EEG source localization Based on Dynamic Bayesian Estimation Techniques

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Abstract. In this paper, we propose a solution to the EEG source localization problem considering its time-dynamic behavior. From the dynamic probabilistic model of the problem, we formulate the Kalman Filter and Particle Filter solutions for the dipolar and non-linear approach. In order to test the algorithms, we designed an experimental protocol based on error-related potentials. During the experiments, our dynamic solutions have allowed the dynamic estimation of a source varying in position and moment within the brain volume. Results confirm the activation of the anterior cingulate cortex which is one of the brain structures associated with error processing. Furthermore, the dynamic methods produce a smooth tracking of the EEG sources. These findings demonstrate the good performance of the dynamic solutions for estimating and tracking the EEG neural generators.

Keywords: EEG Source Localization; Dynamic Tracking; Kalman Filter; Particle Filter; Error-Related Potential

1. Introduction

Electroencephalography (EEG) is a brain imaging technique that gives a unique access to the electric neural activation, furthermore it has very good temporal resolution, and it is non-invasive, very cheap and portable. Hence, EEG is one of the most preferable technologies to study the brain. However clinical and functional interpretations of EEG signals imply the speculation of the possible active areas within the brain that generate those signals. For this reason, the solution to the EEG source localization problem attempts to find from EEG signals which regions of the brain are active. There are two approaches to solve the EEG source localization problem, dipolar methods and distributed methods [Darvas et al., 2004 and Baillet et al., 2001]. Despite their differences, the large majority of these methods share the property of providing static solutions, they only use information from one time instant, whereas the EEG sources and signals clearly have a time varying nature. To account for this limitation, in this paper we propose a methodology for solving the so-called inverse problem in EEG considering its dynamic behavior. We use Kalman Filters (KF) and the Particle Filters (PF) for tracking the EEG sources. Both KF and PF provide a recursive methodology for estimating from noisy measurements, the state of a dynamical system that cannot be observed. The EEG inverse problem fulfills this characteristic. In this work, we first formulate the dynamic model of the EEG source localization problem in a probabilistic framework. Subsequently, we assume a dipolar approach where sources positions and moments have to be estimated which makes the problem highly non-linear. Under this context we derive the Extended Kalman Filter (EKF) and PF solutions. The advantage of these solutions in that they produce a direct tracking of the sources, which are varying in position and time. An alternative advantage of PF is that it can explicitly represent and track simultaneously many likely solutions. In order to validate these formulations, we design a neurophysiological protocol based on error-related potentials (ErrP) where the main focus of neural activity is assumed to be known as reported by other studies. We validate the methods by using these real EEG recordings. The overall result is that we successfully apply the EKF and PF in real settings to track the EEG sources, where the advantage of taking into account the dynamic nature of the problem turns to be clear.

2. Dynamic model of the EEG source localization problem

Let be $\Phi_{0 \ldots t}$ the set of EEG signals up to time t acquired with N_e electrodes attached at the scalp. The neural generators of these signals are N_s sources, each one characterized by six parameters, position and moment (with $r=[r_x,r_y,r_z]$ being the position and $J=[J_x,J_y,J_z]$ being the moment). The goal is to estimate the sources parameters $X=[r_1,J_1 \ldots r_{Ns},J_{Ns}]$ at time t using the measurements up to time t.

For this, two equations are needed: a transition model $X_t=G(X_{t-1})+ w_t$ that describes the knowledge of how the neural sources evolve over the time, and a measurement model $\Phi_t=F(X_t)+v_t$ that allows to compute measurements given the sources, where $X_t{:}R^{6\times Ns}$ is the source space, and $\Phi_t{:}R^{Ne}$ is the measurement space. Furthermore, the random variables w_t and v_t represent the noise in the process and in the measurements respectively. Alternatively, the dynamic equations can be expressed in the probabilistic framework:

