Operation Planning of Elective Patients in an Orthopedic Surgery Department

Daniel Clavel*, Cristian Mahulea*, Jorge Albareda[†], Manuel Silva*

*Aragón Institute of Engineering Research (I3A), University of Zaragoza, Maria de Luna 1, 50018 Zaragoza

{clavel,cmahulea,silva}@unizar.es.

[†]Orthopedic Surgery Department, University Hospital "Lozano Blesa", Zaragoza, Spain

jcalbareda@salud.aragon.es

Abstract—This paper considers the operation scheduling and planning of elective patients in the Orthopedic Department of the "Lozano Blesa" Hospital in Zaragoza. We assume an ordered list of patients that should be planned for surgery in two available rooms, each room being possible to be used for a specific duration per day. Based on the average durations of surgeries that have been computed by considering historical information, we propose a Mixed Integer Linear Programming (MILP) problem to obtain a specific utilization rate per room. We have developed a Decision Support System (DSS) base on MILP that helps doctors in their daily planning. The results are tested on some real data from the hospital and some simulation results are provided.

I. INTRODUCTION

Linear programming models were developed during World War II to make plans or proposals of time for training, logistics or deployment of combat units. After the war, many industries began to use it in their daily planning. Subsequently, it was observed that, through proper system modeling, linear programming can be applied to different fields.

In this paper, we use mathematical programming to model the planning of non-urgent surgeries in a hospital department. This can be seen as the planning of a production system: (a) there is a waiting list of patients representing the system demand and (b) there exists a limited number of surgeons and a limited number of operating rooms (OR) representing the capacity of the production system. For our application, the bottleneck of the resources are the OR. In particular, there exists two OR for non-emergency surgeries in the Orthopedic Department, each one being available only a certain number of hours per day. Furthermore, the operating room is the most expensive and limited material resource in a surgical service, being therefore extremely important to obtain its maximum performance. The operating rooms have permanently human resources and if the maximum performance is not obtained, the mentioned staff do not have committed labor but they have nevertheless economic resources consumption. The purpose of the mathematical model that we propose is to optimize the use of the OR. The different surgeries have associated averages durations, but there are uncertainties due to uncontrollable

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factors such as unforeseen events or the different nature of each body. For this reason it is considered that an acceptable performance is obtained when the ORs are scheduled near 80 percent of occupancy. The actual performance in the Orthopedic Department is 76 percent (in 2015), being the actual objective to perform an operation planning with a performance of about 80 percent. A defect as an excess in the occupation rate of OR respect the objective programing is a system default. Defects imply the consumption of human resources without their utilization, and excess means that staff could lengthen their working day. In order to solve this problem, we obtain a mathematical model based on integer programing that allows doctors to plan the surgeries with a determined occupation rate respecting, as much as possible, the order of patients in the waiting list.

This mathematical model will be used to propose a Decision Support System (DSS) to be used in the management of the Orthopedic Department of the "Lozano Blesa" Hospital in Zaragoza. The DSS can be used to: (a) optimize the use of OR and computerizing the surgeries allocation method, (b) estimate the necessary ORs to perform all surgeries in the waiting list and, (c) dynamic updating the input parameters of the model to improve the solution.

The operation planning and scheduling of elective patients is a problem studied in literature by many researchers. For a state of the art of the problem we can refer the reader to the survey [1] and the references herein. Moreover, some works combine the planning and scheduling problem of elective patients with the urgent ones [2], [3] by using stochastic models. The scheduling and planning of resources have been studied for other problems, as for example home care services [4], [5]. Petri net models have been used for modeling and management of healthcare systems, see for example [6]–[9]. The contributions of this paper with respect to the previous results are: (1) the application of the mathematical programming models to the particular problem in the "Lozano Blesa" Hospital, (2) simulation results using real data; and (3) the DSS for the scheduling and planning of operations of the elective patients.

The paper is organized as follows. Sec. II describes the organizational structure in the Orthopedic Department of the "Lozano Blesa" Hospital in Zaragoza. Sec. III shows the proposed MILP problem for planning the surgeries room. In Sec. IV some results obtained by implementing the model in

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a computer software (CPLEX) are analyzed. The structure, operation and features of DSS are explained in Sec. V. Finally, in Sec. VI, we provide some conclusions and future works.

