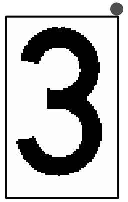
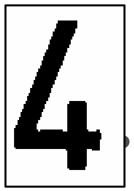
Lesson 5: Morphology

1. Introduction
2. Expansion and contraction
3. Dilation and erosion
4. Opening and closing
5. Skeletons
6. Distance maps

1/6. Introduction

- **Morphology:** analysis of the **shape** of connected components.
- **Algebraic** operators applicable to binary images in order to extract components useful in representing shape
  - Contours
  - Convex hull
  - Skeletons
- Goals:
  - Image simplification
  - Elimination of irrelevances
  - Preservation of useful characteristics

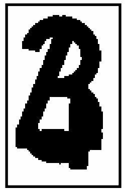
A: 

B: 

Parallelizable techniques

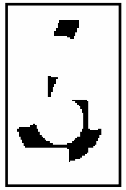
- **Definitions:** Let A and B be binary images, p and q two pixels with indices `[i,j]` y `[k,l]` respectively, and Ω the universal binary image.
- **Union:**

$$A \cup B = \{p | p \in A \vee p \in B\}$$



- **Intersection:**

$$A \cap B = \{p | p \in A \wedge p \in B\}$$

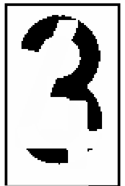


- **Complement:**

$$\overline{A} = \{p | p \in \Omega \wedge p \notin A\}$$

- **Difference:**

$$A - B = A \cap \overline{B}$$



- **Translation:**

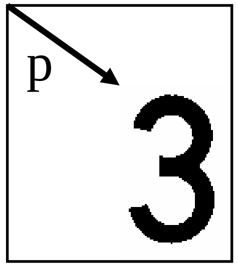
$$A_p = \{a + p | a \in A\}$$

  - **Vectorial sum:**

$$p + q = [i + k, j + l]$$

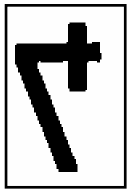
  - **Vectorial difference:**

$$p - q = [i - k, j - l]$$



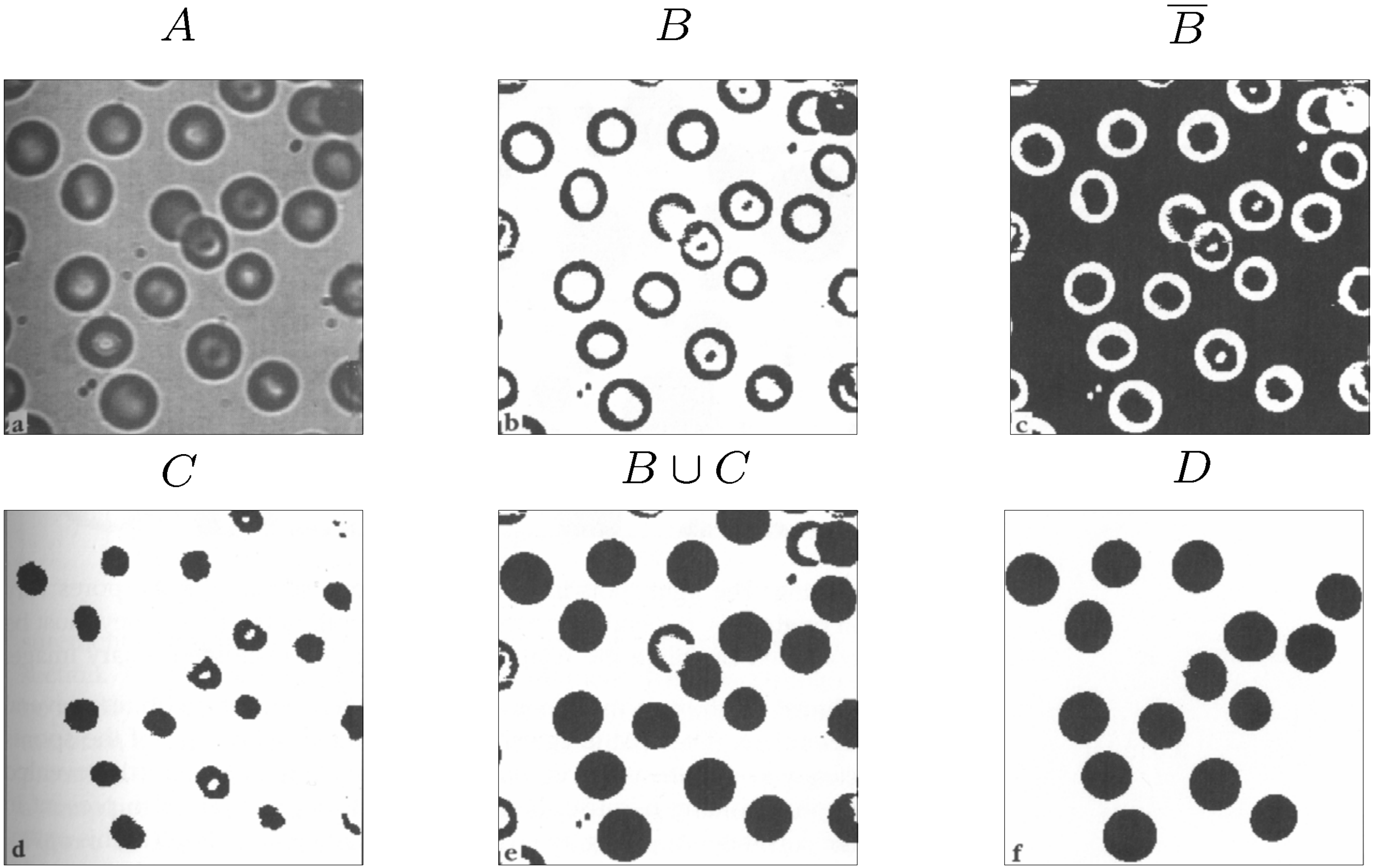
- **Reflex:**

$$A' = \{-p | p \in A\}$$



Introduction

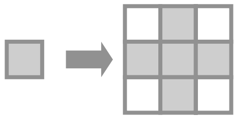
- Example: thresholding



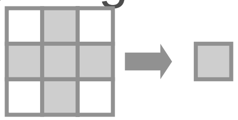
Not touching borders **Size filter**

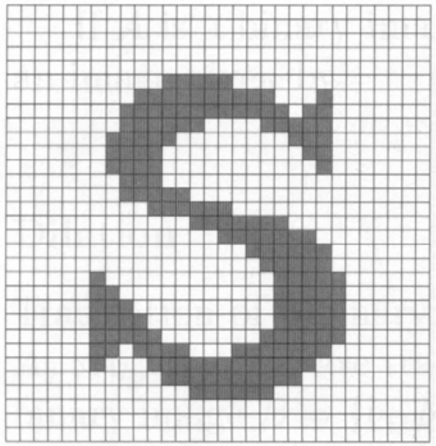
2 / 6. Expansion and contraction

- Transformations to turn foreground pixels into background and vice-versa.
- **Expansion**: turn a pixel from 0 to 1 if any neighbor is 1.

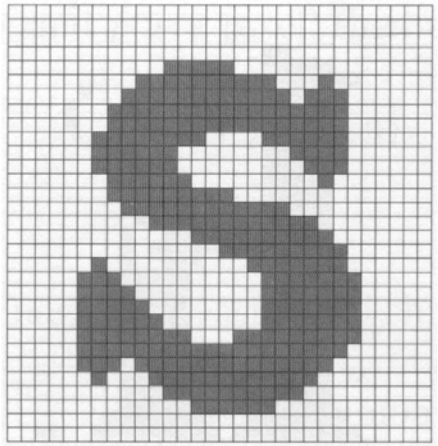


- **Contraction:** turn a pixel from 1 to 0 if any neighbor is 0.

