Lesson 1: Thresholding

1. Introduction

2. Thresholding using the histogram
   - P-tile
   - Modal methods
   - Iterative methods
   - Adaptive methods
   - Variable methods

3. Histogram + image
   - Double
   - Weszka

1. Introduction: binary images

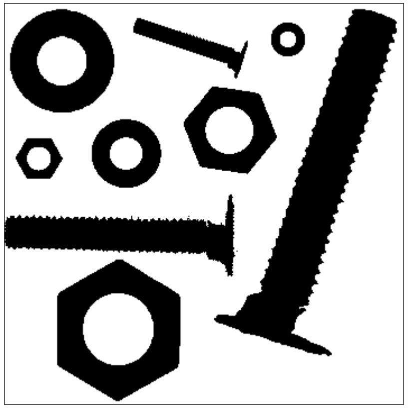
- **Problem:** limited processing capacity

Higher resolution and quantization levels, imply more data

- **Advantages of binary images:**
  - More efficient algorithms
  - Special codifications to reduce storage
  - Logical operators instead of integer arithmetic
  - Techniques applicable to gray level vision
- **Examples:**
  - Planar object recognition
  - Character recognition
  - Chromosome analysis

- **Premise:** the silhouette of the objects must describe them sufficiently

Gray level

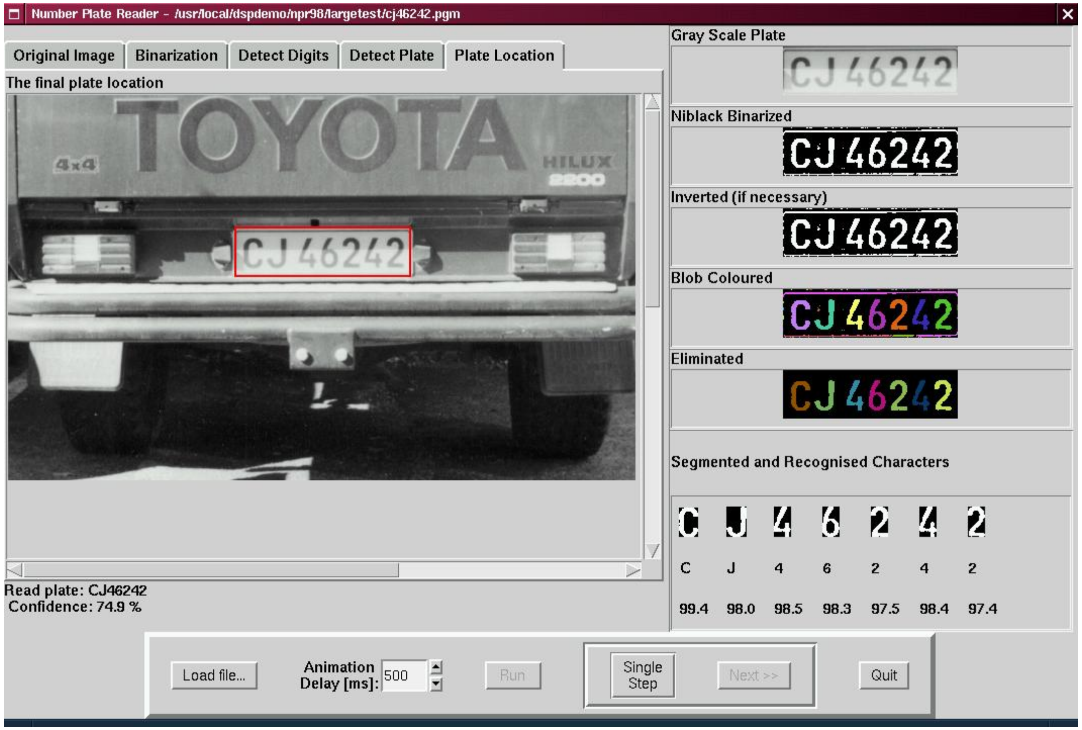


Binary

How can we determine the thresholding level to use?

Example: London Toll Video System

- 688 video cameras in 203 places
- 40,000 vehicles per hour



Histogram

- **Definition:** the histogram of a digital image **RxC**, with gray levels **k** ∈ **[0..L-1]**, is a discreet function:

  n_k : no. of pixels of level k
  n : total amount of pixels

- **p(k)**: relative frequency of gray level **k** (it can be considered an approximation to the probability of occurrence of **k**)

$$n = \sum n_k = R \times C$$

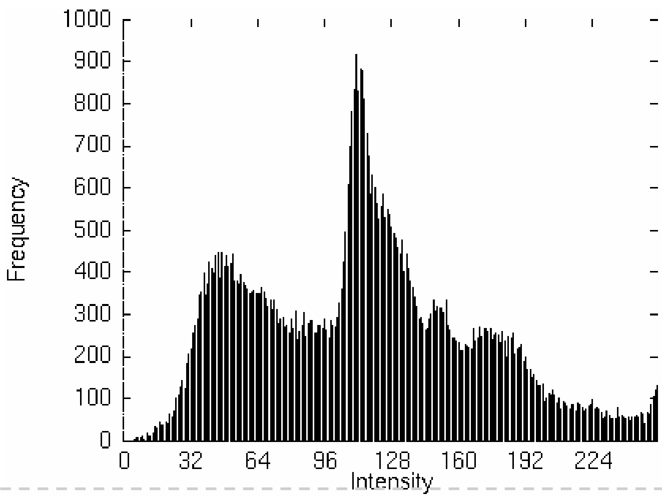
$$\sum p(k) = 1$$

- **Representation:** vector **h** of range **[0..L-1]** where:

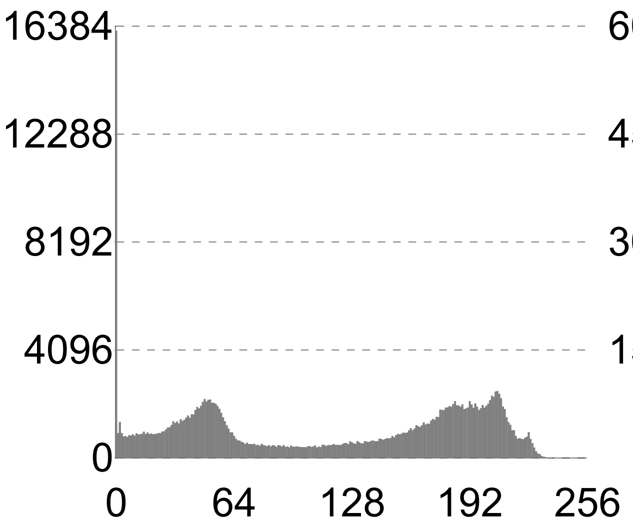
$$h[k] = \#\{(i,j):I(i,j)=k\}$$

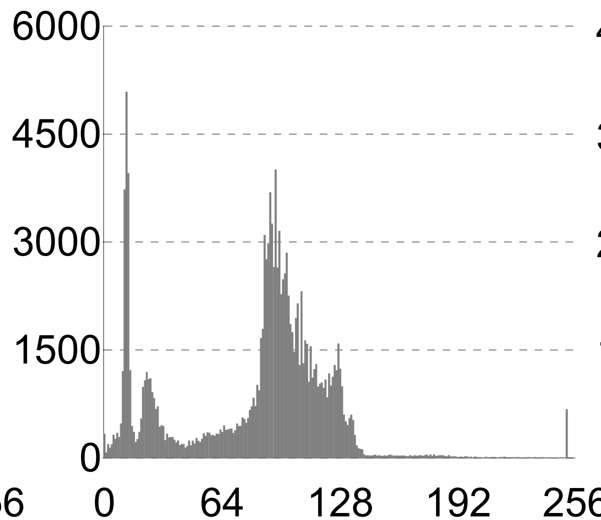
$$i \in [0..R-1]$$
$$j \in [0..C-1]$$
$$k \in [0..L-1]$$

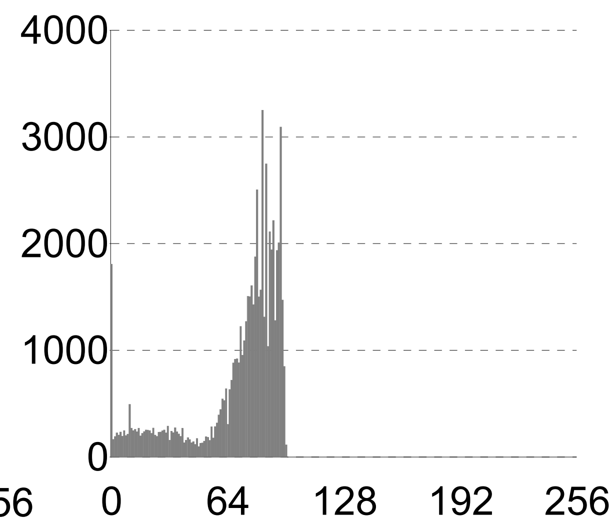
- Global description of the image
  - Information loss?
  - Unicity?



