Distributed Control for Urban Traffic System

C. R. Vazquez, H. Sutarto, R. Boel, M. Silva

Universidad de Zaragoza
University of Ghent

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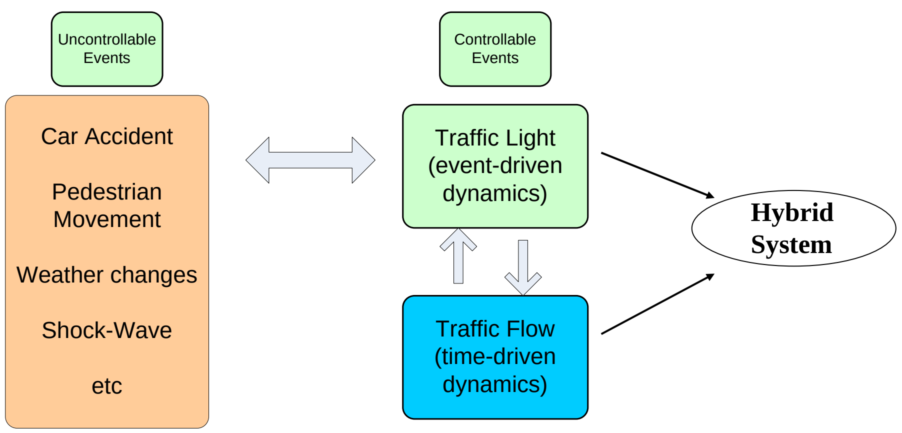
Outline:

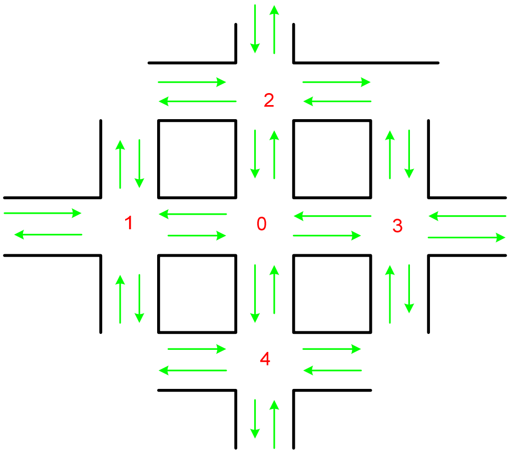
- General Ideas on Traffic Systems.
- Model of 1-Intersection.
- Optimization of 1-Intersection green periods.
- Model of 4-Intersections: Distribuited Control.
- Future work.

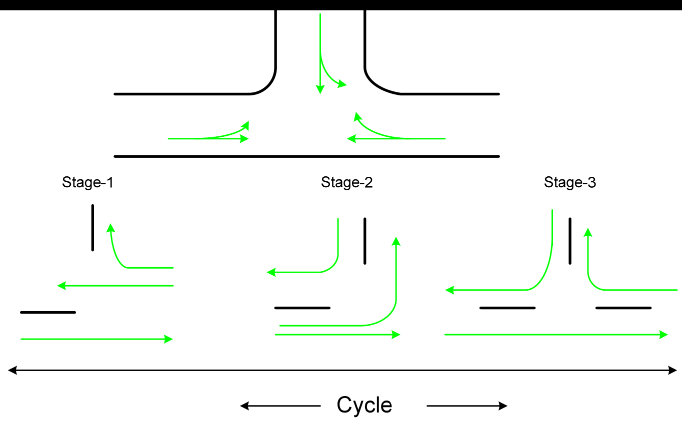
- General Ideas

General Model of an Urban Traffic System



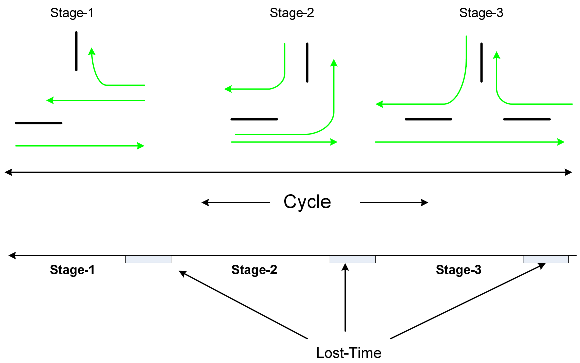
Urban Traffic System





Two streams/flows are called compatible when they can safely cross the junction simultaneously, otherwise they are called incompatible/conflicting