II. ORGANIZATIONAL STRUCTURE OF THE ORTHOPEDIC DEPARTMENT

The Orthopedic Department of the "Lozano Blesa" Hospital in Zaragoza, is studied. Despite this, the proposed solution can be extended and used to other departments of the same or other hospitals.

This department is composed by medical doctors (specialists and residents) divided in five medical teams. Each team has a coordinator (the more experienced doctor), each doctor having his own waiting list of patients waiting for surgery. The patients belonging to the waiting lists of the doctors belonging to a given team compose the patient waiting list of the team.

The Orthopedic Department have two OR available per day for non-urgent surgeries. Each of these OR have an active schedule from 8am to 3pm (7 hours). The Orthopedic Department is organized in such way that during a given day each OR is used by a unique team. The head of the Orthopedic Department (that is a medical doctor as well) is responsible for assigning teams: (a) to the OR, (b) to the external consultations and (c) to the emergency service. The assignment is made with a time horizon of two months, that is, all teams knows two months in advance the days in which they can use the OR.

Actually there no exists an automatic method for operating planning, so each team coordinator is guided by his own intuition and experience to plan the surgeries. During last years, using this manual planning method an occupation rate of ORs of 76 percent has been obtained. This is an excellent rate that can be improved, although not by the manual planning. The organizational structure of the surgical service of the Orthopedic Department of the "Lozano Blesa" Hospital is given in in Fig. 1.

The main objective of this work is to provide an approach based on mixed programing to help the coordinators of the medical teams in the planning task. In addition, the approach will allow the medical doctors to plan the operations with a determined occupation rate (by default it is 80 percent but this is an input parameter in our problem), respecting as far as possible the position in the waiting list (this position is obtained based on some priorities given by the medical doctors but also on the waiting time of patients in the list). It is important that the utilization of the OR does not exceed 3pm because in this case medical staff could lengthen their working day. With the propose of preventing these situations, the medical managers consider appropriated an occupancy rate of 80 percent, being the remaining 20 percent use for cleaning of operating rooms after each surgery and, on the other hand, absorb possible surgeries delays.

In this way, the problem that we solve is the following:

Problem 1: Given a list of patients that should be scheduled for surgeries and a duration d of daily time of using an operating room, schedule the next m working days of ORs

SURGICAL SERVICE OF THE ORTHOPEDIC DEPARTMENT OF THE "LOZANO BLESA" HOSPITAL

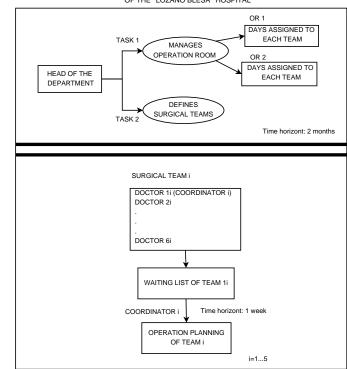


Fig. 1. Organizational structure of the surgical service in the Orthopedic Department of the "Lozano Blesa" Hospital.

with an occupancy rate of about p percent, respecting as far as possible the order of the patients in the waiting list.

In the following we illustrate the problem by an example. The team that uses an OR should operate patients from the waiting list of doctors that compose the team. In the following we assume that the waiting lists of patients of each team are known. The patients in the list are ordered according to the preference of their surgeries. In Tab. I is given an example of two patients of a waiting list. Notice that, each entry in the table has four elements:

- Preference order indicates the preference order of the surgery;
- Patient name the patient name;
- Pathology indicates the type of surgery;
- Medical Doctor surgeon who will perform the surgery.

 TABLE I

 Example of ordered waiting list for surgery.

Order	Patient	Pathology	Medical Doctor
1	Perez, Juan	KNEE ARTHROPLASTY	Pedro Suarez
2	Garcia, Maria	WALLUX VALGUS	Raquel Arrellano
n			

On the other hand, by using the history data from the last two years, for each pathology we compute the average

durations of surgeries. A duration is the time from the moment when the patient enters to the OR until he leaves the OR. Initially, these values are computed by using the data from all surgeons of the orthopedic department. However, the developed software application will update these average durations differentiate them by each medical doctor. Let us define the input numerical data of the problem V_e as a matrix of dimension $2 \times n$, where n is the size of the waiting list. The first row of V_e is the preference order (denoted also as V_{e1}) and the second row is the average duration associated with the corresponding pathology (V_{e2}). For example, if the knee arthroplasty has an average duration of 133 minutes and wallux valgus has an average duration of 112 minutes, then the first two columns of V_e matrix for the list in table I is:

$$\boldsymbol{V_e} = \begin{bmatrix} 1 & 2 & \dots & n \\ 133 & 112 & \dots & \dots \end{bmatrix} \xrightarrow{\rightarrow} \boldsymbol{V_{e1}}$$

Furthermore, the other three input parameters that will allow us to generalize the model are,

- *m*: number of days to plan;
- p: occupancy rate (by default p = 80);
- d: duration of the OR working day (by default d = 7[hours]).