Histogram-image correspondence?







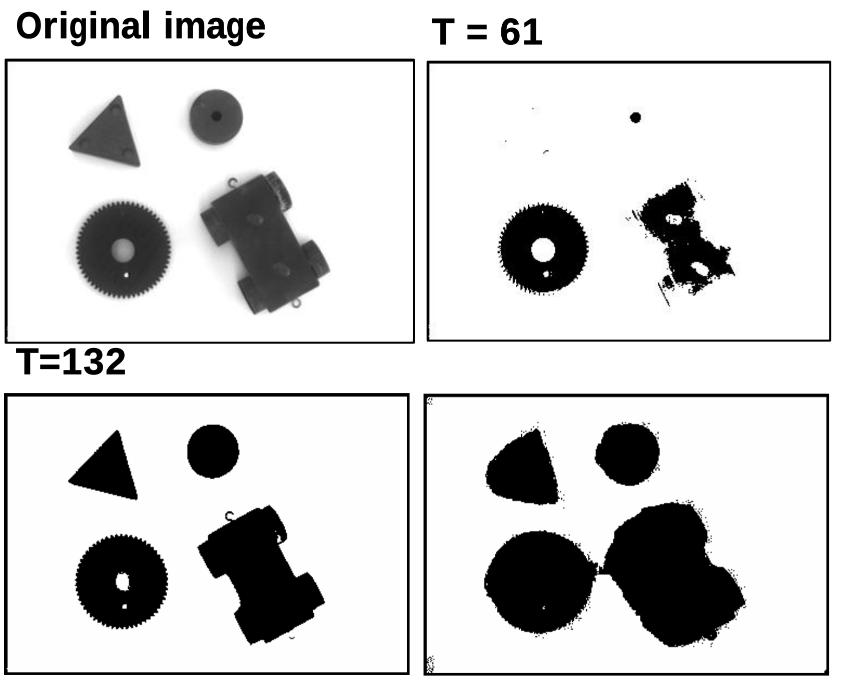
2. Thresholding using the histogram

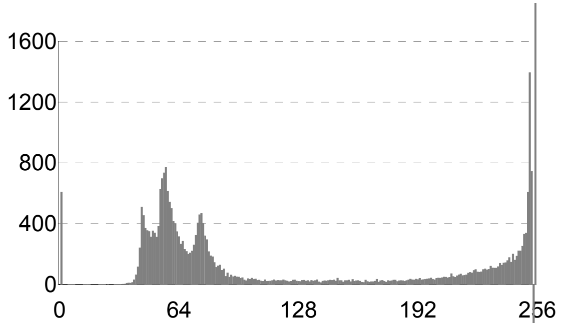
- **Goal:** acquisition of images of 256 levels of gray; separate those belonging to objects from those corresponding to the background.

- **Convention:**
  - **0** : object pixels (black objects)
  - **1** : background (white background)

One threshold T :

$$B(i,j) = \begin{cases} 0, & I(i,j) \le T \\ 1, & \text{otherwise} \end{cases}$$

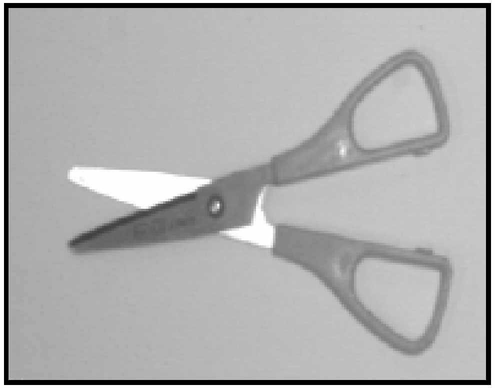


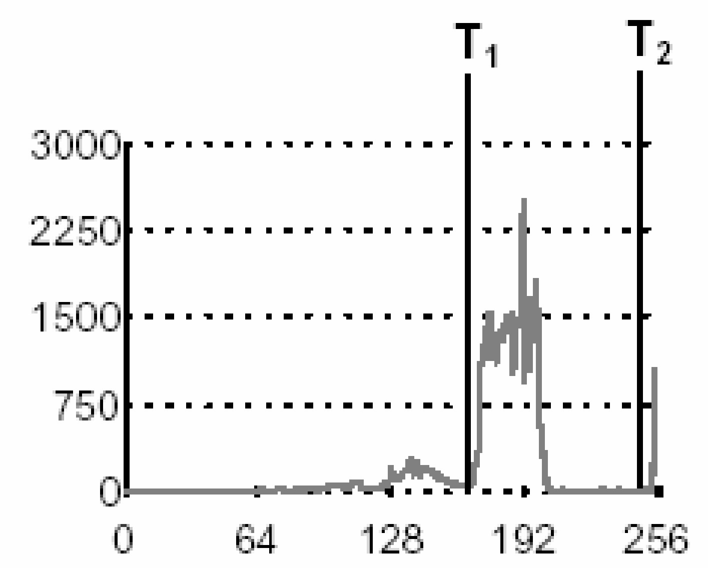


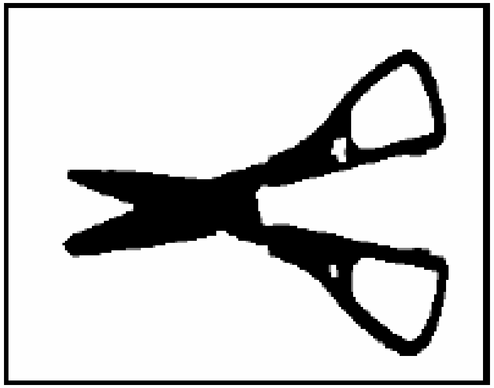
2. Thresholding using the histogram

interval $[T_1, T_2]$:

$$B(i,j) = \begin{cases} 0, & T_1 \leq I(i,j) \leq T_2 \\ 1, & \text{otherwise} \end{cases}$$







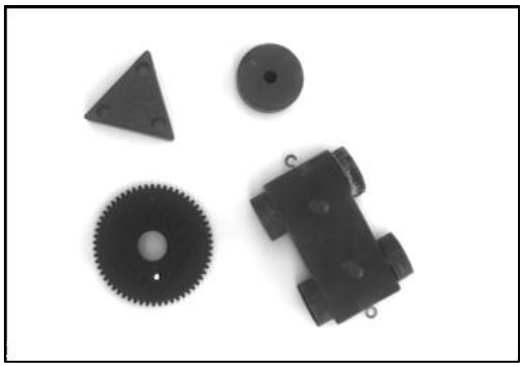
Several intervals Z:

$$B(i,j) = \begin{cases} 0, & I(i,j) \in Z \\ 1, & \text{otherwise} \end{cases}$$

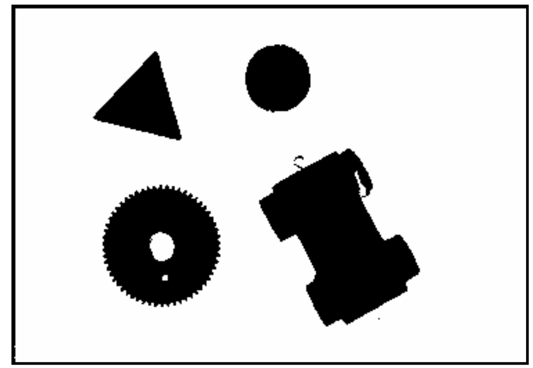
$$Z = \{(T_i, T_j)\}$$

Manual .vs automatic computation of thresholds

- **Manual computation.-** specific prefixed value
  - Controlled illumination
  - Constant background
  - Contrast between objects and background