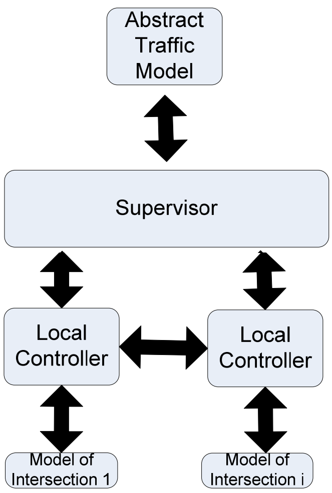
Split is the relative green duration of each stage and should be optimized according to the demand of the involved streams



Constant lost-time (transition from red-green and acceleration/decelaration of cars) are interposed between stages in order to avoid interference between incompatible stream of consecutive stages.

Offset is the time difference between cycles for successive junctions that may give rise to a green wave

Control Structure of the Urban Traffic Network



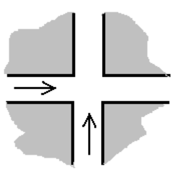
Supervisor:
-Priority for previleged vehicles.
-Can force the local controller to set a phase timing which guarantess the fulfillment of a priority request .
-Allocates different weights for different traffic streams (weight in optimization criterion).

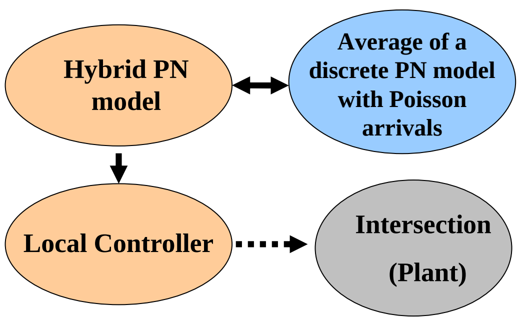
Local Controller:
-Which applies a responsive plan and acts at a single intersection with the objective of minimizing the weighted queue length for each incoming direction.

- Model of 1-Intersection

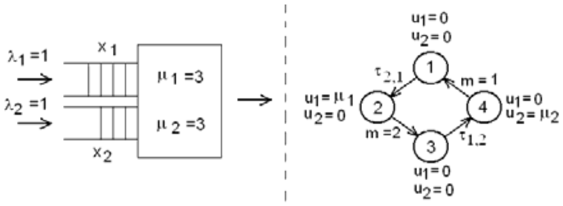
Traffic intersection model (for the local controller).

- Model of 1 intersection of two one-way streets being controlled by a traffic light.
- Assume free-flow conditions (no-congestion).
- Traffic light considers yellow periods that include the lost of time due acelerations.

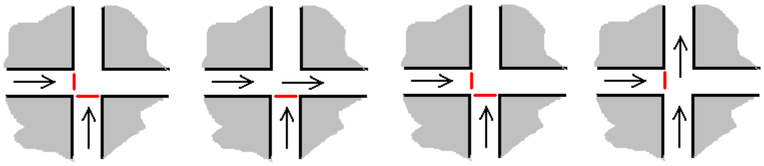




Petri net model of 1-Intersection



E. Lefeber & J.E. Rooda (2006). Controller design for switched linear systems with setups.



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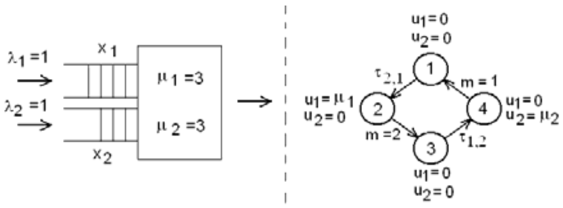

