Toward a Metric-Based Scan Matching Algorithm for Displacement Estimation in $3D$ Workspaces

Lydia Biota   L. Montesano   J. Minguez
I3A, Dpto. de Informática e Ing. de Sistemas
Universidad de Zaragoza, Spain
{*biota,montesano,jminguez*}*@unizar.es*

Florent Lamiraux
LAAS-CNRS
Toulouse, France
florent@laas.fr

***Abstract*— This paper addresses the scan matching problem in three dimensional workspaces. The novel concept is to modify the metric used in the *Iterative Closest Point* (ICP) framework. The new distance compensates sensor translation and rotation simultaneously. The contribution here is the development of all the mathematical tools required to formulate the ICP with this new metric. Furthermore, we show preliminary results of $3D$ scan alignment to validate the development.**

I. Introduction

A key issue in mobile robotics is to keep track of the robot position using on-board sensors. To deal with this problem, one solution is scan matching. The principle is to compute the sensor displacement between two consecutive configurations by maximizing the overlap between the range measurements obtained at each configuration. This paper describes a technique to solve this problem in 3D workspaces (Figure 1).

The most popular scan matching methods usually follow the Iterative Closest Point (ICP) algorithm (principle borrowed from the computer vision community [2]). The ICP algorithm addresses this problem with an iterative process in two steps. At each iteration:

1) **matching:** establishment of correspondent points in between scans with a *closest point* criterion,
2) **minimization:** computation of the sensor displacement by a least square minimization of the error of the correspondences.

In two dimensions, a common feature of most versions of ICP is the usage of the Euclidean distance to establish the correspondences and to estimate the displacement [6], [3], [1]. As pointed out by [4], the limitation of this distance is the difficulty to capture the sensor rotation. Recently, a new metric was proposed to compensate translation and rotation simultaneously improving the performance of previous methods in 2D [5]. Despite many geometric ICP variants have been proposed in the vision community to deal with the registration problem (see [7] for a survey), the use of a metric capturing translation and rotation has not been explored.

This paper describes the extension of this new metric to three dimensional workspaces. The emphasis of the work is on the development of all the mathematical formulation required to address the scan matching problem in three dimensional workspaces with this new metric. Furthermore, we also show preliminary results of $3D$ scan alignment.

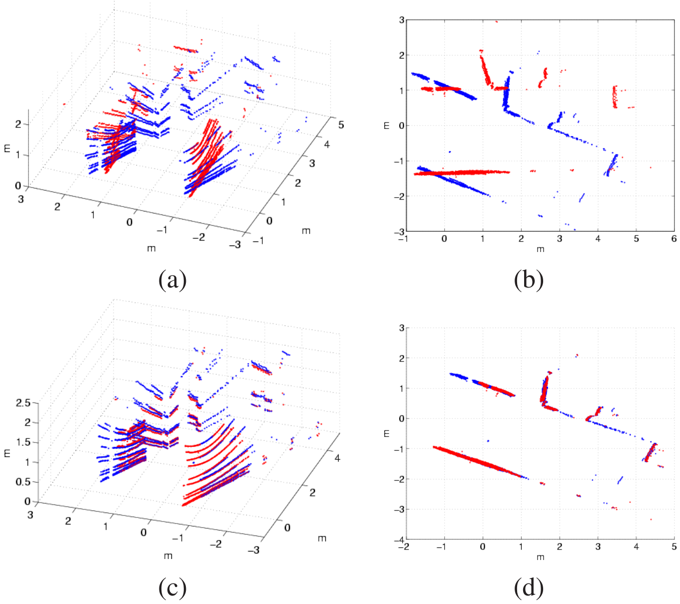

Fig. 1. Example of the scan matching problem. (a) Three dimensional and (b) projection to the XY plane views of the two sensory measurements at different locations. The solution of the scan matching problem is to compute the relative motion between both locations that maximizes the overlap between both measurements ((c) and (d)). This alignment has been obtained with the method proposed in this paper.

