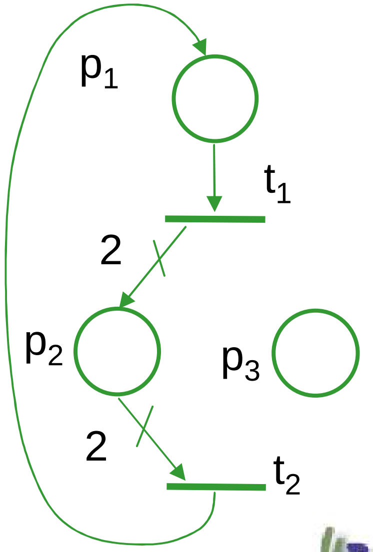
R_1



Canonical PN

Non-canonical PN

R'_1



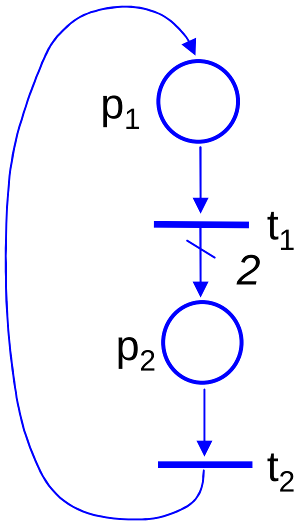
Non-connected place

2. Decision problem based on an undefined Petri net

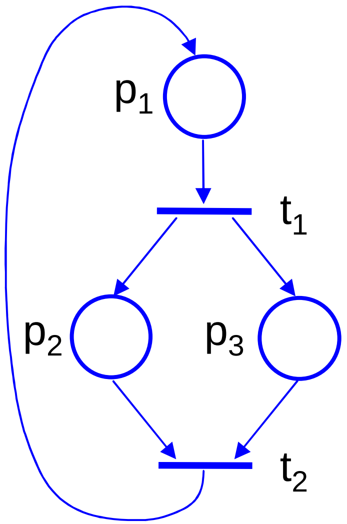
Properties of an undefined Petri net

Example of **non**-matching alternative Petri nets

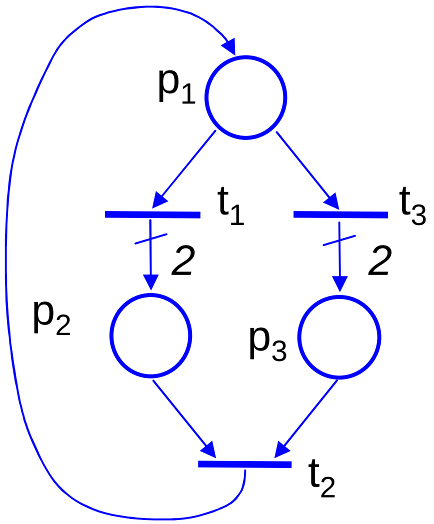
R_1



R_2



R_3



2. Decision problem based on an undefined Petri net

Properties of an undefined Petri net

Matching alternative Petri nets

A set of matching alternative Petri nets can be transformed in a compound PN by means of a single step (incidence matrices of same sizes).

A set of non-matching alternative PN (for example canonical ones) can be transformed into a set of matching PN:

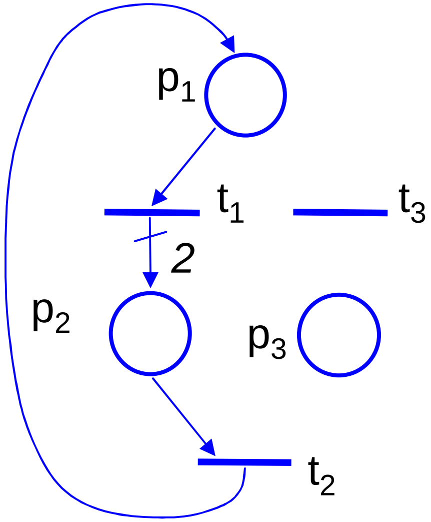
- Adding or removing rows or columns to make equal the size of the incidence matrices.
- Swapping the order of rows and columns according to a certain criterion (for example to minimize the number of undefined parameters finding blocks in the incidence matrices that fit).

2. Decision problem based on an undefined Petri net

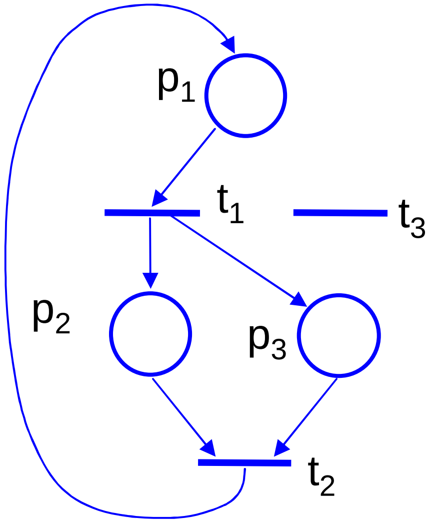
Properties of an undefined Petri net

Example of matching alternative Petri nets

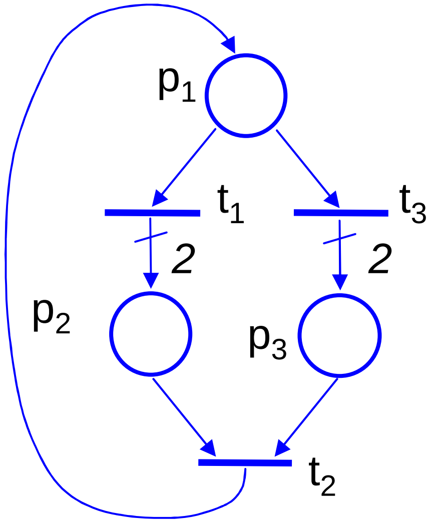
R_1



R_2



R_3



2. Decision problem based on an undefined Petri net

Properties of an undefined Petri net

Merging of matching alternative PN

The ovelapping/merging of PN is the inverse operation to the partition of a compound PN.

It may be applied to simple alternative PN and/or to compound alternative PN.

Requires that the PN to be merged have the same set of parameters (no matter if they are defined or undefined).

The Petri nets to be merged should be matching Petri nets (they have the same subsets of parameters of every different type).

2. Decision problem based on an undefined Petri net

Properties of an undefined Petri net

Number of equivalence classes

It is named order.

Maximal order ***n*** corresponds to: $S_R = \{R^s_1, \ldots, R^s_n\}$

Set of simple alternative Petri nets.

Minimal order ***1*** corresponds to: compound PN R^c.

Intermediate orders correspond to:

Mixed sets of compound alternative and simple alternative PN.

The order is proportional to the refinement (fragmentation) of the partition.

2. Decision problem based on an undefined Petri net

Properties of an undefined Petri net

Example. Number of equivalence classes

The set of three elements {*a*, *b*, *c*} can lead to 5 different partitions with the following orders:

{*a*}, {*b*}, {*c*} presents order ***3*** (three simple alternative PN)

{*a*}, {*b*, *c*} order ***2*** (one simple and one compound alternative PN)

{*b*}, {*a*, *c*} order ***2*** (one simple and one compound alternative PN)

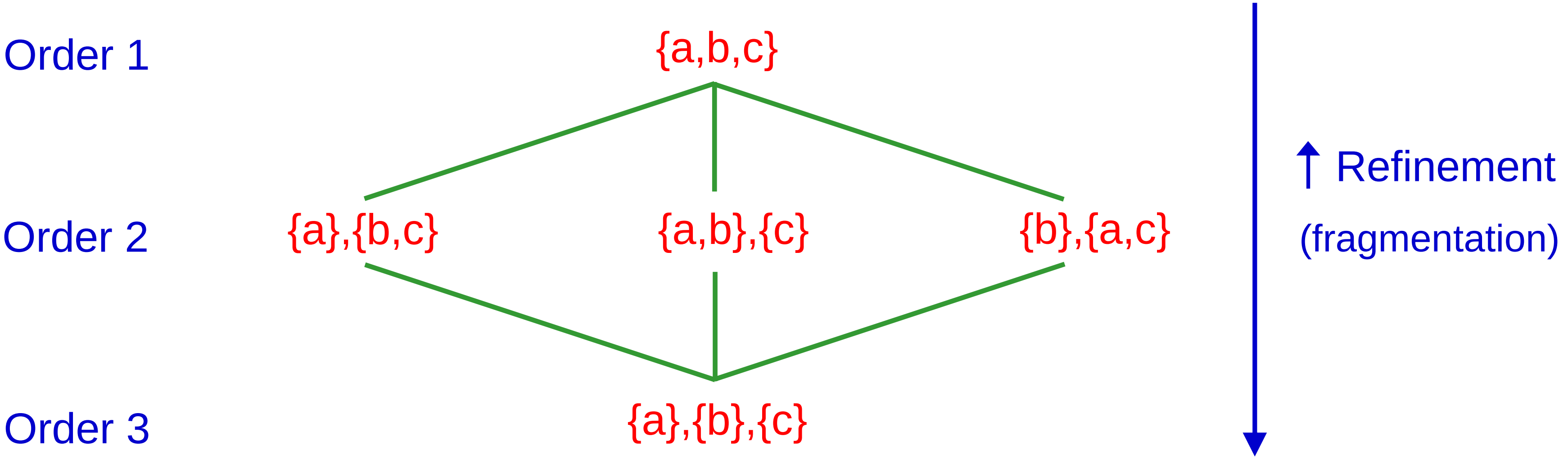
{*c*}, {*a*, *b*} order ***2*** (one simple and one compound alternative PN)

{*a*, *b*, *c*} order ***1*** (one compound PN)

2. Decision problem based on an undefined Petri net

Properties of an undefined Petri net

Hasse diagram


Order 1

Order 2

Order 3

{a,b,c}

{a},{b,c}

{a,b},{c}

{b},{a,c}

{a},{b},{c}

↑ Refinement

(fragmentation)

2. Decision problem based on an undefined Petri net

Properties of an undefined Petri net

Number of possible decompositions of R^c in equivalence classes.

Number of Bell:

$$B_{n+1} = \sum_{k=0}^{n} \binom{n}{k} B_k.$$

Meaning of the number of Bell:

Number of possible equivalent representations of an undefined Petri net, with ***n+1*** = number of feasible combinations of values for the undefined structural parameters.

2. Decision problem based on an undefined Petri net

Properties of an undefined Petri net

Number of Bell

card($S_{valstr\alpha}(R^c)$)		Number of possible partitions.
1	**(simple PN)**	**1**
2	**(compound PN)**	**2**
3	**(compound PN)**	**5**
4	**…**	**15**
5		**52**
10		**115975**
20		**5.1 x 10^{13}**
50		**1.86 x 10^{47}**

2. Decision problem based on an undefined Petri net

Properties of an undefined Petri net

Equivalent representations

- Compound Petri net.
- Set of simple alternative Petri nets:
  - Canonical, matching or others.
- Sets of compound alternative Petri nets.
- Mixed set of compound alternative and simple alternative PN.

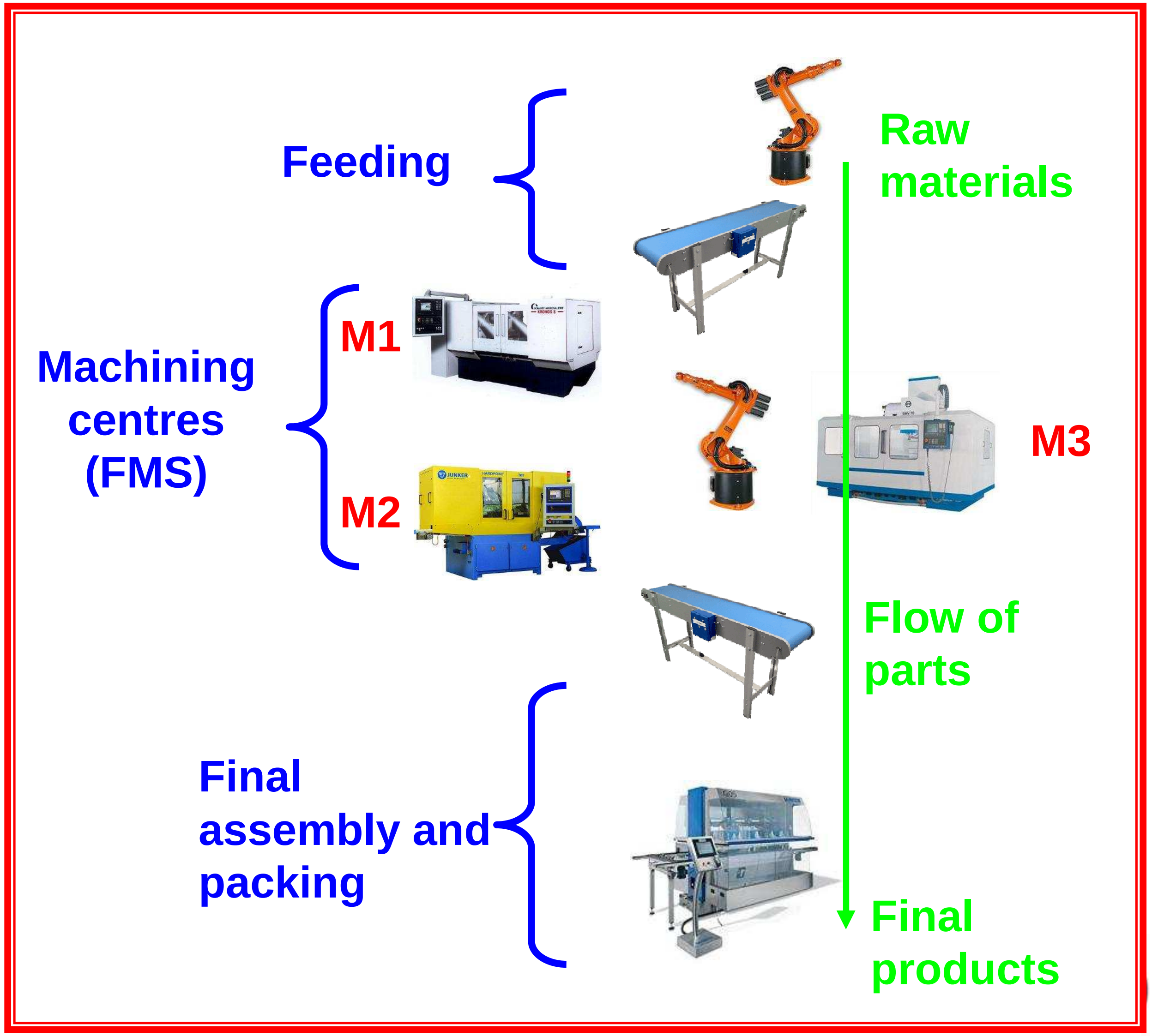
2. Decision problem based on an undefined Petri net