Threshold = 109

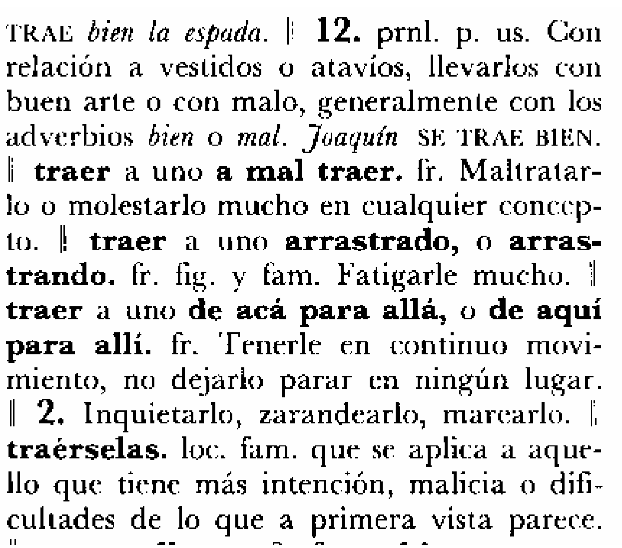


- **Automatic computation**
  - Robustness
  - Some prior knowledge required
    - » Object luminosity
    - » Object size
    - » Occupied fraction of image
    - » Number of objects present

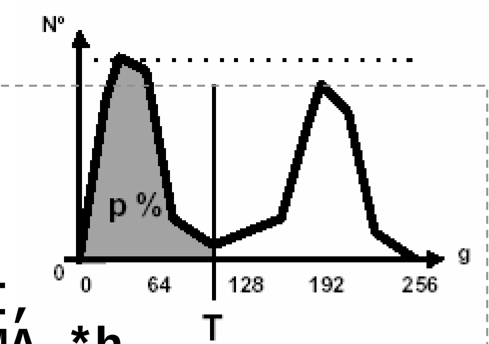
¿?

P-Tile

- If it is known that **p%** of the image is occupied by the objects of interest, the threshold is chosen so that the **p%** darkest pixels are classified as object pixels.



TRAE *bien la espada*. ‖ **12.** prnl. p. us. Con relación a vestidos o atavíos, llevarlos con buen arte o con malo, generalmente con los adverbios *bien* o *mal*. *Joaquín* SE TRAE BIEN. ‖ **traer** a uno **a mal traer.** fr. Maltratarlo o molestarlo mucho en cualquier concepto. ‖ **traer** a uno **arrastrado,** o **arrastrando.** fr. fig. y fam. Fatigarle mucho. ‖ **traer** a uno **de acá para allá,** o **de aquí para allí.** fr. Tenerle en continuo movimiento, no dejarlo parar en ningún lugar. ‖ **2.** Inquietarlo, zarandearlo, marearlo. ‖ **traérselas.** loc. fam. que se aplica a aquello que tiene más intención, malicia o dificultades de lo que a primera vista parece.




```
#define L 256
...

int p_tile (IMAGEN *I,
            HISTOGRAMA *h,
            float p)
{
  int i = 0,
      count = 0,
      n = ROWS (*I) * COLS (*I) * p;

  while ((i < L) && (count < n)) {
     count += h[i];
     i++;
  }
  return i-1;
}
```

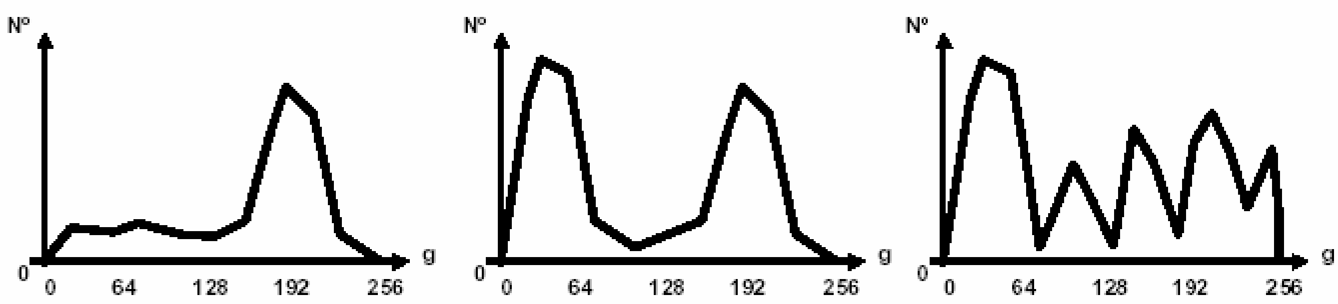
Select the n darkest

Number of pixels That make p%

- In a printed page, between 92% and 95% of the page is empty.
- Usefulness is limited to systems of recognition of printed characters.

Modal methods

- **Unimodal distribution:** There is little contrast between the objects and the background. The histogram contains all the information and there is only one dominant peak (usually the background).
- **Bimodal distribution:** Objects appear in clear contrast with respect to the background, and all have the same gray level distribution.
- **Multimodal distribution:** Several highly contrasted objects with different gray level distributions. The shape of the histogram contains several hills and valleys of separation.



Histograma Unimodal

Histograma Bimodal

Histograma Multimodal

Modal methods

- If all objects are the same color, the background has uniform color, and the image noise is Gaussian $N(0,\sigma^2)$:

- Object pixels are $N(\mu_1,\sigma_1^2)$
- Background pixels are $N(\mu_2,\sigma_2^2)$

A possible algorithm:

1. Find the highest peak:

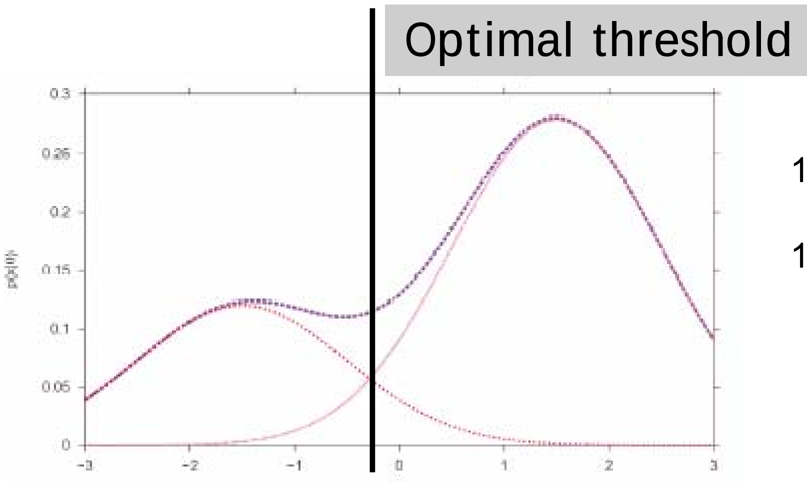
$$p = k{:}\max\{h[k]\}$$

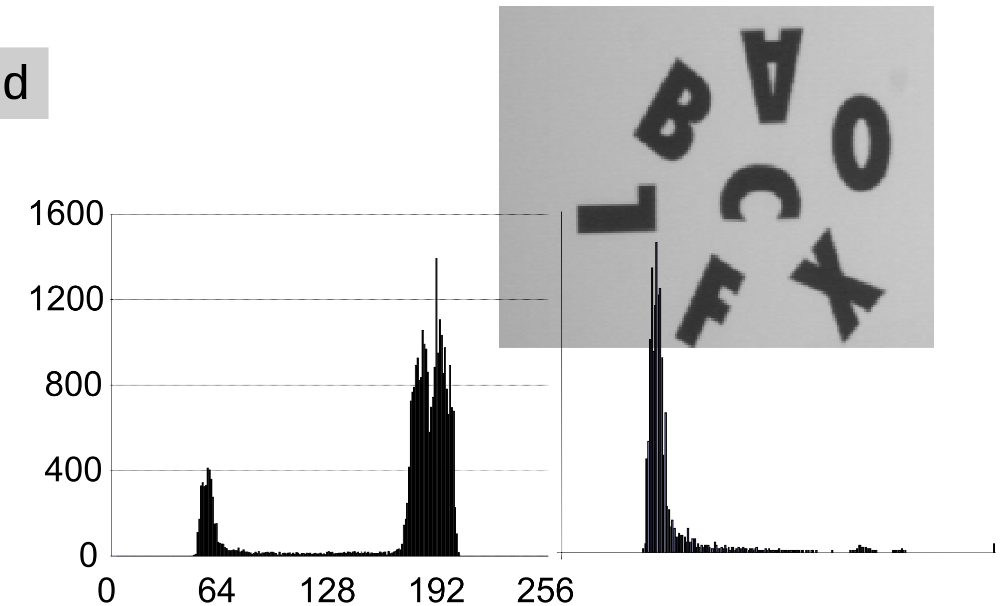
2. Find the second:

$$q = k{:}\max\{(k-p)^2 h[k]\}$$

3. Find the valley between them

Avoids values Close to p





- The purpose is to find the valley between the two main peaks.