II. Mathematical Tools

A. Expression of the Metric distance

A rigid body transformation in $\mathbf{R}^3$ can be decomposed into a rotation of angle θ ($-\pi < \theta < \pi$) about a unit vector $n = (n_x, n_y, n_z)$ and a translation of vector (x, y, z), and thus uniquely defined by vector $q = (x, y, z, \theta\, n_x, \theta\, n_y, \theta\, n_z)$. We define the norm of q as :

$$\|q\| = \sqrt{x^2 + y^2 + z^2 + L^2\theta^2} \tag{1}$$

where L is a positive real number homogeneous to a length. Given two points $p_1 = (p_{1x}, p_{1y}, p_{1z})$ and $p_2 = (p_{2x}, p_{2y}, p_{2z})$ in $\mathbf{R}^3$, we define the distance between p_1 and p_2 as the minimum norm among the rigid body transformations that move p_1 to p_2:

$$d_p(p_1, p_2) = \min\{\|q\| \text{ such that } q(p_1) = p_2\} \tag{2}$$

where

$$q(p_1) = R(n, \theta)p_1 + T \tag{3}$$

with T the translation vector (x, y, z) and $R(n, \theta)$ the matrix of rotation of angle θ about the unit vector $n = (n_x, n_y, n_z)$. Unfortunately there is no closed form expression of d_p with respect to the coordinates of the points. However, we can compute an approximation valid for transformations with small rotations. Linearizing (3) about $\theta = 0$, we have $\cos\theta \approx 1$ and $\sin\theta \approx \theta$. Developing, we obtain:

$$d_p^{ap}(p_1, p_2) = \sqrt{\|\delta\|^2 - \frac{\|p_1 \times \delta\|^2}{||p_1||^2 + L^2}} \tag{4}$$

where $\delta = p_2 - p_1$. This is the expression of an approximation of our new metric distance in $3D$. One can demonstrate that the isodistance surfaces are ellipsoids centered on p_1 (the isodistance surfaces of the Euclidean distance are spheres). Furthermore, their dimensions depend on $||p_1||$ and the value of L. In fact, L balances the trade-off between translation and rotation. When $L \to \infty$, the new distance tends to the Euclidean distance (see equation (4)).

B. Expressions for the Correspondence Step

In this section, we use our new distance to establish pairs in the matching step of the ICP algorithm. To cope with the discrete nature of the data (Figure 1), we assume a local structure in the reference scan. In two dimensions this implies building segments in between points of the reference scan [4]. However, in three dimensions we need to build a mesh composed by *patches* (in our case triangles among neighbour points (Figure 2)). In order to derive the expression of the distance point to patch, we need to introduce first the distance point to segment and point to plane.

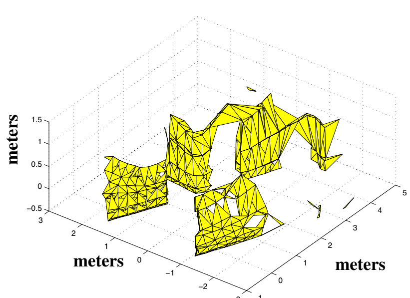

Fig. 2. Example of the mesh for the reference scan of Figure 1.

The distance d_{ps}^{ap} of a point p_1 to a segment $[s_1 \quad s_2]$ with the new metric is:

$$d_{ps}^{ap}(p_1, [s_1 \quad s_2]) \approx \begin{cases} d_p(p_1, s_1), & if \quad \lambda < 0 \\ d_p(p_1, s_2), & if \quad \lambda > 1 \\ \sqrt{\frac{-n^2 + mt}{m}}, & if \quad 0 \leq \lambda \leq 1 \end{cases} \tag{5}$$

where $\lambda = \frac{n}{m}$ and

$$\begin{aligned} m &= \|u_2\|^2 - \frac{\|p_1 \times u_2\|^2}{k} \\ n &= -\delta_1^T u_2 + \frac{(p_1 \times \delta)^T (p_1 \times u_2)}{k} \\ t &= \|\delta_1\|^2 - \frac{\|p_1 \times \delta_1\|^2}{k} \end{aligned}$$

with $u_2 = s_2 - s_1$, $\delta_1 = s_1 - p_1$, and $k = \|p_1\|^2 + L^2$.

The distance d_{pp}^{ap} of a point p_1 to the plane $[v_1 \quad v_2 \quad v_3]$ with the new metric is:

$$d_{pp}^{ap}(p_1, [v_1 \quad v_2 \quad v_3])^2 = \frac{ae^2 + bd^2 - 2cde}{c^2 - ab} + f \tag{6}$$

where the coefficients are:

$$\begin{aligned} a &= \|u_1\|^2 - \frac{\|p_1 \times u_1\|^2}{k} \\ b &= \|u_2\|^2 - \frac{\|p_1 \times u_2\|^2}{k} \\ c &= u_1^T u_2 - \frac{(p_1 \times u_1)^T (p_1 \times u_2)}{k} \\ d &= -\delta_1^T u_1 + \frac{(p_1 \times \delta_1)^T (p_1 \times u_1)}{k} \\ e &= -\delta_1^T u_2 + \frac{(p_1 \times \delta_1)^T (p_1 \times u_2)}{k} \\ f &= \|\delta_1\|^2 - \frac{\|p_1 \times \delta_1\|^2}{k} \end{aligned}$$