Example 1

Decision problem

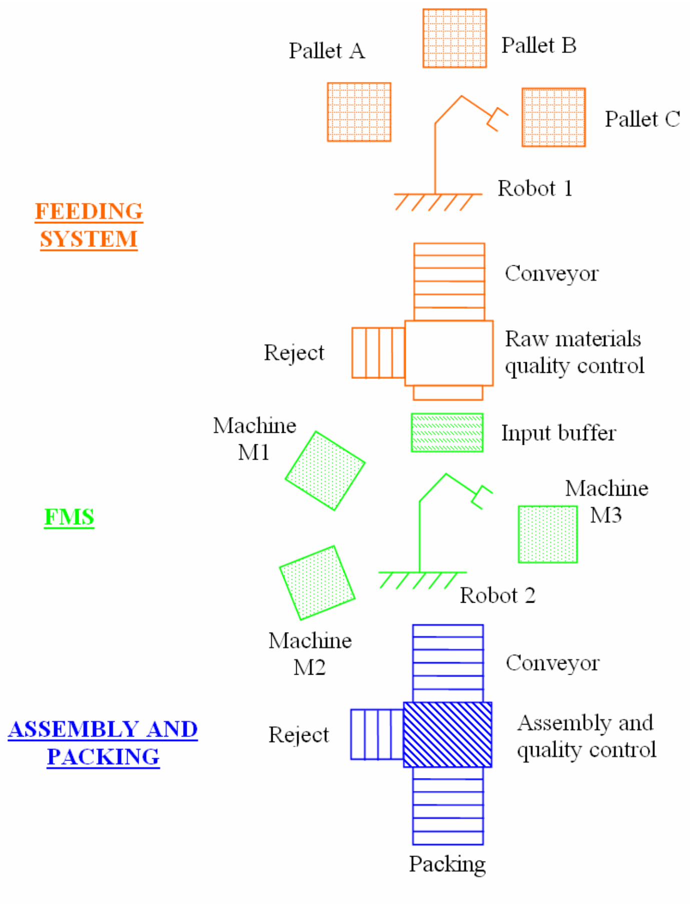
Design of an industrial facility

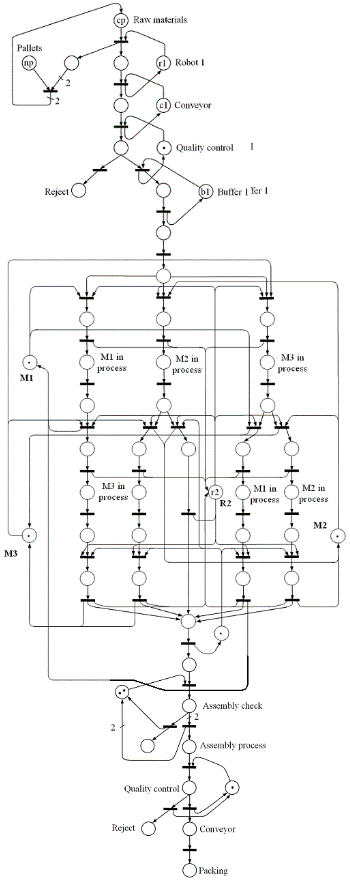
Two alternative suppliers



2. Decision problem based on an undefined Petri net

Example 1 **Simple alternative Petri net R_1**





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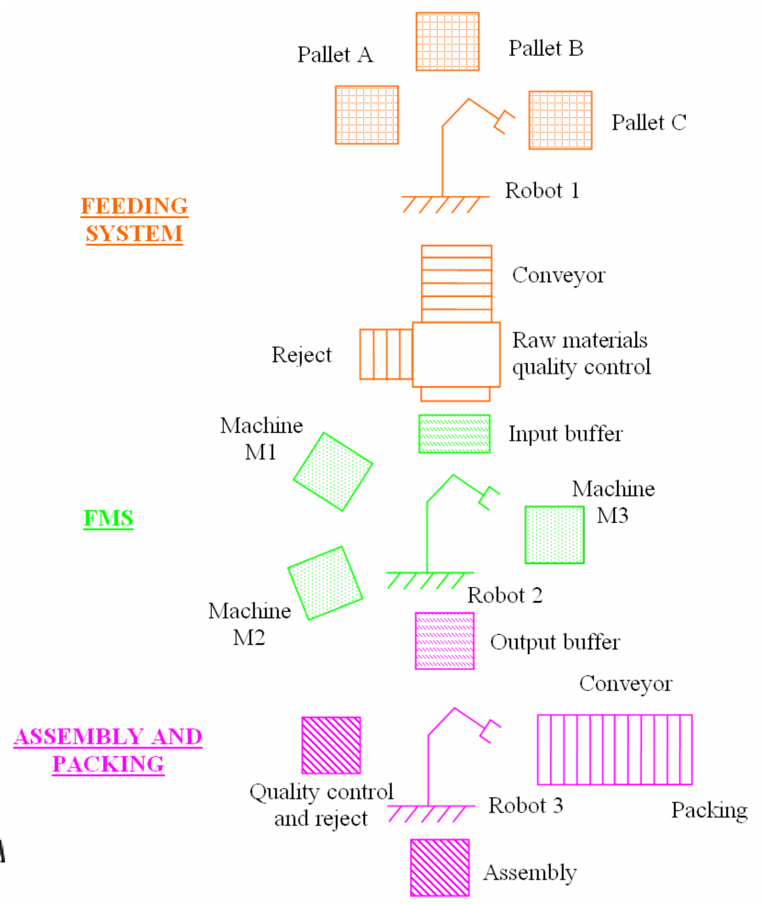
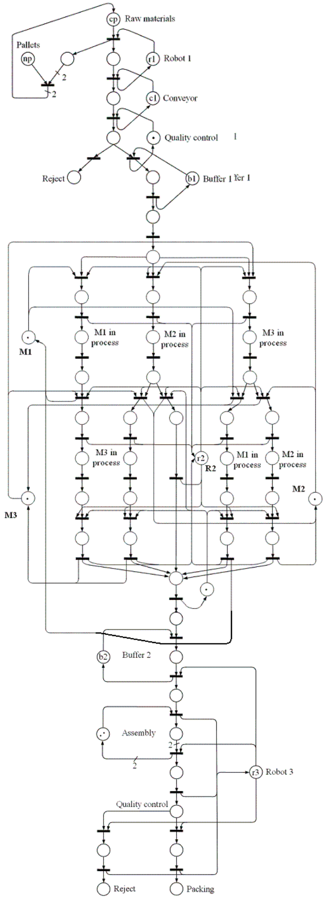
2. Decision problem based on an undefined Petri net

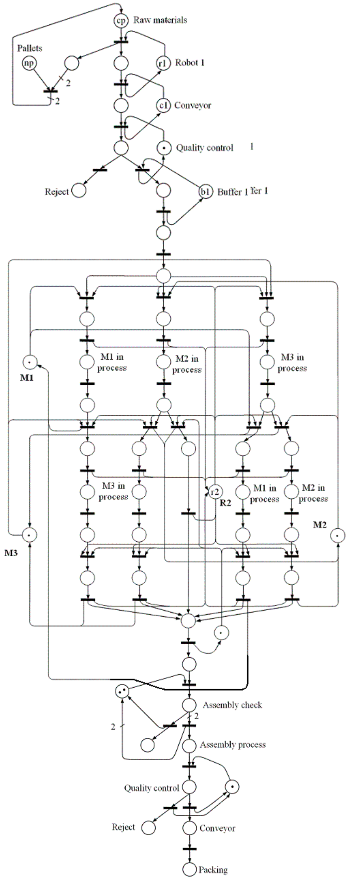
Example 1 | Simple alternative Petri net R_2

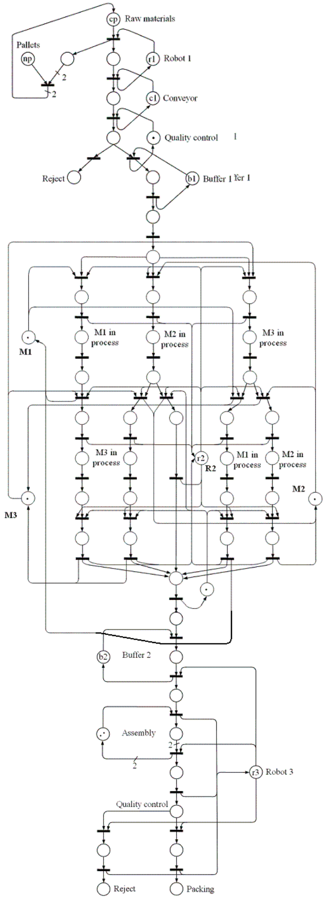




2. Decision problem based on an undefined Petri net

Example 1 **Set of simple alternative Petri nets**





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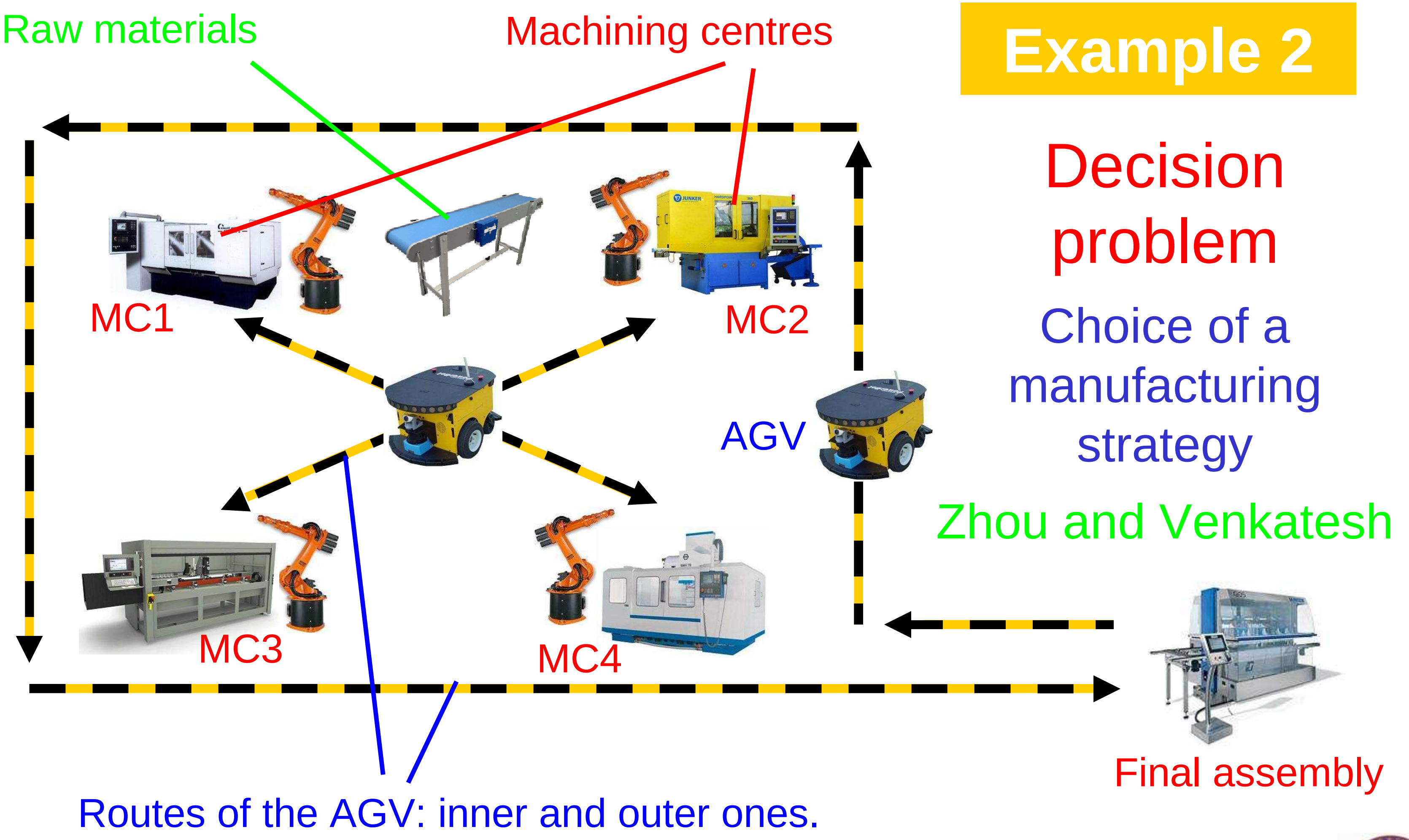
2. Decision problem based on an undefined Petri net



Example 2

Decision problem

Choice of a manufacturing strategy

Zhou and Venkatesh

2. Decision problem based on an undefined Petri net

Example 2 Characteristics and parameters

Design characteristics:

Number of AGV in the system.

- Modelled as the number of tokens in the places that represent the AGV.
- Undefined marking parameters belonging to the set {1,…,5}.

Operation characteristics:

Manufacturing strategy.

- Modelled by means of the weight of arcs that represent work orders.
- Undefined structural parameteres belonging to the set {0,…,12}.

Manufacturing and transport lot size.