Let us assume now the ordered list containing 22 patients given in Tab. II, 5 days to schedule (i.e., m = 5), a duration of the OR working day of 7 hours (i.e., d = 7[hours]= 420[minutes]) and an objective occupancy rate of 80 (i.e., p = 80).

TABLE II AN ORDER LIST OF PATIENTS WAITING FOR SURGERY GIVEN AS $V_e.$

V_{e1}	1	2	3	4	5	6	7	8	9
$oldsymbol{V}_{e2}$	111	133	145	81	150	72	121	137	97
V_{e1}	10	11	12	13	14	15	16	17	18
$oldsymbol{V}_{e2}$	150	137	121	111	111	150	97	150	149
V_{e1}	19	20	21	22					
V_{e2}	133	97	81	111					

Tab. III shows a possible solution of this particular problem. Each row of this table represents the operation planning of a working day. The first column represent the number of OR working day, the next three columns indicate the preference order of surgeries that should be operated in this working day. Finally, the last column is the occupancy rate of the OR in the corresponding working day.

This solution has been computed by solving a mixed integer linear programing (MILP) problem.

III. MILP FORMULATION

To solve the problem described in the previous section, we proposed an MILP. Let us define the following variables,

S₁, S₂, S₃,..., S_m vectors of binary variables where m is the number of working days to schedule. Each S_i ∈ {0,1}ⁿ is a vector where n is the size of the waiting list.

TABLE III

Operation planning of the list of patients given in Tab. II for an objective occupation rate of 80% and a timing horizon of 5 days.

Day	Surgery 1	Surgery 2	Surgery 3	Occupancy
1	2	4	7	79,76
2	1	5	6	79,29
3	3	9	16	80,71
4	8	12	21	80,71
5	13	14	22	79.28

 $S_i[j] = 1$ means that surgery j should be operated in the i^{th} working day;

 α ∈ R^m_{≥0} is a vector of absolute deviations (in minutes) of each day with respect to the objective occupation.

Notice that the total number of variables is $n \times m + m = (n+1) \times m$.

There exist two sets of constraints in this problem. The first constraint set is related to the definition of variables α_i , while the second set of constraints impose that each surgery is performed no more than once.

Since α_i is the absolute deviation (in minutes) of day *i* with respect to the objective occupation, assuming an occupancy rate of p = 80% and a duration of a working day of d = 7 hours, we can write:

where k_i is the total number of surgeries scheduled in the working day i and τ_{ij} is the theoretical duration of the j^{th} surgery scheduled in the working day i. Using the input array of theoretical durations (V_{e2}), we can equivalently rewrite (1) as

$$\alpha_{i} = \left| \sum_{j=1}^{k_{i}} \tau_{ij} - 336 \right| = |\boldsymbol{V}_{e2} \cdot \boldsymbol{S}_{i} - 336|$$
(2)

That is equivalent with the minimum α_i that fulfill the following constraint

$$\begin{cases} \boldsymbol{V}_{e2} \cdot \boldsymbol{S}_i - 336 \leq \alpha_i \\ \boldsymbol{V}_{e2} \cdot \boldsymbol{S}_i - 336 \geq -\alpha_i \end{cases} \quad \forall i = 1, 2, \dots, m \qquad (3)$$

Two constraints will be necessary to define each variable α_i . Since the number of variable α_i is equal to the timing horizon (i.e., m), we need $2 \times m$ constraints to define all variables α_i .

The other set of constraints ensures that each surgery will be planned at most once:

$$\sum_{i=1}^{m} \boldsymbol{S}_{i}[j] \leq 1 \quad \forall j = 1, 2, \dots, n$$

$$\tag{4}$$

Finally, the MILP has the following size

- number of variables $(n+1) \times m$;
- number of constrains $2 \times m + n$.