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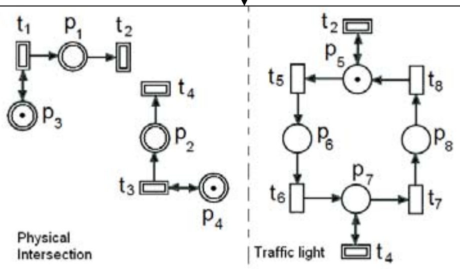
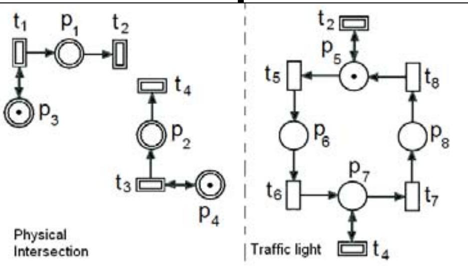


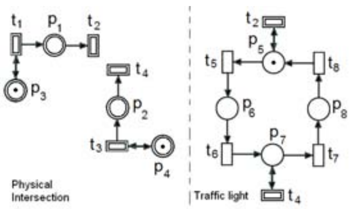
Petri net model of 1-Intersection

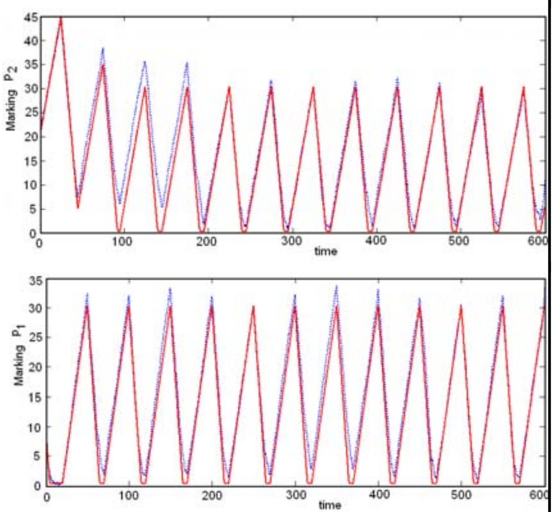
$$\dot{m}(p_1) = \lambda_1 m(p_3) - \lambda_2 \min(m(p_1), m(p_5))$$

$$\dot{m}(p_2) = \lambda_3 m(p_4) - \lambda_4 \min(m(p_2), m(p_7))$$



Petri net model of 1-Intersection

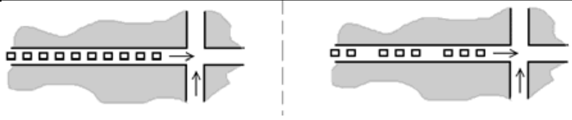


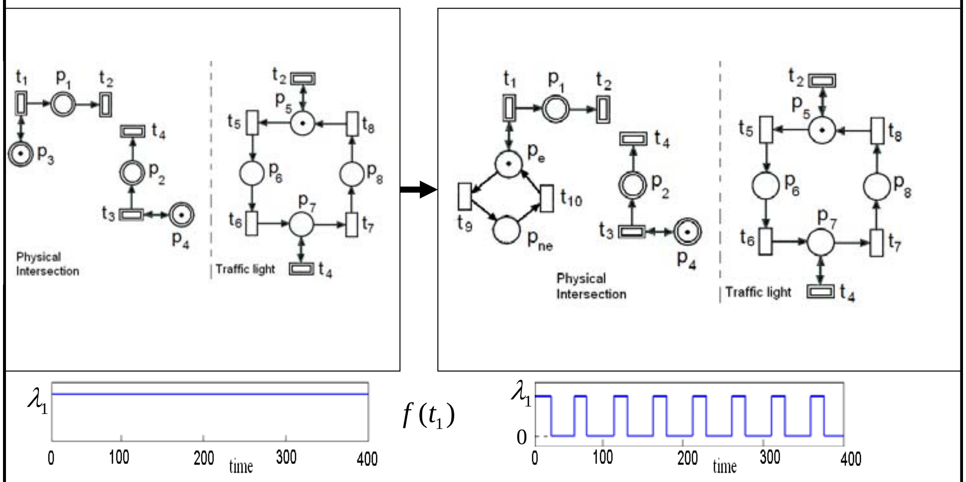


Comparison of the discrete stochastic system and the hybrid model.