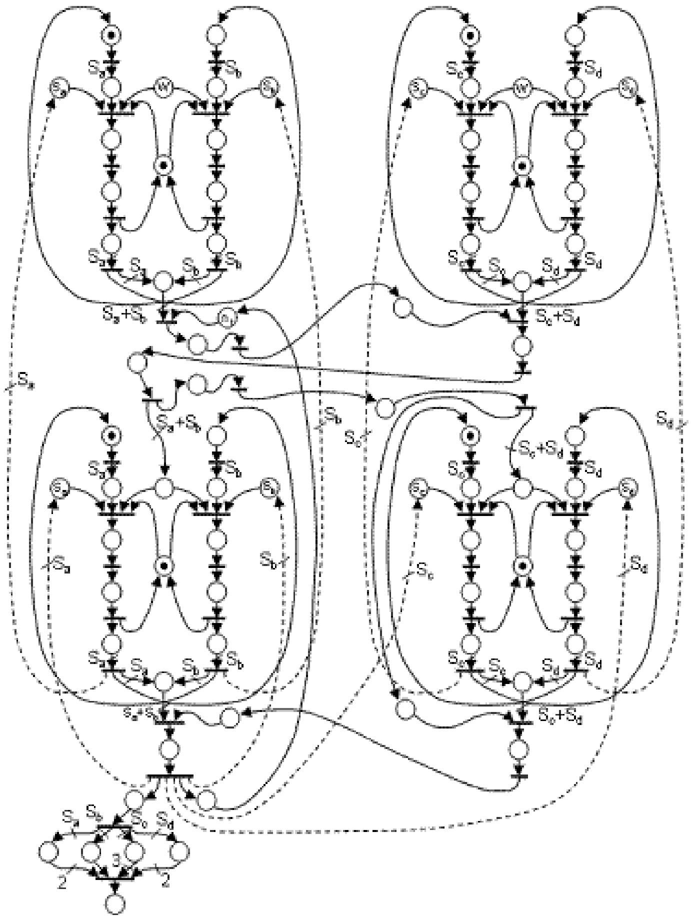
- Modelled by the weight of the arcs that represent the flow of parts.
- Undefined structural parameteres belonging to the set {1,…,24}.

Number of independent routes for the automatic guided vehicles (AGV).

- Modelled by means of different subnets.
- Undefined structural parameteres belonging to the set {0,1}.

2. Decision problem based on an undefined Petri net

Example 2 **Compound alternative Petri net**



$R^c{}_1$

Manufacturing strategy pull.

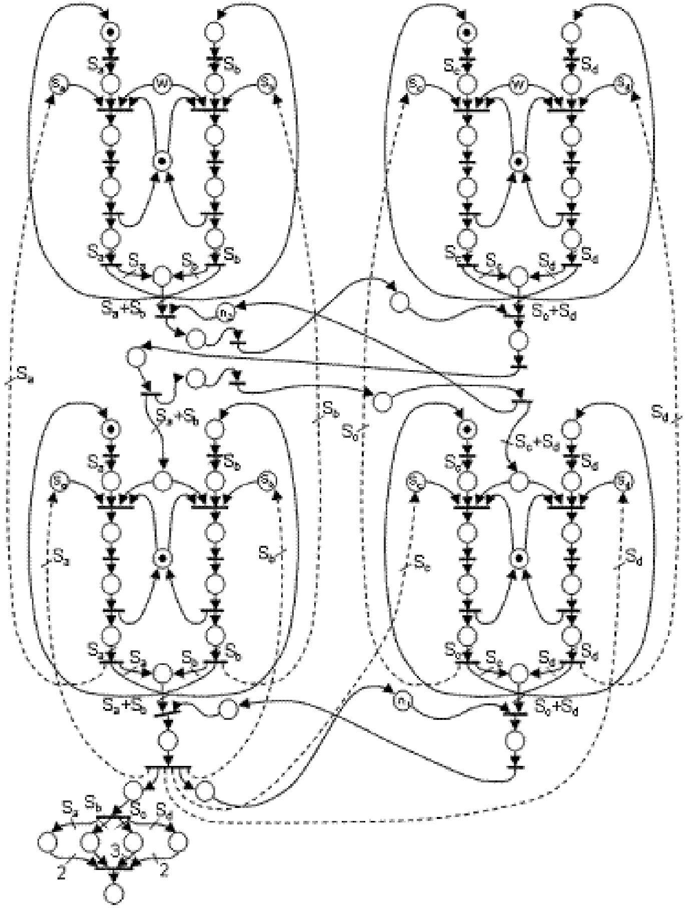
1 route of AGV.

Undefined structural parameters

Lot sizes.

2. Decision problem based on an undefined Petri net

Example 2 **Compound alternative Petri net**



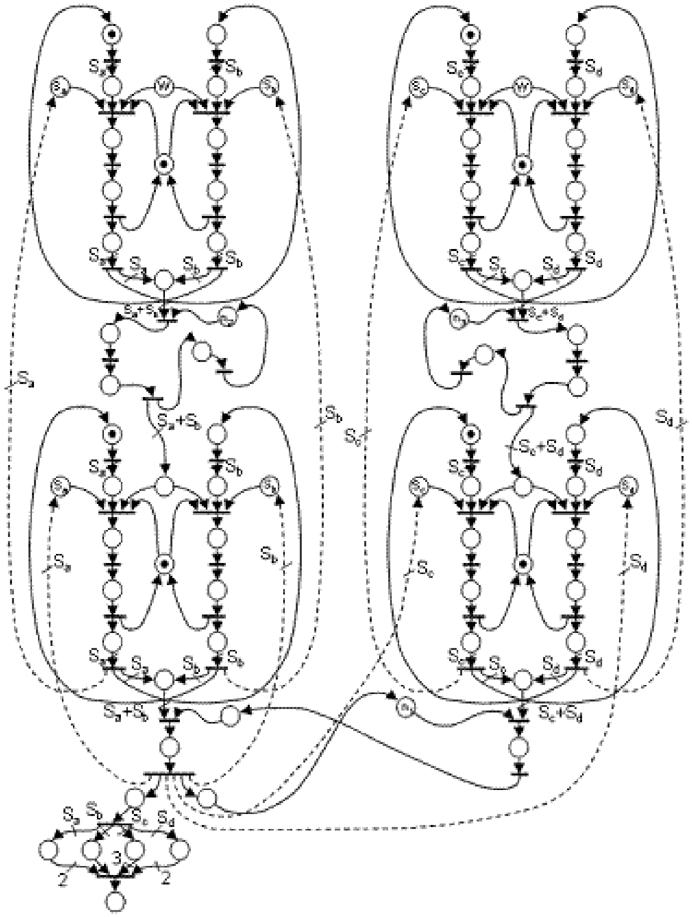
R^{c}_{2}

Manufacturing strategy pull.

2 routes of AGV.

2. Decision problem based on an undefined Petri net

Example 2 **Compound alternative Petri net**



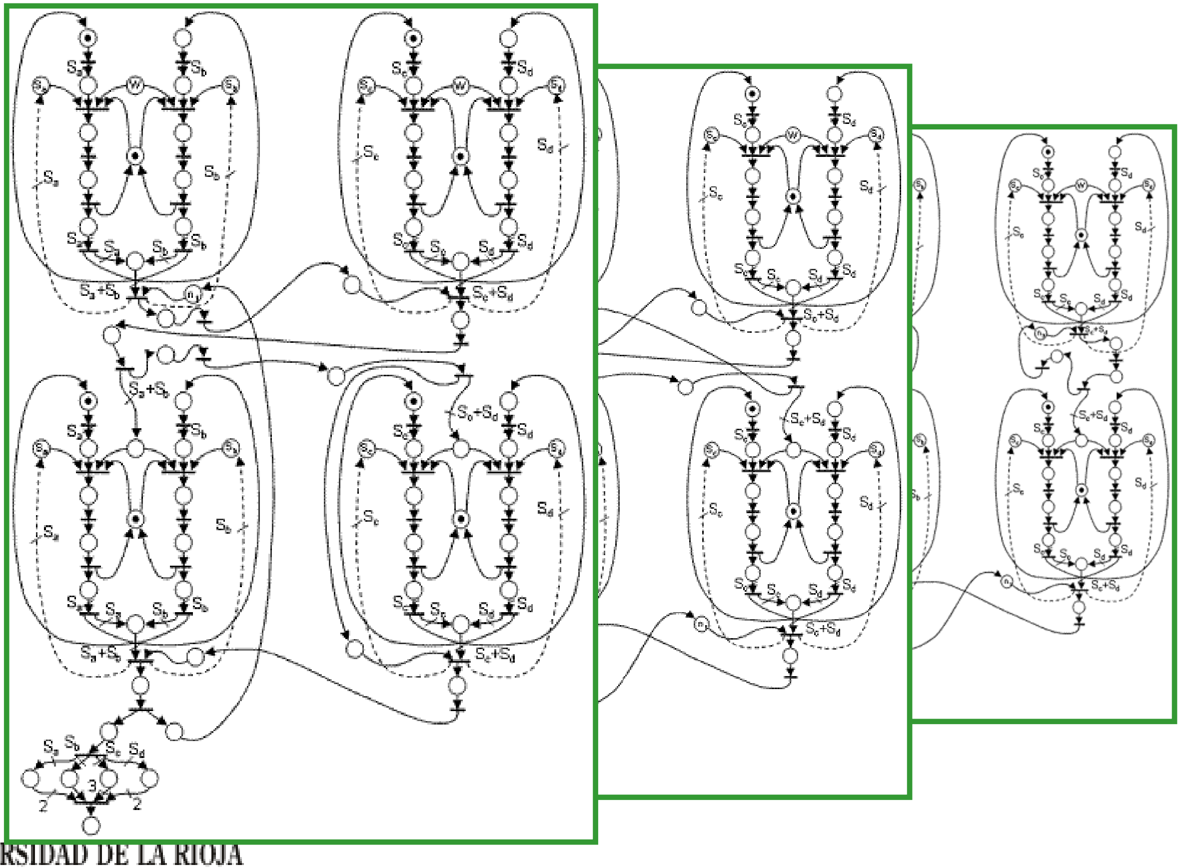
R^{c}_{3}

Manufacturing strategy pull.

3 routes of AGV.

2. Decision problem based on an undefined Petri net

Example 2 | **Compound alternative Petri net**



R^c_4 , R^c_5 , R^c_6

Manufacturing strategy push.

1, 2 of 3 routes of AGV respectively.

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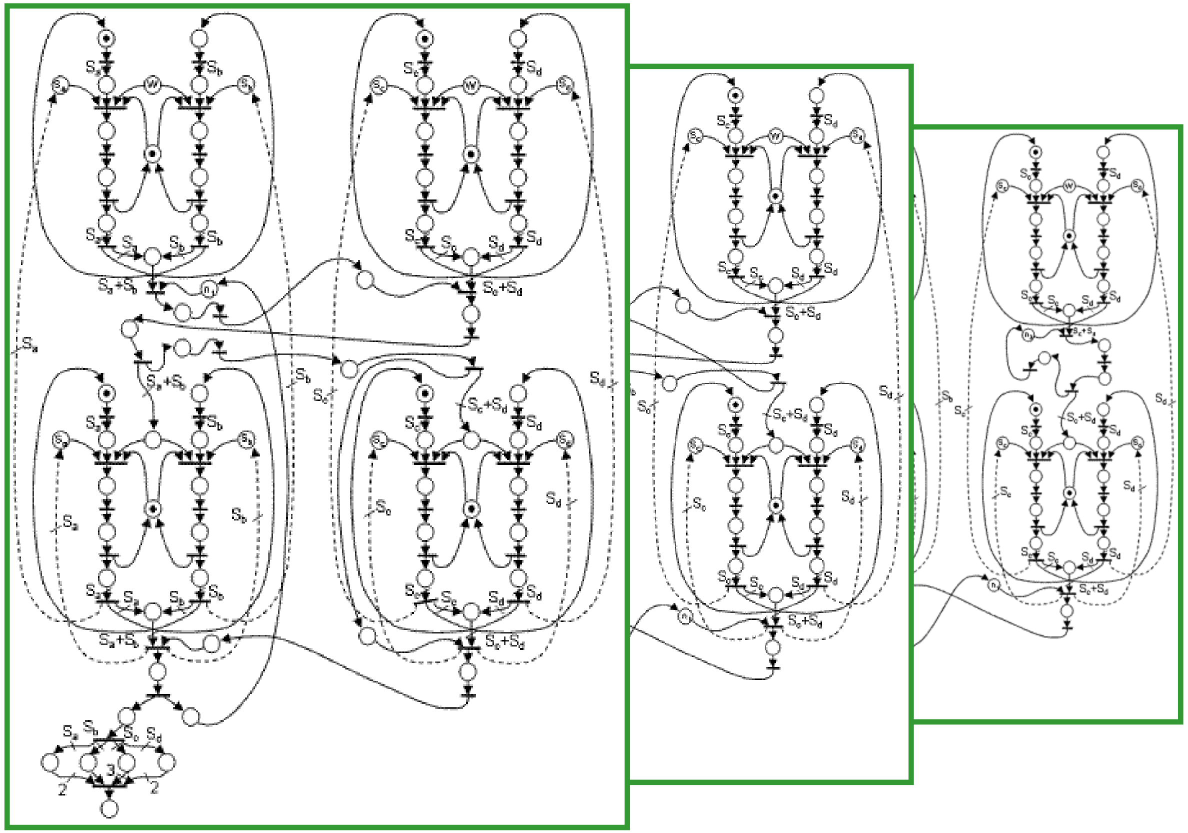
2. Decision problem based on an undefined Petri net