- Generalisable to n objects $N(\mu_\iota,\sigma_\iota^2)$ with background $N(\mu_0,\sigma_0^2)$

Gaussian search

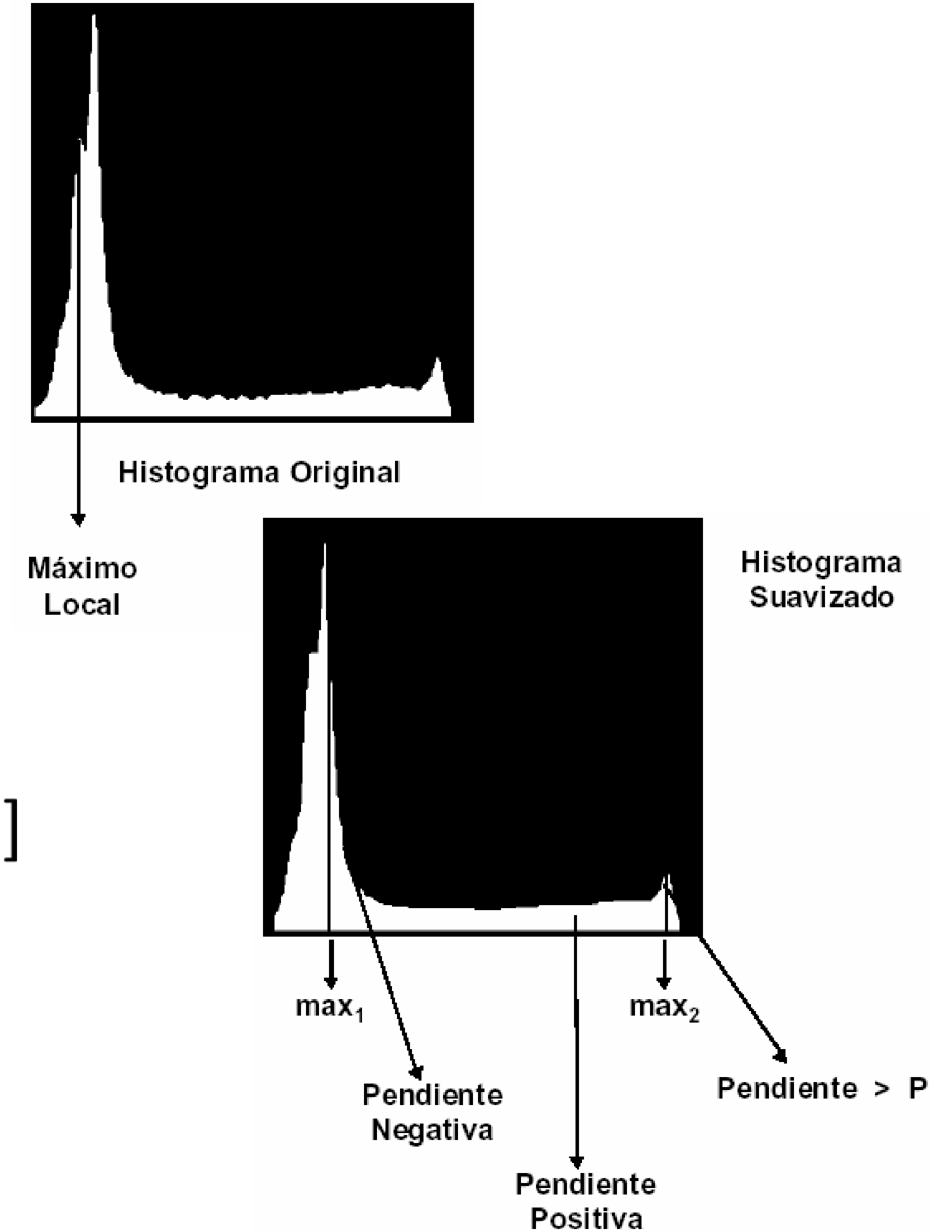
- **Step 1:** smoothing to eliminate local peaks and noise. Local maxima should not change.

- A simple algorithm: mean of the histogram values within a window of size W =3 o 5 (odd).

$$h_s[b] = \frac{1}{W} \sum_{w=-V}^{V} h[b-w]$$

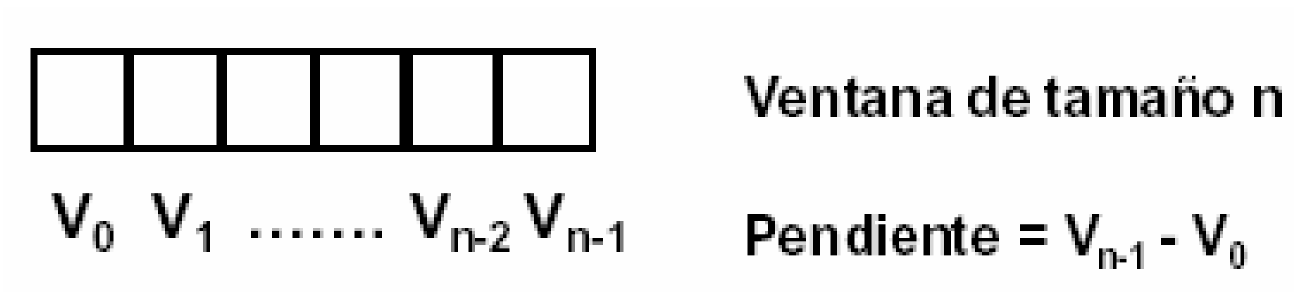
$$V = (W-1)/2$$

$$W \quad \text{odd}$$



Gaussian search

- **Step 2:** starting from the leftmost maximum, traverse the histogram computing the slope in a window of a given size, until the slope becomes larger than a certain threshold P of positive slope.
- **Step 3:** When the slope becomes higher than the threshold P, the final threshold T is the mean position of the window at that instant.

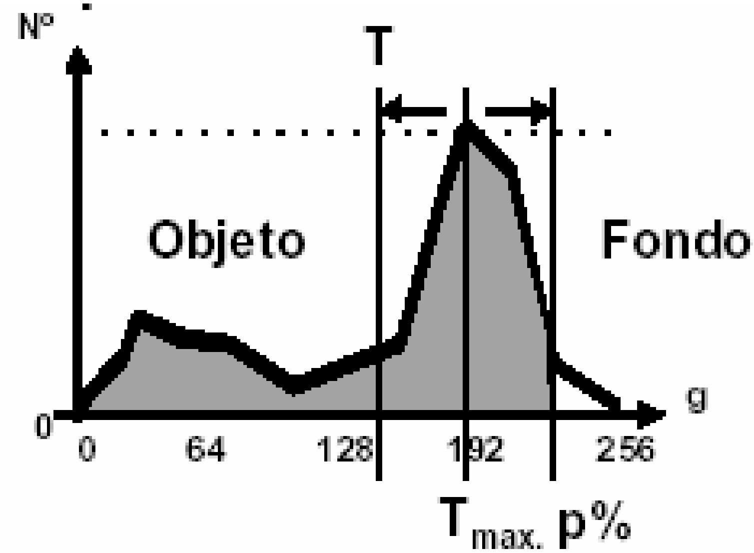


Background simmetry

Assumes that the background forms a symmetric and dominant peak in the histogram.

1. Smooth the histogram.
2. Obtain the global histogram maximum **Tmax** (background).
3. Search from the right (the side opposite to the objects) for the point corresponding to **p%** of the histogram (f.e. **95%**).
4. Given that the background peak is assumed symmetric, take as threshold the maximum minus a displacement equal to that of the point of **p%**:

```
T = Tmax-(p% - Tmax)
```

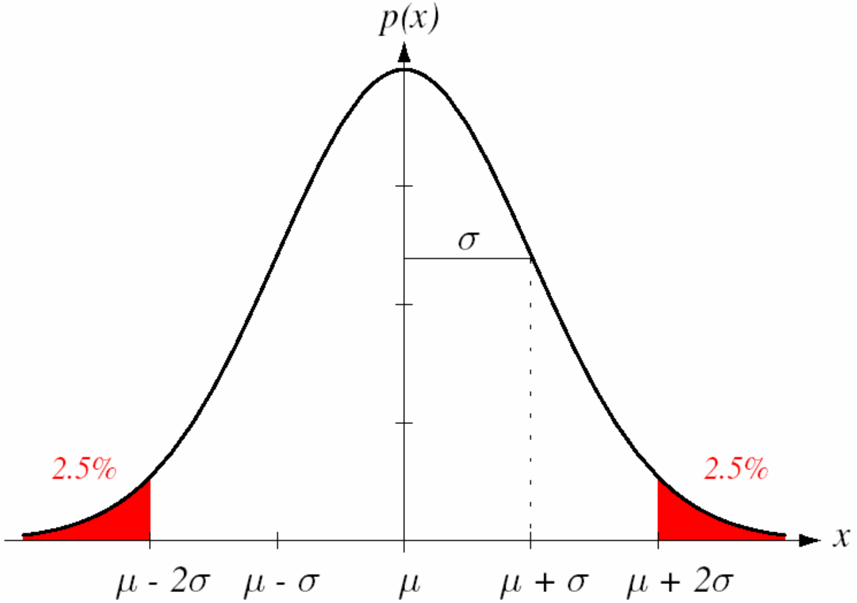


- Assumption: dark objects over light background (you can also assume the opposite).
- It can be adapted to cases in which the objects dominate over the background.