$$P(X_t \mid X_{t-1}, w_t) \tag{1}$$

$$P(\Phi_t \mid X_t, v_t) \tag{2}$$

With this probabilistic representation, the goal is to compute a posterior probability $P(X_t|\Phi_{0...t})$ to make inference about the sources given the measurements. However, EEG signals and sources are changing over the time whereby a recursive update equation has to be obtained to deal with this dynamic behavior. Using the Bayes theorem and assuming Markovian process we finally obtain the recursive update equation known as the Bayes filter for the dynamic EEG source localization problem:

$$P(X_t \mid \Phi_{0...t}) = \eta \cdot P(\Phi_t \mid X_t) \cdot \int P(X_t \mid X_{t-1}) P(X_{t-1} \mid \Phi_{t-1}) dX_{t-1} \tag{3}$$

Where η is a normalizing factor. This equation gives at time t the probability of the EEG sources given EEG signals. To implement the Bayes filter we need three distributions, the initial posterior $P(X_0)$ that characterizes the prior knowledge about the sources, the transition model $P(X_t|X_{t-1})$ that represents the time evolution of the neural sources, and the likelihood $P(\Phi_t|X_t)$ or measurement model that allows to obtain measurements given a source space. The goal now for solving the dynamic EEG source localization problem is to derive a solution to the Bayes filter.

2.1. Transition model

This mathematical model describes how the EEG sources evolve over time which is unknown. Therefore we can assume the transition as a *random walk* in the source space [Somersalo 2003], whereby the specific form of the transition model is $X_t=X_{t-1}+w_t$ or $P(X_t|X_{t-1})=N(X_t|X_{t-1},Q)$. This assumption imposes that the transition model is a zero-mean Gaussian density with a diagonal covariance matrix Q whose elements represent the expected time evolution of each source parameter.

2.2. Deriving the dynamic solutions to the EEG source localization problem

The dynamic EEG source localization problem can be categorized depending on whether the location of the sources is fixed or not, and on whether the type of the noise in the measurements and in the sources is assumed to be Gaussian or not. If the location of the sources is fixed the problem is lineal (typical assumption of distributed methods). Then, if the noise is Gaussian the KF gives the optimal estimator, otherwise the problem could be addressed by PF. In the case that the location of the sources is not fixed the problem is not linear (typical assumption of dipolar methods). Then, if the noises are Gaussian, the EKF or PF can be used, otherwise for non-Gaussian noises one has to use the PF.

We focus on the case where the location of the sources is not fixed (non-linear situation) and we assume Gaussian noise. This is because we are interested in modeling focal areas of the brain (as mentioned before, the evaluation process will be focused on the ErrP that are known to be elicited by specific brain areas). For this case the unknown source space is given by the set of positions and moments for each source $X=[r_1,J_1...r_{Ns},J_{Ns}]$, the transition model is given by $X_t=X_{t-1}+w_t$ or $N(X_t|X_{t-1},Q)$ with $P(w)\sim N(0,Q)$, and the measurements model is given by $\Phi_t=F(X_t)+v_t$ or $P(\Phi_t|X_t)$ with $P(v)\sim N(0,R)$ where Q and R are the process and measurements covariance respectively. We will derive now the EKF and PF dynamic solutions to the EEG source localization problem.

Aplying Extended Kalman Filter (EKF) to solve the dynamic EEG source localization problem

In order to estimate X_t at time t, the EKF performs recursively two steps, *i)* a time update step which estimates the next state $\bar{X}_t$ using the linear transition equation, and *ii)* a measurement update step which adjust the estimated state X_t by using the current measurements Φ_t [Welch and Bishop 2006].

To implement this nonlinear dynamic solution, we further need: *(i)* the covariance matrices, Q which can be determined based on physiological basis and R that can be computed taking some off-line sample measurements; *(ii)* the initial estimation X_0 which can be computed using a static solution such as Beamforming LCMV [van Veen et al., 1997]; and *(iii)* the Jacobian of the non-linear measurement equation which can be calculated once the head model is defined.

