

Introduction

In omnidirectional cameras, straight lines are projected onto curves called line-images. The shape of these curves is strongly dependent of the particular camera configuration. In this paper, we present a novel method to extract line-images in uncalibrated images which is valid for radially symmetric central systems. When projecting 3D points in radially symmetric central systems, the azimuth angle of a 3D point is directly mapped on the image and the radius of the projected point depends on the elevation angle $r = f(\phi)$.



The projection surface of a 3D line is a plane which has two degrees of freedom (DOF). If the line-image is a curve with more than two degrees of freedom, the extra DOFs implicitly encodes the projection model and the line-image can be used to compute the calibration [1],[2]. In radially symmetric central systems [3] the line-image can be encoded by the following,

$$n_x \hat{x} \pm n_y \hat{y} - n_z \hat{\alpha}(\hat{r}) = 0.$$

	\hat{r}	$\hat{\alpha}$	$\frac{\partial \hat{\alpha}}{\partial \hat{r}}$	\hat{r}_{vl}
Perspective	$f \tan \phi$	f	0	∞
Paracatadioptric	$2fp \tan\left(\frac{\phi}{2}\right)$	$\frac{\hat{r}^2}{4fp} - fp$	$\frac{1}{2fp}$	$2fp$
Hypercatadioptric	$\frac{f \sin \chi \sin \phi}{\cos \phi + \cos \chi}$	$-\frac{f + \cos \chi \sqrt{\hat{r}^2 + f^2}}{\sin \chi}$	$\frac{\cot \chi}{\sqrt{\hat{r}^2 + f^2}}$	$f \tan \chi$
Equiangular-Fisheye	$f \phi$	$-\hat{r} \cot \frac{\hat{r}}{f}$	$\frac{1}{f} \left(1 - \frac{f}{\hat{r}} \cot \frac{\hat{r}}{f} + \cot^2 \frac{\hat{r}}{f}\right)$	$f \frac{\pi}{2}$
Stereographic-Fisheye	$2f \tan\left(\frac{\phi}{2}\right)$	$\frac{\hat{r}^2}{4f} - f$	$\frac{1}{2f}$	$2f$
Orthogonal-Fisheye	$f \sin(\phi)$	$-\sqrt{f^2 - \hat{r}^2}$	$\frac{1}{\sqrt{f^2 - \hat{r}^2}}$	f
Equisolid-Fisheye	$2f \sin\left(\frac{\phi}{2}\right)$	$\frac{2\hat{r}^2 - f^2}{2\sqrt{f^2 - \hat{r}^2}}$	$\frac{\hat{r}(3f^2 - 2\hat{r}^2)}{2(f^2 - \hat{r}^2)^{3/2}}$	$f \frac{\sqrt{2}}{2}$

Straight-Line and Gradient-Based Constraints

In radially symmetric central systems any 3D line lying on the horizontal plane is projected on the vanishing line. This line is always mapped in a circle centred on the principal point and with radius \hat{r}_{vl} which is proposed as unified main calibration parameter as a result of it encodes calibration information in different types of camera systems, dioptric and catadioptric. With three points lying on the line-image it is possible to obtain the main calibration parameter \hat{r}_{vl} .

$$l_1 \hat{\alpha}_1 + l_2 \hat{\alpha}_2 + l_3 \hat{\alpha}_3 = 0,$$

$$\text{where } l_1 = \hat{x}_2 \hat{y}_3 - \hat{x}_3 \hat{y}_2, \quad l_2 = \hat{x}_3 \hat{y}_1 - \hat{x}_1 \hat{y}_3 \quad \text{and} \quad l_3 = \hat{x}_1 \hat{y}_2 - \hat{x}_2 \hat{y}_1$$

The line-image constraint is analytically solved for each camera system allowing us to extract simultaneously the projection plane of the line and the main calibration parameter from three points of the line-image.