Like in many optimization problems there are two contradictory objectives: a) obtain the objective occupation rate of ORs b) and respect the order of patients in the waiting list. We solve this problem by a linear cost function composed by two balanced terms: the first one is related to the objective occupancy rate while the second one with the preference order of patients in the list. The two objectives are balanced by a parameter β . In particular, the objective is to minimize,

$$\sum_{i=1}^{m} \left[\alpha_i \cdot (m-i+1) + \beta \cdot \boldsymbol{V}_{e1} \cdot \boldsymbol{S}_i \cdot (m-i+1) \right]$$
 (5)

Variable α_i of the first term penalizes the deviation of the occupancy of the OR with respect to the objective p(e.g., 80%). Since (m - i + 1) is multiplying α_i , it gives more importance to obtain a better occupancy rate in the first workings days. In this way, if there are not enough surgeries for all working days, the last days remain free.

The second term is related to the order in the waiting list. The result of multiplying $V_{e1} \cdot S_i$ is the sum of the preference order of surgeries scheduled the day *i*. In this way, we give preference to the first patients of the waiting list over the patients with higher order number (in general, patients with a lower order number have a longer time in the waiting list). Again we multiply the second term by (m-i+1), this implies that the patients with lower preference order will be scheduled the first days.

Parameter β is a design parameter and it is used to give more importance of respecting the order of the patients in the waiting list or to the occupancy rate of the OR.

The full MILP is as follows:

$$\min \sum_{i=1}^{m} [\alpha_i \cdot (m-i+1) + \beta \cdot \boldsymbol{V}_{e1} \cdot \boldsymbol{S}_i \cdot (m-i+1)]$$

Subject to:
$$\begin{cases} \boldsymbol{V}_{e2} \cdot \boldsymbol{S}_i - 336 \leq \alpha_i, \quad \forall i = 1, 2, \dots, m \\ -\boldsymbol{V}_{e2} \cdot \boldsymbol{S}_i + 336 \leq \alpha_i, \quad \forall i = 1, 2, \dots, m \\ \sum_{i=1}^{m} \boldsymbol{S}_i[j] \leq 1, \quad \forall j = 1, 2, \dots, n \end{cases}$$

(6)

IV. SIMULATION RESULTS USING CPLEX

In this section we are going to test the behavior of the MILP (6). In particular we validate the model and we show the influence of the parameter β on the resulted planning. The simulations have been obtained by using the IBM ILOG CPLEX Optimization Studio which is often referred as CPLEX [10] which is an commercial solver designed to tackle (among others) large scale (mixed integer) linear problems. CPLEX is now actively developed by IBM and it is one of the fastest software solution for MILP problems [11].

Numeric input data (i.e., matrix V_e) has been randomly generated using some real data form hospital. In particular, we used a lot of average durations for different pathologies that have been operated in the Orthopedic Department in the last two years. We have generated randomly only the pathologies of the patients but we use some probabilities computed bases on the real data. In the first simulation, a waiting list of 300 patients should be scheduled in the following 60 days. MILP (6) has the following dimensions:

- number of variables: $(n + 1) \times m = (300 + 1) \times 60 = 18060$;
- number of constraints: $2 \times m + n = 2 \times 60 + 300 = 420$.

Although CPLEX is one of the most powerful tool, the computational time and memory usage of solving this instance of the problem is too high. Using a computer with an Intel Core i3 and 4 GB of memory, after 6 hours of calculation, the computation stops being out of memory.

After some simulations with different values of number of patients (n) and horizon time (m), we observed that the variable that influence more the computational time is m. Moreover, the computational time depends also on the value of the design parameter β . Against smaller is beta (that is more importance is the occupation rate), greater is the computational time. Using a value of $\beta = \frac{1}{3}$ and m = 7 working days, the average calculation time has been of 62 seconds.

In order to be able to plan all patients in the waiting list, our solution consists in solving MILP (6) iteratively (similar with receiding horizon control strategy [12], [13]): from the waiting list of patients, we are planning the first 7 working days and we consider the scheduling of the first 5 days. The patients who surgeries have been planned in these 5 workings days are removed from the waiting list and the process is repeated until the waiting list becomes null.