Example 2 **Compound alternative Petri net**



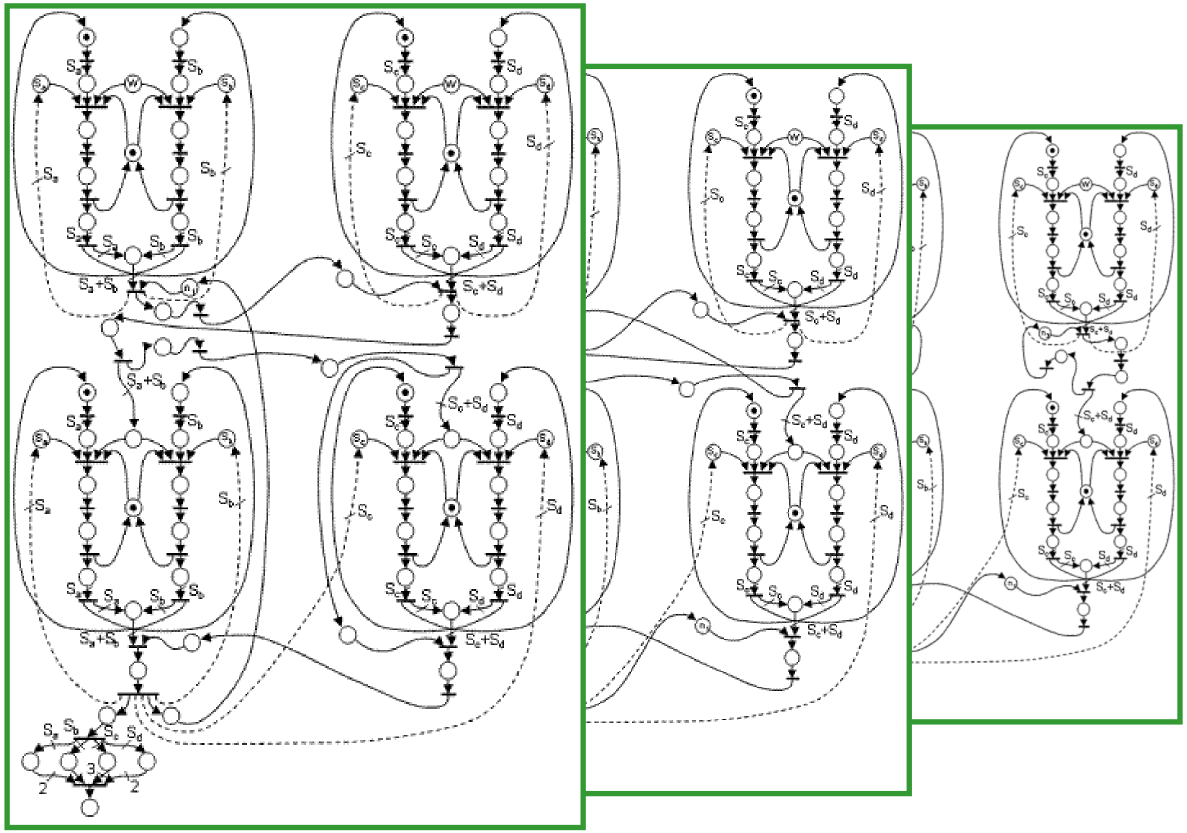
$R^{c}_{7}, R^{c}_{8}, R^{c}_{9}$

Hybrid manufacturing strategy pull-push.

1, 2 of 3 routes of AGV respectively.

2. Decision problem based on an undefined Petri net

Example 2 **Compound alternative Petri net**



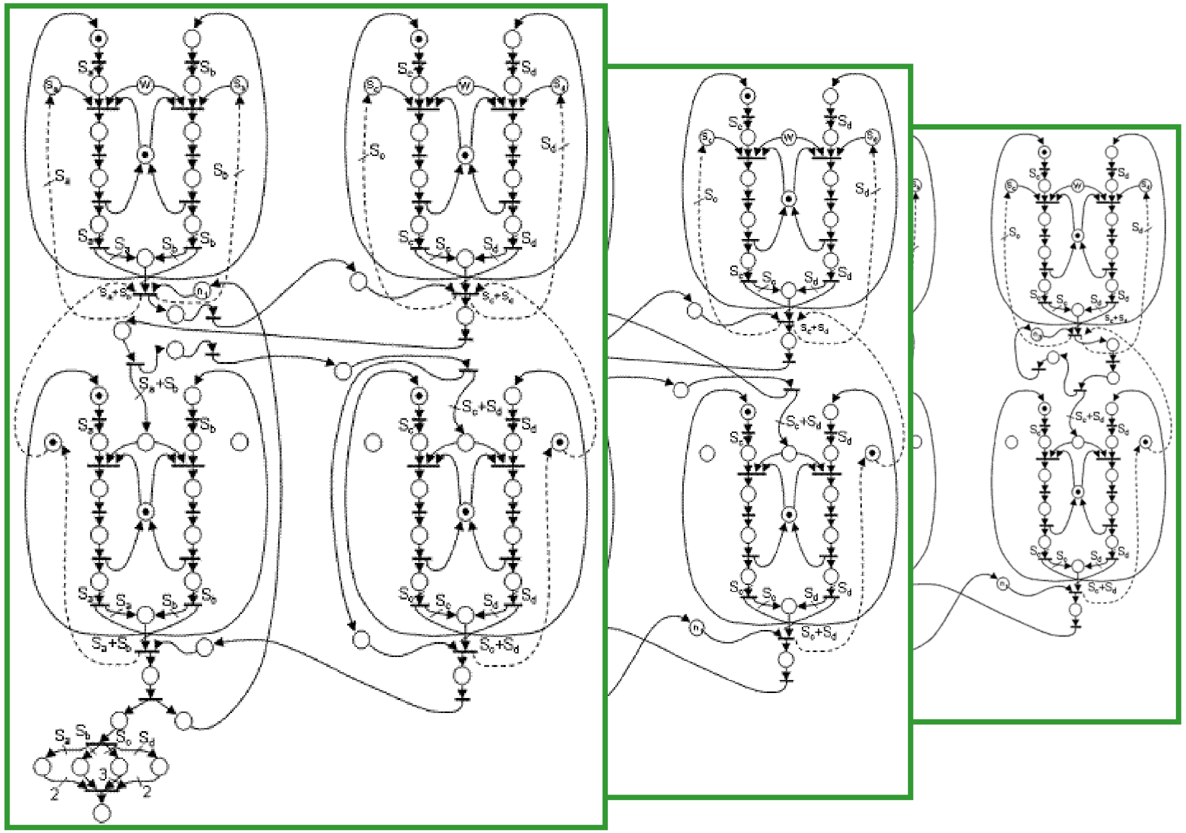
R^{c}_{10} , R^{c}_{11} , R^{c}_{12}

Hybrid manufacturing strategy push-pull.

1, 2 of 3 routes of AGV respectively.

2. Decision problem based on an undefined Petri net

Example 2 **Compound alternative Petri net**



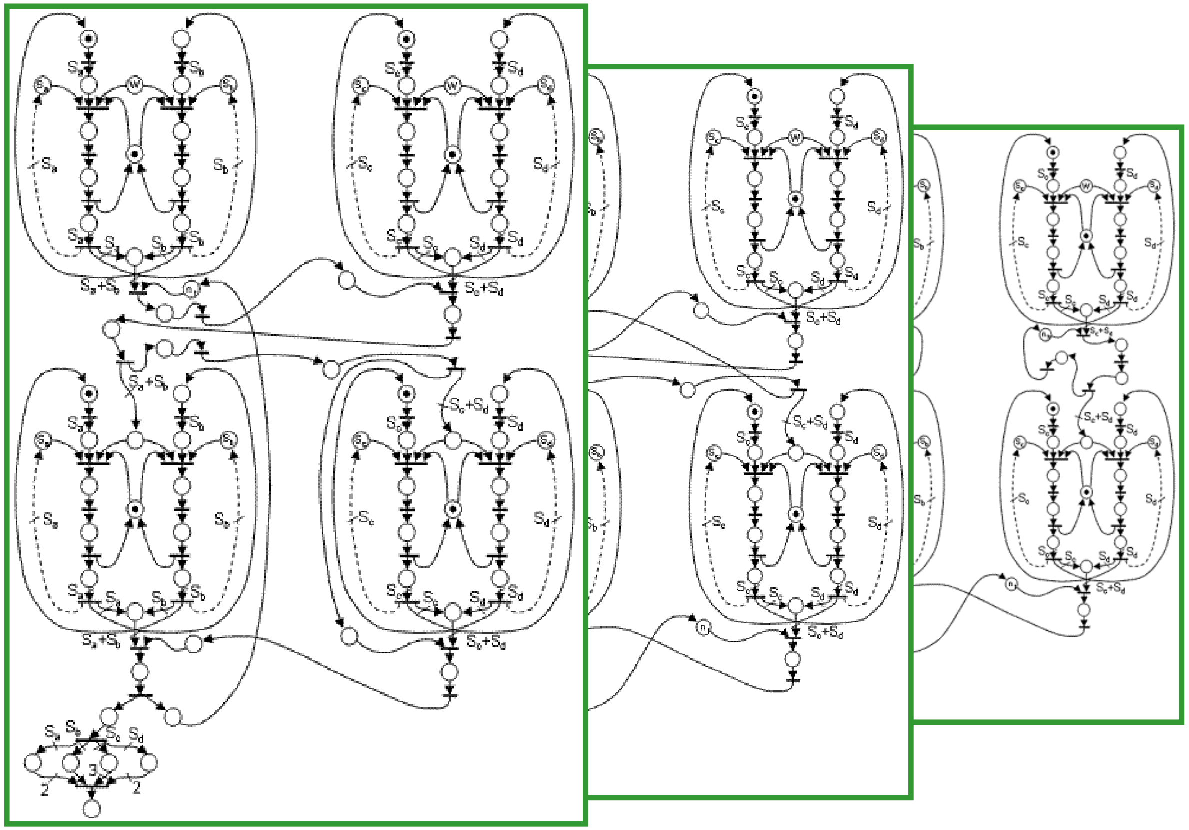
R^c_{13} , R^c_{14} , R^c_{15}

2nd. Manufacturing strategy pull.

1, 2 and 3 routes of AGV respectively.

2. Decision problem based on an undefined Petri net

Example 2 **Compound alternative Petri net**



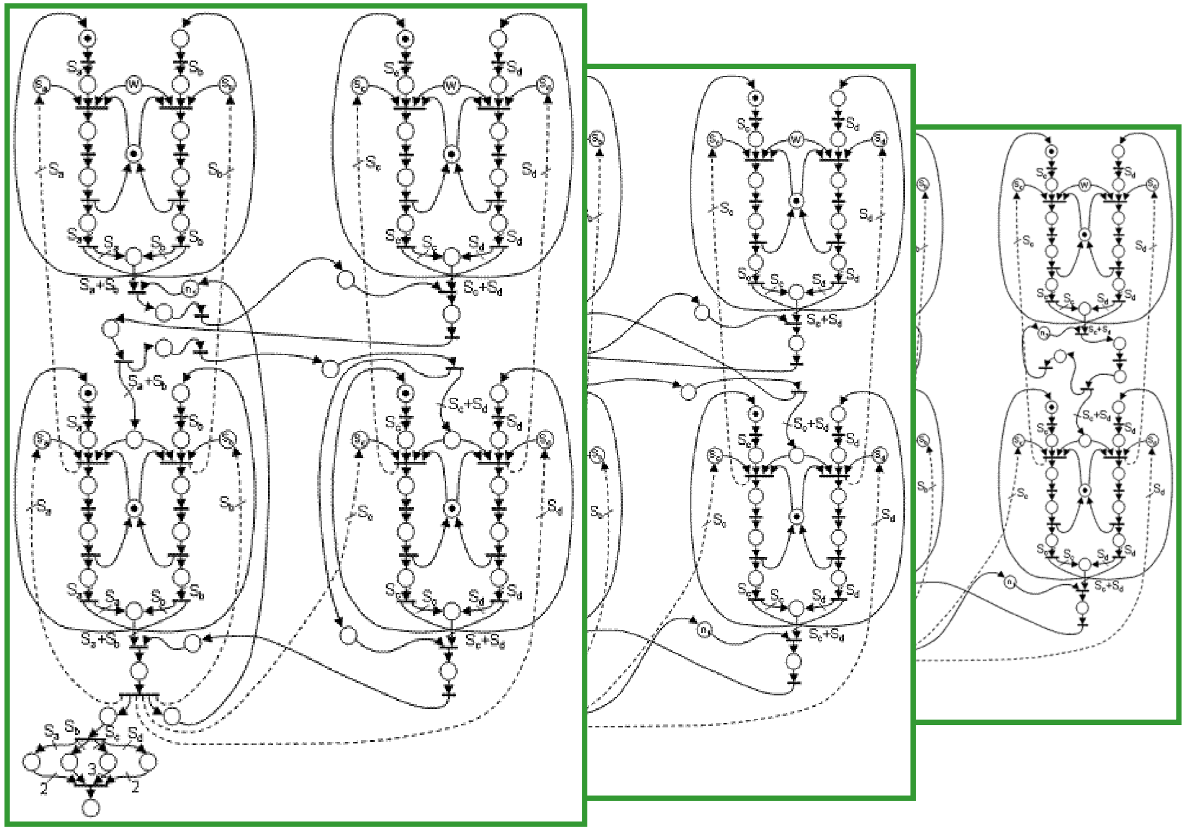
$R^{c}_{16}, R^{c}_{17}, R^{c}_{18}$

2nd. manufacturing strategy push.

1, 2 and 3 routes of AGV respectively.