Aplying Particle Filters (PF) to solve the dynamic EEG source localization problem

The goal in this solution is to get a set of N samples named particles $\{X_t^{(i)}\}_{i=1...N}$ that represents the posterior distribution $P(X_t|\Phi_{0...t})$. In order to estimate X_t at time t, we need a set of particles $\{X_{t-1}^{(i)}\}_{i=1...N}$ distributed according to $P(X_{t-1}|\Phi_{0...t-1})$, then, by using the transition model we obtain a new set of particles $\{\cancel{X}_t^{(i)}\}_{i=1...N}$ which is distributed according to $P(\cancel{X}_t|\Phi_{0...t-1})$. Then, importance weights $\{w_t^{(i)}\}_{i=1...N}$ are computed through the likelihood function $\{w_t^{(i)}\}_{i=1...N} \sim P(\Phi_t|\cancel{X}_t)$. Afterward, a resampling-selection step [Doucet et al., 2001] is applied to the weighted sample $\{\cancel{X}_t^{(i)},w_t^{(i)}\}_{i=1...N}$ discharging/multiplying particles with low/high importance weights finally obtaining a new sample set $\{X_t^{(i)}\}_{i=1...N}$ distributed according to the posterior $P(X_t|\Phi_{0...t})$ which can be used to make inference about the sources space X_t at time t.

To implement this solution, we further need: *(i)* the initial density $P(X_0)$, which can be uniformly distributed; *(ii)* the covariance matrices which are obtained similarly to the case of the EKF solution; *(iii)* the likelihood function which is assumed to be a zero-mean Gaussian function; and *(iv)* the inference of the sources from the posterior $P(X_t|\Phi_{0...t})$, which can be made by selecting the mean value.

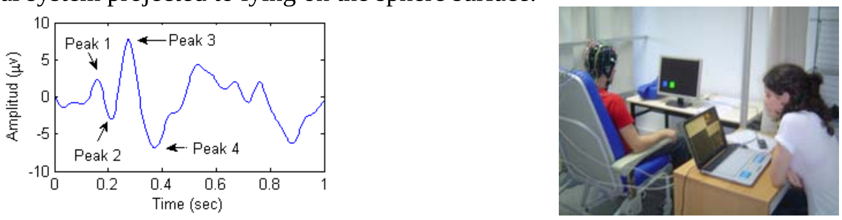
3. Validation with real EEG data

The major problem to assess in real settings the performance of algorithms for the EEG source localization problem is the unknown of the real EEG neural generators. For this reason, we have designed a protocol to elicit error-related potentials since there is evidence of the brain regions that generate those potentials [van Schie et al., 2004]. For the real EEG data, the main neural source was estimated using the EKF and the PF. To evaluate these solutions we have also computed the static dipolar solution given by the beamforming LCMV algorithm [van Veen et al., 1997], and the static, linear and distributed solution given by sLORETA [Pacual-Marqui 2002].

3.1. Neurophysiological protocol, instrumentation and head model

Recently, it has been probed during brain-computer interaction experiments that an interaction error-related potential (ErrP) is evoked after a person is aware of the occurrence of an error [Ferrez and del R. Millan 2008]. Roughly, this ErrP has a waveform with four prominent peaks as shown in figure 1a. The importance of this ErrP in our context is that for these peaks the main focus of neural activity is expected to be mainly in the anterior cingulate cortex or ACC (Brodmann area 24, 32) and in the pre-supplementary motor area or pre-SMA (Brodmann area 6). To elicit the ErrP we have followed the Ferrez's protocol [Ferrez and del R. Millan 2008], where a subject facing a computer screen is concentrated in a green block that is moving from right to left (machine task). While the machine is executing the task, sometimes the block moves to the left (which emulates a machine error) evoking the ErrP. We performed this experiment with one subject while EEG signals were acquired. The whole experiment consisted of 11 sessions with 5 errors trials each one. Figure 1b shows the experimental setting.