Usually 3 patients are scheduled for operation in a working day. So, the expected computation time to plan all patients in a waiting list of size n using the iterative approach is defined as following:

Total time =
$$\frac{n[\text{patients}]}{3\left[\frac{\text{patients}}{\text{day}}\right]} \times \frac{62[\text{sec}]}{5[\text{days}]} = \frac{n \times 62}{15}[\text{Sec}]$$

Assuming the same list of 300 patients and different values of β we have used the iterative method to plan 60 working days. The results obtained of occupation rate have been analyzed. Table IV shows a statistic analysis of planned occupation rate including the average, standard deviation and the extreme values.

TABLE IV STATISTIC ANALYSIS OF OCCUPANION RATE

β	Average	Deviation	Minimum	Maximum
3	79.278	1.554	70.952	81.667
2	80.340	1.033	77.619	81.905
1.5	80.340	0.932	77.857	81.905
1	80.340	0.719	78.333	81.667
$\frac{2}{3}$	80.183	0.703	78.333	81.667
0.5	80.238	0.624	78.333	81.190
$\frac{1}{3}$	80.238	0.621	78.571	81.905

It can be seen that by decreasing the parameter β better results of occupation rate are obtained: the standard deviation decreases and consequently the data are more concentrated around the average value. Unfortunately, this improvement is achieved by allowing a greater disorder in the operations planning. Table V shows the planning done for the first 5 working days of OR with different values of β . In these days at most 3 surgeries per day are scheduled but this is not always true and more surgeries could be scheduled.

TABLE V 5 First ORs Working Days of the planning of a same list with different values of parameter β

β	Day	Op1	Op2	Op3	Occupation
- r-	1	1	2	4	77.857
3	2	3	5	6	80.714
	3	7	8	10	80.714
	4	9	17	0	70.952
	5	12	13	14	80.714
	1	1	2	6	80.714
	2	3	5	9	81.666
2	3	7	8	10	80.714
	4	4	13	18	80.238
	5	11	12	15	79.524
	1	1	2	6	80.714
	2	3	5	9	81.666
1.5	3	7	8	10	80.714
	4	4	13	18	80.238
	5	11	12	15	79.524
	1	1	2	6	80.714
	2	3	7	10	80.714
1	3	5	8	9	81.666
	4	4	13	18	80.238
	5	11	12	15	79.524
	1	1	2	6	80.714
	2	3	7	10	80.714
$\frac{2}{3}$	3	5	8	9	81.666
5	4	4	13	18	80.238
	5	11	12	15	79.524
	1	1	2	6	80.714
	2	3	7	10	80.714
0.5	3	5	11	15	79.524
	4	4	13	18	80.238
	5	8	14	16	80.714
	1	1	2	6	80.714
	2	3	7	10	80.714
$\frac{1}{3}$	3	5	11	15	79.524
э	4	4	13	18	80.238
	5	8	9	29	80

V. DECISION SUPPORT SYSTEM FOR OPERATION PLANNING

In order to perform a rapid, efficient and dynamic operation planning we propose a decision support system (DSS). The core of the DSS is the MILP presented in Sec. III for operation planning, but also it includes other features that enable a) updating the waiting list, b) dynamic planning and, c) improving the input data by updating the average durations.

A. Updating the waiting list

In general, a new patient is added at the end of the waiting list but the surgeon, depending on the priority of the pathology of the patient, could decide to put him in a higher position of the waiting list. The DSS, based on medical criteria, automatically create the ordered waiting list of patients. Each patient have 2 parameters that influence directly in his/her position in the waiting list.

- 1) The first and the most important one is the time waiting for surgery. This time is calculated as the difference in days between the actual day and the day that the patient was introduced in the list. The patient with highest number of waiting days, have a score of 10 while the newest patient has a score of 0. The other patients have a proportional score between 10 and 0. This score denoted as S_1 have a weight in the calculation of total score (denoted S_T) of p_1 .
- 2) The second parameter has to do with the priority of the surgeries. Although the DSS schedules non-urgent surgeries, there exist 3 levels of priority 1, 2 and 3 with a corresponding score (S_2) of 0, 5 and 10, respectively. The weight of S_2 in the computation of S_T is p_2 .

Assuming $p_1 = 0.7$ and $p_2 = 0.3$, the final score that allows to order the waiting list is obtained as follow:

$$S_T = p_1 \cdot S_1 + p_2 \cdot S_2 = 0.7 \cdot S_1 + 0.3 \cdot S_2.$$
(7)

Finally the patients are ordered according to their total score. The patient who has the highest total score will be the first in the waiting list, while the patient who has the lowest punctuation will be the last one in the waiting list.