Perspective	$(l_1 + l_2 + l_3) = 0$
Paracatadioptric	$\hat{r}_{vl} = \sqrt{\frac{l_1 \hat{r}_1^2 + l_2 \hat{r}_2^2 + l_3 \hat{r}_3^2}{l_1 + l_2 + l_3}}$
Hypercatadioptric	$\hat{r}_{vl} = \sqrt{\left(\frac{l_1 \sqrt{\hat{r}_1^2 + f^2} + l_2 \sqrt{\hat{r}_2^2 + f^2} + l_3 \sqrt{\hat{r}_3^2 + f^2}}{l_1 + l_2 + l_3}\right)^2 - f^2}$
Equiangular-Fisheye	$l_1 \hat{r}_1 \cot\left(\frac{\pi}{2} \frac{\hat{r}_1}{\hat{r}_{vl}}\right) + l_2 \hat{r}_2 \cot\left(\frac{\pi}{2} \frac{\hat{r}_2}{\hat{r}_{vl}}\right) + l_3 \hat{r}_3 \cot\left(\frac{\pi}{2} \frac{\hat{r}_3}{\hat{r}_{vl}}\right) = 0$
Stereographic-Fisheye	$\hat{r}_{vl} = \sqrt{\frac{l_1 \hat{r}_1^2 + l_2 \hat{r}_2^2 + l_3 \hat{r}_3^2}{l_1 + l_2 + l_3}}$
Orthogonal-Fisheye	$l_1 \sqrt{\hat{r}_{vl}^2 - \hat{r}_1^2} + l_2 \sqrt{\hat{r}_{vl}^2 - \hat{r}_2^2} + l_3 \sqrt{\hat{r}_{vl}^2 - \hat{r}_3^2} = 0$
Equisolid-Fisheye	$l_1 \frac{\hat{r}_1^2 - \hat{r}_{vl}^2}{\sqrt{2\hat{r}_{vl}^2 - \hat{r}_1^2}} + l_2 \frac{\hat{r}_2^2 - \hat{r}_{vl}^2}{\sqrt{2\hat{r}_{vl}^2 - \hat{r}_2^2}} + l_3 \frac{\hat{r}_3^2 - \hat{r}_{vl}^2}{\sqrt{2\hat{r}_{vl}^2 - \hat{r}_3^2}} = 0$

Using information of the gradient in the defining points we obtain additional constraints which allow us to extract simultaneously the projection plane of the line and the main calibration parameter from only two points of the line-image.

In practice, we also approximate the parameter $\hat{\alpha}(\hat{r}) = \frac{\partial \hat{\alpha}(\hat{r}_{vl})}{\partial \hat{r}} (\hat{r} - \hat{r}_{vl})$ linearly obtaining a generic way of computing the main calibration parameter.

$$\begin{pmatrix} \hat{x} & \pm \hat{y} & -\hat{r} & 1 \\ -\nabla I_y & \pm \nabla I_x & \frac{\hat{x} \nabla I_y - \hat{y} \nabla I_x}{\hat{r}} & 0 \end{pmatrix} \mathbf{m} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad \text{where} \quad \mathbf{m} = \lambda \left(n_x, n_y, n_z \frac{\partial \hat{\alpha}(\hat{r}_{vl})}{\partial \hat{r}}, n_z \frac{\partial \hat{\alpha}(\hat{r}_{vl})}{\partial \hat{r}} \hat{r}_{vl} \right)^T$$