where $u_1 = v_3 - v_1$, $u_2 = v_2 - v_1$, $\delta_1 = v_1 - p_1$ and $k = \|p_1\|^2 + L^2$. Furthermore, the closest point p_0 on the plane is given by:

$$p_0 = v_1 + \lambda_{01} u_1 + \lambda_{02} u_2 \tag{7}$$

with $(\lambda_{01}, \lambda_{02}) = \begin{pmatrix} a & c \\ c & b \end{pmatrix}^{-1} \begin{pmatrix} d \\ e \end{pmatrix}$.

Finally, the distance from a point p_1 to the patch $\triangle_{v_1, v_2, v_3}$ formed by v_1, v_2 and v_3 depends on the relative position location between the closest point to p_1 in the plane $[v_1, v_2, v_3]$ (point p_0 computed by (7)) and the the patch $\triangle_{v_1, v_2, v_3}$ on this plane. Next figure summarizes the distance.

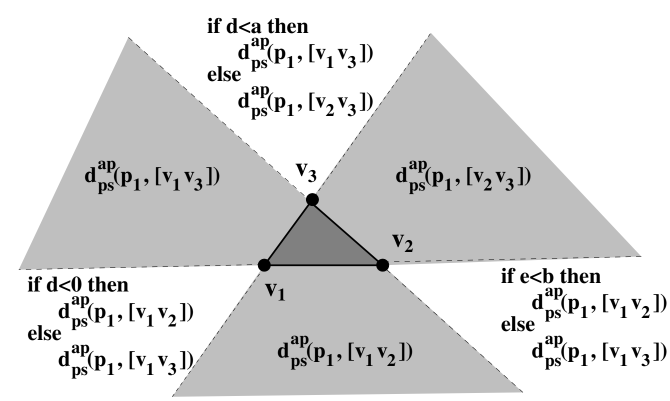

Fig. 3. This Figure depicts the different zones where the closest point to p_1 in the plane defined by $[v_1 \quad v_2 \quad v_3]$ could lie, and the expression of the distance point to patch $d_{p\triangle}^{ap}(p_1, \triangle_{v_1, v_2, v_3})$ in each of these cases. When p_0 is in the triangle, the distance is $d_p^{ap}(p_1, p_0)$.

C. Expression for the minimization

Let be $p_i = (p_{ix}, p_{iy}, p_{iz})$ and $p_i'' = (p_{ix}'', p_{iy}'', p_{iz}'')$ two correspondent points. The expression to minimize is:

$$E_{dist}(q) = \sum_{i=1}^{n} d_{pp}^{ap}(p_i, q(p_i''))^2 = \delta_i^T(q) M \delta_i(q) \tag{8}$$

where $\delta_i(q) = p_i - q(p_i'') \approx p_i - p_i'' + U(p_i'')r - T$ and

$$U(p_i'') = \begin{pmatrix} 0 & -p_{iz}'' & p_{iy}'' \\ p_{iz}'' & 0 & -p_{ix}'' \\ -p_{iy}'' & p_{ix}'' & 0 \end{pmatrix}$$

$$M = \begin{pmatrix} p_{ix}^2 + L^2 & p_{ix}p_{iy} & p_{ix}p_{iz} \\ p_{ix}p_{iy} & p_{iy}^2 + L^2 & p_{iy}p_{iz} \\ p_{ix}p_{iz} & p_{iy}p_{iz} & p_{iz}^2 + L^2 \end{pmatrix}$$

Finally, the q that minimizes expression (8) is:

$$q_{min} = \left(\sum_{i=1}^{n} \begin{pmatrix} M & -MU(p_i'') \\ -U^T(p_i'')M & U^T(p_i'')MU(p_i'') \end{pmatrix} \right)^{-1} \cdot \sum_{i=1}^{n} M\delta$$

In summary, in this section we have presented the mathematical tools in order to introduce the new metric in all the steps of the ICP framework.

III. Experimental Results

This section describes preliminary experimental results obtained over a data set collected with a *TRC* $3D$ laser sensor mounted on a robot (Figure 1). The sensor has a field of view of $240°$ and a range of 6.5m and each scan gathers 4800 points with pan and tilt resolution of $5°$ and $0.5°$. The data corresponds to a travel of approximately 20 meters.