2. Decision problem based on an undefined Petri net

Example 2 **Compound alternative Petri net**



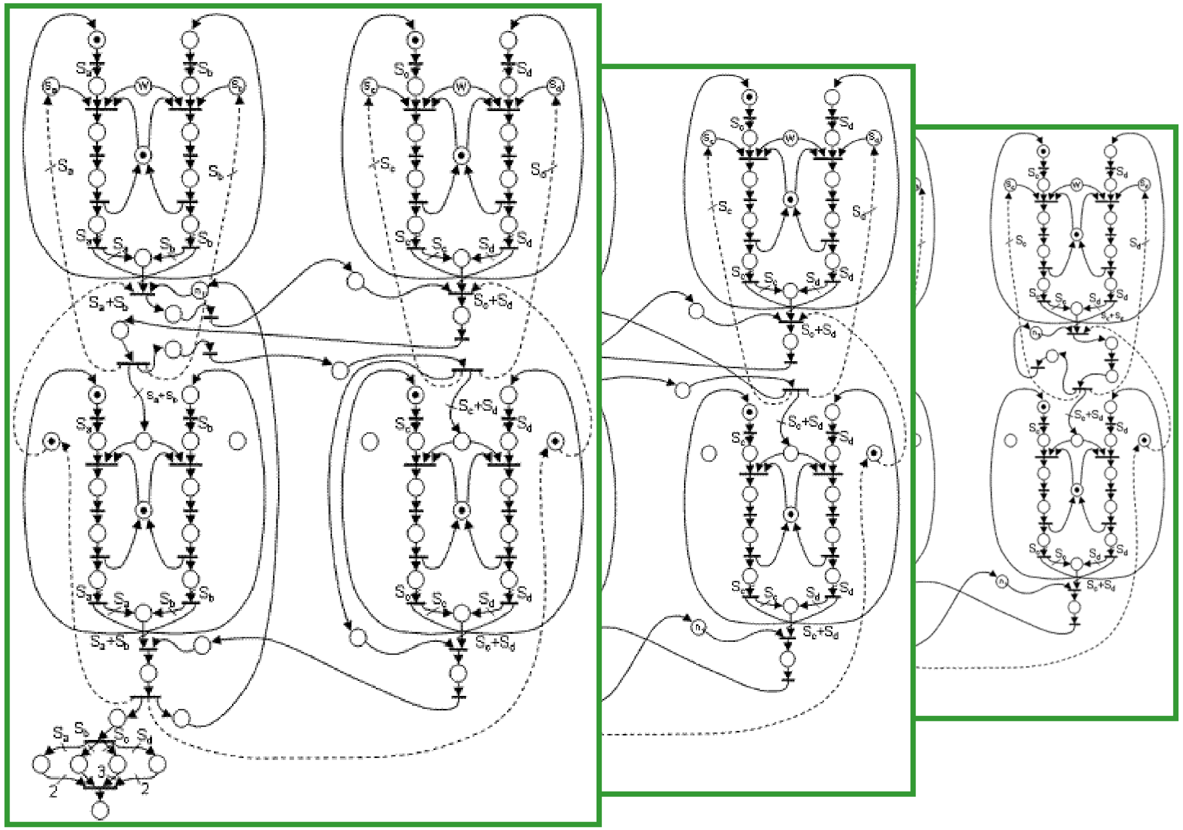
$R^{c}_{19}, R^{c}_{20}, R^{c}_{21}$

2nd. hybrid manufacturing strategy pull-push.

1, 2 and 3 routes of AGV respectively.

2. Decision problem based on an undefined Petri net

Example 2 **Compound alternative Petri net**



R^c_{22}, R^c_{23}, R^c_{24}

3rd. manufacturing strategy pull.

1, 2 and 3 routes of AGV respectively.

2. Decision problem based on an undefined Petri net

Alternatives aggregation Petri nets (AAPN)

Motivation

Is it possible to build up other type of PN from a partition of $S_{valstr\alpha}(R_1)$ different to the alternative PN?

Is it possible to merge the alternative PN to build up a PN which is not a compound PN?

Is it possible to profit from the shared subnets of the alternative PN?

Is it possible to decide wether no undefined structural parameter should appear in the new PN?

2. Decision problem based on an undefined Petri net

Alternatives aggregation Petri nets (AAPN)

Definition

- Let R^c be a compound PN and $\{ S_{valstr\alpha}(R_1), S_{valstr\alpha}(R_2), \ldots , S_{valstr\alpha}(R_{nr})\}$ a partition of $S_{valstr\alpha}(R^c)$.

- Let $S_A = \{a_1, a_2, \ldots, a_{nr} \mid \exists\ a_i=1,\ 1 \le i \le n_r \wedge \text{if } a_i=1 \Rightarrow a_j=0\ \forall\ j\neq i\}$ be a set of choice variables, where

card($\{ S_{valstr\alpha}(R_1), S_{valstr\alpha}(R_2), \ldots , S_{valstr\alpha}(R_{nr})\}$) = card($S_A$)

The choice of $a_i=1$ is the result of a decision.

An alternatives aggregation Petri net system is defined as the 8-tuple:

R^A = <*P*, *T*, Pre, Post, m_0, S_A, $S_\alpha(R_i)$, $S_{val\alpha}(R_i)$>

2. Decision problem based on an undefined Petri net

Alternatives aggregation Petri nets (AAPN)

Construction algorithm from a compound PN

Given: R^c (compound PN)

$\{ S_{1valstr\alpha}, S_{2valstr\alpha}, \ldots , S_{2valstr\alpha} \}$ (partition of $S_{valstr\alpha}(R^c)$)

$S_A = \{a_1, a_2, \ldots, a_{nr} \mid \exists\, a_i=1, 1 \leq i \leq n_r \wedge \text{if } a_i=1 \Rightarrow a_j=0\ \forall\, j \neq i\}$ (set of choice variables)

a) Create a bijection between S_A and the partition of $S_{valstr\alpha}(R^c)$.

b) Replicate every transition t_i into a set $\left\{t_i^1, t_i^2, \ldots, t_i^{nr}\right\}$ where n_r=card(S_A) and

Pre(p_j,t_i^q), Post(t_i^q, p_k) $\in S_{qval\alpha}(R_q) \cup S_{qval\beta}(R_q)$

The choice variable a_q is associated to the transition t_i as a boolean condition.

c) Applicate reduction and simplification rules.

2. Decision problem based on an undefined Petri net

Alternatives aggregation Petri nets (AAPN)

Property

- Let R^c be a compound Petri net.
- Let R^A be an alternatives aggregation Petri net obtained from R^c.
- Let us consider an equivalence relation which states that a PN is equivalent to another one when their reachability graph is the same.
- It is possible to state that:

R^c and R^A are equivalent.

R^A will not have any undefined structural parameter if

$$\text{card}(S_A)=\text{card}(S_{valstr\alpha}(R^c)).$$

- As a consequence, R^A is a valid representation of an undefined PN.

2. Decision problem based on an undefined Petri net

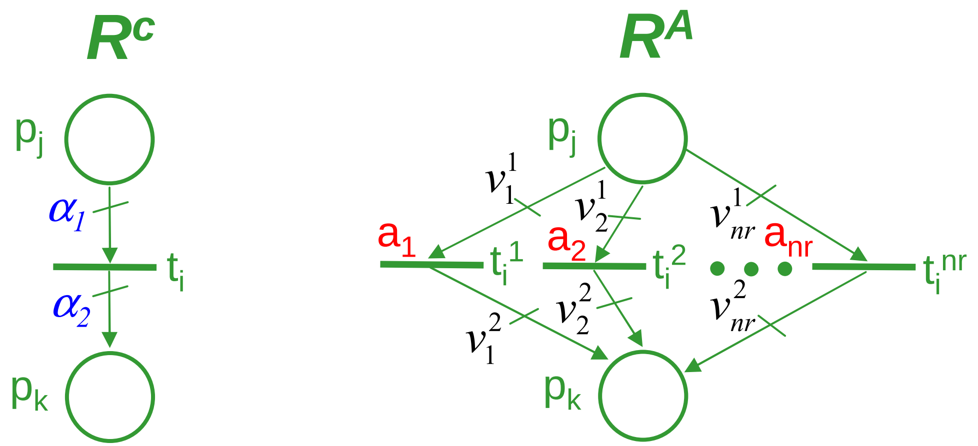
Alternatives aggregation Petri nets (AAPN)

Replication of the transitions of R^c

$$\text{card}(S_{valstr\alpha}(R^c)) = n_r$$

$$S^E_{val\alpha 1}(R^c) = \{v^1_1, v^1_2, \ldots, v^1_{nr}\}$$

$$S^E_{val\alpha 2}(R^c) = \{v^2_1, v^2_2, \ldots, v^2_{nr}\}$$



The choice of v^1_q and v^1_q in R^c is equivalent to the choice of a_q in R^A.

2. Decision problem based on an undefined Petri net

Alternatives aggregation Petri nets (AAPN)

Choice variables

- Allow to impose the condicion of exclusive behaviour of the different subsets of a partition of $S_{valstr\alpha}(R^c)$.

  To activate a choice variable is equivalent to choose an alternative PN in a set of alternative PNs or to choose a feasible combination of values for the undefined structural parameters in a compound PN.

- A function of choice variables can be associated to a transition.

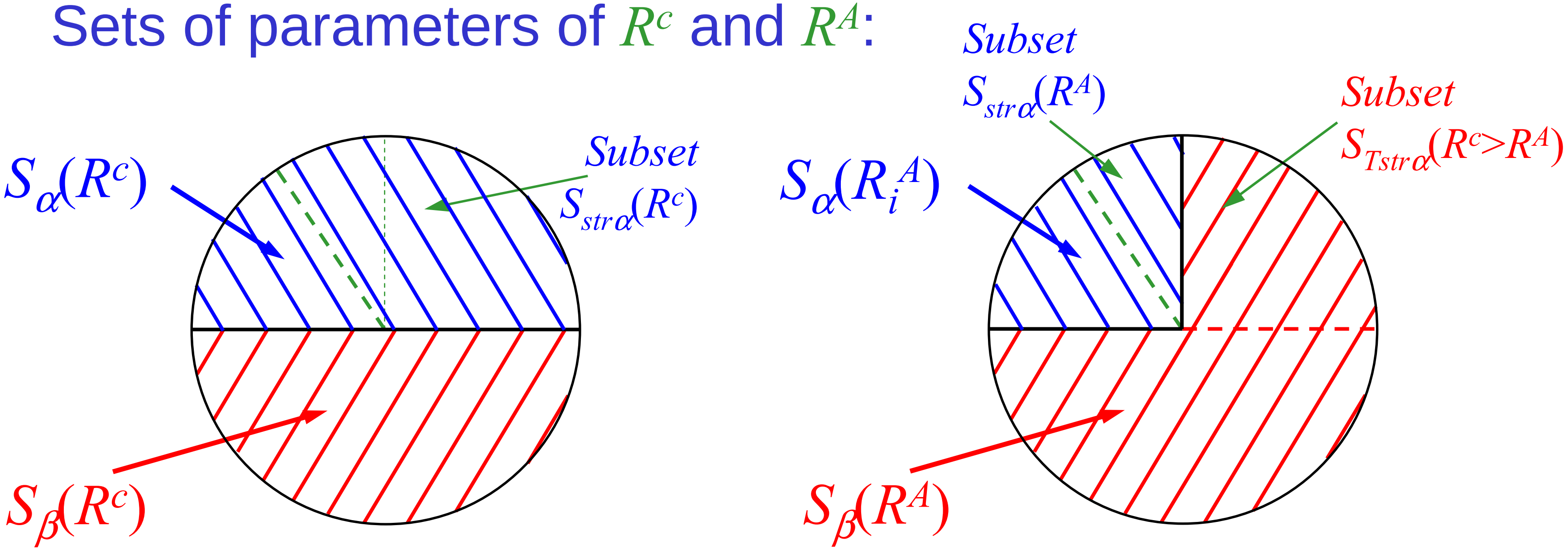
- Before the beginning of a simulation (“token game”) a choice variable must be chosen as a result of a decision.

2. Decision problem based on an undefined Petri net

Alternatives aggregation Petri nets (AAPN)

Compound PN → Alternatives aggregation PN

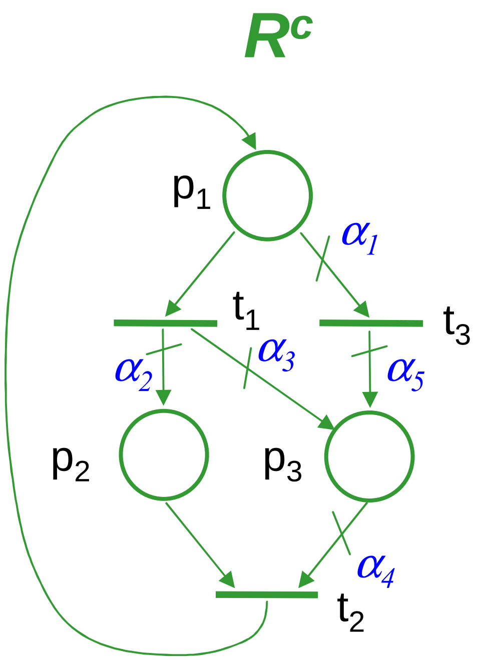
Sets of parameters of R^c and R^A:



2. Decision problem based on an undefined Petri net

Ejemplo 3 Obtention of an AAPN

Original compound Petri net



$$W(R^c) = \begin{array}{c c} \begin{array}{ccc} t_1 & t_2 & t_3 \end{array} & \\ \left(\begin{array}{ccc} -1 & 1 & -\alpha_1 \\ -\alpha_2 & -1 & 0 \\ \alpha_3 & -\alpha_4 & \alpha_5 \end{array}\right) & \begin{array}{c} p_1 \\ p_2 \\ p_3 \end{array} \end{array}$$

Feasible combinations of values for the undefined structural parameters (α_1 , α_2 , α_3 , α_4 , α_5):

$S_{valstr\alpha}(R^c)$ = { (0,2,0,0,0) , (0,1,1,1,0) , (1,2,0,1,2) }

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2. Decision problem based on an undefined Petri net

Ejemplo 3 Obtention of an AAPN

Step 1.