B. Iterative planning

Using the average duration for each surgery, the coordinator of each medical team perform the operation planning for the next m working days (this is done by solving MILP (6)). Next, the team coordinator assigns the planned working days to the available dates assigned by the head of the department. Then the secretary calls the patients scheduled in the following mdays. Once all patients have been called, the secretary give back to the team coordinator the list of patients that have been confirmed and the ones that cannot be contacted (or they cannot be hospitalized in the following days due to external reasons). In this moment, the team coordinator should schedule again the empty gaps. This process is repeated until the next m workings days are completely scheduled.

Once the first planning has been computed and secretary confirms the attendance or the absence of patients, constraints will be added to the MILP (6) and the planning will be iterated. This addition of constraints can be seen also as a reduction of number of variables with respect to the initial problem since the new constraints fix the values of some variables. If a patient with preference order j confirms the attendance in day i, then the following constraint is added:

$$\boldsymbol{S}_i[j] = 1. \tag{8}$$

However, in case that a patient with preference order j cannot be contacted or he/she cannot be hospitalized, then the following m constraints are added:

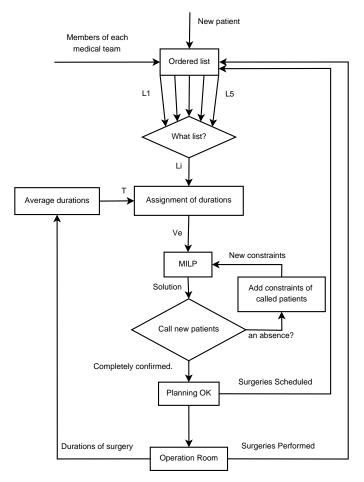


Fig. 2. Flowchart of the DSS for operation planning.

$$\boldsymbol{S}_i[j] = 0, \forall i = 1, \dots, m. \tag{9}$$

C. Updating and customizing the average durations

The average duration of each type of surgery has been computed using historical data obtained during last two years in the hospital department. During a period of two years, it is possible to obtain a sufficiently high number of surgeries and the average durations are representative. However, there exist significant differences between the different surgeons. Moreover, for each surgeon, these average durations are improved each year because after performing the same surgery several times the surgeon has more experience. Therefore, it is very important to dynamic update also these input values.

After each surgery, the time spent by the surgeon that performed the surgery, will be registered in a database. The DSS updates the average durations of the operations of the surgeon.

D. Overview of the DSS

The flowchart of the DSS is given in Fig. 2 and starts by adding a new patient to the waiting list. Each surgeon has his own waiting list while the waiting list of the medical team

is composed by the fusion of the lists of the surgeons that compose the team. Each surgeon is responsible for introducing their patients in the DSS. The method to add a patient belonging to a determinate surgeon is as follows: the DSS recognizes the surgeon (using a personal password) and he/she enters the name of the patient, the pathology, the priority of the surgery and the employment status. Additionally, the DSS saves the information of the actual date in order to compute the waiting time in the list and the surgeon that have to perform the surgery. Medical teams are not always composed by the same surgeons, so they should be periodically updated. When a team coordinator decides to plan the next m working days, he selects in the DSS the waiting list of his team and automatically the tool assigns average theoretical durations to each surgery based on the pathology and on the surgeon. In this way, the vector V_e is generated and the DSS performs an operation planning in an iterative way (as is described in V-B). The states of patients that have been scheduled change from pending to schedule. Once a specific surgery is performed, the surgeon introduces the operating time in the tool. This new input data is used to update the average duration (as is described in V-C). Additionally, the tool removes the patients that have been operated from the waiting list. If finally a scheduled surgery is not performed, the DSS changes the state of this surgery from scheduled to pending.

VI. CONCLUSIONS

By modeling and solving MILP (6) it is possible to perform surgical operations planning of elective patients with the objective of maximizing the occupation rate of the operating rooms. The simulations performed conclude that the computational time of solving MILP (6) is affected mainly by the number of planning days. To schedule more than 8 days in *a reasonable time* it is necessary to solve the problem iteratively. The design parameter β establishes a compromise between the occupation rate and the order of patients in the waiting list. Moreover, this paper presents a DSS, that by using MILP (6), helps the managers in the operation planning and scheduling. It includes features enabling dynamic planning and automatic improvement of the input data. The DSS has been tested by using real data from the hospital and as future work we plan to implement the DSS and integrate it in the hospital department.

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