Discrete average trajectories are obtained after 20 simulations.
The hybrid PN model can be used for quantitative analysis of the original discrete system.

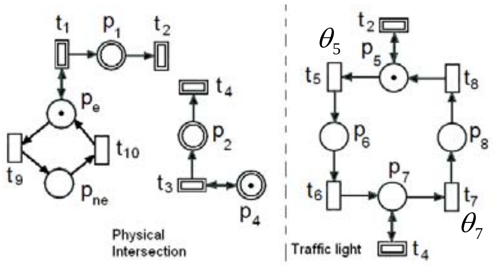
Platoon arrivals.





- Optimization of 1-Intersection green periods

Optimal green periods



Goal:

Obtain the pair $(\theta_5^{opt}, \theta_7^{opt})$ that minimizes

$$J(T, \theta_5, \theta_7) = \frac{1}{T}\int_0^T w \cdot \begin{bmatrix} x_1(\tau) \\ x_2(\tau) \end{bmatrix} \cdot d\tau$$

Optimal green periods

Fool strategy:

1) Define the set of possible combinations

$$CS = \left\{ (\theta_5, \theta_7) \in \mathbb{R} \times \mathbb{R} \mid \theta_5^{\min} \le \theta_5 \le \theta_5^{\max}, \theta_7^{\min} \le \theta_7 \le \theta_7^{\max} \right\}$$

2) Evaluate the cost function for any pair

$$J(T, \theta_5, \theta_7) = \frac{1}{T}\int_0^T w \cdot \begin{bmatrix} x_1(\tau) \\ x_2(\tau) \end{bmatrix} \cdot d\tau$$

3) Chose the pair that gives the minimum cost

$$(\theta_5^{opt}, \theta_7^{opt})$$

Optimal green periods

2) Evaluate the cost function for any pair

$$J(T,\theta_5,\theta_7)=\frac{1}{T}\int_0^T w\cdot\begin{bmatrix}x_1(\tau)\\ x_2(\tau)\end{bmatrix}\cdot d\tau$$

Can be computed very efficiently by using discrete event simulation-like algorithm!

The hybrid model is determinist, then, given a pair (θ_5,θ_7) , it is easy to compute the time at which discrete event occurs, and also, since the queues evolve as a continuous PN system when the discrete state is fixed, it is also easy to obtain expresions for computing the queues and cost fuction at each discrete event ocurrence in parametric form.

Initialize $\tau=0$ $J_{ac}=0$

While $\tau=0\le T$ do

Compute the remaining time at the current discrete state: $\Delta\tau$

Compute the queues at $\tau+\Delta\tau$ by evaluating the expressions obtained off-line.

Compute the incremental cost ΔJ by evaluating the expressions obtained off-line.

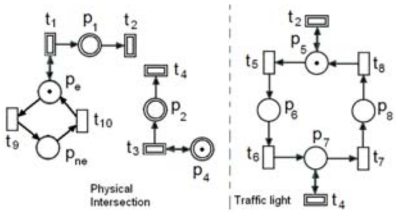
Add the incremental cost: $J_{ac}=J_{ac}+\Delta J$

Actualize the time: $\tau=\tau+\Delta\tau$

Fire the next discrete transition

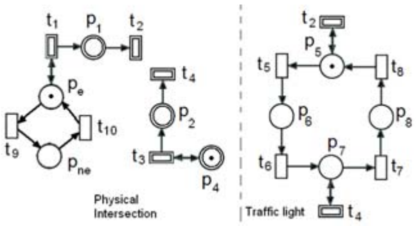
End while

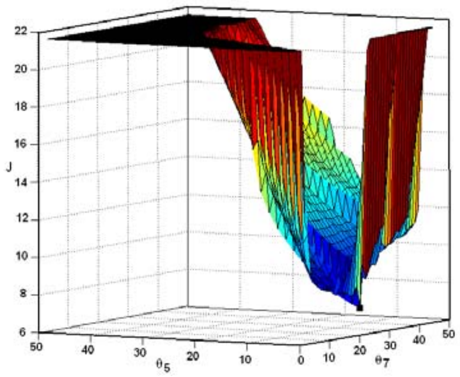
The total cost fuction is: $J(\tau)=\frac{1}{\tau}J_{ac}$