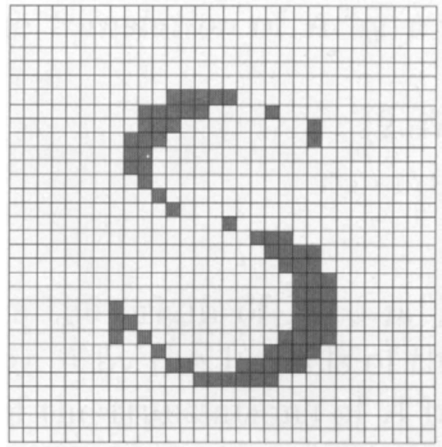




Expansion (4v)



Contraction (4v)



Expanding a blob is equivalent to contracting the background

S^k ***S*** expanded ***k*** times

S^{-k} ***S*** contracted ***k*** times

- Non-commutative

$$(S^m)^{-n} \neq (S^{-n})^m$$

- Neither invertible:

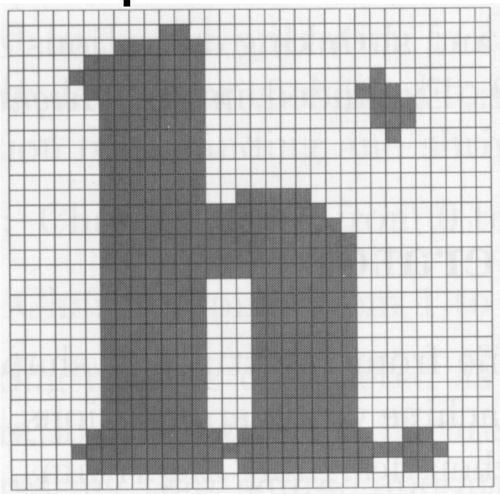
$$\neq S^{m-n}$$

$$S \subset (S^k)^{-k}$$

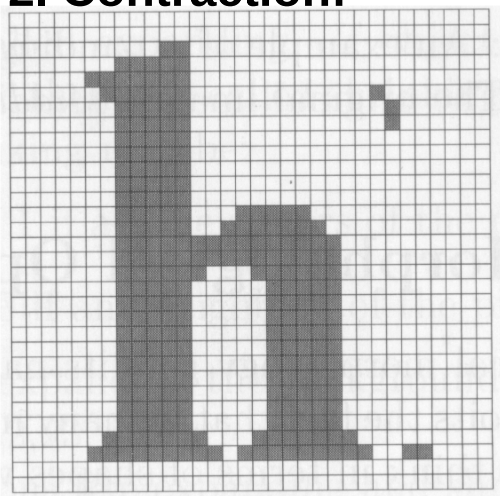
$$S \supset (S^{-k})^k$$

Expansion y contraction

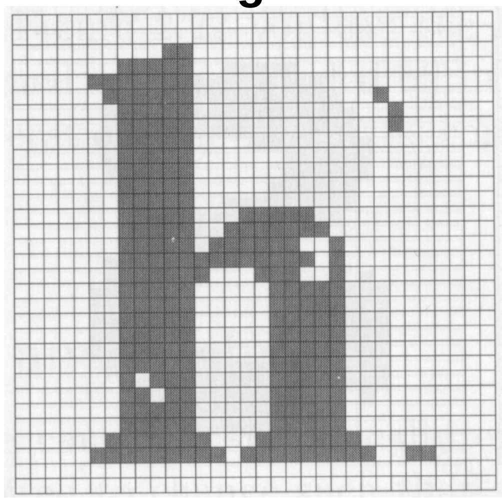
1. Expansion:



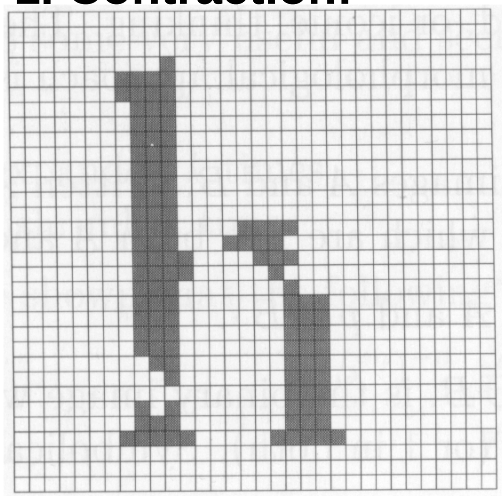
2. Contraction:



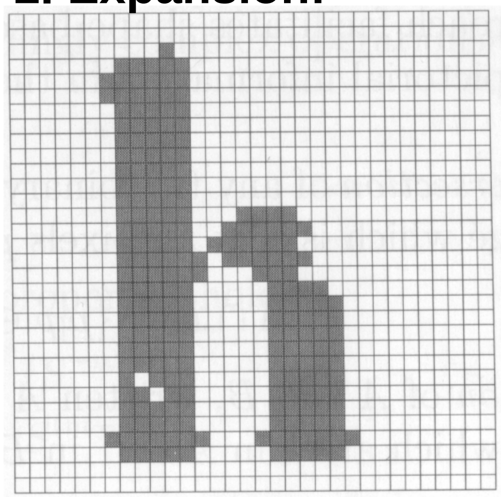
Original:



1. Contraction:



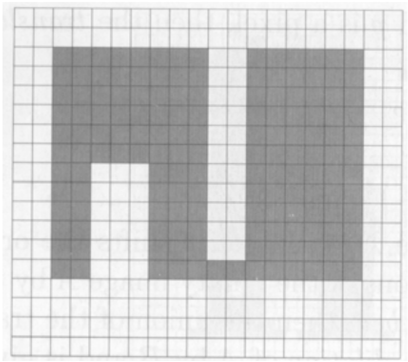
2. Expansion:



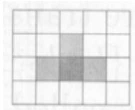
- **Expansion + Contraction**: elimination of undesired holes (salt noise).
- **Contraction + Expansion:** Elimination of noise (pepper noise).

3 / 6. Dilation y erosion

A

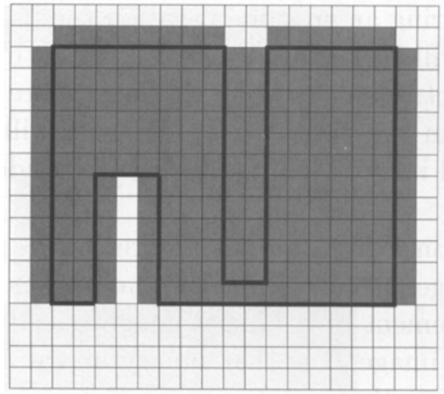


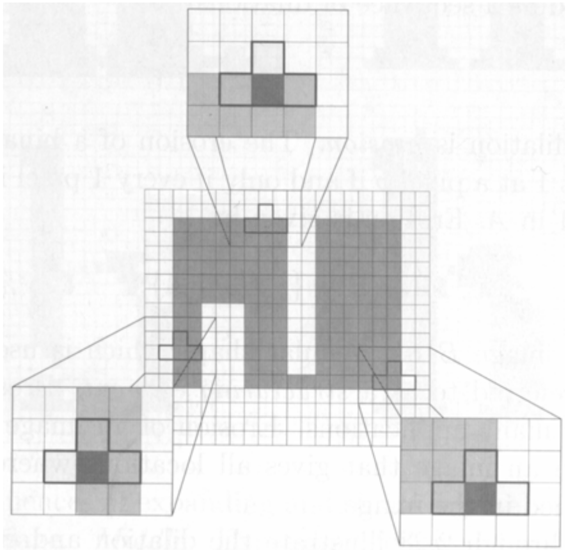
B



- **Dilation:** union of translations of an image ***A*** by each pixel of image ***B***, called ***structural element***:

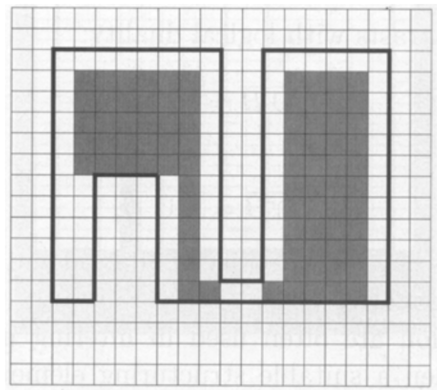
$$A \oplus B = \bigcup_{b_i \in B} A_{b_i}$$





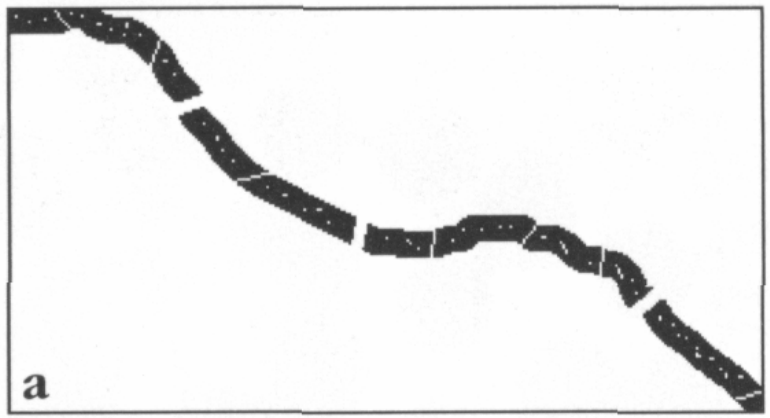
- **Erosion** inverse operation

$$A \ominus B = \{p | B_p \subseteq A\}$$

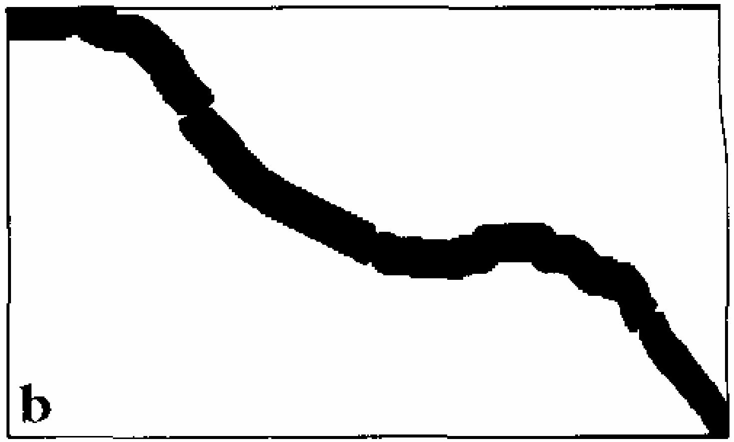


Asociative y commutative?