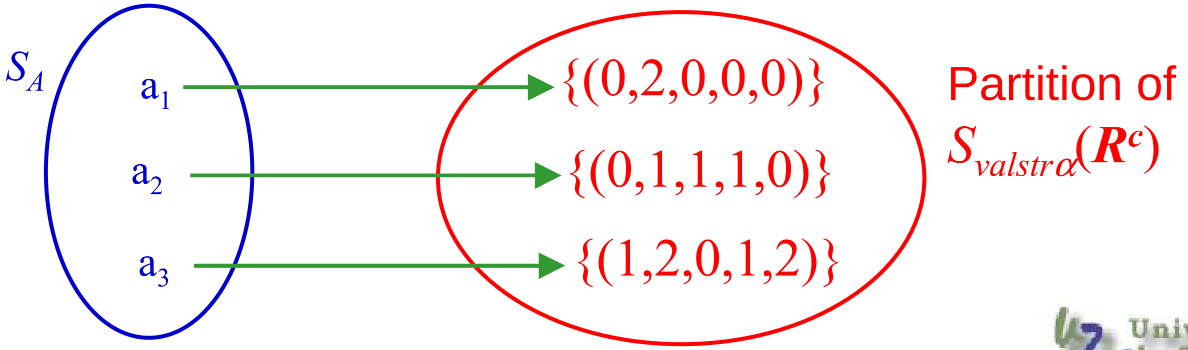
Partition of $S_{valstr\alpha}(\boldsymbol{R}^c)$ of order 3: $\{(0,2,0,0,0)\},\{(0,1,1,1,0)\},\{(1,2,0,1,2)\}$

Notice that $n_r = \mathrm{card}(\{(0,2,0,0,0)\},\{(0,1,1,1,0)\},\{(1,2,0,1,2)\}) = 3$

Definition of the set of choice variables

$$S_A = \{a_1, a_2, a_3 \mid \exists\, a_i=1,\ 1 \le i \le n_r \wedge \text{if } a_i=1 \Rightarrow a_j=0\ \forall\, j \ne i\}$$

Creation of a bijection between S_A and $\{(0,2,0,0,0)\}$, $\{(0,1,1,1,0)\}$, $\{(1,2,0,1,2)\}$



S_A

a_1 → $\{(0,2,0,0,0)\}$

a_2 → $\{(0,1,1,1,0)\}$

a_3 → $\{(1,2,0,1,2)\}$

Partition of $S_{valstr\alpha}(\boldsymbol{R}^c)$

2. Decision problem based on an undefined Petri net

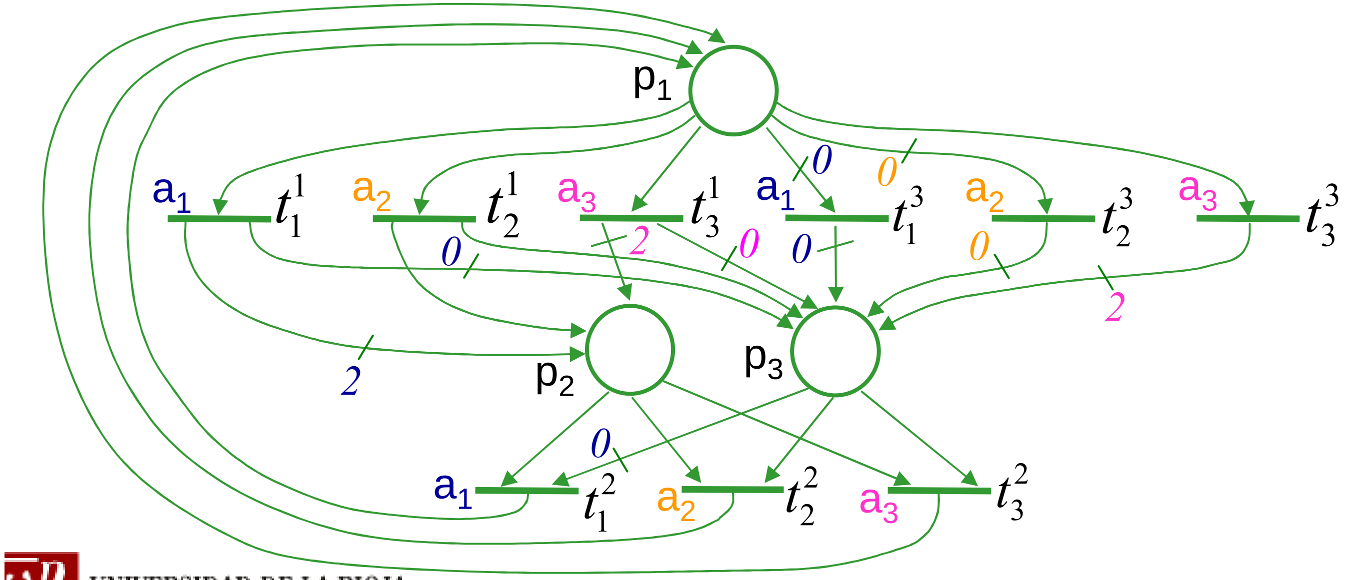
Ejemplo 3 | **Obtention of an AAPN**

Step 2.

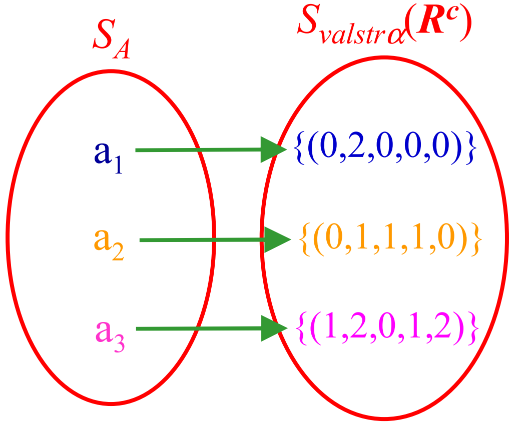
Replication of every transition into n_r new transitions:

Associate the Pre and Post functions to the subsets of the partition.

Associate tha appropriate choice variable to every transition.



Partition of $S_{valstr\alpha}(\boldsymbol{R}^c)$



2. Decision problem based on an undefined Petri net

Ejemplo 3 Obtention of an AAPN

Step 2.

Replication of every transition into n_r new transitions:

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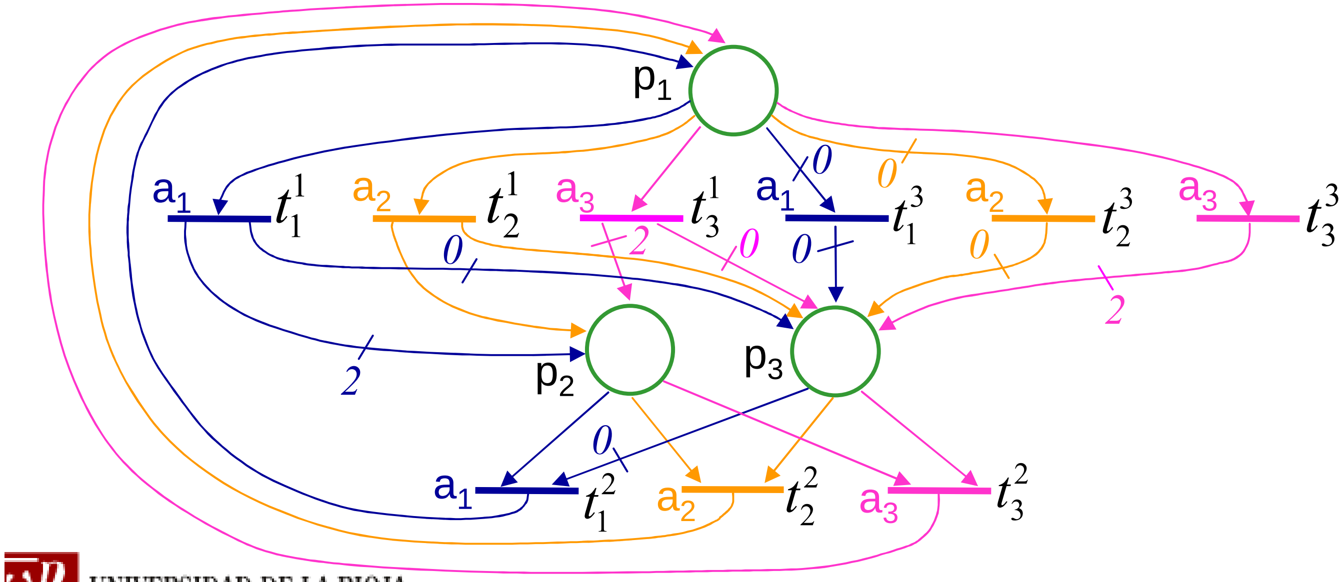
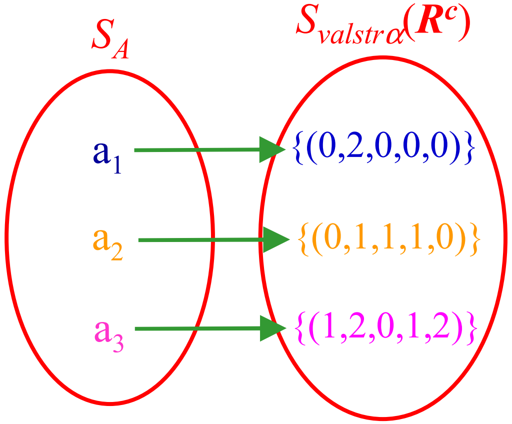


Partition of

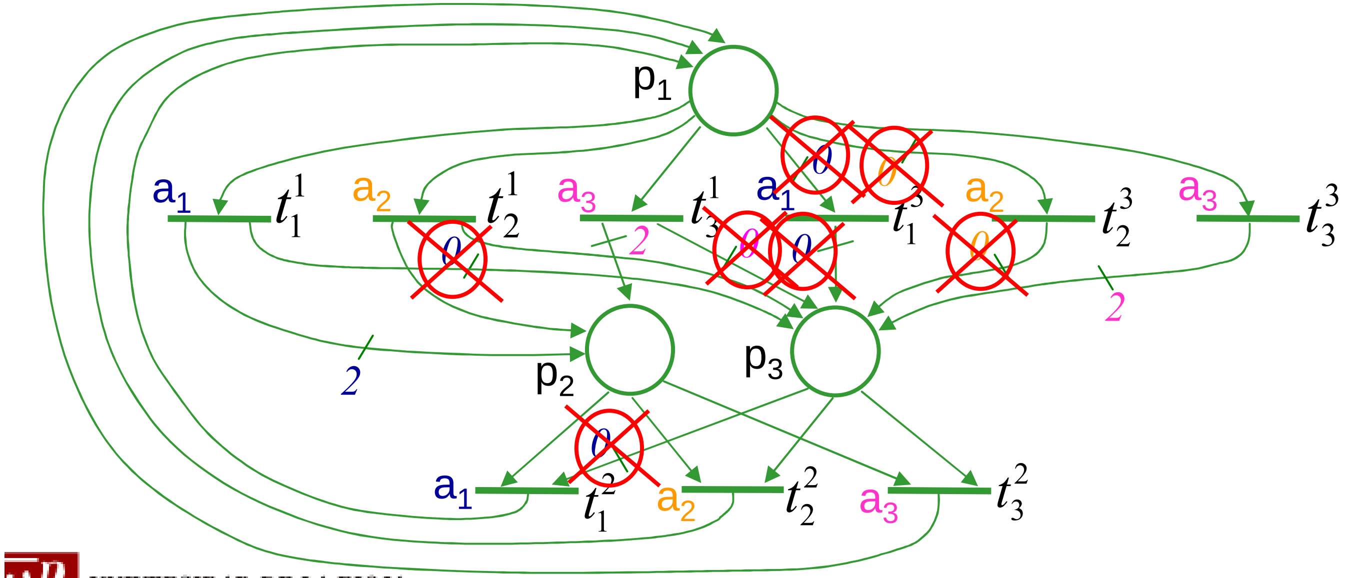


2. Decision problem based on an undefined Petri net

Ejemplo 3 **Obtention of an AAPN**

Step 3.

The Petri net is redrawn without the arcs of weight 0.

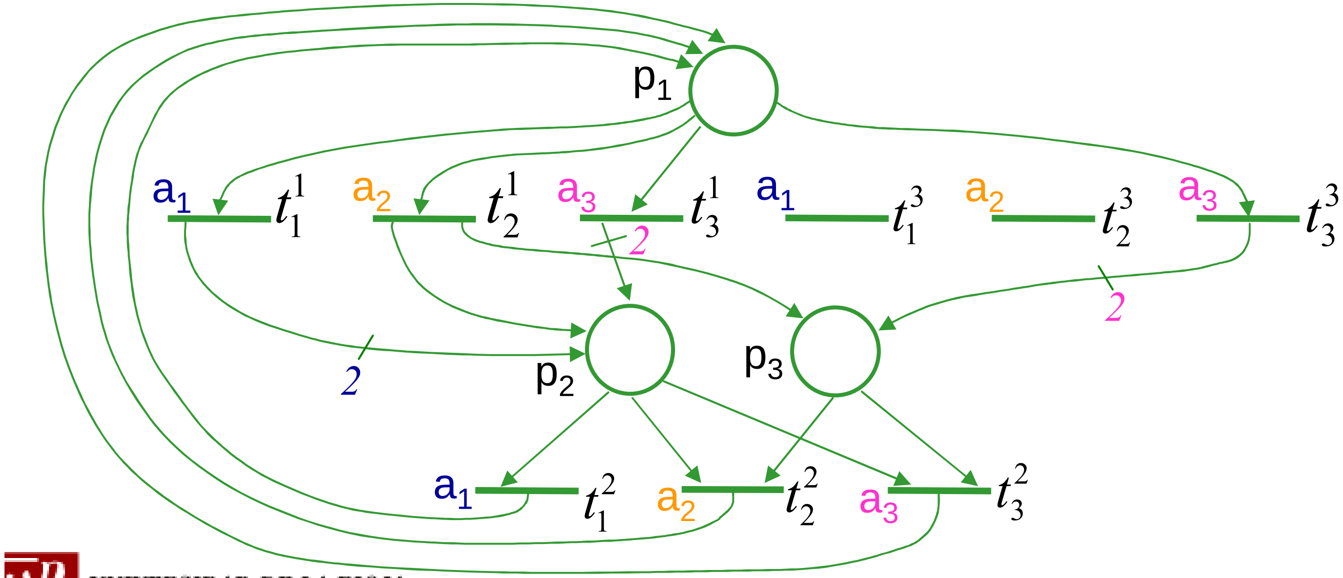


2. Decision problem based on an undefined Petri net

Ejemplo 3 **Obtention of an AAPN**

Step 3.

The Petri net is redrawn without the arcs of weight 0.