The general instrumentation is a commercial gTec EEG system (an EEG cap, 32 electrodes and two gUSBamp amplifiers). EEG signals were recorded with a sampling rate of 256Hz. After the recording sessions, the EEG signals were average referenced and band pass filtered from 1 to 10Hz. A time window of one second was selected after the ErrP stimulus onset. The head was modeled by a homogeneous sphere of radius 96mm. The measurement space is a set of 32 electrodes from the 10-10 international system projected to lying on the sphere surface.



(a) (b)

Figure 1. *a) Interaction error-related potential at electrode Cz recorded during one of the error trials of the experimental protocol. b) Snapshot of the experimental setting.*

3.2 Results

Error potentials are expected to be acquired with more magnitude at the scalp area enclosed by electrodes FCz, Cz and their neighbors (this is because this scalp area is the closest area to the brain structures associated with error processing). In this sense, to verify the elicitation of the ErrP, figure 2 shows the scalp potential topography for one of the ErrP at the occurrence of the four prominent peaks. As expected, first and third peaks (which are positives) reveal a front-central positivity, and the second and fourth peaks (which are negatives) reveal a frontro-central negativity. In consequence, these results seems to indicate the elicitation of the error-related potentials.

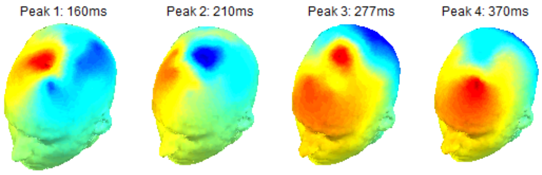


Figure 2. *Scalp topography at the prominent peaks for one of the ErrP.*

For all the ErrP's, we apply the sLORETA, LCMV, EKF and the PF source localization methods. Table 1 shows the estimated Brodmann area and brain structure at the occurrence of the peaks obtained in one of the ErrP's. Note that the result of sLORETA is far different of the results of the other methods, which are very similar in some periods of time. This could be explained by the different nature of the methods, the former is distributed whereas the others are dipolar. Although these results have to be taken in consideration carefully, one could infer that three methods of very different nature (one static method and two dynamic solutions with different hypotheses) estimate closely the same solution. Furthermore, the mean of the location of the solution over half a second in these methods also agrees with the location where neurological studies [van Schie et al., 2004] locate the origin of ErrPs (Brodman areas 24, 32).

Table 1. *Source estimation given by sLORETA, LCMV, EKF and PF at the occurrence of the peaks.*

Peak	*sLORETA*	*LCMV*	*EKF*	*PF*
1	Brodmann area 4 Paracental Lobule	Brodmann area 25 Anterior Cingulate	Brodman area 25 Anterior Cingulate	Brodmann area 25 Anterior Cingulate
2	Brodman area 6 Superior Frontal Gyrus	Brodman area 24 Cingulate Gyrus	Brodman area 32 Cingulate Gyrus	Brodmann area 32 Medial Frontal Gyrus
3	Brodman area 6 Superior Frontal Gyrus	Brodman area 31 Cingulate Gyrus	Brodman area 23 Cingulate Gyrus	Brodmann area 23 Cingulate Gyrus
4	Brodman area 6 Anterior Cingulate	Brodman area 28 Parahippocampal Gyrus	Brodman area 28 Uncus	Brodmann area 28 Uncus
Mean (0.5s)	Brodman area 6 Medial Frontal Gyrus	Brodman area 24 Cingulate Gyrus	Brodmann area 24 Cingulate Gyrus	Brodmann area 33 Anterior Cingulate