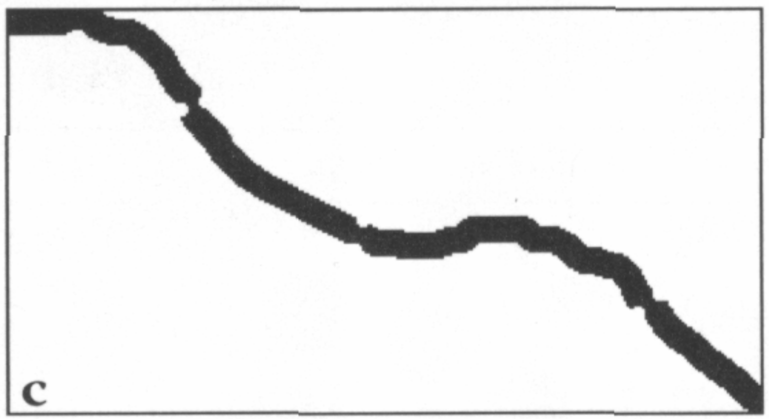
Object connection



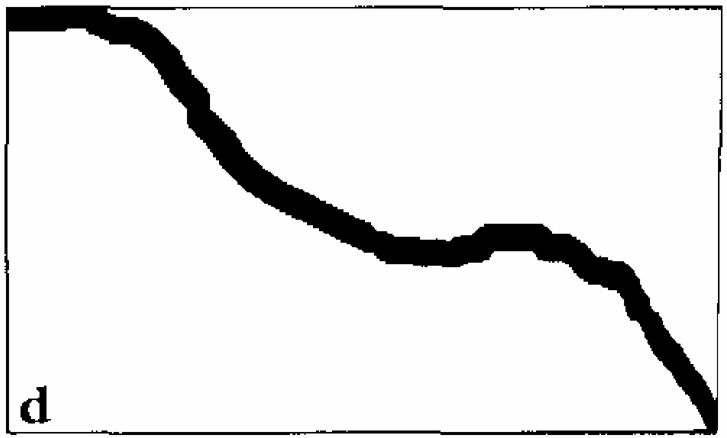
Original



Dilated twice



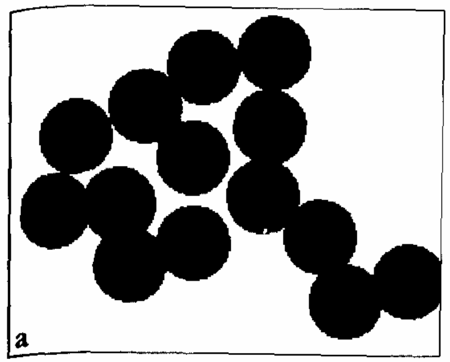
Eroded twice



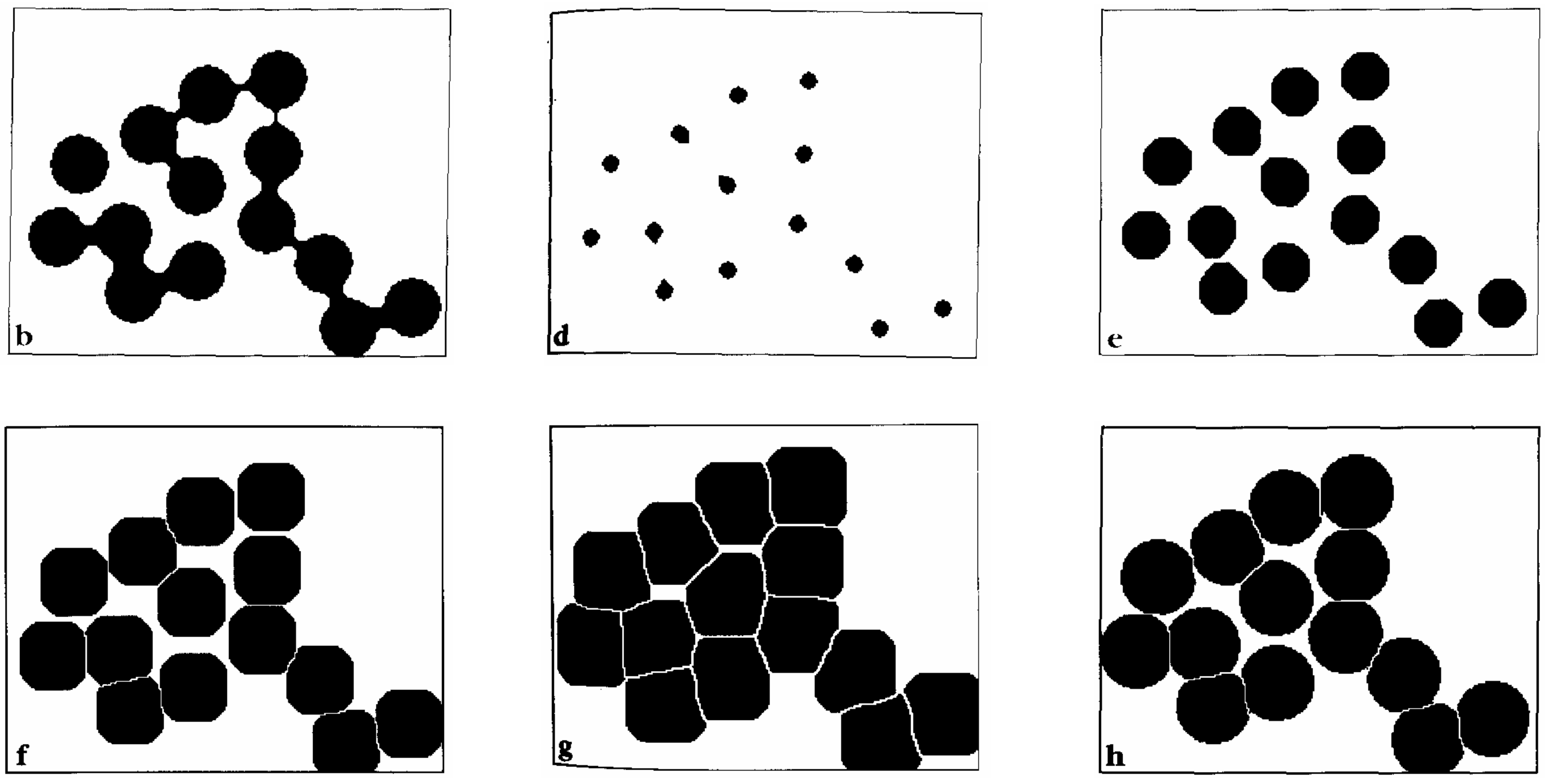
Dilated and eroded four times

Object separation

a. Original
b. Eroded twice
d. Eroded 7 times

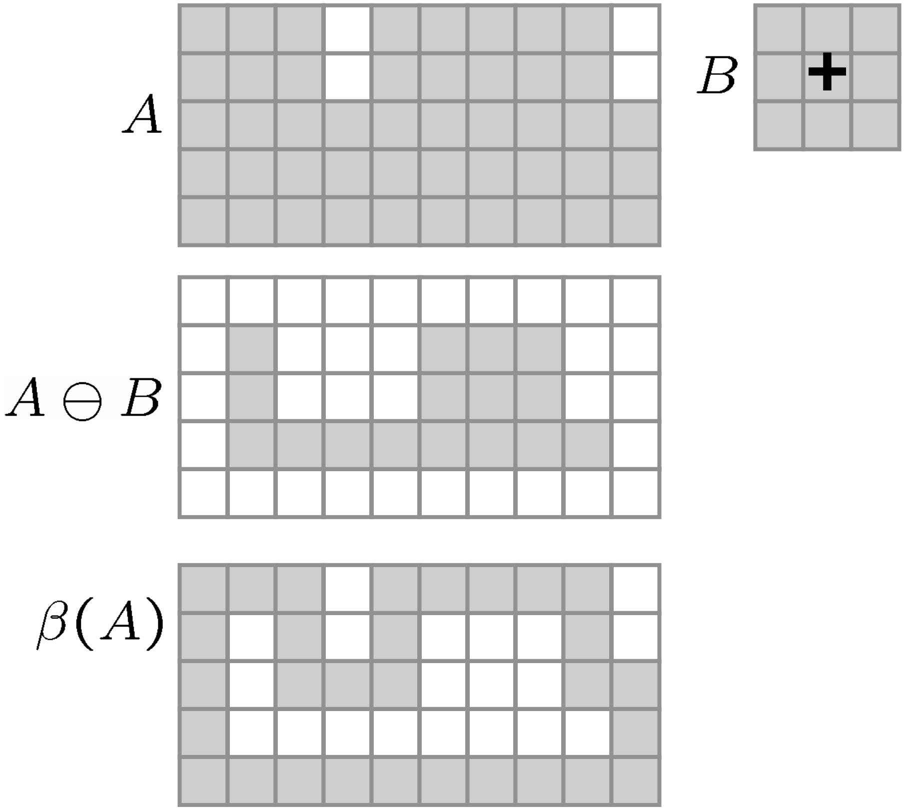


e. Dilated four times with XOR
f. Dilated seven times with XOR
g. Dilated nine times with XOR
h. AND with original image



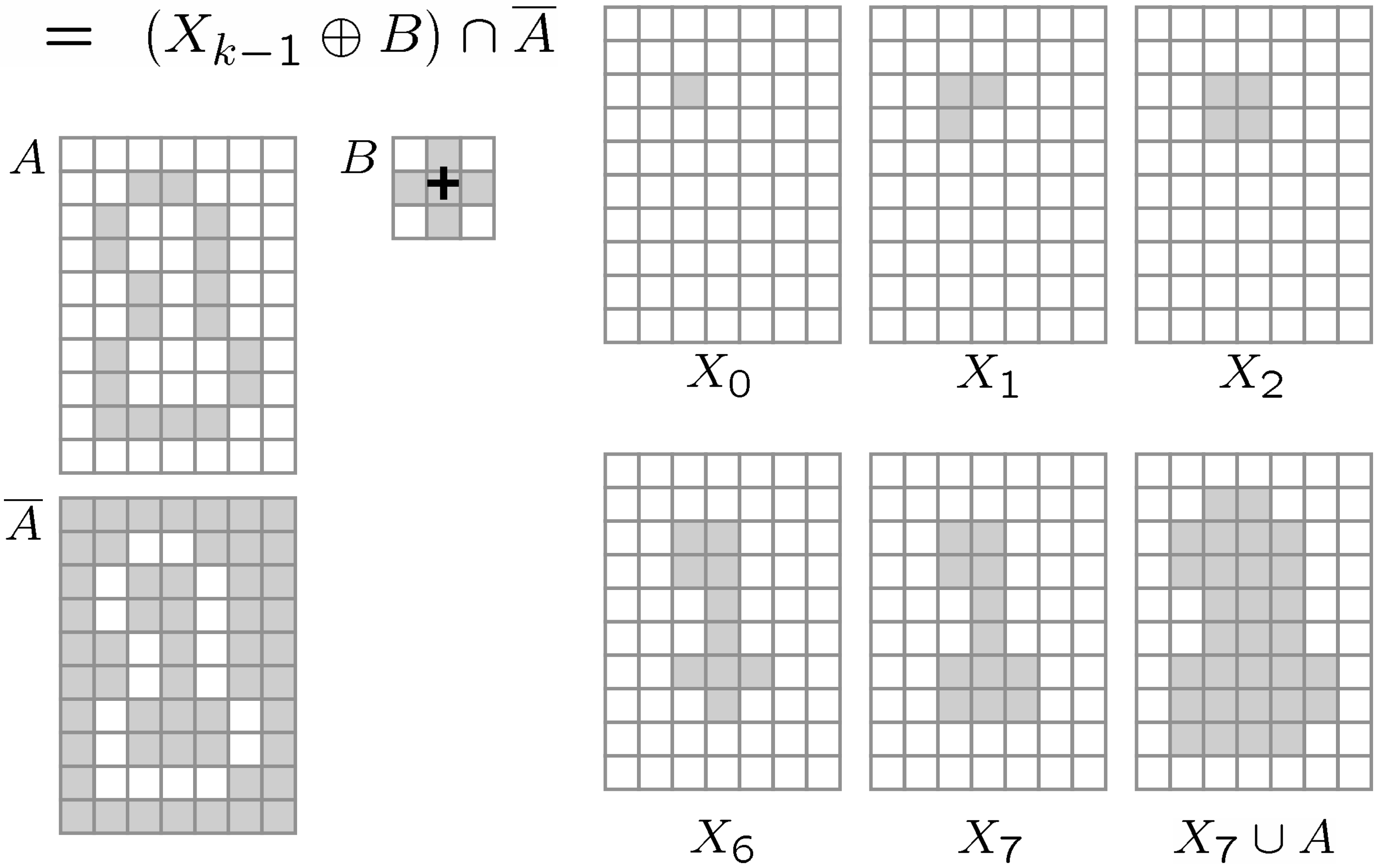
Contour extraction

$$\beta(A) = A - (A \ominus B)$$



Region filling

$$X_0 = p$$
$$X_k = (X_{k-1} \oplus B) \cap \overline{A}$$



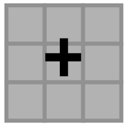
$$X_k = X_{k-1}$$

Holes

M