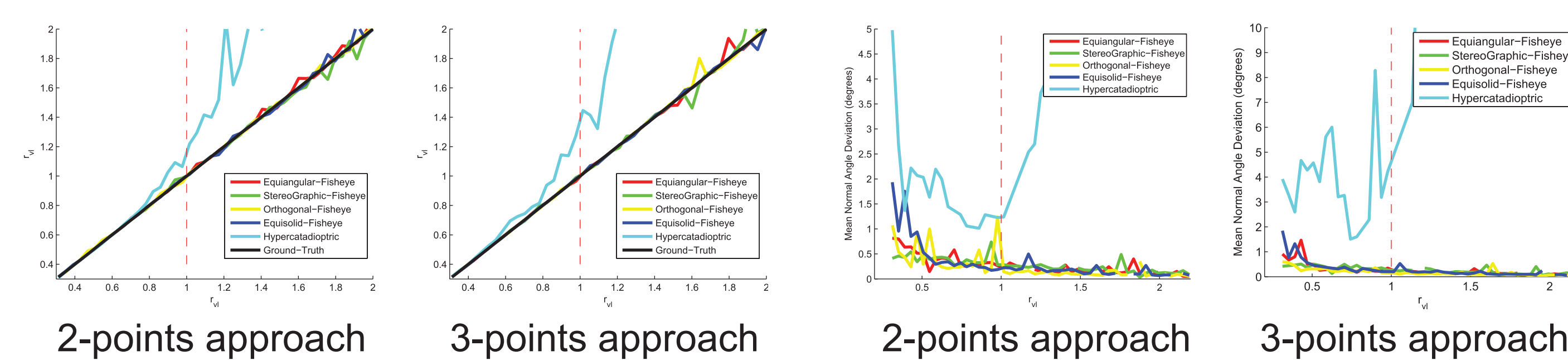
Uncalibrated Line-Image Extraction and Experiments

The inputs of the method are the edges and their gradients obtained from Canny detector. These edges are stored in connected components called boundaries and the gradient orientation is low-pass filtered to reduce noise. A subset of these sub-boundaries is selected following a heuristic criterion to avoid line-images which do not contain self-calibration information. Then a RANSAC-based approach is used for the extraction. Depending on the type of constraints, 2 points or 3 points are needed.

Both are compared in simulation using synthetic images with known ground-truth. The criteria for the comparison are the error in \hat{r}_{vl} estimation and the angle deviation of the normals.