Background symmetry: variation

$$x \sim N(\mu, \sigma^2)$$

$$p(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$



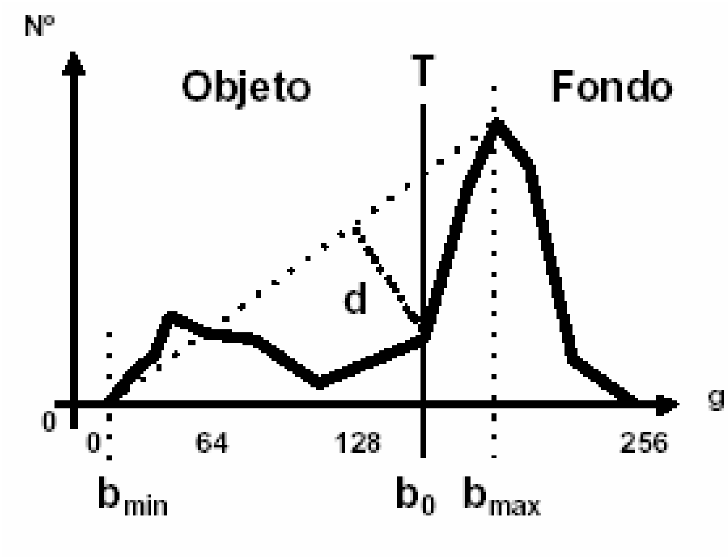
- Use the standard deviation of the background.
- Computer the standard deviation (σ) of the background to the right of the maximum.

T = Tmax - 1.96 σ

Triangle algorithm [Zack, ?]

1. Draw a line between the maximum value of the histogram (to gray level **bmax**) and the minimum level in the image **bmin** = (**p=0%**).
2. Compute the distance from the line to histogram **h[b]** for all values of **b = bmin..bmax**.
3. Choose the gray level value $\mathbf{b_0}$ for which the distance between $\mathbf{h[b_0]}$ and the line is maximal, being the threshold $\mathbf{T = b_0}$.

- This technique is particularly effective when the object pixels produce a weak peak in the histogram.



Difficulties in attaining optimality

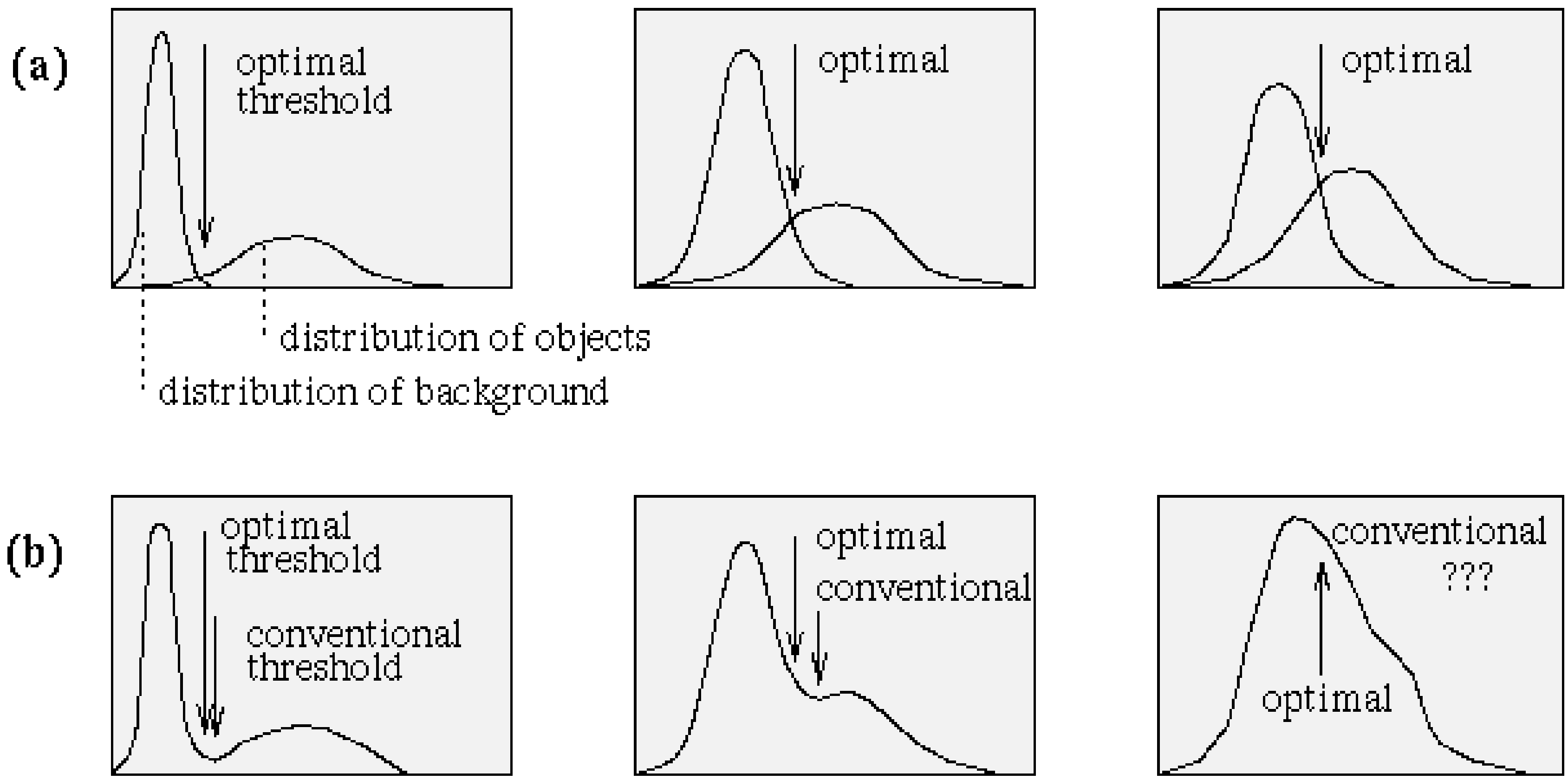


Figure 5.4 *Grey level histograms approximated by two normal distributions; the threshold is set to give minimum probability of segmentation error: (a) Probability distributions of background and objects, (b) corresponding histograms and optimal threshold.*

Iterative methods

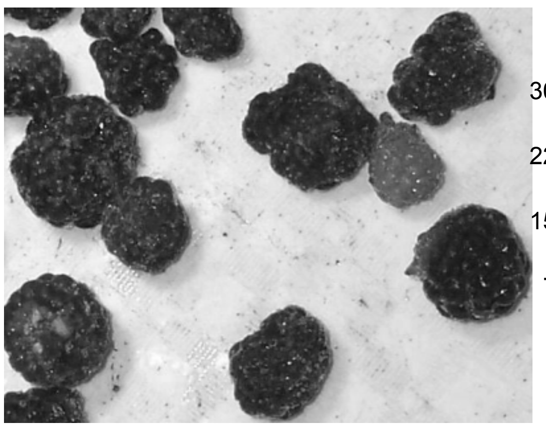
- Start with an approximate value, and refine it iteratively using some sub-image property.