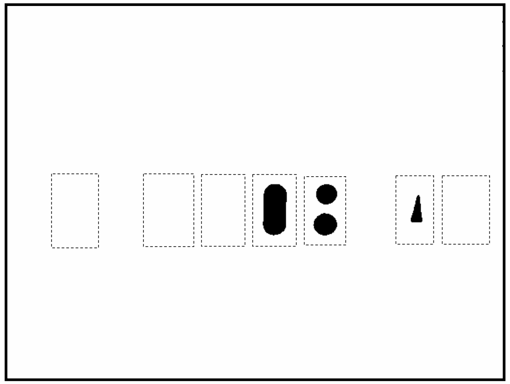
$A = \overline{M}$

B



X_0

X_n



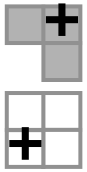
The total area might be less sensitive to noise, although the Euler number might be more discriminating.

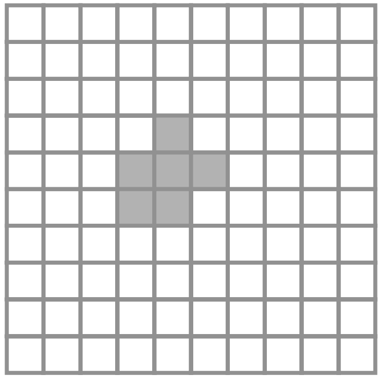
Hit or Miss

- Selection of pixels with some particular properties (corners, isolated, border).

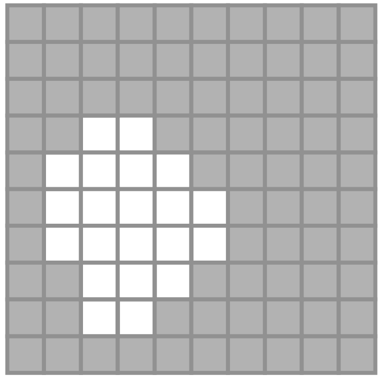
$$B = (J, K), \;\; J \cap K = \emptyset$$

$$A \otimes B = (A \ominus J) \cap (\overline{A} \ominus K)$$





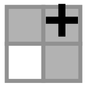
$A \ominus J$



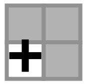
$\overline{A} \ominus K$

- Example: sup. rig. corners
- ***J***: description of object pixels
- ***K***: description of background pixels