2. Decision problem based on an undefined Petri net

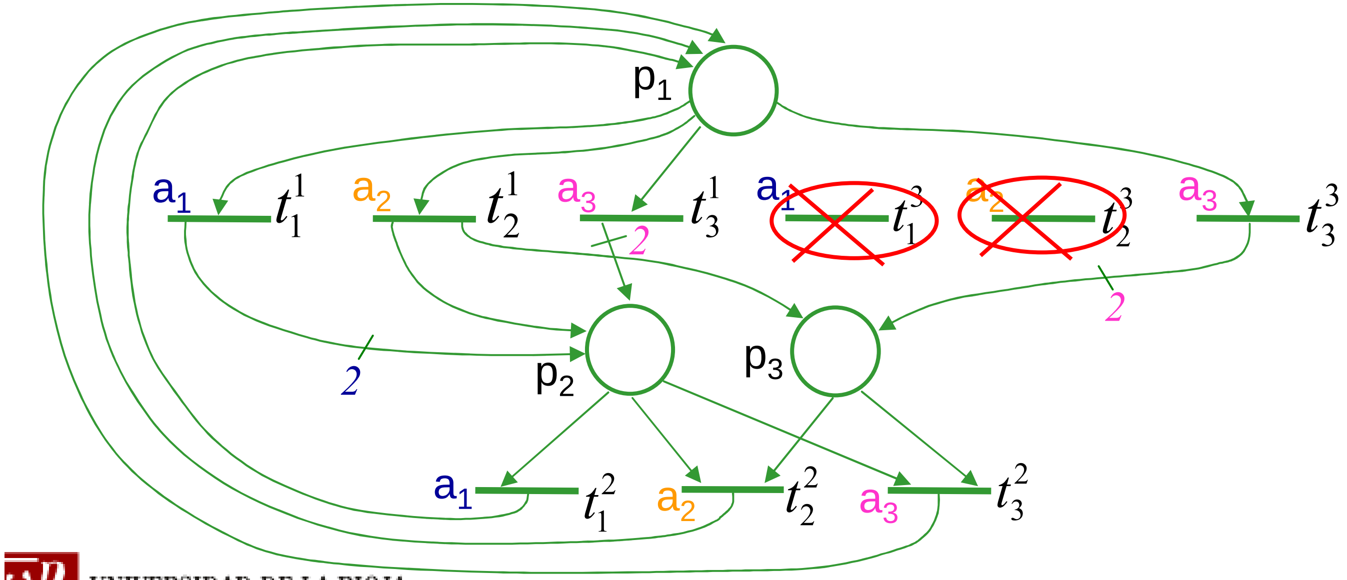
Ejemplo 3 **Obtention of an AAPN**

Step 4.

An equivalent Petri net is obtained by removing the rows and columns of 0:

Non-connected places: none

Non-connected transitions: t_1^3 and t_2^3



2. Decision problem based on an undefined Petri net

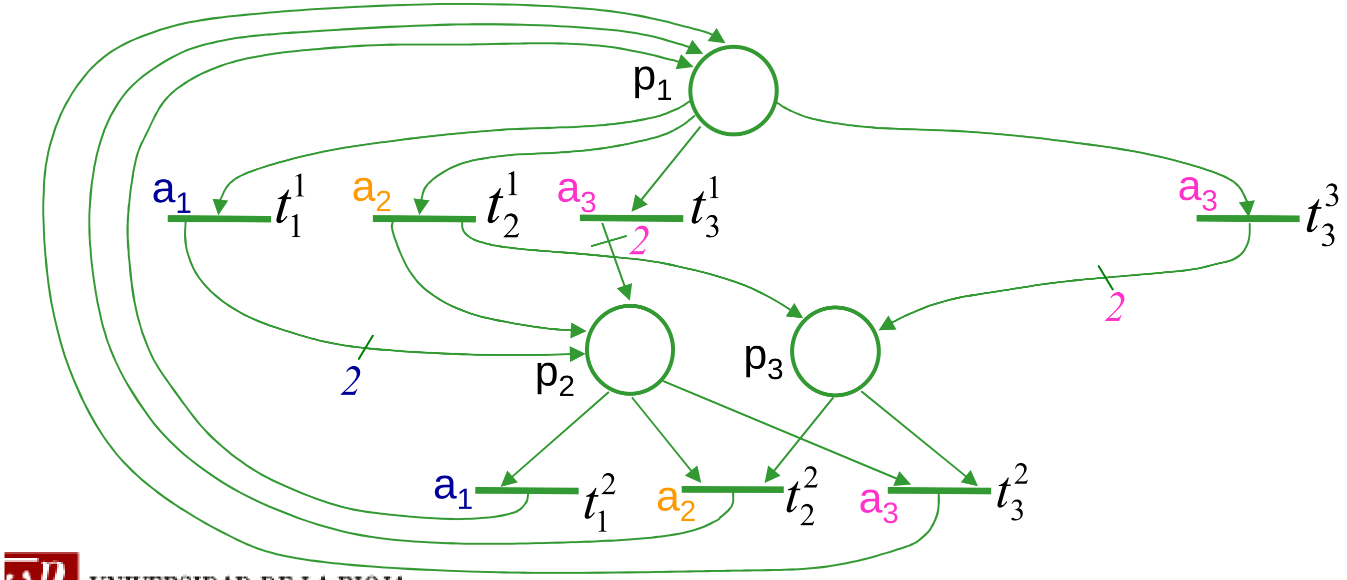
Ejemplo 3 **Obtention of an AAPN**

Step 4.

An equivalent Petri net is obtained by removing the rows and columns of 0:

Non-connected places: none

Non-connected transitions: t_1^3 and t_2^3



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2. Decision problem based on an undefined Petri net

Ejemplo 3 | **Obtention of an AAPN**

Step 4.

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$$\begin{array}{cc} & \begin{array}{ccccccccc} t_1^1 & t_2^1 & t_3^1 & t_1^2 & t_2^2 & t_3^2 & t_1^3 & t_2^3 & t_3^3 \end{array} \\ W(R^A) = & \left(\begin{array}{ccccccccc} \mathbf{-1} & \mathbf{-1} & \mathbf{-1} & \mathbf{1} & \mathbf{1} & \mathbf{1} & \boxed{\mathbf{0}} & \boxed{\mathbf{0}} & \mathbf{-1} \\ \mathbf{2} & \mathbf{1} & \mathbf{2} & \mathbf{-1} & \mathbf{-1} & \mathbf{-1} & \boxed{\mathbf{0}} & \boxed{\mathbf{0}} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{-1} & \mathbf{-1} & \boxed{\mathbf{0}} & \boxed{\mathbf{0}} & \mathbf{2} \end{array}\right) \begin{array}{c} \mathbf{p_1} \\ \mathbf{p_2} \\ \mathbf{p_3} \end{array} \\ & \underbrace{\begin{array}{ccccccccc} a_1 & a_2 & a_3 & a_1 & a_2 & a_3 & a_1 & a_2 & a_3 \end{array}}_{\text{Choice variable associated to every transition}} \end{array}$$

2. Decision problem based on an undefined Petri net

Ejemplo 3 | **Obtention of an AAPN**

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$$W(R^A) = \begin{array}{c|ccccccccc|c} & t_1^1 & t_2^1 & t_3^1 & t_1^2 & t_2^2 & t_3^2 & \cancel{t_1^3} & \cancel{t_2^3} & t_3^3 & \\ & -1 & -1 & -1 & 1 & 1 & 1 & \boxed{\cancel{0}} & \boxed{\cancel{0}} & -1 & p_1 \\ & 2 & 1 & 2 & -1 & -1 & -1 & \boxed{\cancel{0}} & \boxed{\cancel{0}} & 0 & p_2 \\ & 0 & 1 & 0 & 0 & -1 & -1 & \boxed{\cancel{0}} & \boxed{\cancel{0}} & 2 & p_3 \end{array}$$

$$\underbrace{a_1 \quad a_2 \quad a_3 \quad a_1 \quad a_2 \quad a_3 \quad a_1 \quad a_2 \quad a_3}_{\text{Choice variable associated to every transition}}$$

2. Decision problem based on an undefined Petri net

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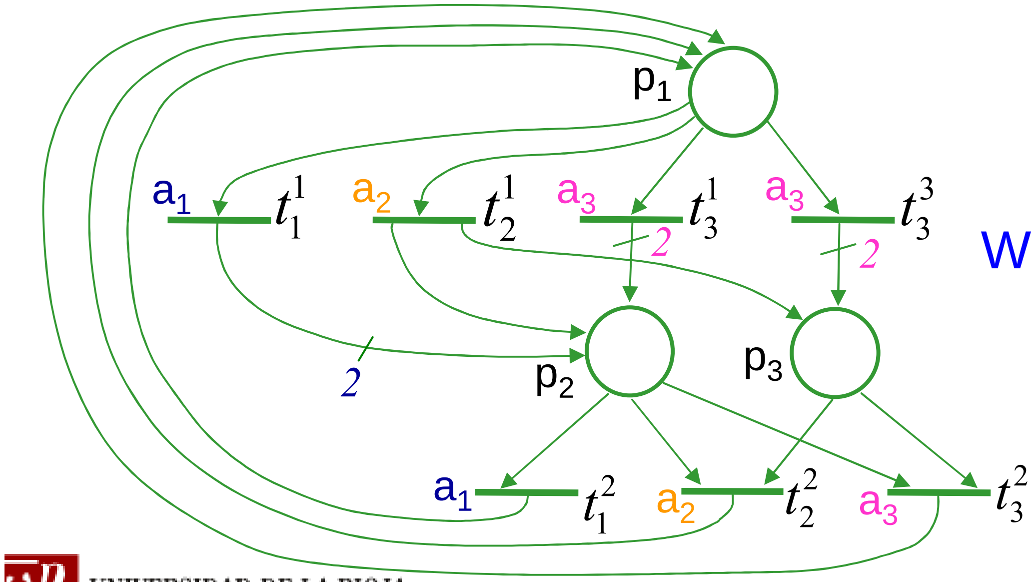
2. Decision problem based on an undefined Petri net

Ejemplo 3 | Obtention of an AAPN

Step 5.

Application of a reduction rule to identical transitions:

Function of choice variables associated to the new transitions.



$$W(R^A)=\begin{array}{c}\begin{array}{ccccccc} t_1^1 & t_2^1 & t_3^1 & t_1^2 & t_2^2 & t_3^2 & t_3^3 \end{array} \\ \left(\begin{array}{ccccccc} -1 & -1 & -1 & 1 & 1 & 1 & -1 \\ 2 & 1 & 2 & -1 & -1 & -1 & 0 \\ 0 & 1 & 0 & 0 & -1 & -1 & 2 \end{array}\right)\begin{array}{c} p_1 \\ p_2 \\ p_3 \end{array} \\ \begin{array}{ccccccc} a_1 & a_2 & a_3 & a_1 & a_2 & a_3 & a_3 \end{array}\end{array}$$

2. Decision problem based on an undefined Petri net

Ejemplo 3 | Obtention of an AAPN

Step 5.

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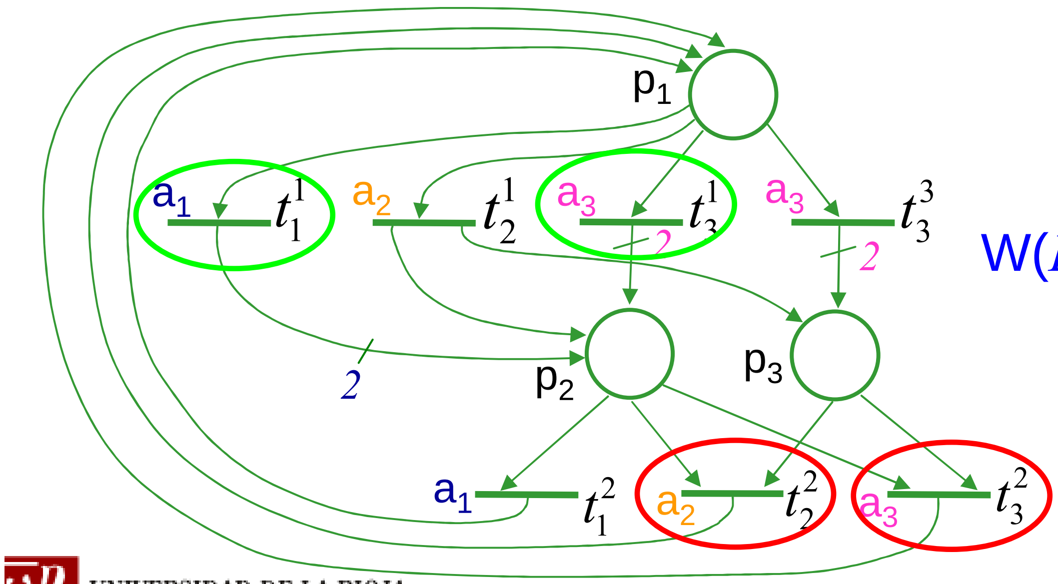
Function of choice variables associated to the new transitions.



$$W(R^A)=\begin{array}{c|ccccccc|c} & t_1^1 & t_2^1 & t_3^1 & t_1^2 & t_2^2 & t_3^2 & t_3^3 & \\ & -1 & -1 & -1 & 1 & 1 & 1 & -1 & p_1 \\ & 2 & 1 & 2 & -1 & -1 & -1 & 0 & p_2 \\ & 0 & 1 & 0 & 0 & -1 & -1 & 2 & p_3 \\ & a_1 & a_2 & a_3 & a_1 & a_2 & a_3 & a_3 & \end{array}$$

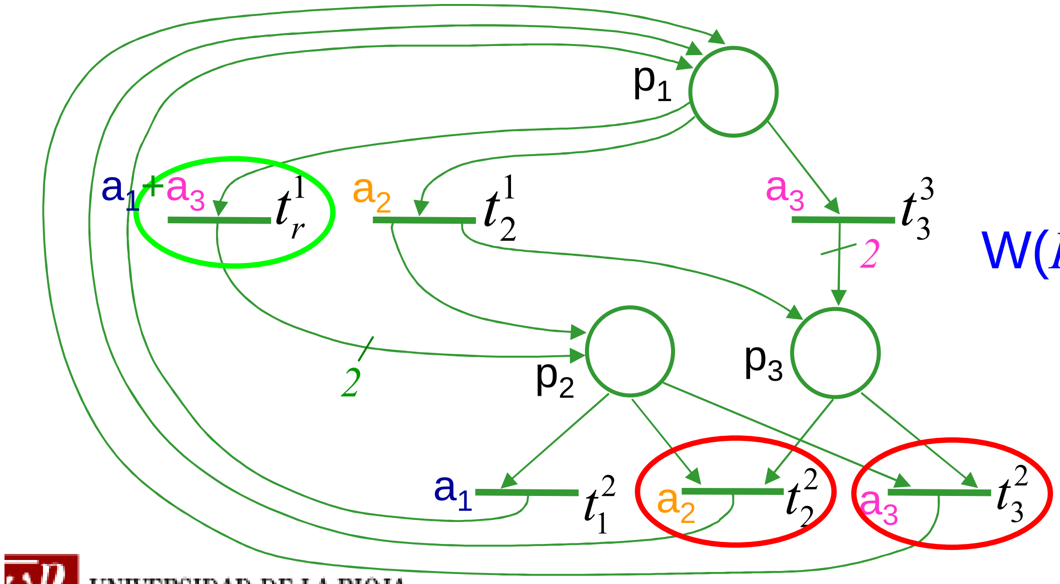
2. Decision problem based on an undefined Petri net

Ejemplo 3 | **Obtention of an AAPN**

Step 5.