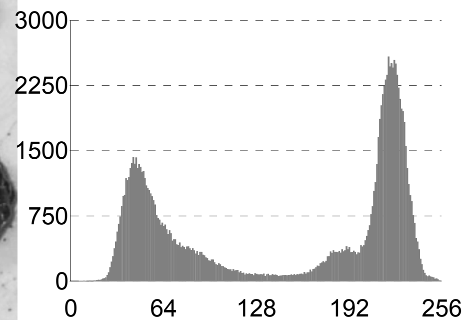
Isodata (Ridler & Calvard, 1978) :

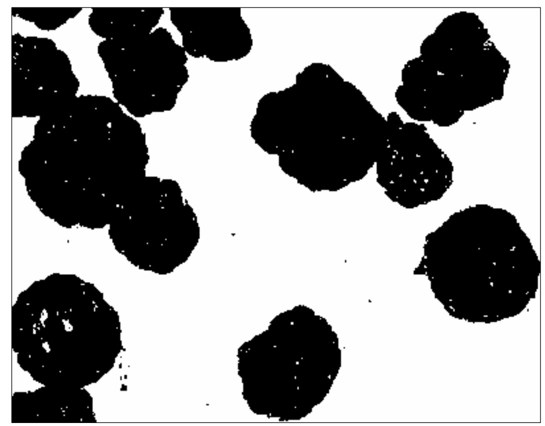
1. Choose an initial threshold T (f.e. the mean intensity).
2. Partition the image into two groups R_1 and R_2 using T.
3. Compute the mean values of each region μ_1 **and** μ_2

4.
$$T = (\mu_1 + \mu_2)/2$$

5. Repeat steps 2-4 until the value T does not change.



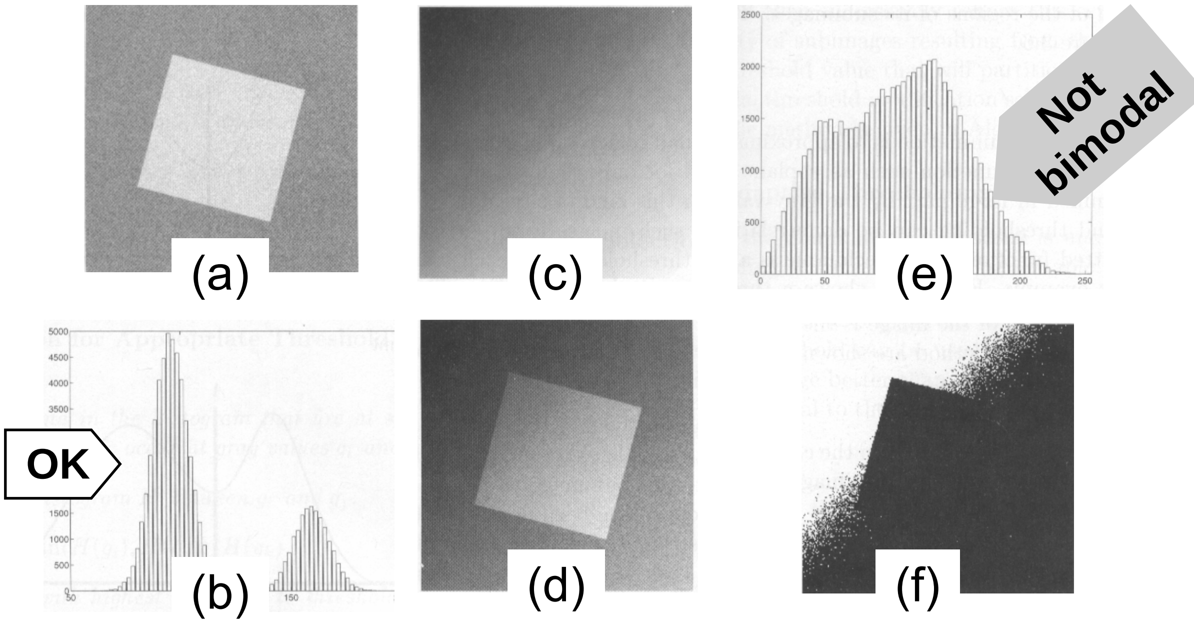




Tries to find mean values of objects and background.

Adaptive methods

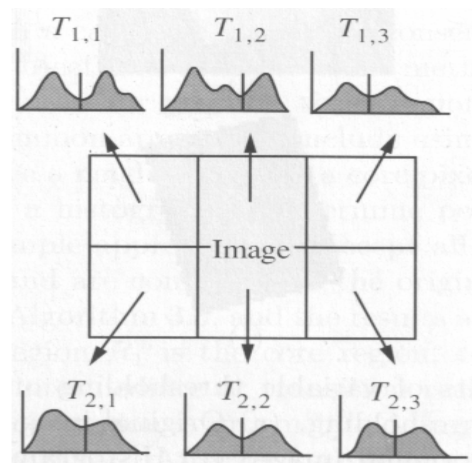
- If illumination is not uniform in the whole image, we cannot use a single value T.



- (a) Original image
- (b) histogram
- (c) Non-uniform illumination
- (d) Non-uniform illuminated image
- (e) Histogram
- (f) result

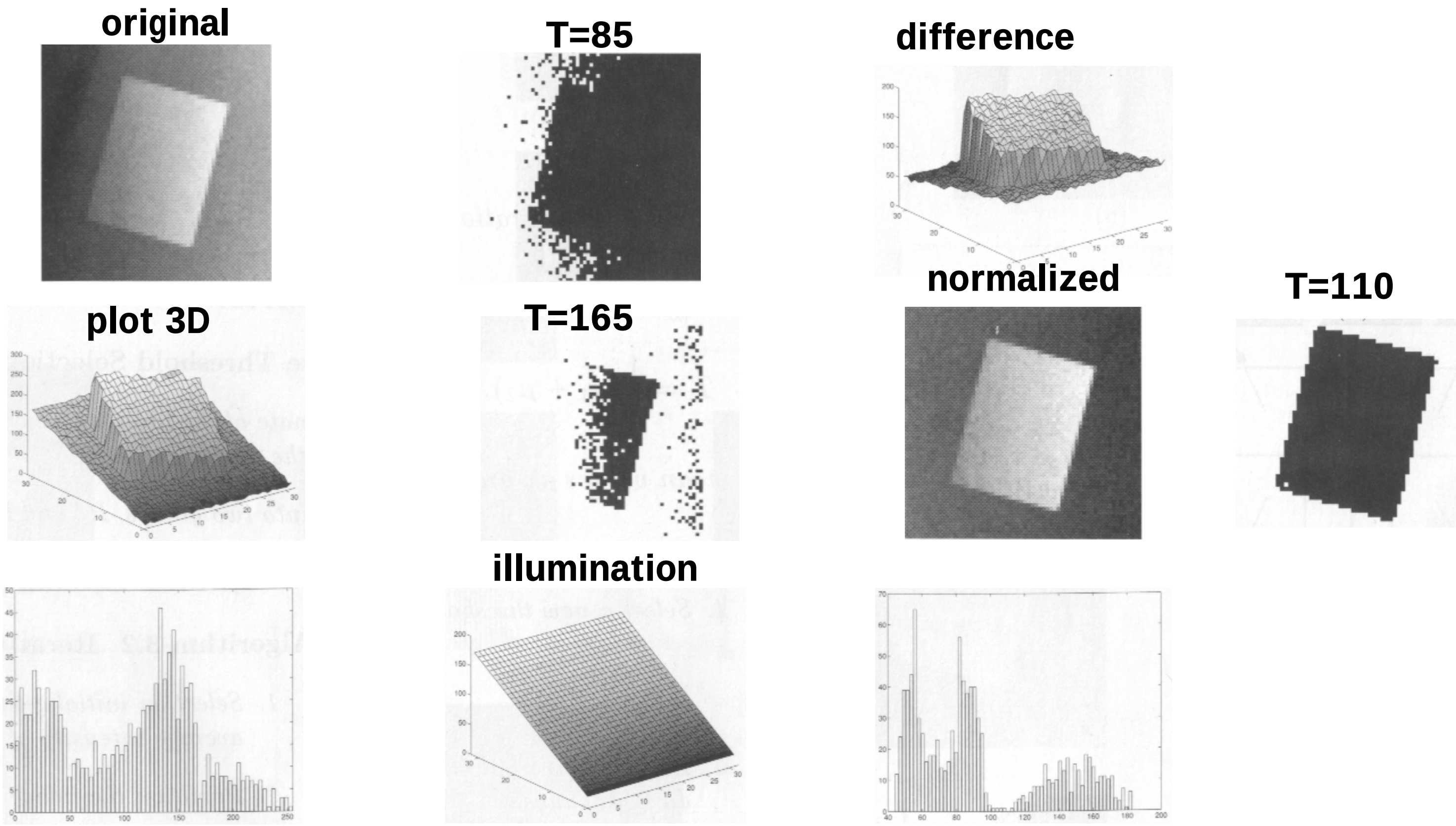
Adaptive algorithm:

1. Partition the image into $\mathbf{I}_{ij}$
2. For each region $\mathbf{I}_{ij}$ compute $\mathbf{T}_{ij}$
3. Threshold each region $\mathbf{I}_{ij}$
4. The final result is the union



Variable methods

- Non uniform illumination, but you can model it (linear or quadric variation). Try to eliminate non uniformity:
  1. Compute an approximation (linear or quadric) of the image.
  2. Compute the threshold relative to this approximation.