We have created a prototype of the new scan matching technique and tested it with these data (the best results were obtained with $L = 3$). Figure 4 shows the reconstruction of the scenario projected to the XY plane using only the odometry and using the new algorithm. The figure shows how the scan matching technique ameliorates the odometry since (i) the walls are thinner and (ii) it better aligns the walls of the building. This is the expected result that validates the mathematical formulation proposed in this paper. Notice that although the vehicle moves in a $2D$ world, random noise was added to the odometry readings to simulate error in the six degrees of freedom. In other words, the algorithm works in a fully three dimensional world and has to estimate the six coordinates of q.

Alternatively, we have compared the performance of this technique with the standard ICP algorithm. We have done this by matching each scan with itself adding random noise up to 0.4m in translation and $15°$ in the rotation vector (like this we know the ground truth $(0, 0, 0)$). We repeated this procedure with 20 scans and 1000 times for each scan. The preliminary results show that the new technique slightly ameliorates the ICP in terms of robustness, precision and convergence rate. However, the improvement is not as large as that reported in the two dimensional case. We think that this is due to the nature of the data collected (the scenario is very structured and not dense).

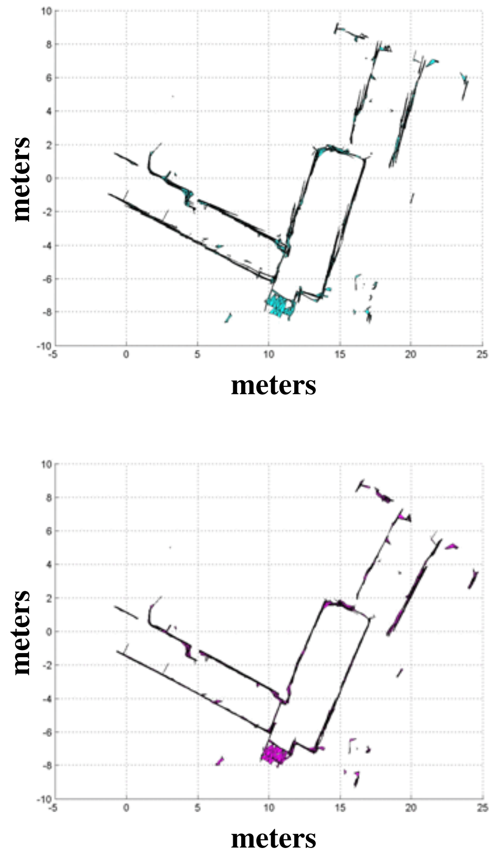

Fig. 4. Projection to the XY plane of the reconstruction using the vehicle odometry (top) and using the new scan matching method (bottom).

IV. Conclusions and Future Work

This paper describes the extension of $2D$ metric-based scan matching to three dimensional workspaces. The emphasis of the work is on the development of all the mathematical formulation required to address the scan matching problem in three dimensional workspaces with these new metric.

The future work is to improve the experimental validation of the method. Although we find the results very encouraging, we plan to collect a more complete data set to provide a more rigorous comparison with other methods as was shown in the two dimensional case by [5].

References

[1] O. Bengtsson and A-J. Baerveldt. Localization by matching of range scans - certain or uncertain? In *EURobot'01*, Lund, Sweden, 2001.

[2] P.J. Besl and N.D. McKay. A method for registration of 3-d shapes. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 14:239–256, 1992.

[3] J.-S. Gutmann and C. Schlegel. Amos: Comparison of scan matching approaches for self-localization in indoor environments. In *1st Euromicro Workshop on Advanced Mobile Robots*, 1996.

[4] F. Lu and E. Milios. Robot pose estimation in unknown environments by matching 2d range scans. *Intelligent and Robotic Systems*, 18:249–275, 1997.

[5] J. Minguez, L. Montesano, and F. Lamiraux. Metric-based iterative closest point scan matching for sensor displacement estimation. *IEEE Transaction on Robotics (to appear)*, 2006.

[6] S.T. Pfister, K.L. Kreichbaum, S.I. Roumeliotis, and J.W. Burdick. Weighted range sensor matching algorithms for mobile robot displacement estimation. In *In Proceedings of the IEEE International Conference on Robotics and Automation (ICRA)*, pages 1667–74, 2002.

[7] S. Rusinkiewicz and M. Levoy. Efficient variants of the icp algorithm. In *International Conference 3DIM*, 2001.