Application of a reduction rule to identical transitions:

Function of choice variables associated to the new transitions.



$$W(R^A)=\begin{array}{cccccccc} & t_r^1 & t_2^1 & t_1^2 & t_2^2 & t_3^2 & t_3^3 & \\ \left(\right. & -1 & -1 & 1 & 1 & 1 & -1 & p_1 \\ & 2 & 1 & -1 & -1 & -1 & 0 & p_2 \\ & 0 & 1 & 0 & -1 & -1 & 2 & \left. p_3 \right) \\ & \uparrow & a_2 & a_1 & a_2 & a_3 & a_3 & \\ & a_1+a_3 & & & & & & \end{array}$$

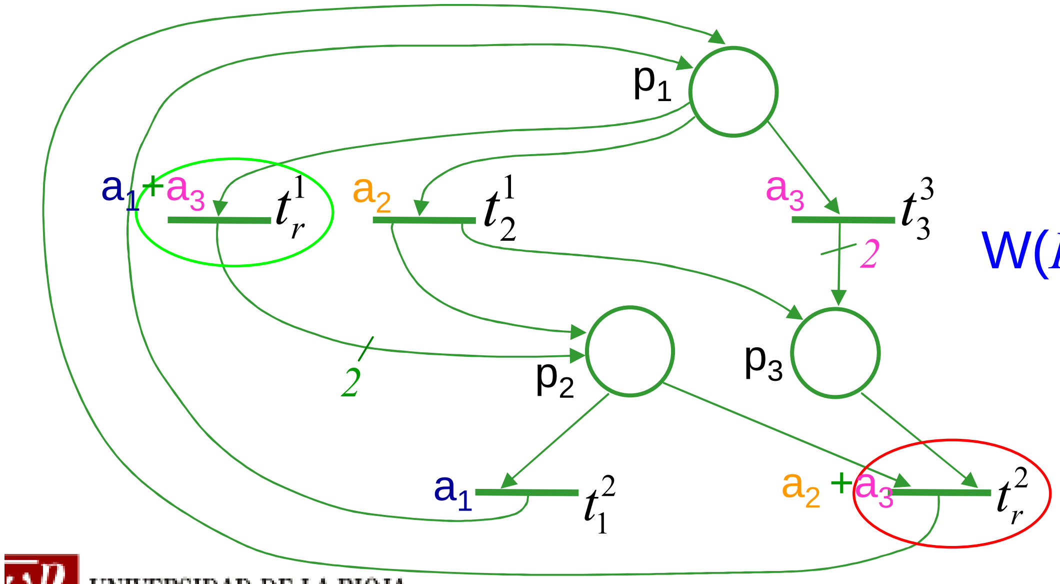
2. Decision problem based on an undefined Petri net

Ejemplo 3 | Obtention of an AAPN

Step 5.

Application of a reduction rule to identical transitions:

Function of choice variables associated to the new transitions.



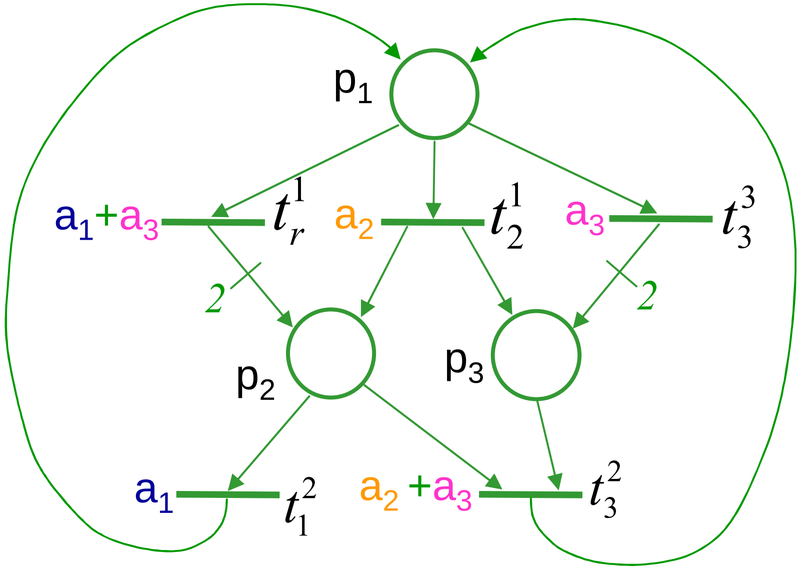
$$W(R^A)=\begin{array}{cccccc} t_r^1 & t_2^1 & t_1^2 & t_r^2 & t_3^3 & \\ -1 & -1 & 1 & 1 & -1 & p_1 \\ 2 & 1 & -1 & -1 & 0 & p_2 \\ 0 & 1 & 0 & -1 & 2 & p_3 \\ \uparrow & a_2 & a_1 & \uparrow & a_3 & \\ a_1+a_3 & & & a_2+a_3 & & \end{array}$$

2. Decision problem based on an undefined Petri net

Ejemplo 3 | **Obtention of an AAPN**

Step 6.

Redrawing of the alternatives aggregation Petri net.



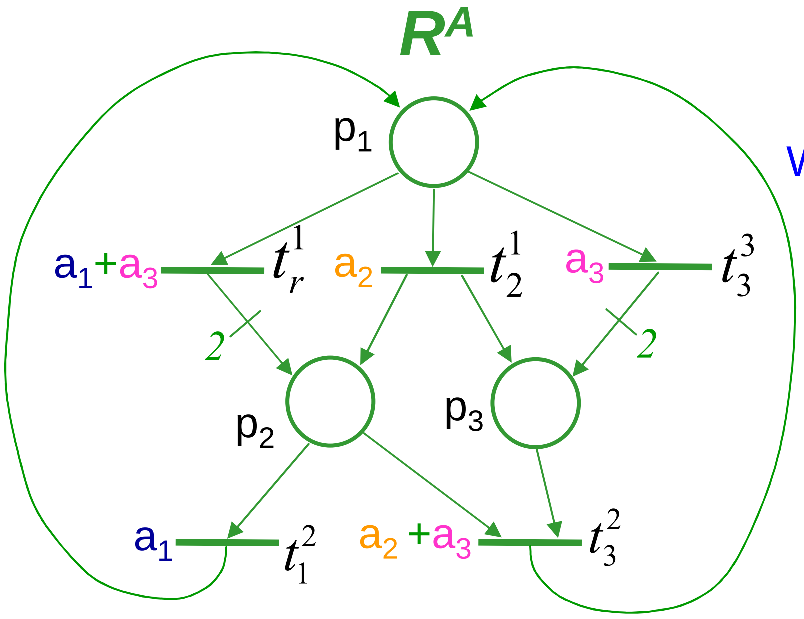
$$W(R^A)=\begin{array}{cccccc} t_r^1 & t_2^1 & t_1^2 & t_r^2 & t_3^3 & \\ \hline -1 & -1 & 1 & 1 & -1 & p_1 \\ 2 & 1 & -1 & -1 & 0 & p_2 \\ 0 & 1 & 0 & -1 & 2 & p_3 \\ \uparrow & a_2 & a_1 & \uparrow & a_3 & \\ a_1+a_3 & & & a_2+a_3 & & \end{array}$$

2. Decision problem based on an undefined Petri net

Ejemplo 3 **AAPN and compound PN**

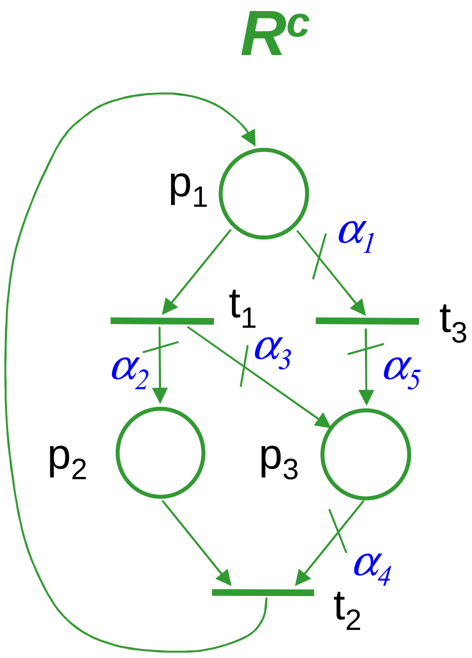
Comparison.

Graphical and algebraic representations:



$$W(R^A)=\begin{array}{ccccccc} & t_r^1 & t_2^1 & t_1^2 & t_r^2 & t_3^3 & \\ & \left(\begin{matrix} -1 \\ 2 \\ 0 \end{matrix}\right. & \begin{matrix} -1 \\ 1 \\ 1 \end{matrix} & \begin{matrix} 1 \\ -1 \\ 0 \end{matrix} & \begin{matrix} 1 \\ -1 \\ -1 \end{matrix} & \left.\begin{matrix} -1 \\ 0 \\ 2 \end{matrix}\right) & \begin{matrix} p_1 \\ p_2 \\ p_3 \end{matrix} \\ & a_1+a_3 & a_2 & a_1 & a_2+a_3 & a_3 & \end{array}$$

$$W(R^C)=\begin{array}{cccc} t_1 & t_2 & t_3 & \\ \left(\begin{matrix} 1 \\ -\alpha_2 \\ \alpha_3 \end{matrix}\right. & \begin{matrix} 1 \\ -1 \\ -\alpha_4 \end{matrix} & \left.\begin{matrix} -\alpha_1 \\ 0 \\ \alpha_5 \end{matrix}\right) & \begin{matrix} p_1 \\ p_2 \\ p_3 \end{matrix} \end{array}$$



$S_{valstr\alpha}(R^c) = \{ (0,2,0,0,0) , (0,1,1,1,0) , (1,2,0,1,2) \}$

2. Decision problem based on an undefined Petri net

AAPN and compound Petri nets

Comparison

- Size of the incidence matrices: greater in the AAPN.
- Size of feasible combinations of values for the undefined structural parameters: greater in the compound PN.
- Computer time needed to impose the initial conditions for a simulation: greater in the compound PN.
- Computer time needed to compute the incidence matrix: greater in the AAPN.
- Complexity of the software: higher in the compound Petri net (imposition of the initial conditions). Reuse of CPN software for AAPN.

2. Decision problem based on an undefined Petri net

Alternatives aggregation Petri nets (AAPN)

Property

- Let R^A be an alternatives aggregation Petri net.
- Let us consider an equivalence relation which states that a PN is equivalent to another one when their reachability graph is the same.
- There exists a coloured Petri net R such that:

R^A and R are equivalent.

if $S_{str\alpha}(R^A)=\emptyset \Rightarrow S_{str\alpha}(R)=\emptyset$

- As a consequence, the coloured Petri net R is a valid representation of an undefined PN.

2. Decision problem based on an undefined Petri net

Alternatives aggregation Petri nets (AAPN)

Construction algorithm of a CPN from an AAPN

Given: R^A (Alternatives aggregation Petri net)

$S_A = \{a_1, a_2, \ldots, a_{nr} \mid \exists\, a_i=1, 1 \leq i \leq n_r \wedge \text{if } a_i=1 \Rightarrow a_j=0\ \forall\, j \neq i\}$ (set of choice variables)

a) Create a set of choice colours $S_C = \{c_1, c_2, \ldots, c_{nr}\}$.

b) Create a bijection between S_A and S_C.

c) Impose the condition of monochrome initial choice colours (mutual exclusion):

$\exists\, c_i=1, 1 \leq i \leq n_r \wedge \text{if } c_i=1 \Rightarrow c_j=0\ \forall\, j \neq i$.

d) In every transition of R^A substitute every a_i by c_i. As a result, the coloured PN R will be obtained.

e) Associate a choice colour to every initial marking.

2. Decision problem based on an undefined Petri net

Alternatives aggregation Petri nets (AAPN)

Coloured PN obtained from an AAPN

- As a consequence of the equivalence between an AAPN and a CPN:
  - Results from AAPN can be applied to CPN.
  - Results from CPN can be applied to AAPN.
  - Software developed for CPN can be applied to AAPN.
  - Optimization procedures for the design of DES developed for AAPN can be applied using CPN and simulation / optimization software for CPN.

2. Decision problem based on an undefined Petri net

Alternatives aggregation Petri nets (AAPN)

Coloured PN obtained from an AAPN

- The simulation of the evolution of an AAPN can be performed through its equivalent CPN. Therefore it is not necessary any specific software.
- There are equivalent representations of an undefined PN:
  - Compound PN.
  - Set of alternative PN (simple, compound or mixed) in an amount given by the number of Bell.
  - Alternatives aggregation PN.
  - Coloured Petri net.