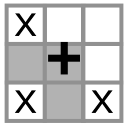
J



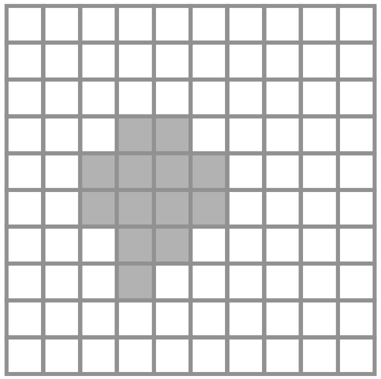
K



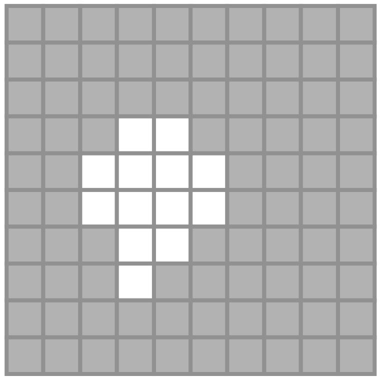
B

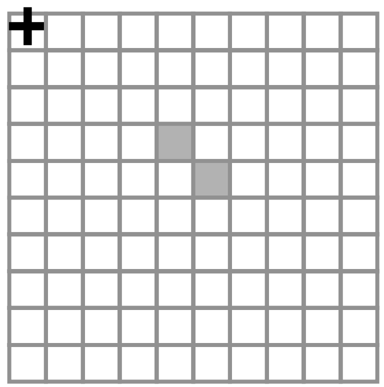


A



$\overline{A}$



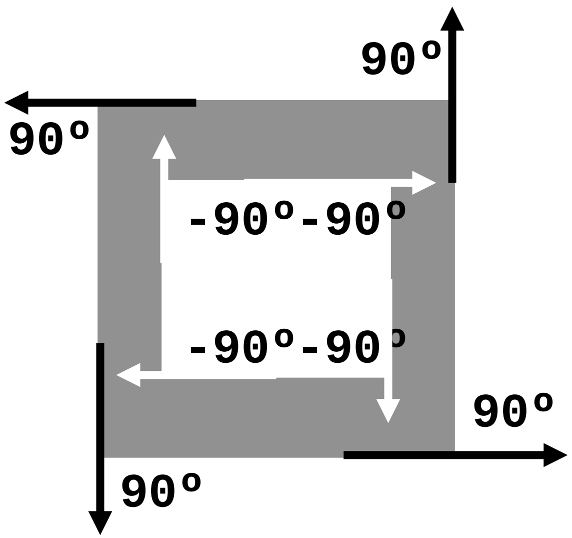


$(A \ominus J) \cap (\overline{A} \ominus K)$

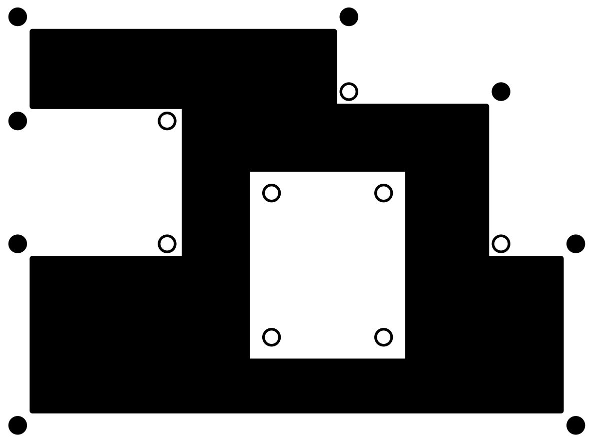
Computation of Euler number

$$Eu = C - H$$

- Given a closed polygonal line, the sum of its angles should be +-360º



- The Euler number is equal to the number of convex corners minus the number of concave corners, all divided by four:



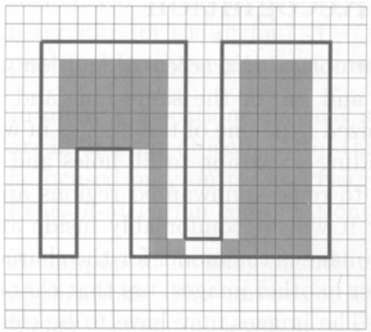
$$\frac{N_{\bullet} - N_{\circ}}{4} = 0$$

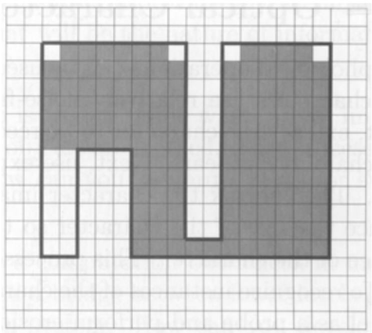
C **and** ***H*** **cannot be computed separately**

4 / 6. Opening and closing

- **Opening:** erosion + dilation with the same element

$$A \circ K = (A \ominus K) \oplus K$$



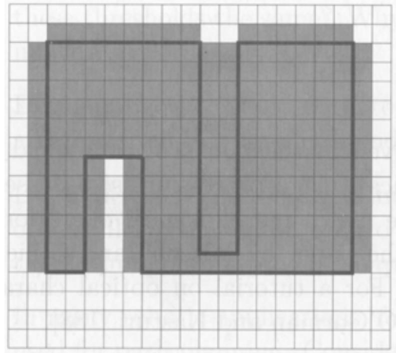


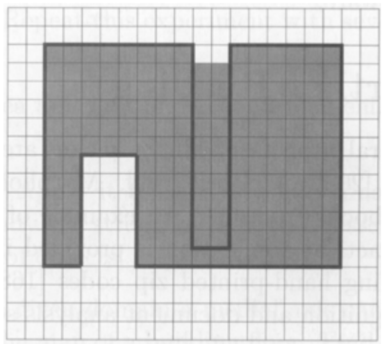
- Eliminates all regions too small to contain the structural element