Does NOT get rid of shadows

3. Histogram + image: double thresholdiing

- First, simple threshold, which is refined with this algorithm:

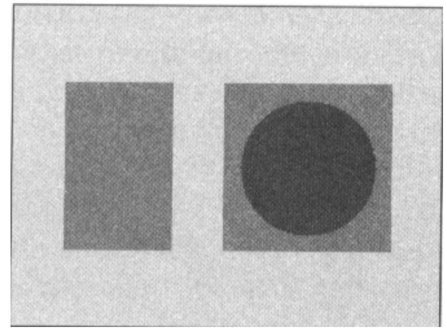
1. **Choose two thresholds T_1 y T_2.**
2. **Partition the image into three sets:**

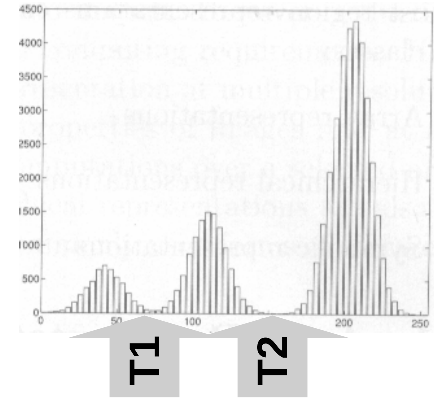
Object $R_1 < T_1$

Doubtful $T_1 <= R_2 <= T_2$

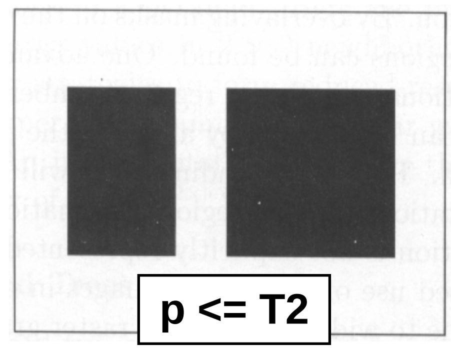
Background $T_2 < R_3$

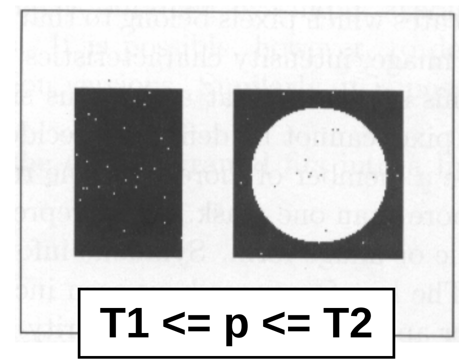
3. **Visit ea/pixel in R_2; if it has a neighbor belonging to R_1, move it to R_1.**
4. **Repeat 3 until no pixel is reassigned.**
5. **Remaining pixels in R_2 go to R_3.**



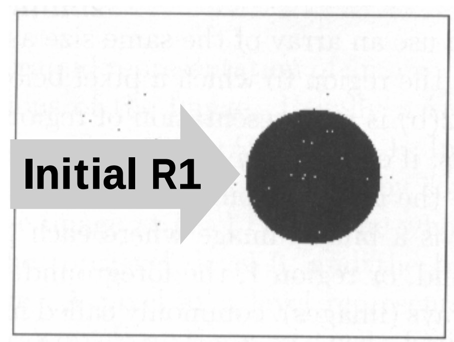


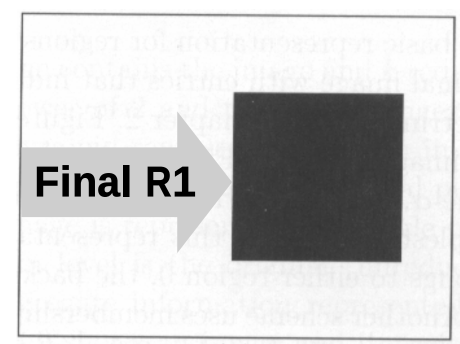
Normal threshold:





Double threshold:



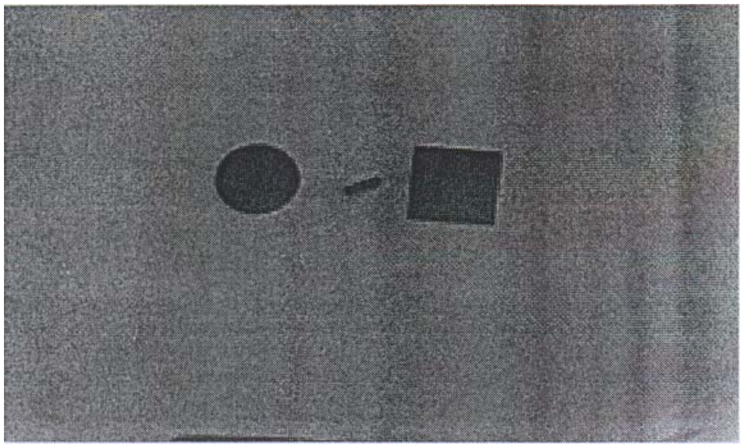


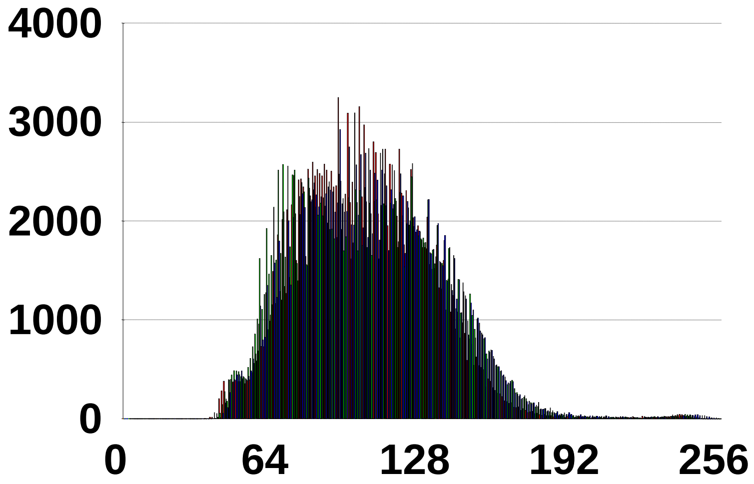
Histogram + image (Weszka, 1973)

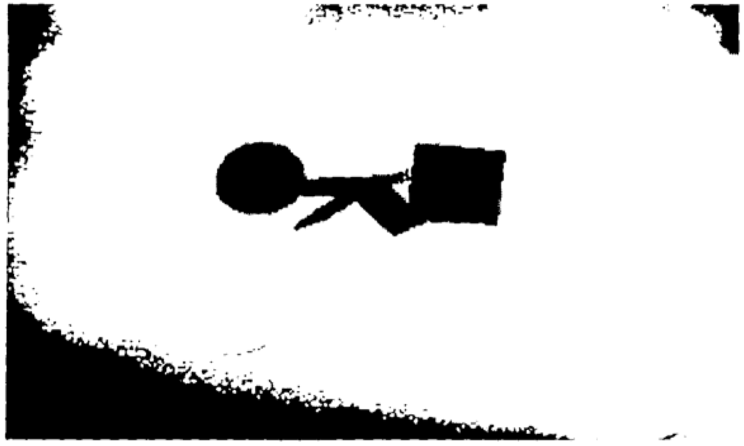
- The mean is a simple threshold to compute:

$$T = \frac{\sum_{i=0}^{L-1} i \cdot h(i)}{\sum_{i=0}^{L-1} h(i)}$$

- If the amount of foreground and background pixels is not similar, this threshold is not appropriate.