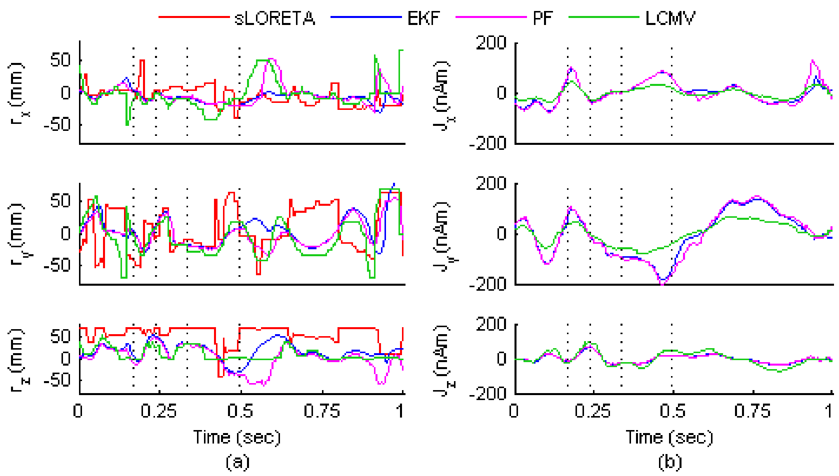


Figure 3. *Source parameters estimated for one of the ErrP. a) Position estimations. b) Moment estimations.*

Figure 3 displays the estimated position and moment using the four methods for the entire ErrP time window (dotted lines are in the occurrence of the peaks that agree with the locations of Table 1). Notice that there are abrupt changes in position of the static estimations. This is because they do not address the dynamic nature of the problem. In contrast, the EKF and PF methods produce position estimations with coherent time tracking for the whole time window. This is the expected behavior of the neural source that explains the neurological phenomenon in this area of the brain (see the smooth variation of the ErrP in Figure 1). On the other hand, regarding the estimation of the moment, notice that the EKF and PF solutions produce very similar estimations. Although it is no possible to validate this result, it is a good indicator of the validity of the solution process that the solution of both methods is very similar, and also quite similar to the LCMV method.

Figure 4 shows the magnitude of the position first derivative given by the algorithms. This result agrees with the Figure 3 and explicitly shows the sudden variations in the estimation for the methods that do not address the dynamic nature of the problem. In particular, it can be observed that in many times the position changes abruptly for the static sLORETA and LCMV solutions, on the other hand the position always change gradually for the EKF and PF estimations. Again, we can state that the dynamic methods show the better performance in terms of smooth position tracking given the neurological phenomenon in this area of the brain (see the smooth variation of the ErrP in Figure 1).

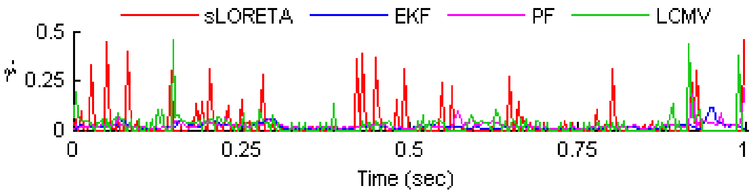


Figure 4. *Magnitude of the position first derivative given for the sLORETA, EKF, PF and LCMV solutions.*

4. Conclusions

We have described a methodology to solve the EEG source localization problem considering the dynamic behavior of the EEG sources and signals. From the dynamic probabilistic formulation of the problem we derived the EKF and PF solutions for the dipolar and non-linear approach. To assess the performance of the algorithms we designed a neurophysiological protocol based on error-related potentials. We demonstrated throughout real EEG recordings that the application of EKF and PF for tracking the EEG sources is very promising. The advantages of the proposed dynamic solutions rely on the ability for estimating time-varying sources that are moving in the brain volume, and the smooth tracking in the position, orientation and strength of the sources. These properties can be explained by the fact that the EKF and PF solutions take into account the non-linearity of the problem and the presence of noise in the measurements and in the EEG neural generators. In the future work we will extend the application of the PF for the estimations of various sources and for obtaining various likely solutions in realistic head models.

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