$$A \circ K \circ K = A \circ K$$

- **Closing:** dilation + erosion with the same element

$$A \bullet K = (A \oplus K) \ominus K$$





- Fills all holes and cavities smaller than the structural element

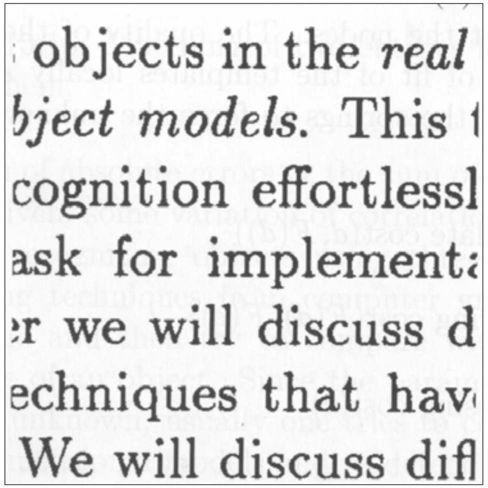
$$A \bullet K \bullet K = A \bullet K$$

Idempotent

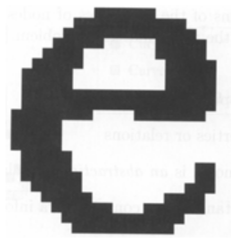
Applications

- **Template matching**

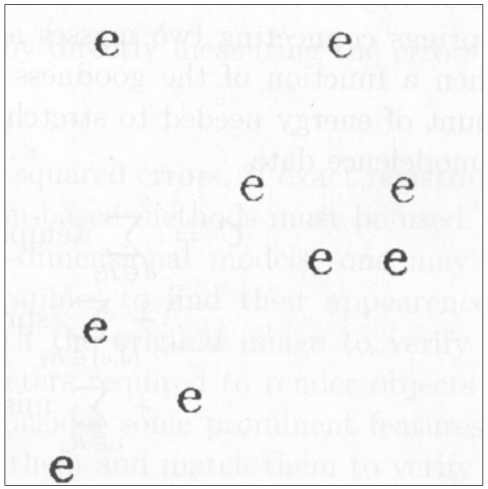
A



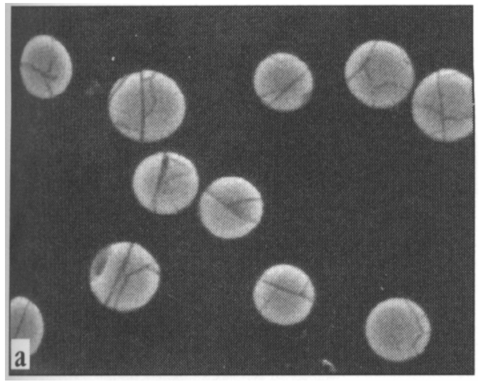
B



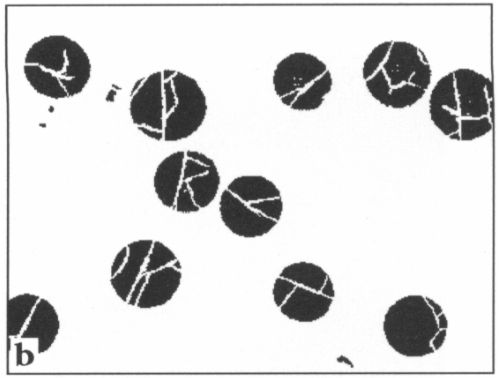
$A \circ B$



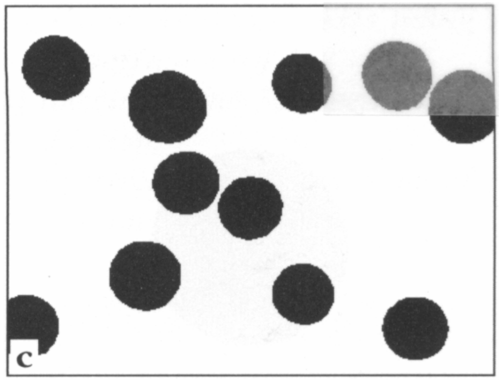
- **Reconstruction**



Original



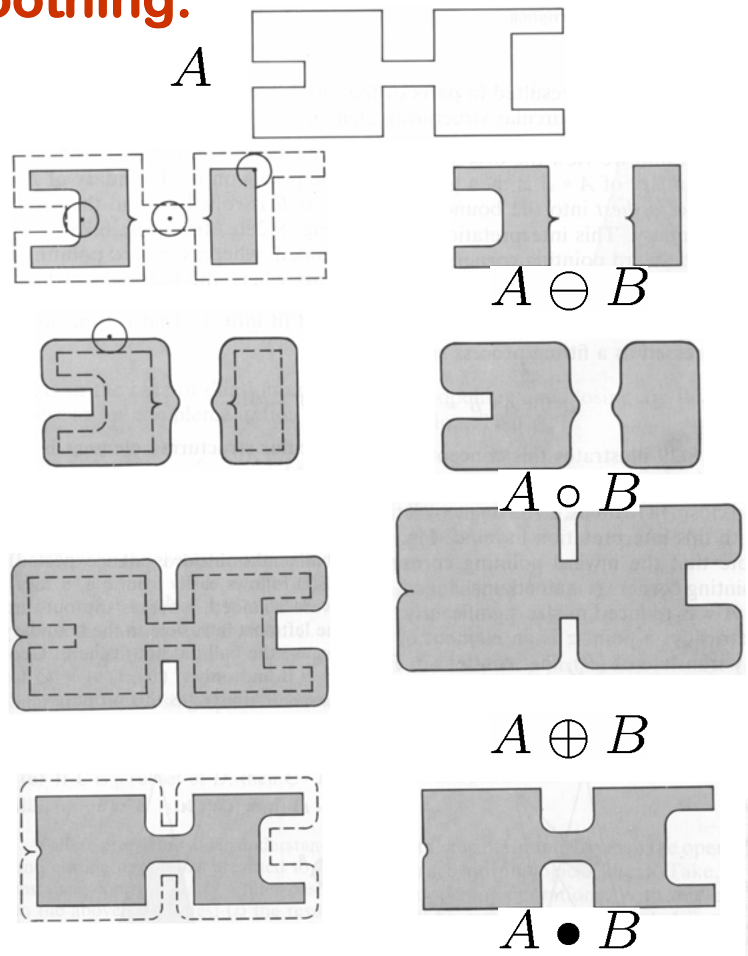
Thresholded



Closing

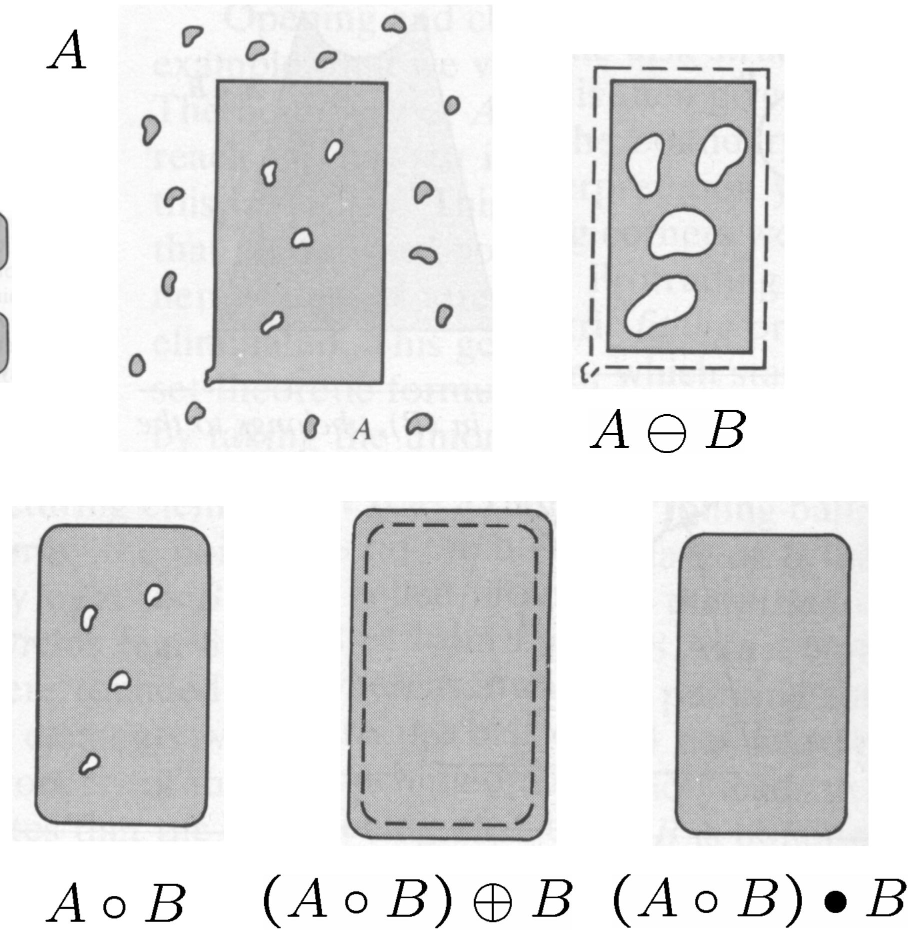
Applications

- **Smoothing:**



- **Morphologic filtering:** B is a disc of size >= all noise components.

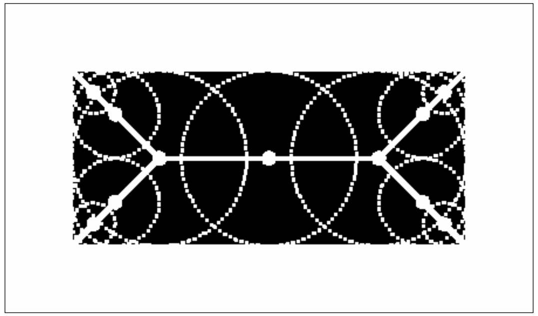
$$(A \circ B) \bullet B$$

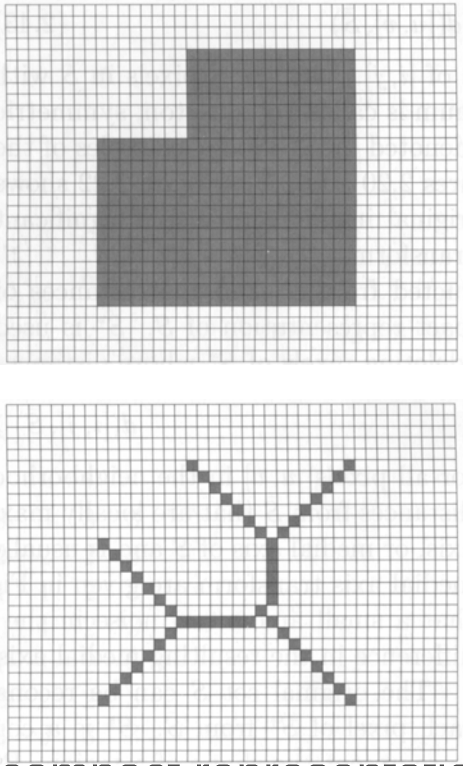


- **Opening:** smooth convex corners.
- **Closing:** smoot concave corners

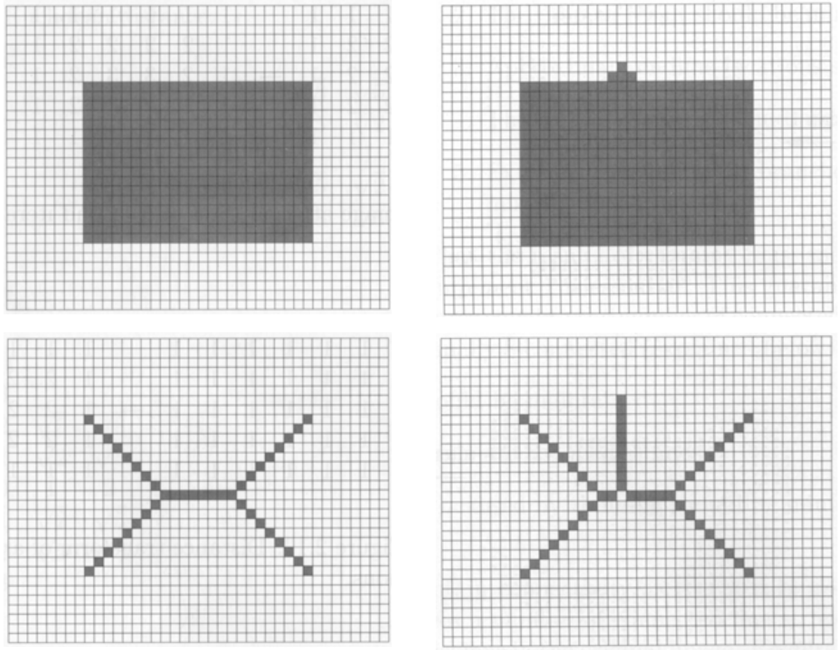
5 / 6. Skeletons

- **Skeletons, axis of symmetry S*:** geometric place of the centers of all at least bi-tangent circles.