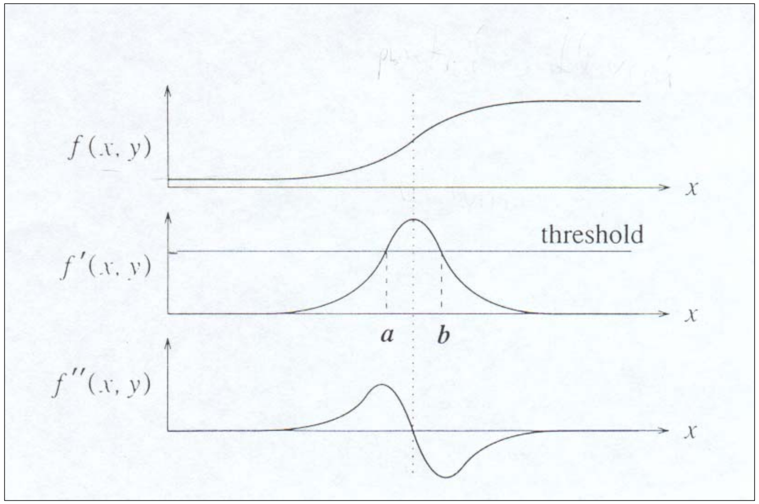






Weskza

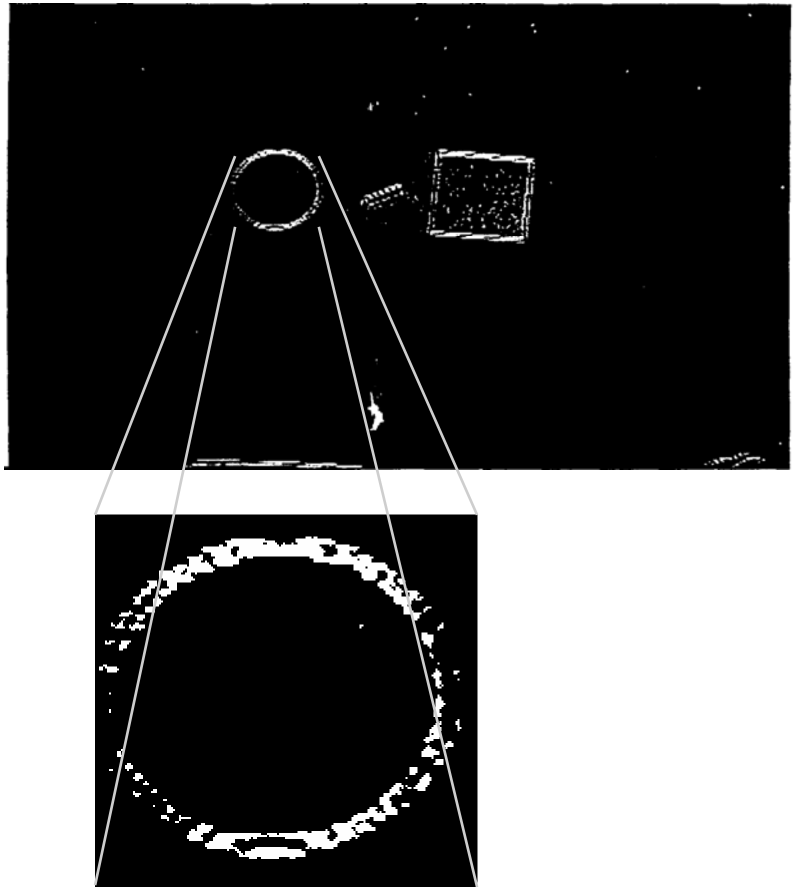
- **Laplacian:** non directional operator of segment detection



- Approximation: convolution

0	1	0
1	-4	1
0	1	0

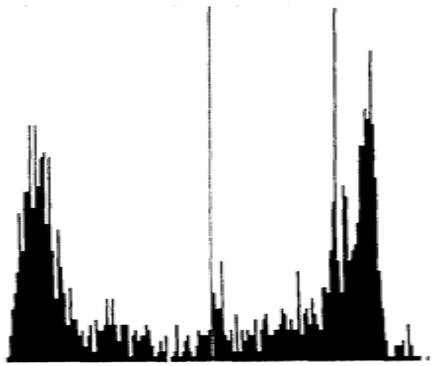
1	4	1
4	-20	4
1	4	1



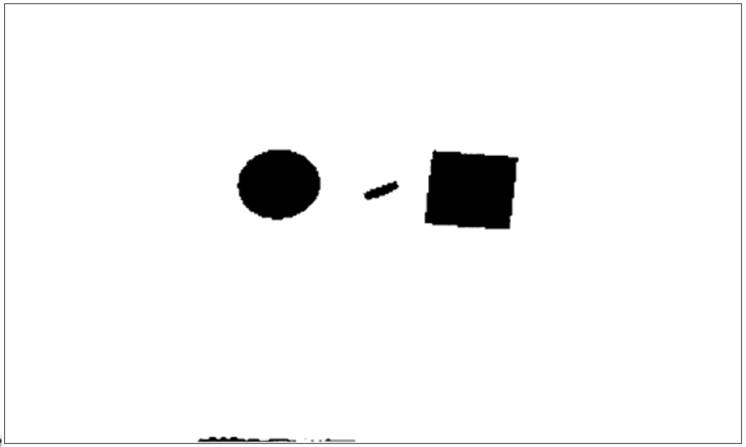
- **15%** of more brilliant pixels constitute the contour of the objects (at both sides)

Weskza

- Contour pixels belong uniformly to foreground and background



- Considering only contour pixels, the mean can be used as threshold



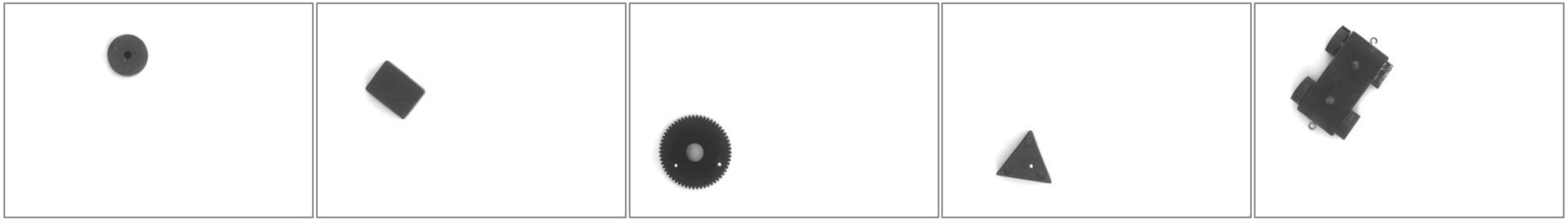
Thresholding

Limitations:

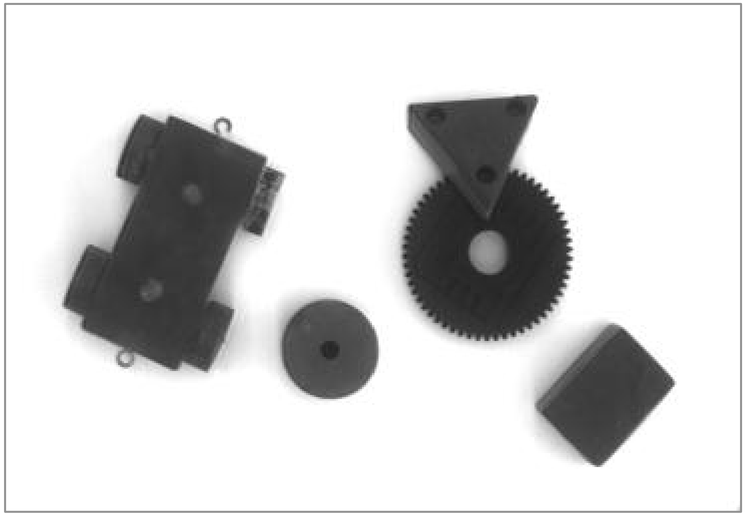
- Each application requires a specific threshold/algorithm.
- These techniques degrade when the complexity of the images increases.
- The most efficient methods use the histogram only. They are however limited because the spatial information is ignored.

Labs 1-5

- Planar object recognition:
- Learning phase:

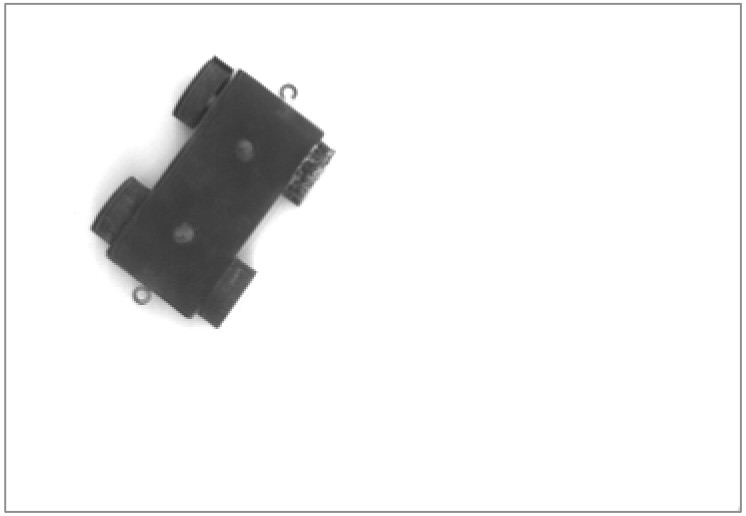


- Explotation phase:



P1: Thresholding

- Implement Isodata



```
i: 1, threshold: 173
i: 2, threshold: 164
```