Estimation of \hat{r}_{vl}

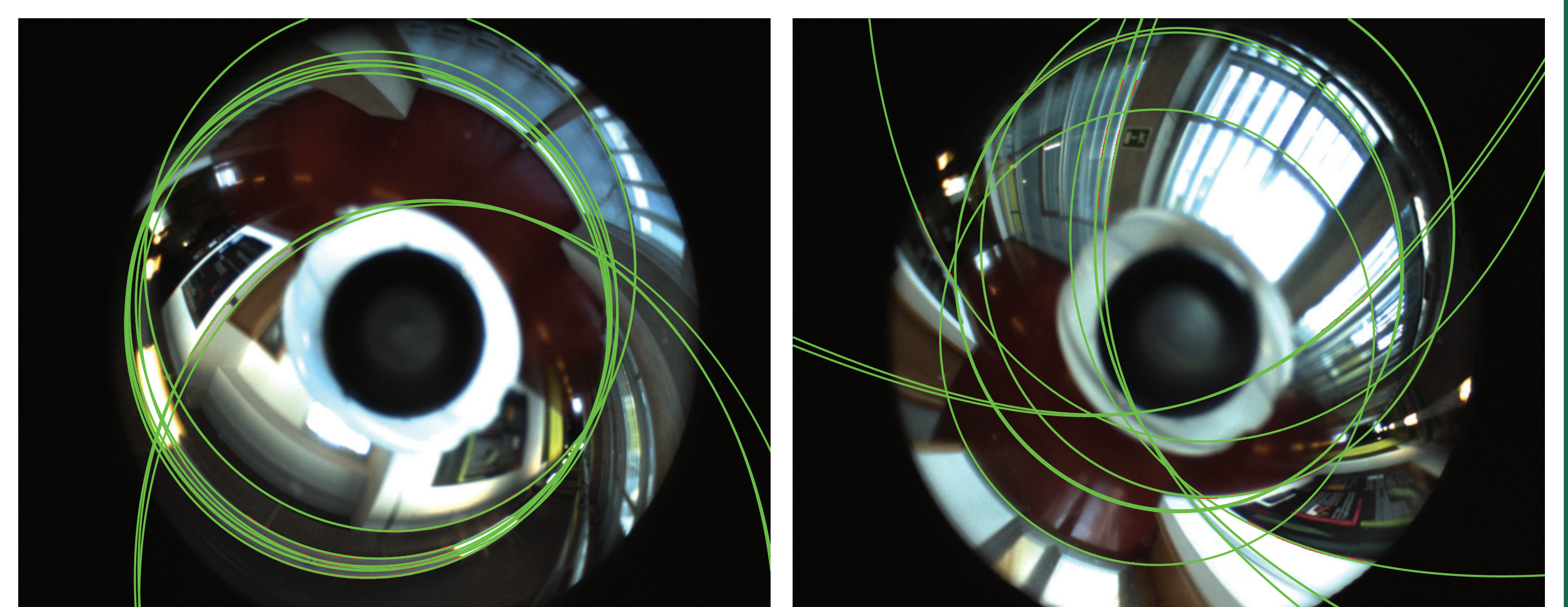
Angle deviation



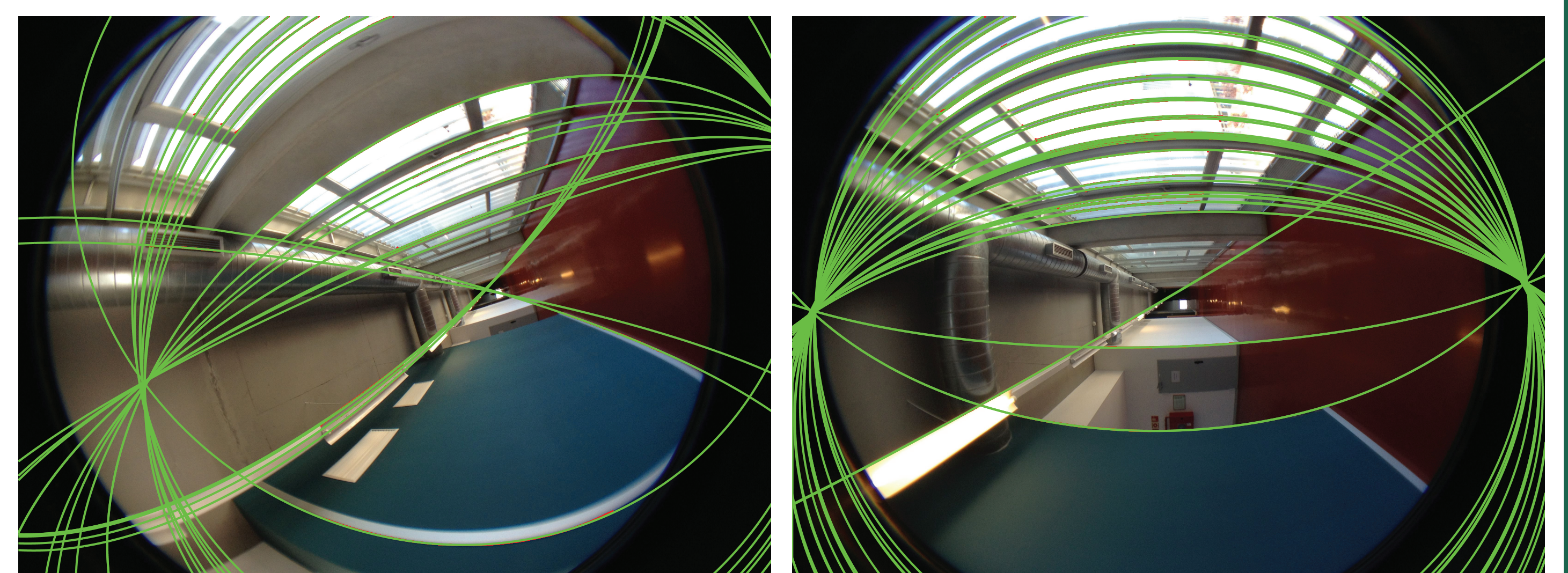
To show the behaviour of the method in a real scenario the algorithm has been tested with real images acquired with an hypercatadioptric system mounted in a helmet and with an equiangular fisheye mounted in an Iphone-4s.

We have also applied the algorithm to an image sequence taken with a camera in hand. The sequence has been acquired with a Nexus 4 camera using an equiangular fisheye.

The video of the sequence with the extracted line-images has been attached as supplementary material.



Hypercatadioptric



Equiangular-Fisheye