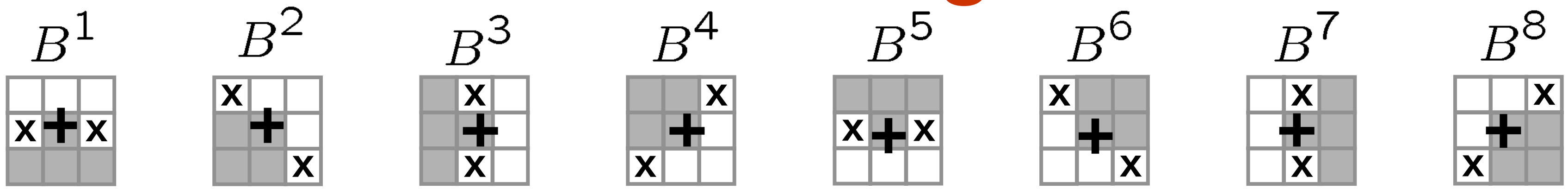
- Highly sensitive to noise.



- S* is a compact representation of S; it represents the *shape* of the region.

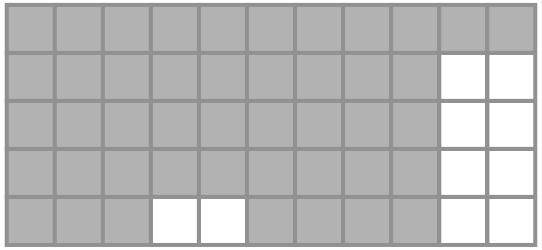
The frontier is also a compact representation of shape.

Thinning

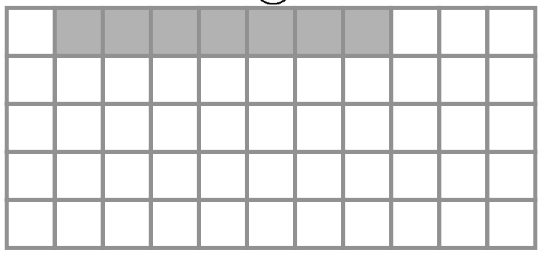


$$A \odot B^i = A - (A \otimes B^i)$$

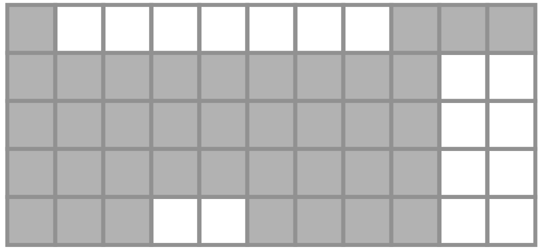
A



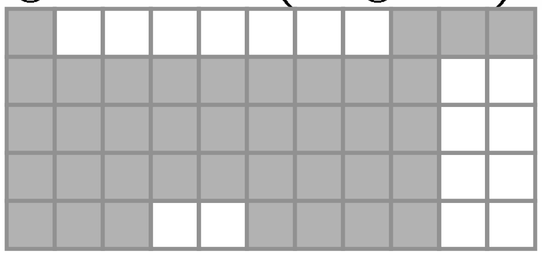
$A \otimes B^1$



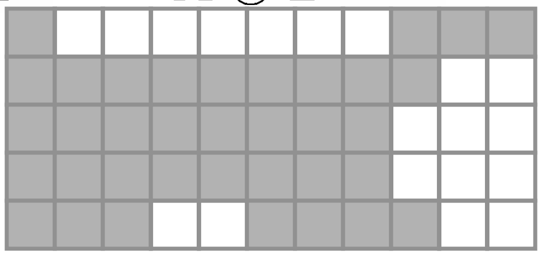
$A \odot B^1$



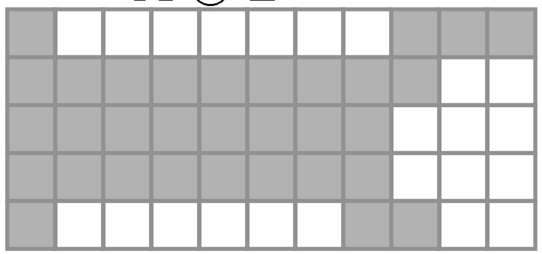
$A \odot B^{1,2} = (A \odot B^1) \odot B^2$