$$x_2(\tau+\Delta\tau)=x_2(\tau)+\Delta\tau\cdot\lambda_3$$

Example





Given rates $[1,3,1,3]$ for $\{t_1, t_2, t_3, t_4\}$, delays $[10,30]$ for $\{t_9, t_{10}\}$

and $[5,5]$ for $\{t_6, t_8\}$

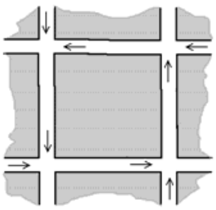
A time horizont $T = 1200$ and weights $w = [1,1]$

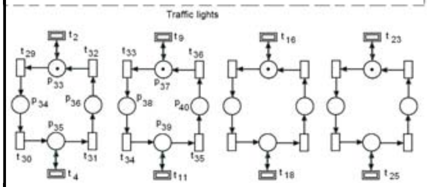
An optimal pair was obtained $\left(\theta_5^{opt}, \theta_7^{opt}\right) = (4, 27)$ with a cost of $J = 7.18$

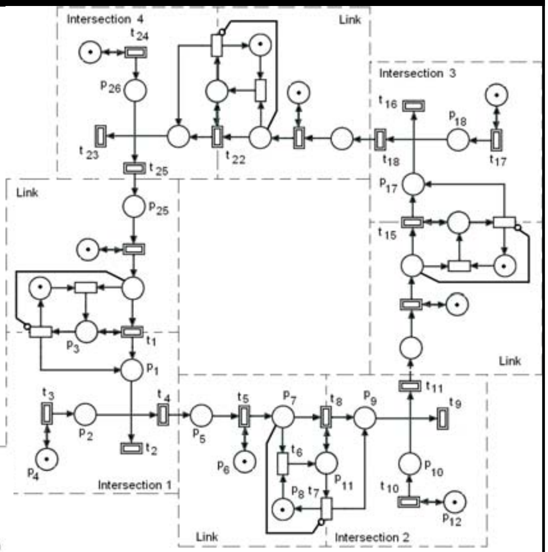
- Model of 4-Intersections:
  Distribuited Control.

4-Intersections

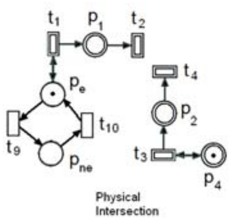
Four intersections connected by links (streets) that introduce pure delays

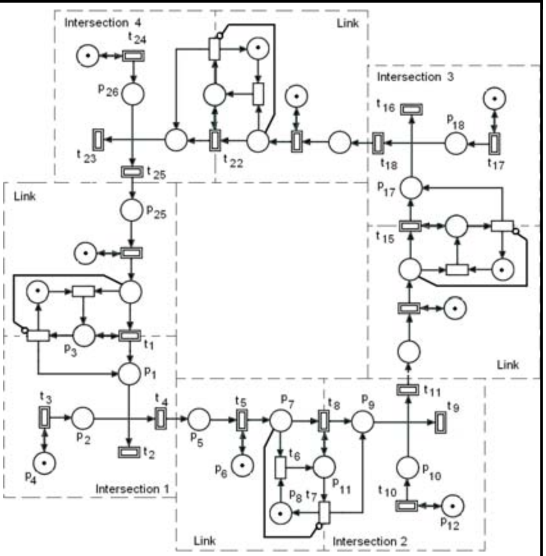




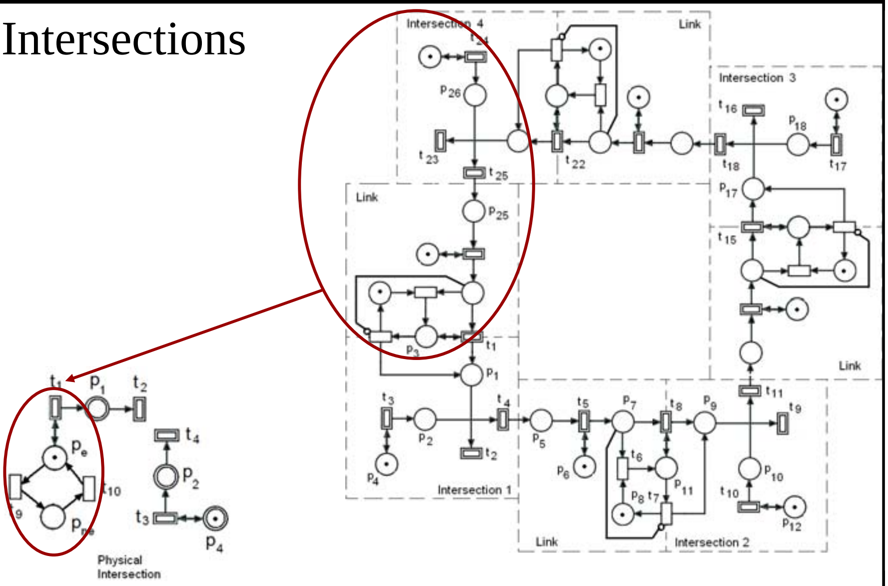


4-Intersections

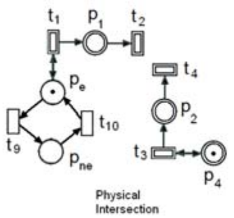


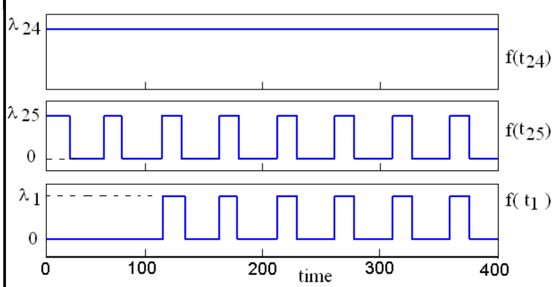


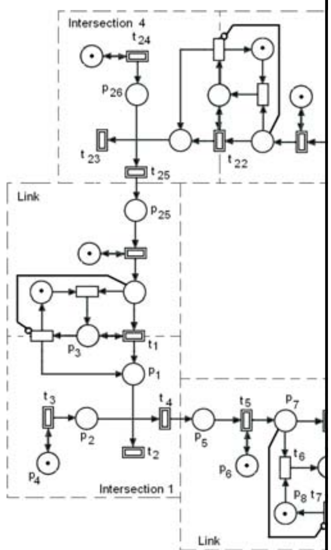
4-Intersections



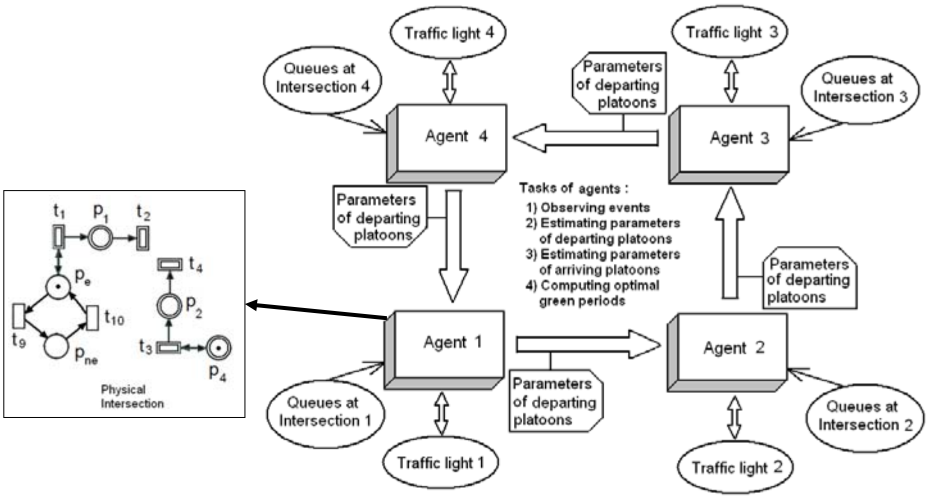
4-Intersections







Distribuited control strategy



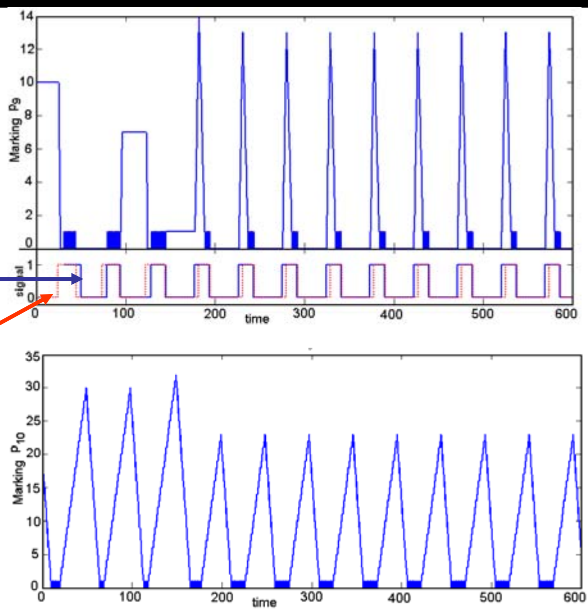
Example 1

Controlling the traffic light of intersection 2. Others remain with constant delays.

Incoming platoon

Green period

Control law was computed and applied every 5 t.u. (each computation takes 2.9 sec), with an horizont of 110 time units.



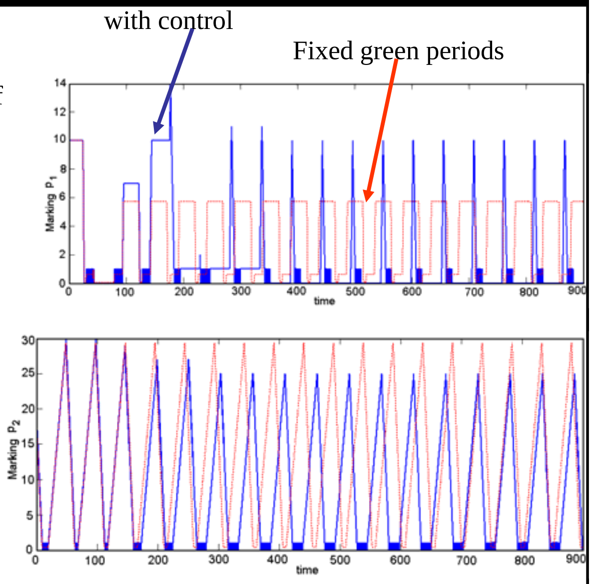
Example 2

Controlling the traffic lights of all intersections.

Control law was computed and applied simultneaosly every 5 t.u. (each computation takes 3.8 sec), with an horizont of 150 t.u.

The cost with T=900 and w=[1,1] for each intersection was 11.39. As a comparison, the cost with fixed green periods is 16.57.

$$J(T,\theta_5,\theta_7) = \frac{1}{T}\int_0^T w \cdot \begin{bmatrix} x_1(\tau) \\ x_2(\tau) \end{bmatrix} \cdot d\tau$$



Future work

1) Improve the optimization at each intersection.
2) Improve the estimation of parameters of incoming platoons.
3) Extended the 1-Intersection model in order to consider incoming platoons to both queues.
4) In a distributed control scheme of several intersections, investigate the efects of computing and aplying green periods with delays (not simultaneosly).
5) Simulate more realistic models:
   a) With perturbations.
   b) With real data (parameters).
   c) With several intersections.

- **Thanks for your attention!**

Questions?