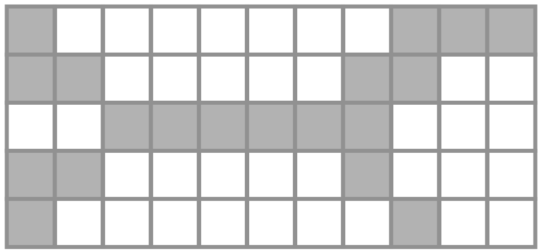
$A \odot B^{1,2,3}$



$A \odot B^{1,2,3,4,5}$

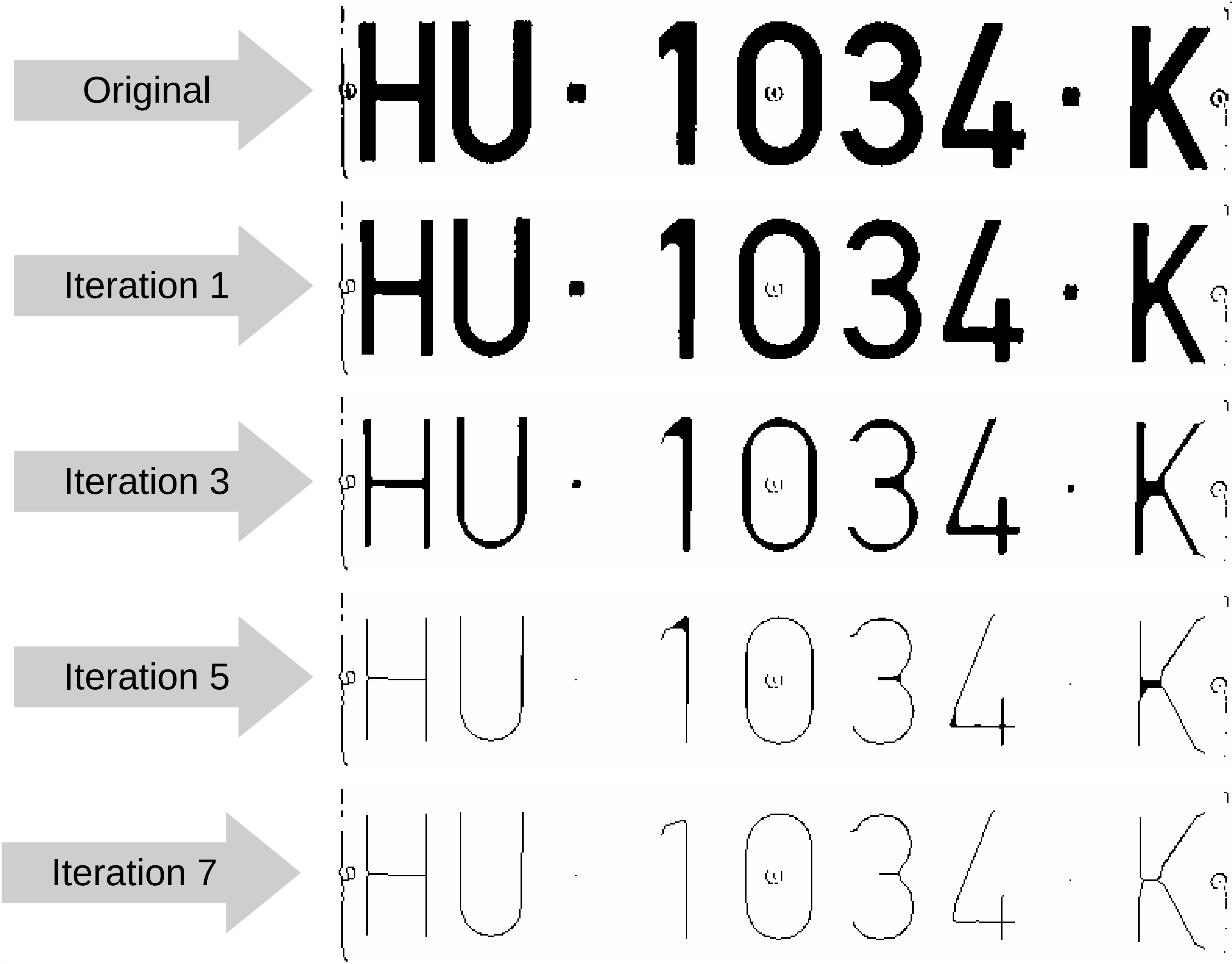


$$X_0 = A$$
$$X_k = X_{k-1} \odot B^{\{1,\cdots,n\}}$$



Final result

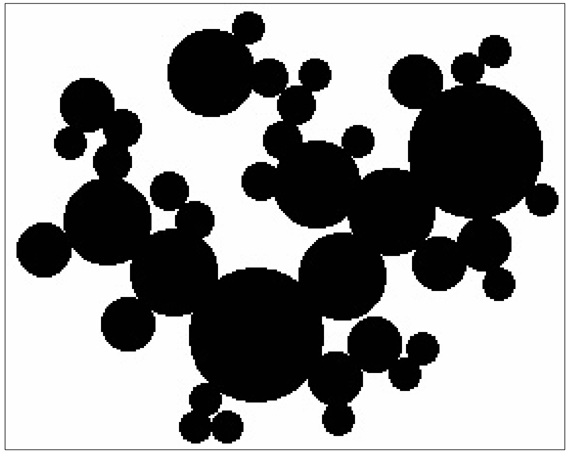
Thinning

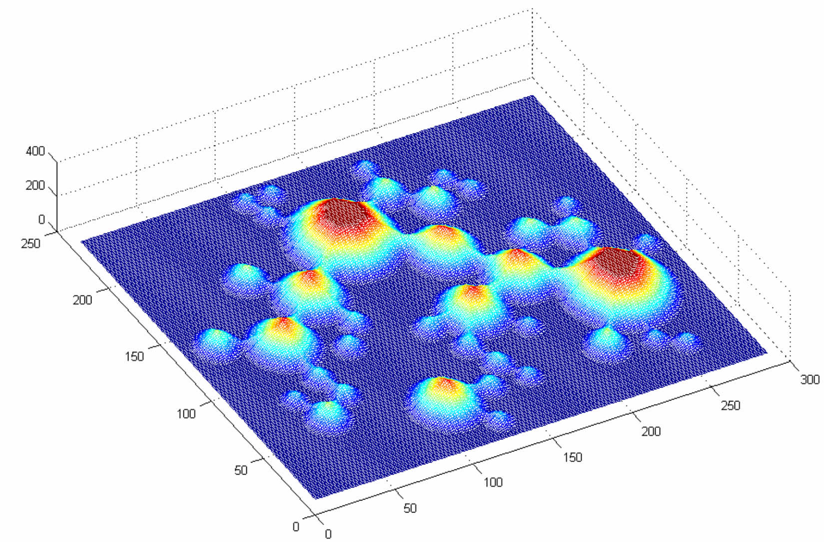

Original
HU 1034 K
Iteration 1
Iteration 3
Iteration 5
Iteration 7

6 / 6. Euclidean distance maps (EDM)

- Image representing the smallest distance of e/pixel to the background.

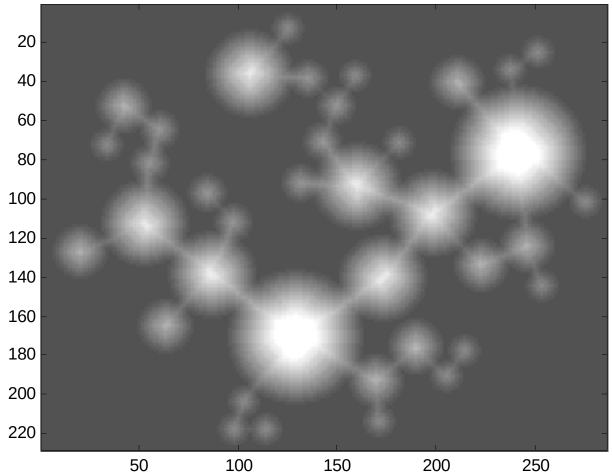
A





$D(3d)$

D



There are several possible definitions for distance

Distance measurements

- Fundamental properties:

$\forall p, q, r:$

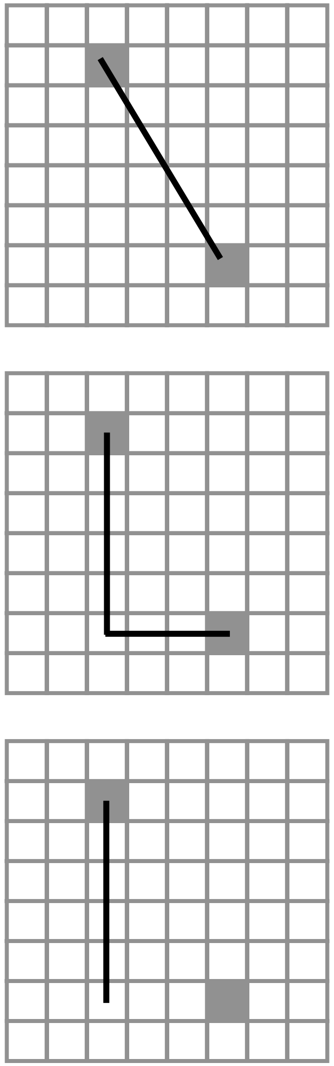
1. $d(p,q) \geq 0,$
   $d(p,q) = 0 \Leftrightarrow p = q$
2. $d(p,q) = d(q,p)$
3. $d(p,r) \leq d(p,q) + d(q,r)$

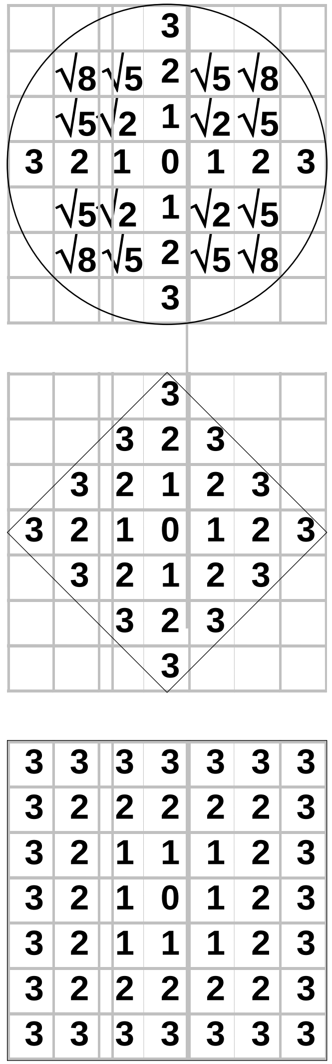
Discs: pixels at distance <= k.

$$d([i_1, j_1], [i_2, j_2]) =$$

1. Euclidean:
$$\sqrt{(i_1 - i_2)^2 + (j_1 - j_2)^2}$$
2. Manhattan:
$$|i_1 - i_2| + |j_1 - j_2|$$
3. Chess:
$$\max(|i_1 - i_2|, |j_1 - j_2|)$$

1. 2. 3.





Euclidean is closest to the real case; Mosr costly to compute

Obtaining distance maps

$$f^0[i,j] = B[i,j]$$
$$f^m[i,j] = f^0[i,j] + \min\left(f^{m-1}[u,v]\right)$$

$$\forall[u,v] : d([u,v],[i,j]) = 1$$

1	1	1	1	1	1
1	1	1	1	1	1
1	1	1	1	1	1
1	1	1	1	1	1
1	1	1	1	1	1
1	1	1	1	1	1

1	1	1	1	1	1
1	2	2	2	2	1
1	2	2	2	2	1
1	2	2	2	2	1
1	2	2	2	2	1
1	1	1	1	1	1

1	1	1	1	1	**1**
1	**2**	2	2	**2**	1
1	2	**3**	**3**	2	1
1	2	**3**	**3**	2	1
1	**2**	2	2	**2**	1
1	1	1	1	1	**1**

- **Iteration 0:** original image.
- **Iteration 1:** All pixels not adyacent to background change to 2.
- **Next iterations:** pixels farther from background change.
- No pixels changes when the distances to all have been computed.